

“Three Grades of Modal Involvement”

Box and Diamond

- In modal logic, the symbols \Box and \Diamond are used to express “modes” of truth. $\Box P$ is typically taken to mean *that it is necessary that P*, and $\Diamond P$ is taken to mean *that it is possible that P*.
- As operators (see below), \Box and \Diamond are widely thought to be inter-definable, $\Box P \leftrightarrow \sim \Diamond \sim P$, so \Box or \Diamond can then be taken to be an abbreviation for a construction out of the other and \sim .
- We can symbolize *that it is impossible that P* as $\sim \Diamond P$, and *that it is contingent that P* as $(\Diamond P \ \& \ \Diamond \sim P)$. We may also define a notion of strict implication, $(P \rightarrow Q)$, which, unlike material implication, $(P \supset Q)$, is not equivalent to $(\sim P \vee Q)$, but rather to *the necessitation of* $(\sim P \vee Q)$. If \Box is an operator (again, see below), then $(P \rightarrow Q) \leftrightarrow \Box(P \rightarrow Q) \leftrightarrow \Box(\sim P \vee Q)$.
 - *Note:* Quine regards the notion of strict implication as arising (like most things!) from a confusion of use and mention. The notion is trying to capture the idea of implication. But this notion attaches to *names of sentences*, not to sentences.

Three Uses of Box

- Quine distinguishes three ways to integrate the symbol \Box (or \Diamond) into a formal language.
 1. We may regard \Box as a monadic predicate. In this case, $\Box P$ is really just a sentence of first-order logic, of the form Fa . P names some sentence (e.g., “The sky is blue.”), and \Box has some extension (e.g., the set of all sentences which are logically true).
 - Quine accepts this understanding of \Box so long as it is taken to mean something like *is a logical truth* (so even $\sim \Box(\text{All bachelors are unmarried})$). However, if \Box taken to mean *is analytic*, then he famously rejects \Box as unintelligible.
 2. We may regard \Box as an operator on sentences. In this case, we expand our grammar so that if P is a *sentence* (not the name of one!), then $\Box P$ is a sentence. Sentences contain no free variables. $\Box(\forall x)(Fx)$ now makes sense, but still $(\forall x)\Box Fx$ does not.
 - This understanding of \Box is translatable into the first, so long as we do not allow for iterated modalities. However, if \Box , understood as a predicate, is taken to mean *is a logical truth*, then $\Box\Box P$ will not, in fact, follow from $\Box P$.

3. We may regard $[]$ as an operator on formulas. In this case, we expand our grammar so that if F is a *formula*, then $[]F$ is too. Now even $(\forall x)[]Fx$ is said to make sense.

Vocabulary

- A singular term, t , occurs purely referentially in a sentence, S , just in case one can substitute any co-referring term, t' , for t , without changing the truth-value of S . For instance, the numeral '2' occurs purely referentially in ordinary arithmetic sentences, such as '2 is prime'.
 - *Intuition*: If one can flip the truth-value of S by replacing an occurrence of t with t' , then that occurrence of t was not about the referent of t at all. For a law of identity, which seems *prima facie* unimpeachable, is that $(\forall x)(\forall y)([x=y \ \& \ F(x)] \rightarrow F(y))$.
- Let C be a context of a sentence, S , in which another sentence, S' , may occur. Then C is referentially transparent just in case, if every occurrence of a singular term in S' is purely referential in S' , then it remains purely referential in S . It is referentially opaque otherwise.
 - *Example*: The context 'P contains three characters' is referentially opaque. While the occurrence of '2' in '2 is prime' is purely referential, the occurrence of '2' in '2 is prime' contains three words' is not. 'The smallest prime number' and '2' are co-referring, but 'The smallest prime number' contains three words' is false.
 - *Note*: One can avoid this problem by using a different convention to name names. One might require of one's language that no symbol is the *proper part* of another (though I doubt this makes sense – we are talking about symbol *types*, not tokens!).
- A context is extensional when, if S has truth-value, V , and S' has truth-value, V' , then the truth-value of S remains V if S' is replaced by any sentence with truth-value V' . For instance, the context 'It is true that P' is evidently extensional, but 'Necessarily, P' is not.

Extensionality and Referential Transparency

- Let us say that a context is L-extensional if and only if, whenever S has truth-value, V , then the truth-value of S remains V if S' is replaced by any logically equivalent sentence. Note that 'Necessarily, P' is apparently an L-extensional context, but not an extensional context.
- *Question*: Can extensionality fail for a context if it is referentially transparent and L-extensional? Quine argues that it cannot. In other words, if a context is L-extensional but

not extensional, then it *must* be referentially opaque. One might take this to show that, in such a context, occurrences of singular terms are not *about* their ordinary referents at all.

1. Suppose, for *reductio*, that ‘Necessarily, P’ is a referentially transparent context.
 2. Suppose that ‘Necessarily, P’ is an L-extensional context.
 3. Suppose that ‘Necessarily, P’ is not an extensional context.
 4. Let ‘P’ be a true sentence making ‘Necessarily, P’ true.
 5. ‘P’ is logically equivalent to ‘ $\{x : x = \{ \} \ \& \ P\} = \{ \{ \} \}$ ’.
 - *Rationale:* If P is true, then ‘the x such that $x = \{ \}$ and P’ is true of exactly one thing – i.e., $\{ \}$. So, the set of all such x is the set containing $\{ \}$ – i.e., $\{ \{ \} \}$. In the other direction, if ‘ $\{x : x = \{ \} \ \& \ P\} = \{ \{ \} \}$ ’ (not $\{ \}$), then P must be true.
 6. By (2), ‘Necessarily, $\{x : x = \{ \} \ \& \ P\} = \{ \{ \} \}$ ’ is true.
 7. Let ‘Q’ be an arbitrary (merely) true sentence.
 8. Then ‘ $\{x : x = \{ \} \ \& \ P\} = \{x : x = \{ \} \ \& \ Q\}$ ’ is true.
 9. By (1), ‘Necessarily, $\{x : x = \{ \} \ \& \ Q\} = \{ \{ \} \}$ ’ is true.
 - *Note:* Remember that ‘ $\{x : x = \{ \} \ \& \ Q\}$ ’ is a *singular term*.
 10. ‘Q’ is logically equivalent to ‘ $\{x : x = \{ \} \ \& \ Q\} = \{ \{ \} \}$ ’.
 11. Hence, by (2), ‘Necessarily, Q’ is true, which contradicts (3).
- *Upshot:* Assuming (2) and (3) – which seem hard to deny – premise (1) must be false.

Quantification and Opacity

- Argument (1) – (11) seems to show that the third use of \square is incoherent. If ‘ $\square P$ ’ is referentially opaque, then ‘ $\square F(a)$ ’ may be true while ‘ $\square F(b)$ ’ is false – *even if ‘a’ and ‘b’ refer to the same thing*. But is not the principle that $(\forall x)(\forall y)[x=y \ \& \ F(x)] \rightarrow F(y)$ sacrosanct? If it is, *and if Universal Instantiation is everywhere valid*, then this situation is impossible.
- Correlatively, suppose that ‘ $\square F(a)$ ’, ‘ $\sim \square F(b)$ ’, and ‘ $a=b$ ’ are all true. Then presumably we may infer ‘ $(\exists x)F(x)$ ’, by Existential Instantiation. But, then, *what is the x of which ‘F(x)’ is true?* Not, evidently, the thing referred to by ‘a’, since that is the thing referred to by ‘b’!
- *Example:* Consider the claims that $\square(9 > 7)$ and $\sim \square(\text{the number of planets} > 7)$. The former is a paradigmatic truth. But what of the latter? Surely the number of planets could have failed to be greater than 7. But $9 =$ the number of planets. Hence, if Universal Instantiation is valid, we may infer a contradiction by letting ‘F(x)’ be ‘ $\square(x > 7)$ ’ in the schema above.

De Re Modality and Essentialism

- Given (1) – (11) and Universal and Existential Instantiation unrestrictedly, then perhaps the moral is that *necessity cannot attach to objects, but only to descriptions of them*. We can talk about *statements* being necessary, but not coherently about *objects being necessarily F*.
- Alternatively, one could accept the legitimacy of necessity applied to objects by *restricting applications of Universal and Existential Instantiation*. But what is the metaphysical picture that remains? According to Quine, it is Aristotelian Essentialism, the view that some of an objects' properties are essential to it, and others are only accidental, *independent of the way it is described*. Can we make sense of such a picture? How can we tell what properties of a thing are essential to it? Do not Quine's examples show that our judgments of a thing's essence are sensitive to the way it is described (even if its essence is not itself so sensitive)?