# Optimal Fiscal Policy with Robust Control*

Justin Svec†

*Columbia University*

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**Abstract**

This paper compares the fiscal policies implemented by two types of government when confronted by consumer uncertainty. Consumers, lacking confidence in their understanding of the stochastic environment, distort their subjective probability model. The government does not face this uncertainty. Through its choice of a labor tax and a supply of one-period bonds, the government manipulates the competitive equilibrium allocation and the consumers' probability distortion. I consider two types of altruistic government. The 'benevolent' government maximizes the consumers' expected utility under the true probability model, whereas the 'political' government maximizes the consumers' expected utility under the distorted probability model. I find that, relative to rational expectations, the benevolent government relies more heavily on labor taxes to finance fluctuations in spending, while the political government depends more on public debt to absorb the fiscal shock. These policies are designed to influence the consumers' savings decisions and probability distortion.

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†Columbia University, Department of Economics, 1022 International Affairs Building, 420 West 118th Street, New York, NY 10027. *E-mail: jcs2104@columbia.edu.*

# 1 Introduction:

Within the optimal fiscal policy literature, it is generally assumed that consumers completely understand the probability model characterizing the equilibrium. That is, the consumers can accurately forecast the state-contingent paths of the endogenous variables, which include prices and policies, as well as the exogenous variables. This ability is critical because one channel through which policy influences the equilibrium outcome is through its effect on consumer beliefs. In particular, each policy considered by the government helps shape the consumers' expectations about future asset returns. These beliefs then determine the incentives faced by the consumers, guiding their decisions about how, for example, to allocate wealth across time and state.

Consumer beliefs are central to the fiscal policy model of Lucas and Stokey (1983). In this rational expectations model, it is optimal for the government to set a smooth profile of labor taxes across states. This policy reduces the fluctuations in the intra-temporal distortion due to the labor tax. Since taxes are less volatile across states than is government spending, the government relies on public debt to finance the difference. This leads to a primary deficit when spending is high and a primary surplus when spending is low. To sustain this equilibrium, the consumers must hold the opposite profile of debt, loaning money to the government in the former case and borrowing money from the government in the latter. For the consumers to choose this pattern of savings, they must hold a particular set of beliefs about how asset returns move across state and time. Fiscal policy is designed to generate these beliefs. It does so by influencing the path of the stochastic discount factor, which in turn alters the consumers' forecasts of asset returns. Thus, the beliefs of the consumers are an integral component of the equilibrium.

Underlying this solution is the assumption that the consumers are correct in their forecasts of the state-contingent path of the economy. This may lead to the worry that the outcome hinges upon the accuracy of consumer beliefs. If, instead, consumers do not possess model-consistent expectations, they might react according to distorted forecasts of future policies and prices. This could lead to consumer behavior that undermines the government's ability to implement a smooth tax rate across states. As a result, consumer uncertainty could potentially call into question the fiscal policy prescriptions of Lucas and Stokey (1983).

The goal of this analysis, therefore, is to determine how different types of government respond to consumer uncertainty. Does consumer uncertainty restrict the government's ability to borrow and lend to the consumers? More critically, how should a fiscal authority balance the distortions associated with the linear labor tax and consumer uncertainty? By examining a number of different objective functions for the government, this paper attempts to disentangle the policy implications of consumer uncertainty from the planner's preferences.

As in Lucas and Stokey (1983), this paper assumes only one source of uncertainty: a shock to government spending. This shock will be interpreted as an extreme and unlikely event, implying a large rise or fall in public expenditures[1]. The consumers and the government are endowed with an approximating model that fully specifies the probabilities over all possible histories of both the endogenous and exogenous variables. The government is confident that this approximating model is an accurate description of the economy. The consumers, however, are not. The consumers are unsure about which probability model truly characterizes the equilibrium.

There is a substantial experimental literature devoted to understanding how individuals respond to this type of uncertainty. Ellsburg (1961), for example, demonstrates that people prefer to place bets on games with known probabilities rather than unknown probabilities. These preferences suggest that people respond to uncertainty as if there was no single measure characterizing the probabilities over events. As described by Camerer and Weber (1992), this aversion to uncertainty holds across a wide variety of environments.

One way to formalize this behavior is through the multiplier preferences of Hansen and Sargent (2005, 2006). I will follow their formulation when developing the consumers' decision problem. The consumers, instead of trusting that the approximating model represents the truth, believe that the true probability measure lies within a range of measures. Given a finite amount of data, they feel that any probability model within this range could potentially characterize the equilibrium. The consumers respond to this type of uncertainty by applying a max / min operator to their decision problems. In doing so, the consumers endogenously distort their subjective probability model away from the approximating model[2]. This process ensures that they choose a 'robust' allocation, one that performs well across all of the probability measures considered by the consumers.

[1] I will refer to the large rise in government spending as 'war' and to the large fall in government spending as 'peace'.

[2] Importantly, this is a model of doubt, not lack of information. The consumers are aware that they are endowed with the same approximating model as the government. However, whereas the government is confident in the accuracy of this model, the consumers are not. Thus, the government has no additional information that it could reveal to the consumers in order to reduce their uncertainty.

The government is able to commit to a path of fiscal policy, chosen at time $t = 0$. Unlike the consumers, the government does not doubt its approximating model. In solving its optimization problem, though, the government does take into account how its choice of fiscal policy affects both the consumers' distorted expectations and decisions[3].

The government is altruistic, maximizing the consumers' welfare. When the beliefs of the government and consumers coincide, this assumption leads to a unique objective function for the government. When consumers face model uncertainty, though, this is no longer the case. The consumers' doubt leads them to optimize according to a distorted probability model, one that the government feels confident is incorrect. In this setting, there are a number of objective functions the government could have. This paper considers two types. First, I assume that the planner maximizes the representative consumer's expected utility under the true probability model. This type of government is labeled 'benevolent.' Second, I assume that a different planner maximizes the representative consumer's expected utility under the consumer's subjective probability model. This type of government is labeled 'political.'

The benevolent government represents a paternalistic planner, one that rejects the consumers' beliefs as distorted and sets policy according to what it believes the consumers should prefer. The political government, although confident in its approximating model, rejects this paternalism. Instead, the political government chooses to maximize the expected utility of the consumers under their own distorted expectation. Depending on the probability model that truly characterizes the spending shock, either government could achieve higher welfare for the consumers. In this paper, I will assume that the true model happens to be the approximating model given to both agents. This multiplicity in planner objective functions allows me to examine the interaction between consumer uncertainty and the preferences of the government.

To foreshadow the results detailed below, the benevolent government relies more heavily on labor taxes to finance government spending than would be optimal if consumers faced no uncertainty. The benevolent government chooses this volatile labor tax rate in order to reduce the distortion in the consumers' savings decisions. By increasing the labor tax during war and decreasing it during peace, the government influences asset returns, which partially realigns the consumers' savings decisions with their rational expectations levels. Conversely, the political government chooses the opposite type of policy, opting to finance more of the shock to spending through public debt. This policy smoothes the consumers' allocation across states,

[3] Given the path of prices and allocation, the consumers' probability distortion is fully revealed to the government.

directly reducing their probability distortion. Each government's policy then determines, among other things, the volatility of the primary deficit, asset returns, and the path of the underlying allocation. As these conclusions make clear, the implications of consumer uncertainty depend critically on the preferences of the planner.

There is a growing literature that examines whether the policy prescriptions derived from rational expectations models are still relevant when agents are uncertain. This literature, however, largely concentrates on a different issue than the one considered in this paper. Whereas this paper analyzes the impact of consumer uncertainty, other papers in this literature generally focus on the policy implications of the government facing uncertainty. For example, Dennis (2007) considers a monetary policy model in which the central bank is unsure about the stochastic process governing the shocks to the Philips' curve and the Euler equation. In addition, the central bank is also unsure about the probability model held by the firms. Given this uncertainty, a discretionary central bank reacts more aggressively to stabilize inflation than would be optimal under rational expectations.

Woodford (2008) studies a different, and novel, type of uncertainty faced by the central bank. In his model, the central bank is confident about its own model of the economy but is unsure about the beliefs entertained by the private sector. Not wanting to implement a policy that performs poorly due to incorrect firm beliefs, the bank applies a max / min operator to its decision problem. He finds that this type of uncertainty leads the central bank to restrict the degree to which cost-push shocks translate into inflation relative to a rational expectations model.

One paper in the literature that discusses the impact of consumer uncertainty on policy is Karantounias, Hansen, and Sargent (2007). They also incorporate model uncertainty into the fiscal policy framework of Lucas and Stokey (1983). The focus of their analysis, however, is considerably different than mine. Their paper examines how the consumers' distrust of their approximating model affects the policy implemented by one type of fiscal authority. Using this setup, they test to see whether the resulting fiscal policy is consistent with the empirical literature on public finance. The goal of my analysis, though, is to determine how different types of government should set policy when confronted with consumers who face uncertainty. By examining a range of preferences for the planner, this paper is better able to isolate the impact of consumer uncertainty on fiscal policy. In my paper, the formulation of the benevolent government's problem overlaps with that of Karantounias, Hansen, and Sargent (2007). In fact, with a redefinition of variables and specific parameter values, my solution replicates theirs. However, the focus of my analysis is to contrast the policies chosen by different fiscal governments facing uncertain consumers.

In Svec (2008), I explore the impact of consumer uncertainty on optimal fiscal policy in a model with capital. The government's objective function is the representative consumer's expected utility under the distorted probability measure. With these preferences, the government optimally relies more heavily on a private assets tax to finance its spending than would be optimal if consumers were confident about their probability model. In addition, the government structures the ex-post capital taxes so that the ex-ante capital tax remains quantitatively near zero. The greater volatility in the private assets tax allows the government to set a relatively smooth labor tax. This policy reduces the fluctuations in the consumers' subjective welfare, lowering their probability distortion.

The outline of the paper is as follows. Section 2 describes the structure of the economy and characterizes the representative consumer's problem. This problem, and the corresponding constraints placed upon the competitive equilibrium, holds for both types of government. Section 3 formulates the 'benevolent' government's problem and discusses the intuition behind the chosen fiscal policy. This exercise is repeated for the 'political' government in section 4. Section 5 compares the two solutions, focusing on how the incentives of each government lead to qualitatively different policy implications. Section 6 concludes.

## 2 The Economy:

Time is discrete in this infinite-horizon model. There are two types of agents: the government and an infinite number of identical consumers. The only source of randomness in this model is a shock to government spending, which can take on a finite number of values. Let $g^t = (g_0, ..., g_t)$ represent the history of shocks up to and including period $t$. The approximating model indicates that the probability of each history is $\pi(g^t)$. In period 0, government spending is known to be $g_0$ with probability 1. The government must finance its expenditure through either a linear tax on labor, $\tau^n$, or through state-contingent one-period debt. In each period, the government will supply the economy with a vector of these state-contingent bonds $b(g_{t+1} \mid g^t)$ at prices $p(g_{t+1} \mid g^t)$, $\forall g_{t+1}, g^t, t \geq 0$. If $b(g_{t+1} \mid g^t)$ is held by a consumer, the government will pay out 1 unit of the consumption good if $g_{t+1}$ occurs in the following period and zero if $g_{t+1}$ does not occur. It is assumed that the government can commit to its history-dependent fiscal policy chosen at time 0. There is no capital in this economy.

Consumers are endowed with one unit of time each period, out of which they choose to work or enjoy leisure, $x\left(g^t\right)$. For every unit of labor supplied, one unit of output is produced. Feasible allocations must therefore satisfy the resource constraint:

$$c\left(g^t\right) + x\left(g^t\right) + g_t = 1 \tag{1}$$

where $c\left(g^t\right)$ represents consumption. Each consumer's wealth is composed of her after-tax labor income and the amount of savings brought into that particular state. Out of her wealth, the consumer chooses an amount of consumption and savings in the state-contingent asset market.

## 2.1 The Consumers' Model Uncertainty:

The fundamental novelty of this model relative to Lucas and Stokey (1983) is that the consumers face model uncertainty. They are endowed with an approximating model that specifies a probability measure over future exogenous and endogenous variables. However, the consumers are uncertain whether this approximating model accurately characterizes the equilibrium. They fear that other probability measures could describe the stochastic nature of the economy. To ensure that these alternative models conform to some degree with the approximating model, restrictions must be placed on what kind of alternative models are allowed.

Following Hansen and Sargent (2005, 2006), it is assumed that each member of the set of alternative probability distributions must be absolutely continuous with respect to the approximating model. This requirement implies that the consumers only fear models that correctly put no weight on events with zero probability. That is, if fiscal policy implies that a certain event will never occur, the consumers must also believe that this is true. The type of alternative model considered, then, allows for different weights as long as the approximating model indicates that the event occurs with a weight in between zero and one. More specifically, the alternative models must be absolutely continuous over finite time intervals. This implies that the alternative models entertained by the consumers cannot be rejected with a finite set of data, even though they could be rejected with an infinite data set. As indicated by Hansen and Sargent (2006), this restriction allows model uncertainty to have consequences for policy in the distant future.

Applying the Radon-Nikodym Theorem, there exists a measurable function, $M_t$, such that the distorted expectation of a random variable, $X_t$, can be rewritten in terms of the true expectation:

$$\widetilde{E}\left[X_t\right] = E\left[M_t X_t\right]$$

where $E[M_t] = 1$ and $\widetilde{E}$ is the distorted expectations operator. This equation allows me to reinterpret the consumers' uncertainty as uncertainty about the underlying shock process, rather than uncertainty about the distribution characterizing the policies and other endogenous variables. Consumers can now be thought of as assigning the correct values of the endogenous variables to each state of the world, even though they do not place true weight on the likelihood that those states occur.

By defining an additional term, one can measure the size of the consumers' probability distortion relative to the approximating model. Let the incremental probability distortion be defined as

$$m_{t+1} = \frac{M_{t+1}}{M_t}, \forall M_t > 0$$

and $m_{t+1} = 1$ otherwise. Then, $E_t m_{t+1} = 1$. This restriction guarantees that the feared probability distributions are indeed legitimate. With this definition, the one-period distance between the alternative and approximating models is measured by relative entropy:

$$\epsilon_t(m_{t+1}) \equiv E_t m_{t+1} \log m_{t+1}$$

This measure is grounded – if $m_{t+1} = 1, \forall g_{t+1}$, then $\epsilon(m_{t+1}) = 0$ – and convex in $m_{t+1}$. Thus, if $\epsilon(m_{t+1})$ is small, the set of alternative models considered by the consumer is also small. This means the equilibrium remains close to rational expectations. As $\epsilon(m_{t+1})$ increases, the set grows larger and the consumers less confident that their approximating model governs the spending shock.

Each period's relative entropy can be aggregated and discounted to form a measure of the total distortion:

$$E_0 \sum_{t=0}^{\infty} \beta^t M_t \epsilon_t(m_{t+1})$$

This distortion measure is used in the multiplier preferences of Hansen and Sargent (2006) and characterizes how the consumers rank their allocations. With these preferences, the consumers choose the allocation that maximizes the following criteria:

$$\min_{m_{t+1}, M_{t+1}} \sum_{t=0}^{\infty} \sum_{g^t} \beta^t \pi\left(g^t\right) M_t \left[u(c_t, x_t) + \beta\theta\epsilon_t(m_{t+1})\right]$$

The coefficient $\theta > 0$ is a penalty parameter that indicates the the degree to which consumers are uncertain about the probability measure. A small $\theta$ implies that the consumers are very unsure about their approximating model, leading to large probability distortions. A high value of $\theta$ means that the consumers have more confidence about the underlying measure, decreasing the size of the distortion. As $\theta \to \infty$, this model collapses to the rational expectations model of Lucas and Stokey (1983).

## 2.2 The Consumer's Problem:

Out of her wealth, each consumer chooses how much to consume and save in state-contingent debt. The consumer's wealth is composed of two elements: the after-tax labor income and the value of assets brought into the period. Given that the production function turns a unit of labor into one unit of output and the wage equals 1, the budget constraint in each period is

$$\sum_{g^{t+1}} p\left(g^{t+1}\right) b\left(g^{t+1}\right) + c\left(g^{t}\right) \leq \left(1-\tau^{n}\left(g^{t}\right)\right)\left(1-x\left(g^{t}\right)\right) + b\left(g^{t}\right) \tag{2}$$

Also imposing the legitimacy constraint, the consumer's problem can be written recursively using the value function, $V(b,g,A)$:

$$V(b,g,A) = \max_{c,x,b'} \min_{m'} \left\{ \begin{array}{c} u(c,x) + \beta \sum_{g'} \pi(g' \mid g)\left[m' V(b',g',A') + \theta m' \log m'\right] \\ -\lambda \left[\sum_{g'} p' b' + c - (1-\tau^{n})(1-x) - b\right] \\ -\beta\theta\Psi \left[\sum_{g'} \pi(g' \mid g) m' - 1\right] \end{array} \right\} \tag{3}$$

where the state variable $A$ represents the set of aggregate state variables that the consumer must track and comes from the government's problem. The consumer takes these state variables as given, believing that her decisions cannot affect their values. In tracking these aggregate state variables, the consumer is able to forecast fiscal policy after every history.

**The Minimization Stage:** The minimization problem determines the probability distortion that minimizes the consumer's expected utility for any given allocation. There are two incentives that must be considered when setting this incremental distortion, $m'$. First, the incremental distortion should be more extreme in order to lower the consumer's subjective welfare. However, these extreme values are penalized by the convex penalty term. The optimal distortion balances the marginal benefit of lowering the consumer's subjective welfare with the marginal cost due to the penalty:

$$V(b',g',A') + \theta\left(1 + \log \hat{m}'\right) - \theta\Psi = 0$$

where $\hat{m}'$ is the probability distortion that solves this first order condition. Using the additional constraint $\sum_{g'} \pi(g' \mid g) m' = 1$, the optimal distortion is

$$\hat{m}' = \frac{\exp\left(\frac{-V(b',g',A')}{\theta}\right)}{\sum_{g'} \pi(g' \mid g) \exp\left(\frac{-V(b',g',A')}{\theta}\right)} \tag{4}$$

This equation depicts the optimal tilting of the subjective probability measure away from the approximating model. The size of this tilting depends upon the consumer's subjective welfare, $V$, in each state in period $t+1$. If the allocation in a particular state results in a large subjective welfare relative to the average across all states, the numerator will be smaller than the denominator, meaning that $m' < 1$. As a result, the consumer places a smaller subjective weight on this state than the approximating model does. The reverse is true for an allocation that yields a small subjective welfare. More generally, uncertainty leads each consumer to increase the subjective weight placed on bad outcomes and decrease the subjective weight placed on good outcomes.

The size of the distortion also depends upon $\theta$, the penalty parameter. A large $\theta$ decreases the probability distortion in all states in $t+1$, meaning that $m'$ is close to 1, $\forall g'$. A small $\theta$, conversely, implies that the probability distortions will diverge from 1, meaning that the decisions of the consumer will drastically differ from the rational expectations setup.

**The Maximization Stage:** In this stage, the consumer, taking as given the prices and fiscal policy, chooses her consumption, leisure, and state-contingent bond holdings. By plugging the optimal distortion into the consumer's value function, the consumer incorporates the forecasted worst-case shock process, determined in the minimization step. The consumer then chooses an allocation, understanding how the feared distortion is influenced. The resulting recursive problem is

$$V(b,g,A) = \max_{c,x,b'} \left\{ \begin{array}{c} u(c,x) - \beta\theta \log \sum_{g'} \pi(g' \mid g) \exp\left(\frac{-V(b',g',A')}{\theta}\right) \\ -\lambda \left[ \sum_{g'} p'b' + c - (1-\tau^n)(1-x) - b \right] \end{array} \right\} \tag{5}$$

The consumer's first order conditions with respect to $c$, $x$, and $b'$ are

$$c : u_c(c,x) - \lambda = 0 \tag{6}$$

$$x : u_x(c,x) - \lambda(1-\tau^n) = 0 \tag{7}$$

$$b' : \frac{\beta \pi \left(g' \mid g\right) V_b \left(b', g', A'\right) \exp\left(\frac{-V\left(b', g', A'\right)}{\theta}\right)}{\sum_{g'} \pi \left(g' \mid g\right) \exp\left(\frac{-V\left(b', g', A'\right)}{\theta}\right)} - \lambda p' = 0 \tag{8}$$

and the envelope condition is

$$V_b \left(b, g, A\right) = \lambda$$

As is standard in models that assume the government has access only to a distortionary labor tax, the first order conditions imply the following intra-temporal tradeoff between consumption and leisure:

$$\frac{u_x \left(c, x\right)}{u_c \left(c, x\right)} = 1 - \tau^n \tag{9}$$

A larger tax increases the intra-temporal wedge. The Euler equation when consumers face model uncertainty is

$$u_c \left(c, x\right) p' = \beta \pi \left(g' \mid g\right) u_c \left(c', x'\right) m' \tag{10}$$

The consumer's fears influence her expected future marginal utility of consumption. For a given price, the consumer will choose different path of consumption and leisure than are optimal if she faced no model uncertainty.

Given these conditions, I can now define a competitive equilibrium:

**Definition 1** *A competitive equilibrium is an allocation* $\left\{c\left(g^t\right), x\left(g^t\right), V\left(g^t\right), b\left(g^{t+1}\right)\right\}_{t=0}^{\infty}$*, probability distortions* $\left\{m\left(g^{t+1}\right), M\left(g^{t+1}\right)\right\}_{t=0}^{\infty}$*, prices* $\left\{p\left(g^{t+1}\right)\right\}_{t=0}^{\infty}$*, and policies* $\left\{\tau^n\left(g^t\right)\right\}_{t=0}^{\infty}$ *such that*

1. Given the consumer's allocation, the probability distortion $\left\{m\left(g^{t+1}\right), M\left(g^{t+1}\right)\right\}_{t=0}^{\infty}$ solves the consumer's minimization problem,

2. Given the government's policy and prices, the allocation $\left\{c\left(g^t\right), x\left(g^t\right), b\left(g^{t+1}\right)\right\}_{t=0}^{\infty}$ solves the consumer's maximization problem, forecasting the response of the malevolent agent, and

3. All markets clear.

With the competitive equilibrium defined, I can now discuss the planner's problem. Following Lucas and Stokey (1983), I will write the problem using its primal representation. This formulation allows the government to directly choose the consumers' allocation, taking into account how the consumers' distortions endogenously evolve. Given this choice, the competitive equilibrium constraints then determine the necessary prices and policies that support the allocation and distortions. It is assumed that each government is able to commit to this fiscal policy at time 0. Each government chooses the best competitive equilibrium given its objective function. In the following sections, I detail each planner's decision problem in turn and analyze the resulting policies and allocations.

# 3 The Benevolent Government:

The government has access to a commitment technology with which it is able to bind itself to a sequence of policies chosen at t=0. In formulating these policies, the government acknowledges and accounts for the consumers' uncertainty. For each possible policy rule, the planner understands how it influences the optimal allocation and feared distribution of the consumers. The objective function for the benevolent government is the consumers' true expected utility. The resulting allocation achieves the highest possible welfare for the consumers.

**Definition 2** *The Ramsey problem of the benevolent government is to maximize the true expected utility of the representative consumer over competitive equilibria. The Ramsey outcome is the competitive equilibrium that attains the maximum.*

**Proposition 1** *The allocation and distortions in a Ramsey outcome solve the following problem:*

$$\max_{c_t, x_t, V_t, b_{t+1}, m_{t+1}} \sum_{t=0}^{\infty} \sum_{g^t} \beta^t \pi\left(g^t\right) u\left(c_t, x_t\right)$$

*subject to*

$$\beta \sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) u_c\left(c_{t+1}, x_{t+1}\right) b_{t+1} m_{t+1} + u_c\left(c_t, x_t\right)\left(c_t - b_t\right) - u_x\left(c_t, x_t\right)\left(1 - x_t\right) = 0 \tag{11}$$

$$m_{t+1} = \frac{\exp\left(\frac{-V_{t+1}}{\theta}\right)}{\sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) \exp\left(\frac{-V_{t+1}}{\theta}\right)} \tag{12}$$

$$V_t = u\left(c_t, x_t\right) + \beta \sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) \left\{m_{t+1} V_{t+1} + \theta m_{t+1} \ln m_{t+1}\right\} \tag{13}$$

$$c_t + x_t + g_t = 1 \tag{14}$$

**Proof.** When setting its policy, the government is restricted in the set of feasible allocations that it can achieve by the competitive equilibrium constraints. The claim is that those restrictions are summarized by the constraints (11) – (14). To demonstrate this, I will first show that any allocation and probability distortion that satisfies the competitive equilibrium constraints must also satisfy (11) – (14). (2) holds with equality in equilibrium. Insert (6), (7), and (8) into (2) to get the constraint (11). The constraint (12) follows directly from the optimality condition in the inner minimization, (13) is the consumer's Bellman equation, and (14) is the resource constraint. Thus, (11) − (14) are necessary conditions that the Ramsey outcome must solve. Going in the other direction, given an allocation and distortions that satisfy (11) − (14), policies and prices can be determined from the representative consumer's first order conditions. ■

The first constraint, a period implementability constraint, describes the transition equation of the marginal value of debt: $u_c(c_t, x_t) b_t$. This equation is similar to the constraint in the rational expectations fiscal policy model, except that the expectation of tomorrow's marginal value of debt is tilted by the probability distortion.

The second implementability constraint (13) captures how the consumer's value function updates across time and states:

$$V_t = u(c_t, x_t) + \beta E_t \left[m_{t+1} V_{t+1} + \theta m_{t+1} \log m_{t+1}\right] \tag{15}$$

This equation is a new constraint that does not appear in the rational expectations model. The planner also faces the resource constraint and the description of the optimal probability distortion. The constraints $\{(11), (12), (13), \text{ and } (14)\}$ fully characterize the competitive equilibrium restrictions by which the government must abide.

**Sequential Formulation of the Benevolent Planner's Problem:** The benevolent government's optimization problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi(g^t) \left\{ \begin{array}{c} u(c_t, x_t) + \mu_t \left[c_t + x_t + g_t - 1\right] \\ +\xi_t \left[\beta \sum_{g_{t+1}} \pi(g_{t+1} \mid g^t) u_c(c_{t+1}, x_{t+1}) b_{t+1} m_{t+1} + u_c(c_t, x_t)(c_t - b_t) - u_x(c_t, x_t)(1 - x_t)\right] \\ +\Gamma_t \left[V_t - u(c_t, x_t) - \beta \sum_{g_{t+1}} \pi(g_{t+1} \mid g^t) \left\{m_{t+1} V_{t+1} + \theta m_{t+1} \ln m_{t+1}\right\}\right] \\ +\beta \sum_{g_{t+1}} \pi(g_{t+1} \mid g^t) \varpi_{t+1} \left[m_{t+1} - \dfrac{\exp\left(\frac{-V_{t+1}}{\theta}\right)}{\sum_{g_{t+1}} \pi(g_{t+1} \mid g^t) \exp\left(\frac{-V_{t+1}}{\theta}\right)}\right] \end{array} \right\}$$

The first order conditions are

$$\begin{aligned} c_t, \forall t &\geq 1: \\ 0 &= u_c(c_t, x_t) + \mu_t - \Gamma_t u_c(c_t, x_t) + \xi_{t-1} u_{cc}(c_t, x_t) b_t m_t \\ &+ \xi_t \left[ u_{cc}(c_t, x_t)(c_t - b_t) + u_c(c_t, x_t) - u_{cx}(c_t, x_t)(1 - x_t) \right] \end{aligned} \tag{16}$$

$$\begin{aligned} x_t, \forall t &\geq 1: \\ 0 &= u_x(c_t, x_t) + \mu_t - \Gamma_t u_x(c_t, x_t) + \xi_{t-1} u_{cx}(c_t, x_t) b_t m_t \\ &+ \xi_t \left[ u_{cx}(c_t, x_t)(c_t - b_t) + u_x(c_t, x_t) - u_{xx}(c_t, x_t)(1 - x_t) \right] \end{aligned} \tag{17}$$

$$\begin{aligned} V_t, \forall t &\geq 1: \\ 0 &= \Gamma_t - \Gamma_{t-1} m_t + \left(\frac{1}{\theta}\right) m_t \left[\varpi_t - E_{t-1} m_t \varpi_t\right] \end{aligned} \tag{18}$$

$$\begin{aligned} m_{t+1}, \forall g^{t+1}, \forall t &\geq 0: \\ 0 &= \xi_t u_c(c_{t+1}, x_{t+1}) b_{t+1} - \Gamma_t \left[V_{t+1} + \theta(1 + \ln m_{t+1})\right] + \varpi_{t+1} \end{aligned} \tag{19}$$

$$\begin{aligned} b_{t+1}, \forall g^{t+1}, \forall t &\geq 0: \\ 0 &= \xi_t m_{t+1} - \xi_{t+1} \end{aligned} \tag{20}$$

The first order conditions (18) and (20) imply that both Lagrange multipliers on the implementability constraints $(\xi_t, \Gamma_t)$ are martingales under the true expectation:

$$\begin{aligned} E_t \xi_{t+1} &= \xi_t \\ E_{t-1} \Gamma_t &= \Gamma_{t-1} \end{aligned}$$

This result, noted by Karantounias, Hansen, and Sargent (2007), implies that the allocation exhibits persistence. This persistence appears because the benevolent government must take into account the effects of the consumers' probability distortion, a martingale. The degree of persistence therefore depends upon the level of the consumers' uncertainty. If the consumers face a large degree of uncertainty, then fiscal policy becomes more persistent[4]. As $\theta \to \infty$, the Lagrange multipliers reduce to constants, eliminating

[4] As noted by Bohn (1998), Soresnen, Wu, and Yosha (2001), and Huang and Lin (1993), fiscal policy persistence is empirically relevant in the United States.

the persistence. As a result of this persistence, consumer uncertainty leads to variations in the shadow values of the marginal utility of debt and the consumer's welfare across time and states. This variation is absent in Lucas and Stokey (1983).

Beyond the inter-temporal persistence, model uncertainty also imparts additional intra-temporal smoothing. Whereas the implementability constraint is the sole link across states in Lucas and Stokey (1983), the probability distortion directly connects the allocation across states. The linkage is most salient in the first order condition for (18), as the movement of $\Gamma_t$ depends upon the difference between one state's characteristics and its expectation. As $\theta \to \infty$ and the model returns to the rational expectations limit, the additional intra-temporal connection disappears.

The time 0 first order conditions are

$$\begin{aligned} c_0 \quad &: \quad 0 = u_c(c_0, x_0) + \mu_0 - \Gamma_0 u_c(c_0, x_0) \\ &\qquad + \xi_0 \left[ u_{cc}(c_0, x_0)(c_0 - b_0) + u_c(c_0, x_0) - u_{cx}(c_0, x_0)(1 - x_0) \right] \end{aligned}$$

$$\begin{aligned} x_0 \quad &: \quad 0 = u_x(c_0, x_0) + \mu_0 - \Gamma_0 u_x(c_0, x_0) \\ &\qquad + \xi_0 \left[ u_{cx}(c_0, x_0)(c_0 - b_0) + u_x(c_0, x_0) - u_{xx}(c_0, x_0)(1 - x_0) \right] \end{aligned}$$

$$V_0(g_0) : \Gamma_0 = 0$$

In order to numerically implement the solution, I will formulate the recursive problem of the government below. This recursive problem will use the fact that the solution is recursive in the Lagrange multipliers on the implementability constraints to determine which variables should be added to the list of state variables.

**Recursive Formulation of the Benevolent Planner's Problem:** In deriving the recursive form of the government's optimization problem, I assume that government expenditures follow a Markov process with transition matrix $\Pi$. Due to the time-inconsistency of the planner's problem, I apply the Marcet and Marimon (1998) procedure to the implementability constraints. The co-state variable on (11) is $\xi_-$ with a state-contingent increment of $\xi_g$. The subscript $g$ implies a state-contingent value in period $t \geq 1$. The co-state variable on (13) is $\Gamma_-$ with a state-contingent increment of $\Gamma_g$. An ex-ante value function is necessary to account for probability distortion, which connects all states in a particular period.

The planner's problem in recursive form is

$$W\left(\xi_-,\Gamma_-,g_-\right)=\min_{\xi_g,\Gamma_g}\max_{c_g,x_g,m_g,V_g,b_g}\sum_g \pi\left(g\mid g_-\right)\left\{\begin{array}{c} u\left(c_g,x_g\right)+\mu_g\left[c_g+x_g+g-1\right]+\xi_-\left[u_c\left(c_g,x_g\right)m_g b_g\right] \\ +\xi_g\left[u_c\left(c_g,x_g\right)\left(c_g-b_g\right)-u_x\left(c_g,x_g\right)\left(1-x_g\right)\right] \\ -\Gamma_-\left[m_g V_g+\theta m_g\ln m_g\right]+\Gamma_g\left[V_g-u\left(c_g,x_g\right)\right] \\ +\varpi_g\left[m_g-\frac{\exp\left(\frac{-V_g}{\theta}\right)}{\sum_g \pi(g|g_-)\exp\left(\frac{-V_g}{\theta}\right)}\right]+\beta W\left(\xi_g,\Gamma_g,g\right)\end{array}\right\}$$

The initial values of the co-state variables are 0, representing the assumption that the planner at time t=0 is not bound by any previous promises.

The first order conditions are

$$\begin{aligned} c_g \quad : \quad & 0=u_c\left(c_g,x_g\right)+\mu_g+\xi_- u_{cc}\left(c_g,x_g\right)m_g b_g-\Gamma_g u_c\left(c_g,x_g\right) \\ & +\xi_g\left[u_{cc}\left(c_g,x_g\right)\left(c_g-b_g\right)+u_c\left(c_g,x_g\right)-u_{cx}\left(c_g,x_g\right)\left(1-x_g\right)\right] \end{aligned} \tag{21}$$

$$\begin{aligned} x_g \quad : \quad & 0=u_x\left(c_g,x_g\right)+\mu_g+\xi_- u_{cx}\left(c_g,x_g\right)m_g b_g-\Gamma_g u_x\left(c_g,x_g\right) \\ & +\xi_g\left[u_{cx}\left(c_g,x_g\right)\left(c_g-b_g\right)+u_x\left(c_g,x_g\right)-u_{xx}\left(c_g,x_g\right)\left(1-x_g\right)\right] \end{aligned} \tag{22}$$

$$m_g: 0=\xi_- u_c\left(c_g,x_g\right)b_g-\Gamma_-\left[V_g+\theta\left(1+\ln m_g\right)\right]+\varpi_g \tag{23}$$

$$V_g: 0=-\Gamma_- m_g+\Gamma_g+\left(\frac{1}{\theta}\right)m_g\left[\varpi_g-\sum_g \pi\left(g\mid g_-\right)m_g\varpi_g\right] \tag{24}$$

$$b_g: 0=\xi_- m_g-\xi_g \tag{25}$$

$$\xi_g: 0=u_c\left(c_g,x_g\right)\left(c_g-b_g\right)-u_x\left(c_g,x_g\right)\left(1-x_g\right)+\beta W_\xi\left(\xi_g,\Gamma_g,g\right) \tag{26}$$

$$\Gamma_g: 0=V_g-u\left(c_g,x_g\right)+\beta W_\Gamma\left(\xi_g,\Gamma_g,g\right) \tag{27}$$

and the envelope conditions are

$$W_\xi\left(\xi_-,\Gamma_-,g_-\right)=\sum_g \pi\left(g\mid g_-\right)\left[u_c\left(c_g,x_g\right)m_g b_g\right]$$

$$W_{\Gamma}\left(\xi_{-},\Gamma_{-},g_{-}\right)=-\sum_{g}\pi\left(g\mid g_{-}\right)\left[m_{g}V_{g}+\theta m_{g}\ln m_{g}\right]$$

The envelope conditions can be combined with (26) and (27) to retrieve the implementability constraints. It can be shown that the other four first order conditions are equivalent to those in the sequential formulation. As in the sequential version, the co-state variables are martingales.

The time 0 value function is

$$W_{0}=\min_{\xi_{0},\Gamma_{0}}\max_{c_{0},x_{0},V_{0}}\left\{\begin{array}{c}u\left(c_{0},x_{0}\right)+\mu_{0}\left[c_{0}+x_{0}+g_{0}-1\right]\\+\xi_{0}\left[u_{c}\left(c_{0},x_{0}\right)\left(c_{0}-b_{0}\right)-u_{x}\left(c_{0},x_{0}\right)\left(1-x_{0}\right)\right]\\+\Gamma_{0}\left[V_{0}-u\left(c_{0},x_{0}\right)\right]+\beta W\left(\xi_{0},\Gamma_{0},g_{0}\right)\end{array}\right\}$$

The first order conditions are

$$c_{0}:0=u_{c}\left(c_{0},x_{0}\right)+\mu_{0}+\xi_{0}\left[u_{cc}\left(c_{0},x_{0}\right)\left(c_{0}-b_{0}\right)+u_{c}\left(c_{0},x_{0}\right)-u_{cx}\left(c_{0},x_{0}\right)\left(1-x_{0}\right)\right]-\Gamma_{0}u_{c}\left(c_{0},x_{0}\right)$$

$$x_{0}:0=u_{x}\left(c_{0},x_{0}\right)+\mu_{0}+\xi_{0}\left[u_{cx}\left(c_{0},x_{0}\right)\left(c_{0}-b_{0}\right)+u_{x}\left(c_{0},x_{0}\right)-u_{xx}\left(c_{0},x_{0}\right)\left(1-x_{0}\right)\right]-\Gamma_{0}u_{x}\left(c_{0},x_{0}\right)$$

$$V_{0}:0=\Gamma_{0}$$

$$\xi_{0}:0=u_{c}\left(c_{0},x_{0}\right)\left(c_{0}-b_{0}\right)-u_{x}\left(c_{0},x_{0}\right)\left(1-x_{0}\right)+\beta W_{\xi}\left(\xi_{0},\Gamma_{0},g_{0}\right)$$

$$\Gamma_{0}:0=V_{0}-u\left(c_{0},x_{0}\right)+\beta W_{\Gamma}\left(\xi_{0},\Gamma_{0},g_{0}\right)$$

### 3.1 Model Solution and Discussion:

In order to understand how consumer uncertainty affects optimal fiscal policy, this section compares the rational expectations equilibrium with the equilibrium under consumer uncertainty. To ease the exposition, assume a simple process for government spending:

$$\begin{aligned}g_{t}&=0,\forall t\neq T\\g_{T}&=\left\{\begin{array}{c}G,\text{with probability }\alpha\\0,\text{with probability }1-\alpha\end{array}\right\}\end{aligned}$$

Additionally, $b_0 = 0$, so that consumers have no debt or assets in the initial period.

I will first discuss the rational expectations solution. As shown in Lucas and Stokey (1983), the allocation depends only upon the current value of the government spending shock. This means that for consumption and leisure, for example, there are two possible values: $\{c(0), x(0)\}$ and $\{c(G), x(G)\}$. Using (9), the tax rate also takes on two positive values: $\tau^n(0)$ and $\tau^n(G)$. As government spending is zero for all periods except T, the government's positive tax $\tau^n(0)$ yields a surplus at each of these dates.

The surplus, equal to $\tau^n(0)(1-x(0))$, is loaned to the consumers in each period $t<T$ at an interest rate of $\frac{1}{\beta}$, in addition to the accumulated assets from the previous period.

In period T-1, the government uses its accumulated assets to obtain insurance from the consumers against the shock at T. The government buys bonds (setting $b_T(G)<0$) at a price $\beta\alpha\frac{u_{c(c(G),x(g))}}{u_{c(c(0),x(0))}}$ that can be redeemed if $g_T=G$. In addition to using its accumulated wealth, the government finances the cost of this insurance through period T-1's primary surplus and through selling debt that pays off if $g_T=0$. The price of this debt is $\beta(1-\alpha)$.

In period T, the allocation depends upon whether or not the shock occurs. If $g_T=G$, the government runs a primary deficit $\tau^n(G)(1-x(G))-G<0$. This deficit is financed partially by the interest repayment on the loan made in the previous period and partially by selling additional debt to the consumers. For each period $t>T$, the government collects tax revenues and borrows money from the consumers to pay for its debt. If $g_T=0$, though, the government uses its primary surplus and additional borrowed money from the consumers to finance its previous debt from that period forward.

Viewing this result from an insurance perspective, the government's chosen profile of tax and debt mitigates the cost of the spending shock by spreading the intra-temporal distortion across time and states. Rather than having no distortion when $g_t=0$ and a large distortion when $g_T=G$, the government charges a positive tax on labor income in all periods leading up to T. In period T-1, these assets are then used to buy insurance from the consumers, who agree to pay a fraction of the cost of the spending shock if it occurs. The additional funds come from tax revenue and from borrowing money from the consumers in period T. Thus, under rational expectations, the primary role of state-contingent debt is to provide insurance, allowing the government to reduce its dependence on the linear labor tax to finance the spending shock.

Model uncertainty breaks this simple relationship. In addition to the insurance incentive depicted above, the benevolent government also seeks to mitigate the impact of the consumers' probability distortion. The government therefore must use fiscal policy to balance the social costs of the linear labor tax and model uncertainty.

To see how these tradeoffs are balanced, I have computed the solution to the benevolent government's problem and have graphed the solutions below. In calculating these solutions, I have assumed that the consumers have the following CRRA preferences:

$$u(c,x) = \frac{c^{1-\sigma}}{1-\sigma} + \eta \frac{x^{1-\delta}}{1-\delta}.$$

Government spending follows the process:

$$g_t = \bar{g} + \rho \left( g_{t-1} - \bar{g} \right) + \epsilon.$$

Depending on the value of $\rho$, this process could resemble an iid shock to government spending or an AR(1) process. Shocks to government spending will be drawn from an approximation to a normal distribution, $\epsilon \sim N\left(0, 0.02^2\right)$. In the numerical calculations, three values of the shock are considered. The probability of being hit by the high spending shock is 17%, which, because of symmetry, is also the probability of being hit by the low shock. In the graphs below, I plot only the solutions that occur in either war or peace. Consumers begin with no assets: $b_0 = 0$. The parameters used in these calculations are listed below in Table 1:

| Parameters: | |
|---|---|
| Utility: | Government Spending: |
| $\sigma = 2$ | $\bar{g} = 0.1$ |
| $\eta = 1$ | $\rho = 0$ |
| $\delta = 2$ | $g_0 = \bar{g}$ |
| $\beta = 0.95$ | |

Table 1: Parameter values

To understand how the solution changes relative to the Lucas and Stokey (1983) benchmark, I have plotted the solutions below. These solutions are the time 1 values of the endogenous variables, plotted for varying levels of consumer uncertainty. When log(theta) is large, the consumers are confident about their approximating model and the equilibrium approaches the rational expectations solution discussed above. As log(theta) falls, however, the consumers are increasingly uncertain about the probability model. Because this larger uncertainty leads to larger distortions, the government becomes more concerned about the costs stemming from model uncertainty. Even though the numerical procedure assumes three values of the shock to government spending, the graphs below plot the solutions given that the shock value is either high or low.

As analytically shown above, the consumers' uncertainty leads them to tilt their subjective probabilities away from their approximating model. This probability distortion can be seen in Figure 1. As log(theta) falls, the consumers place greater weight on the high government spending state (labeled "war") and lower weight on the low government spending state (labeled "peace").

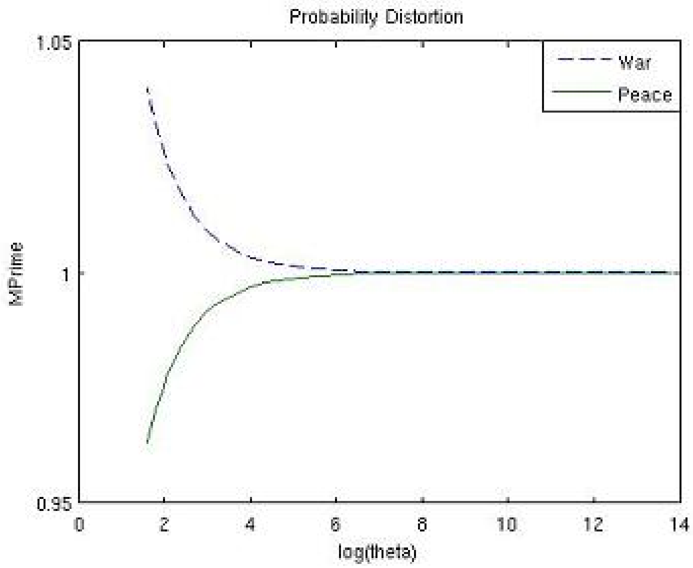


Figure 1: Consumers' incremental probability distortion for different levels of $\theta$

This probability tilting leads the consumers to distort their savings decisions away from the ones exhibited in rational expectations. For the same price level, the increase in the consumers' subjective probability placed on war leads them to increase their holdings of the war-contingent asset. Similarly, because the consumers' place less weight on the event of peace, they desire fewer of these state-contingent assets. These movements have implications for the prices and returns of the assets. As seen in the pricing equation, (10), the increase in demand for war-contingent assets raises the price of these holdings. As a result, the return on this debt falls. The opposite movements occur for peace-contingent assets.

This savings distortion undermines the government's ability to obtain insurance against its spending shock. As in the rational expectations case, the benevolent government would like to buy war-contingent debt (to be paid off by the consumers in the event of war) and sell peace contingent debt (to be paid off by the government in the event of peace). This profile enables the government to set a smooth labor tax rate. However, as indicated above, model uncertainty leads consumers to desire the exact opposite profile of debt. The consumers, in their uncertainty about the true probability distribution, want to save in a manner that makes it difficult for the government to use debt to finance spending.

The fundamental tension facing the benevolent government, then, is to balance these two opposing desires. On the one hand, the government would like to insure itself against the spending shock. On the other hand, the consumers want to rearrange their wealth profile to reflect their distorted beliefs. The government resolves this tradeoff by partially realigning the consumers' savings decisions with their rational expectations values. It does so by manipulating the prices and returns on debt, as seen in Figure 2. Specifically, fiscal policy is designed to raise (lower) the price of war-contingent (peace-contingent) assets. The higher price on war-contingent assets discourages the consumers from holding this type of instrument. This movement allows the government to increase the amount of money it loans to the consumers that they must repay during times of war. The lower price on peace-contingent assets encourages the consumers to loan money to the government that it must repay during times of peace. These decisions mean that the government is able to obtain a degree of insurance from consumers.

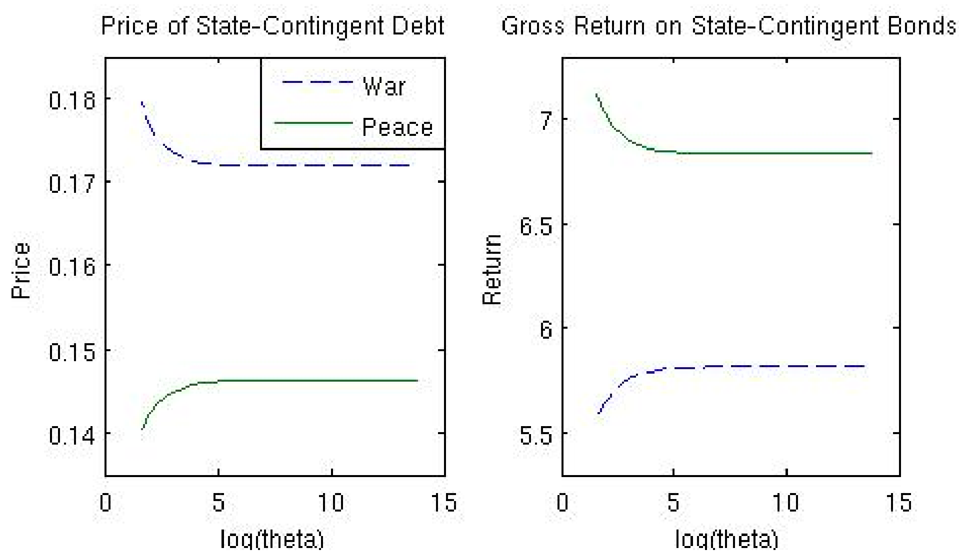


Figure 2: Price and return on debt for different levels of $\theta$

To be clear, two factors drive the movement in state-contingent asset prices. The first factor, noted above, is that the consumers' distorted beliefs over the states affect their demand for each type of asset. This changing demand puts upward pressure on the price of war-contingent assets and downward pressure on the price of peace contingent assets. The second factor is that the government implements a policy that intensifies these price movements, driving the prices on war-contingent (peace-contingent) assets even higher (lower). These two factors are decomposed in Figure 3. This graph plots the war-contingent asset price under two different scenarios. The solid line, labeled "Beliefs", depicts the movement in the price due purely to the distorted beliefs of the consumers. This line is obtained by multiplying the stochastic discount factor that arises when consumers are certain with their probability distortion across different levels of $\theta$[5]. The dotted line, labeled "Policy", depicts the price of the war-contingent asset, taking into account both the consumers' beliefs and the fiscal policy.

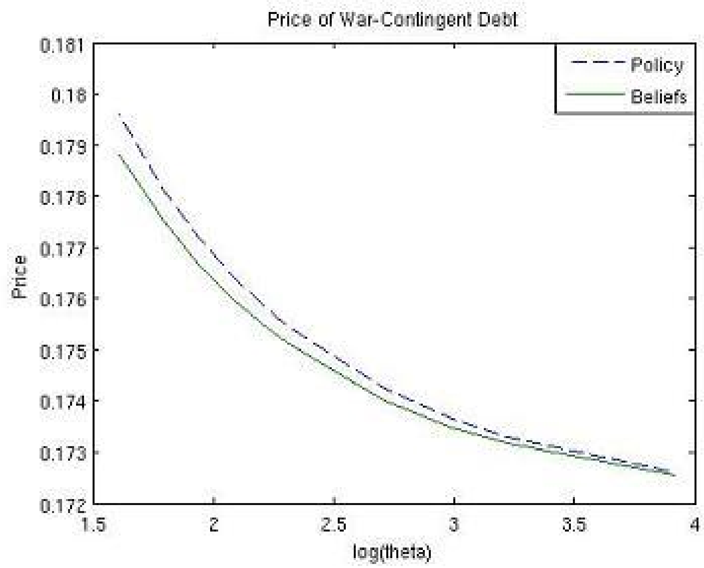


Figure 3: Decomposing price movements into beliefs and policy

The government is able to influence the prices on state-contingent debt by altering the labor tax rate. Relative to the policy chosen when consumers face no uncertainty, model uncertainty leads the government to implement a relatively high (low) labor tax rate conditional on war (peace). This policy affects the allocation chosen by the consumers in each state. As a result, the stochastic discount factor moves in such a way as to raise the war-contingent and lower the peace-contingent asset price.

To elucidate this point, consider the increase in the war-time tax in T+1. The higher tax encourages consumers to enjoy more leisure during wars because the after-tax marginal return on labor has fallen. Due to the decrease in labor income, consumers reduce their consumption. This change then affects the price of the war-contingent debt in period T. For a given marginal utility of consumption at T, the increase in the labor tax rate in war will raise the marginal utility of consumption at T+1, causing

[5] This relative contribution of beliefs in price movements is only approximate. By keeping the stochastic discount factor constant at the $\theta \to \infty$ value, I am assuming that the allocation remains at its "certainty" level even though the consumers' uncertainty is growing. However, the probability distortions and the allocation move together, making it difficult to separate the role of beliefs and policy. This exercise is merely meant to hint at the two factors that drive price movements.

the price of war-contingent debt to rise. Consumers, faced with the increased price and lower return on war-contingent debt, choose to hold less of this debt than if the labor tax had not increased. The opposite profile of incentives holds true in the event of peace. In effect, the government sets fiscal policy to discourage consumers from holding war-contingent debt and from borrowing peace-contingent debt, partially reversing the savings distortion.

The changes in the labor tax rate have important macroeconomic implications. The direct effect, seen in Figure 4, is to increase the state-contingent volatility of the tax revenue. The government, in effect, relies more heavily upon the labor tax rate to finance fluctuations in spending than it would if consumers faced no uncertainty. As a result, the state-contingent fluctuations in the primary deficit become less pronounced. These policies then reduce the state-contingent volatility of output and consumption.

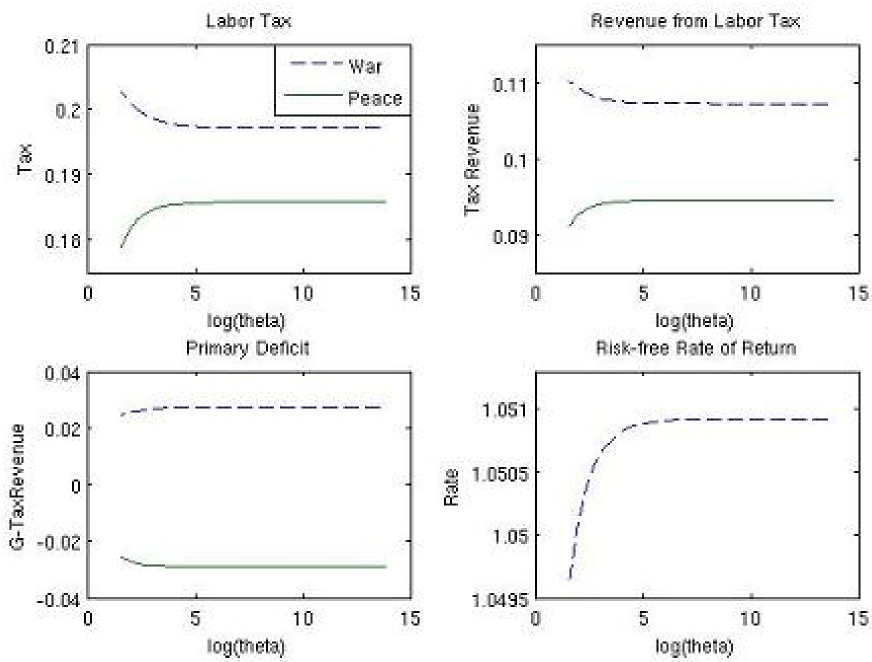


Figure 4: Optimal fiscal policy implemented by the benevolent government for different levels of $\theta$

The conclusions described above characterize the volatility of the solutions at a particular point in time. Below, I compare the impulse response functions under different degrees of model uncertainty. To create these response functions, I assume that government spending is equal to its average, $g_t = \bar{g}$, for all periods except period 1, at which time $g_1 = g_{high}$. The spending profile associated with this one-period war can be seen in the top left graph in Figure 5.

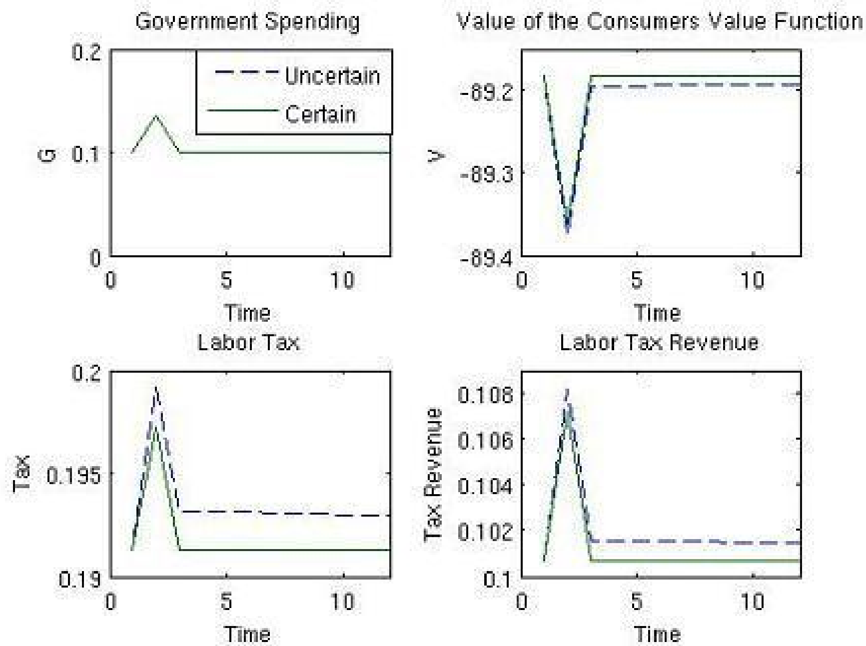


Figure 5: Policy impulse response functions to an increase in government spending when $\theta \to \infty$ and $\theta = 15$

During periods of war, the benevolent government raises the labor tax rate to help finance the spending shock. Relative to the case in which consumers completely understand the shock process, model uncertainty leads to a higher spike in the tax rate. This increase, although partially offset by a decrease in labor supply, results in a rise in labor tax revenues. As indicated above, the benevolent government relies more heavily on labor taxes to finance the spending shock when consumers face model uncertainty.

The impact of this tax change on the consumers' allocation can be seen in Figure 6.

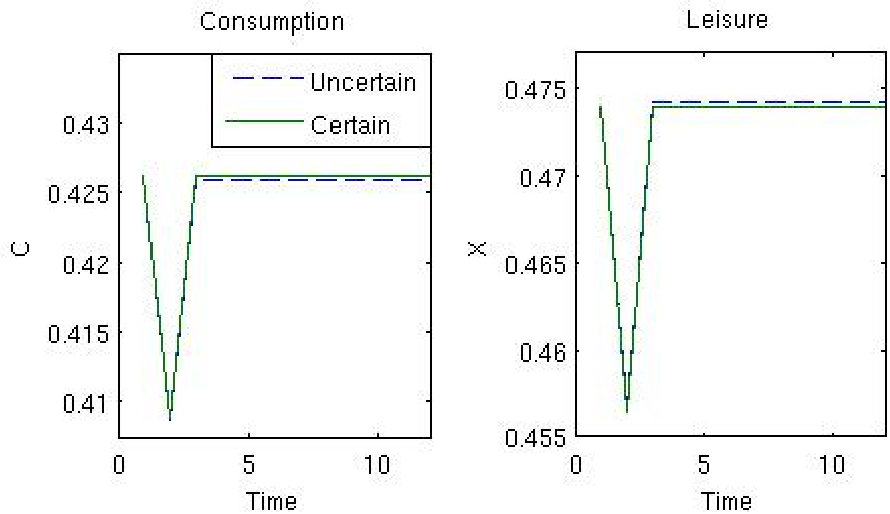


Figure 6: Allocation impulse response functions to an increase in government spending when $\theta \to \infty$ and $\theta = 15$

The shock to government spending implies that the consumers will supply more labor and consume less in the shock period. This accounts for the large drop in leisure and consumption on impact. However, because model uncertainty results in a higher labor tax implemented by the benevolent government, the consumers increase their leisure relatively more than when they are certain. This implies that their labor income is relatively low, leading to a decrease in their consumption levels.

An additional feature of these impulse response functions that is worthy of note is that the levels of persistence differ across the two models. When consumers are certain, the post-war allocation returns to its pre-war levels after the shock ends. This is not the case when consumers face uncertainty. Instead, the war-time labor tax remains higher than its rational expectations value for a number of periods after the one-period war. This persistence translates into a prolonged period of decreased consumption and labor hours for the consumers.

**Summary:** A benevolent government that knows its citizens face model uncertainty encounter a pair of incentives that pull in opposite directions. The government wants to smooth the intra-temporal distortion due to the distortionary tax across time and states by obtaining insurance from the consumers. This means that the government wants to hold debt that pays off if war occurs. Model uncertainty, however, leads the consumers to increase the weight they place on the event of war. This encourages the consumers to save more into this state, undermining the government's ability to insure against the spending shock. The government responds to these incentives by increasing the tax rate in war and lowering it in peace. By influencing the price on state-contingent debt, this policy mitigates the impact of the savings distortion caused by uncertainty. As a result of the tax changes, the variance of the primary deficit has fallen relative to what is optimal when consumers are confident in their approximating model.

# 4 The Political Government:

A central implication of model uncertainty is that the consumers' expectation of the likelihood of events could differ from the true expectation. This distinction leads to some flexibility as to the objective function of the government. The previous section modeled a benevolent government that maximizes the consumers' true expected utility. This objective function leads the government to choose a volatile tax rate that is meant to manipulate the consumers' savings profile. This section describes the decision problem of a political government, which maximizes the distorted expectation of the consumers' utility. This objective function is more aligned with the preferences of the consumers than the paternalistic objective function of the benevolent government.

In studying the equilibrium consequences of this change, I follow the same steps as above. I begin this section by formulating the sequential version of the planner's problem. I show that the first order conditions from this problem are equivalent to those from a recursive setup. Then, I compute the numerical solutions of this model, comparing the equilibrium to that in a rational expectations model.

**Definition 3** *The Ramsey problem of the political government is to maximize the distorted expected utility of the representative consumer over competitive equilibria. The Ramsey outcome is the competitive equilibrium that attains the maximum.*

**Proposition 2** *The allocation and distortions in a Ramsey outcome solve the following problem:*

$$\max_{c_t, x_t, V_t, b_{t+1}, m_{t+1}, M_{t+1}} \sum_{t=0}^{\infty} \sum_{g^t} \beta^t \pi\left(g^t\right) M_t u\left(c_t, x_t\right)$$

*subject to*

$$\sum_{t=0}^{\infty} \sum_{g^t} \beta^t \pi\left(g^t\right) M_t \left[u_c\left(c_t, x_t\right) c_t - u_x\left(c_t, x_t\right)\left(1 - x_t\right)\right] = u_c\left(c_0, x_0\right) b_0$$

$$m_{t+1} = \frac{\exp\left(\frac{-V_{t+1}}{\theta}\right)}{\sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) \exp\left(\frac{-V_{t+1}}{\theta}\right)}$$

$$V_t = u\left(c_t, x_t\right) + \beta \sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) \left\{m_{t+1} V_{t+1} + \theta m_{t+1} \ln m_{t+1}\right\}$$

$$M_{t+1} = m_{t+1} M_t$$

$$c_t + x_t + g_t = 1$$

The proof is similar to the previous one – except one must sum across $t$ to derive the first implementability constraint – and so is suppressed. The transversality condition imposed here is

$$lim_{T\to\infty} \beta^T M_T \lambda_T p_T b_T = 0$$

It is clear from this proposition that the government places the same weights on future events as the representative consumer does. The first constraint is the infinite-time implementability constraint. This constraint differs from the rational expectations version because the distortion $M_t$ affects the perceived probability of each history. In addition, there is one constraint that must be applied to the political government's problem that was not applied to the benevolent government's problem. This constraint tracks the movement in $M_t$.

**Sequential Formulation of the Political Planner's Problem:** The objective function of the political government leads to the following sequential problem, in which the government chooses paths for $(c_t, x_t, V_t, \{m_{t+1}, M_{t+1}\})$ :

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi\left(g^t\right) \left\{ \begin{array}{c} M_t u\left(c_t, x_t\right) + \mu_t \left[c_t + x_t + g_t - 1\right] \\ +\xi M_t \left[u_c\left(c_t, x_t\right) c_t - u_x\left(c_t, x_t\right)\left(1 - x_t\right)\right] - \xi u_c\left(c_0, x_0\right) b_0 \\ +M_t \Gamma_t \left[V_t - u\left(c_t, x_t\right) - \beta \sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) \left\{m_{t+1} V_{t+1} + \theta m_{t+1} \ln m_{t+1}\right\}\right] \\ +\beta M_t \sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) \varpi_{t+1} \left[m_{t+1} - \frac{\exp\left(\frac{-V_{t+1}}{\theta}\right)}{\sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) \exp\left(\frac{-V_{t+1}}{\theta}\right)}\right] \\ +\beta \sum_{g_{t+1}} \pi\left(g_{t+1} \mid g^t\right) \chi_{t+1} \left[M_{t+1} - m_{t+1} M_t\right] \end{array} \right\}$$

The first order conditions for this problem are

$$\begin{aligned} c_t, \forall t &\geq 1 : \\ 0 &= u_c\left(c_t, x_t\right) + \frac{\mu_t}{M_t} - \Gamma_t u_c\left(c_t, x_t\right) \\ &\quad +\xi \left[u_{cc}\left(c_t, x_t\right) c_t + u_c\left(c_t, x_t\right) - u_{cx}\left(c_t, x_t\right)\left(1 - x_t\right)\right] \end{aligned}$$

$$\begin{aligned} x_t\left(g^t\right), \forall t &\geq 1 : \\ 0 &= u_x\left(c_t, x_t\right) + \frac{\mu_t}{M_t} - \Gamma_t u_x\left(c_t, x_t\right) \\ &\quad +\xi \left[u_{cx}\left(c_t, x_t\right) c_t + u_x\left(c_t, x_t\right) - u_{xx}\left(c_t, x_t\right)\left(1 - x_t\right)\right] \end{aligned}$$

$$
\begin{aligned}
m_{t+1}, \forall t &\geq 0 : \\
0 &= -\Gamma_t \left[ V_{t+1} + \theta \left(1 + \log m_{t+1}\right)\right] + \varpi_{t+1} - \chi_{t+1}
\end{aligned}
$$

$$
\begin{aligned}
V_t, \forall t &\geq 1 : \\
0 &= \Gamma_t - \Gamma_{t-1} + \left(\frac{1}{\theta}\right) \left[ \varpi_t - \sum_{g_t} \pi \left(g_t \mid g^{t-1}\right) m_t \varpi_t \right]
\end{aligned}
$$

$$
\begin{aligned}
M_{t+1}, \forall t &\geq 0 : \\
0 &= \chi_{t+1} + u \left(c_{t+1}, x_{t+1}\right) + \xi \left[ u_c \left(c_{t+1}, x_{t+1}\right) c_{t+1} - u_x \left(c_{t+1}, x_{t+1}\right) \left(1 - x_{t+1}\right)\right] \\
&\quad - \beta \sum_{g_{t+2}} \pi \left(g_{t+2} \mid g^{t+1}\right) m_{t+2} \chi_{t+2}
\end{aligned}
$$

Just as in the benevolent government's first order conditions, the multiplier with respect to (15) is a martingale, this time with respect to the distorted expectation. This implies that $\Gamma_{t-1}$ will be a state variable in the recursive version of the problem. Interestingly, the level of the probability distortion, $M_t$, disappears from the first order conditions. This is because the planner does not seek to realign the consumers' expectation with what it feels is correct, since the two agents agree about the probability model. As a result, the planner does not need a third state variable to track the level of the distortion. Thus, the equivalent recursive problem only has two state variables.

**Recursive Formulation of the Political Planner's Problem:** The planner's problem in recursive form is

$$
W \left(\Gamma_-, g_-\right) = \min_{\Gamma_g} \max_{c_g, x_g, m_g, V_g} \sum_g \pi \left(g \mid g_-\right) \left\{ \begin{array}{c} m_g u \left(c_g, x_g\right) + \mu_g \left[c_g + x_g + g - 1\right] \\ + \xi m_g \left[ u_c \left(c_g, x_g\right) c_g - u_x \left(c_g, x_g\right) \left(1 - x_g\right)\right] \\ - \Gamma_- \left[ m_g V_g + \theta m_g \ln m_g \right] + m_g \Gamma_g \left[ V_g - u \left(c_g, x_g\right)\right] \\ + \varpi_g \left[ m_g - \frac{\exp\left(\frac{-V_g}{\theta}\right)}{\sum_g \pi(g|g_-) \exp\left(\frac{-V_g}{\theta}\right)} \right] + \beta m_g W \left(\Gamma_g, g\right) \end{array} \right\}
$$

The first order conditions are

$$c_g \quad : \quad 0 = u_c(c_g, x_g) - \Gamma_g u_c(c_g, x_g) + \frac{\mu_g}{m_g}$$
$$+\xi\left[u_{cc}(c_g, x_g)c_g + u_c(c_g, x_g) - u_{cx}(c_g, x_g)(1 - x_g)\right]$$

$$x_g \quad : \quad 0 = u_x(c_g, x_g) - \Gamma_g u_x(c_g, x_g) + \frac{\mu_g}{m_g}$$
$$+\xi\left[u_{cx}(c_g, x_g)c_g + u_x(c_g, x_g) - u_{xx}(c_g, x_g)(1 - x_g)\right]$$

$$m_g \quad : \quad 0 = u(c_g, x_g) + \xi\left[u_c(c_g, x_g)c_g - u_x(c_g, x_g)(1 - x_g)\right]$$
$$-\Gamma_-\left[V_g + \theta(1 + \ln m_g)\right] + \Gamma_g\left[V_g - u(c_g, x_g)\right] + \varpi_g + \beta W(\Gamma_g, g)$$

$$V_g : 0 = \Gamma_g - \Gamma_- + \left(\frac{1}{\theta}\right)\left[\varpi_g - \sum_g \pi(g \mid g_-) m_g \varpi_g\right]$$
$$\Gamma_g : 0 = V_g - u(c_g, x_g) + \beta W_\Gamma(\Gamma_g, g)$$

where the envelope condition is

$$W_\Gamma(\Gamma_-, g_-) = -\sum_g \pi(g \mid g_-)\left[m_g V_g + \theta m_g \ln m_g\right]$$

These first order conditions are equivalent to those derived in the sequential formulation.

The solution to this problem is indexed by the multiplier $\xi$. For each value of $\xi$, the first order conditions imply an optimal allocation. In order to solve for the correct value of $\xi$, the one that corresponds to the initial level of debt held by the consumers, I will find the value such that the implementability constraint is satisfied with equality. For this $\xi$, the allocation satisfies all constraints and yields the highest subjective welfare for the consumers.

The time 0 planner's problem is

$$W_0 = \min_{\Gamma_0} \max_{c_0, x_0, V_0} \left\{ \begin{array}{c} u(c_0, x_0) + \mu_0\left[c_0 + x_0 + g_0 - 1\right] \\ +\xi\left[u_c(c_0, x_0)(c_0 - b_0) - u_x(c_0, x_0)(1 - x_0)\right] \\ +\Gamma_0\left[V_0 - u(c_0, x_0)\right] + \beta W(\Gamma_0, g_0) \end{array} \right\}$$

and the associated first order conditions are

$$c_0 : 0 = u_c(c_0, x_0) + \mu_0 + \xi \left[ u_{cc}(c_0, x_0)(c_0 - b_0) + u_c(c_0, x_0) - u_{cx}(c_0, x_0)(1 - x_0) \right] - \Gamma_0 u_c(c_0, x_0)$$

$$x_0 : 0 = u_x(c_0, x_0) + \mu_0 + \xi \left[ u_{cx}(c_0, x_0)(c_0 - b_0) + u_x(c_0, x_0) - u_{xx}(c_0, x_0)(1 - x_0) \right] - \Gamma_0 u_x(c_0, x_0)$$

$$V_0 : 0 = \Gamma_0$$

$$\Gamma_0 : 0 = V_0 - u(c_0, x_0) + \beta W_\Gamma(\Gamma_0, g_0)$$

## 4.1 Model Solution and Discussion:

The solution to the political government's problem has been calculated using the same utility function, government spending process, and parameter values as for the benevolent government problem. These results will be compared to the Lucas and Stokey (1983) benchmark. Given that the preferences of the two agents are aligned, the intuition behind the chosen fiscal policy is relatively straight-forward. To illustrate the logic behind the solution, I first describe a small thought experiment. Upon completion, I give a more detailed description that characterizes how the choice of fiscal policy shapes the economy's path and maximizes the consumers' subjective utility.

In Lucas and Stokey (1983), it is shown that public debt should be used as insurance against the spending shock. This insurance allows the government to reduce the fluctuations in the labor tax rate, decreasing the volatility of the intra-temporal distortion. Because of the concavity of the consumers' utility function, a smooth intra-temporal distortion comes with the smallest welfare cost for the consumers. For the same reason, the political government wants to smooth labor taxes across state. However, the political government faces one additional reason to limit the volatility in the tax rate: a smooth labor tax reduces the consumers' probability distortion. This smaller probability tilting means that the consumers are not quite as pessimistic, limiting how much additional weight they place on bad states and how much weight they subtract from good states. As a result, the consumers' subjective expected utility – the objective function of the political government – does not fall by as much when consumers face model uncertainty.

To see this, consider a tax profile that results in a volatile allocation and welfare profile across states. The consumers, in their uncertainty, decrease the weight they place on the high welfare state and increase the weight they place on the low welfare state. This implies that the consumers' subjective welfare is relatively low. Now, consider that the government changes its fiscal policy by lowering the labor tax in war and raising the tax in peace. This tax profile leads to a smoother allocation for the consumers, reducing the volatility of their subjective welfare. As a result, the consumers do not fear the low welfare state as much because reaching that state is no longer as harmful. This means that, for a given level of model uncertainty, the consumers reduce their incremental probability distortion. More generally, by smoothing the labor tax rate across states, the government mitigates the consumers' probability tilting, resulting in a higher subjective welfare.

This intuition can be seen graphically below. As before, I have plotted the equilibrium solutions for different levels of consumer uncertainty. A large $\theta$ implies that the consumers are confident in their approximating model. As $\theta$ falls, the consumers are increasingly uncertain about the true model and so raise their probability distortions.

The logic above suggests that the planner decreases (increases) the labor tax in war (in peace), relative to when consumers face no uncertainty. The goal of this change is to reduce the deleterious impact of model uncertainty. Although the tax rate changes are partially offset by the change in labor supply, the new policy yields a less volatile profile of tax revenues across states. This can be seen in the top two graphs in Figure 7. Because the government obtains less (more) tax revenue from the labor tax, a high (low) spending shock necessarily leads to a larger war-time (smaller peace-time) primary deficit. The bottom right graph in Figure 7 indicates that the volatility of the primary deficit has increased. This means that the political government finances more of the spending shock through debt when $\theta$ is small compared to when $\theta \to \infty$. By narrowing the gap in the labor tax revenues, the government obtains more insurance from the consumers against the spending shock. As a result, the consumers' allocation and welfare profile remain relatively flat and the probability distortion diminished.

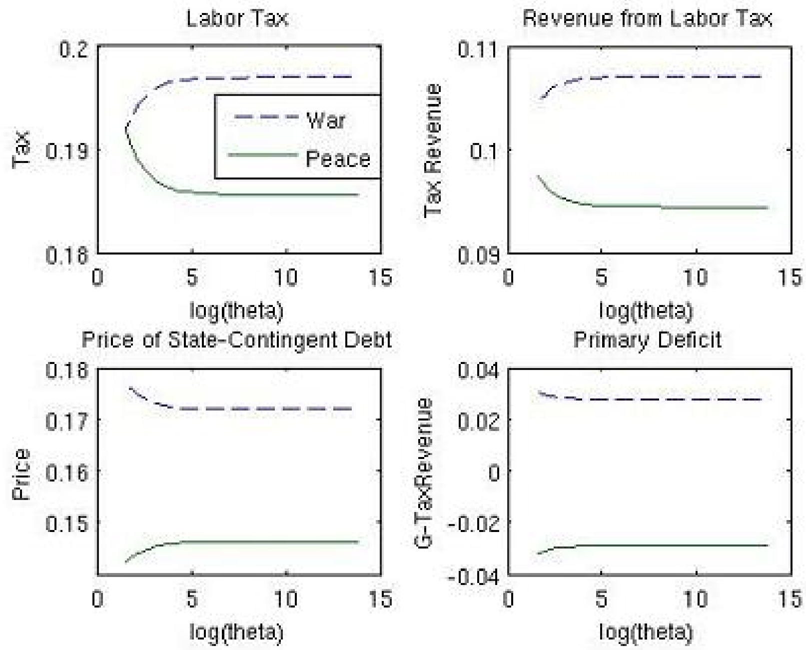


Figure 7: Optimal fiscal policy implemented by the political government for different levels of $\theta$

Critically, this choice of fiscal policy reduces the volatility of the consumers' welfare across states, as shown in Figure 8. The combination of raising the consumers' consumption and labor supply during war evidently leads to a smaller decrease in welfare than the peace-time decrease in consumption and labor supply. Since the welfare levels are not equal across states, the consumer still fears high government spending, leading to an increase in that state's perceived probability. Nevertheless, given the choice of fiscal policy, the size of the distortion is reduced relative to what it would have been without this fiscal policy.

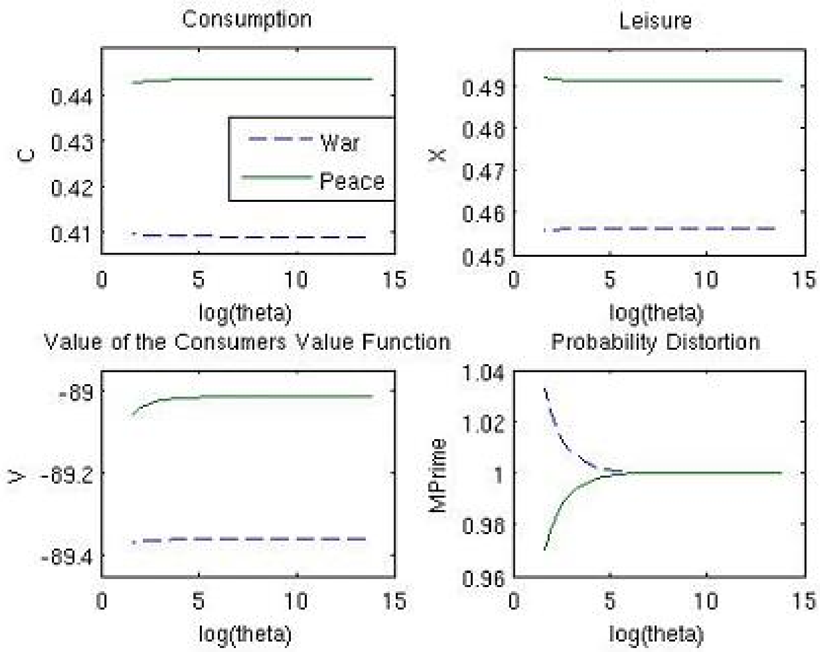


Figure 8: Optimal allocation for different levels of $\theta$

In the graphs below, I plot the impulse response functions to a positive government spending shock under two different degrees of model uncertainty. Again, for all periods except period 1, $g_t = \bar{g}$. In period 1, there is an unexpected increase in government spending so that $g_1 = g_{high}$. The line associated with consumers facing a large degree of model uncertainty is labeled "Uncertain", while the line associated with consumers completely understanding the spending process is labeled "Certain".

As discussed above, a positive, one-period war leads the political government to increase its labor tax. This increase, though, is smaller when consumers face uncertainty. Even after the shock has passed, the labor tax is persistently lower when the consumers face model uncertainty. The same results hold for labor tax revenues, as seen in the bottom right graph of Figure 9. This result suggests that the government is able to finance a larger portion of the war through debt.

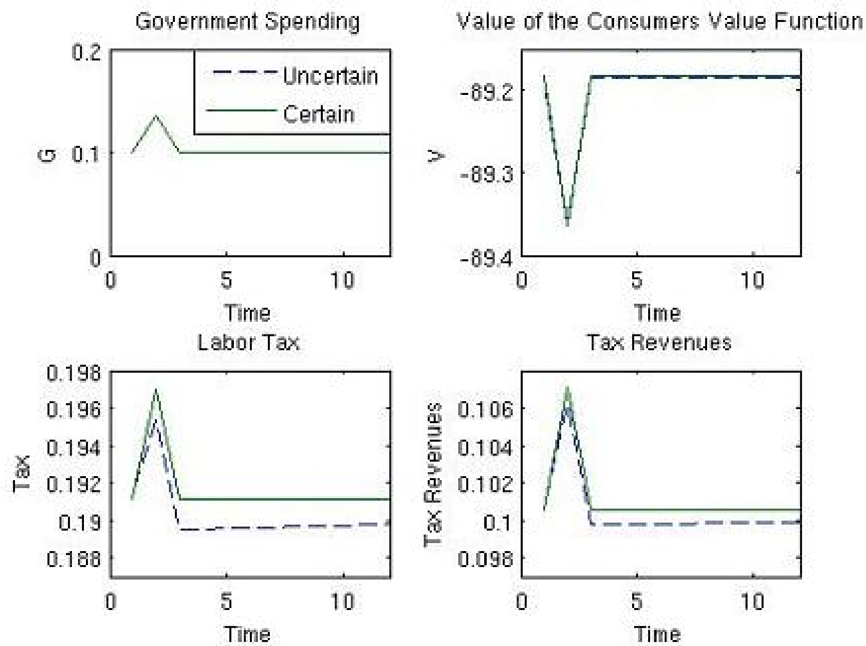


Figure 9: Policy impulse response functions to an increase in government spending when $\theta \to \infty$ and $\theta = 15$

The decrease in labor tax during war means that the consumers supply more labor during this event. With the increase in labor income, consumers increase their purchases of consumption. Both of these effects can be seen in Figure 10. Just as before, the effects are persistent beyond the period of the shock to spending. That is, the political government induces uncertain consumers to purchase more consumption for many periods beyond the shock period. When consumers are certain, though, the consumers return to the allocation levels chosen before the period of the shock.

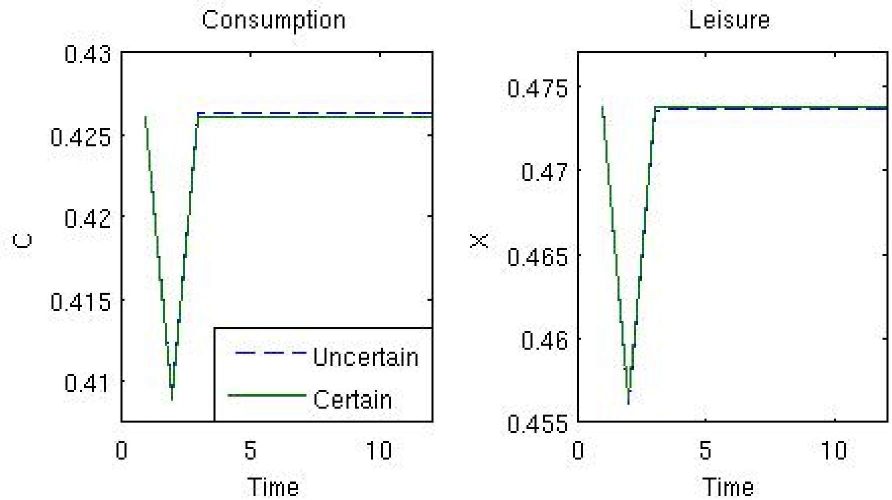


Figure 10: Allocation impulse response functions to an increase in government spending when $\theta \to \infty$ and $\theta = 15$

# 5 Model comparison: the benevolent and political governments

The preferences of the government have dramatic implications for the volatility of the primary deficit, the response of labor supply to fiscal shocks, and the role of tax policy in absorbing fiscal shocks. Fundamentally, these macroeconomic conclusions turn on the degree to which the government finances the spending shock through public deficits. This, in turn, depends on the weight the government places on the savings distortion induced by model uncertainty.

The political and benevolent governments share the following incentive: by smoothing the consumers' welfare across states, their probability distortion shrinks. In reducing this distortion, the government limits the welfare costs of model uncertainty. This incentive leads the political government to decrease the distance between the consumers' subjective welfare across states. This can be seen in Figure 10. In this graph, the solid line represents the consumers' subjective welfare across many different levels of consumer uncertainty, while the dotted line plots the same variable under the political government.

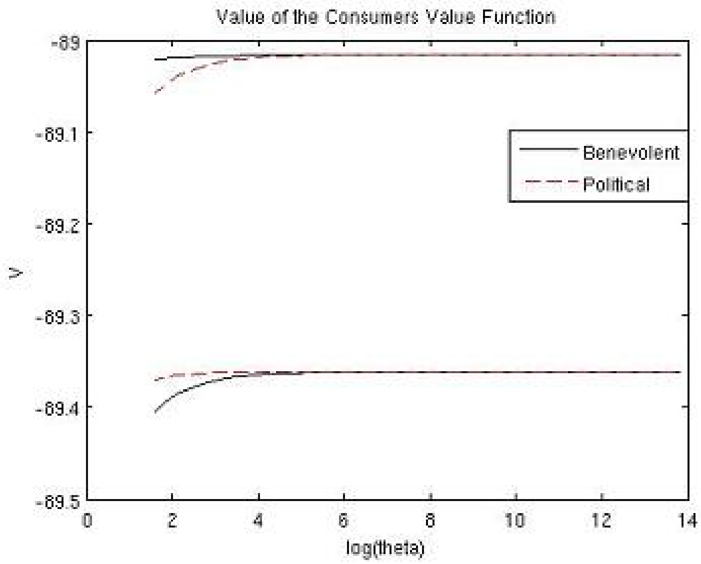


Figure 10: Comparing the consumers' subjective welfare under the benevolent and political governments

As can be seen, the welfare distance under the political government is smaller when consumers are uncertain about the shock process compared to when they are not. This reduction is the key objective of the political government because its preferences are aligned with those of the consumers. This alignment is crucial because it limits the role of the political government. The political government does not attempt to change the decisions of the consumers to reflect the true probability model. Instead, the government merely chooses the allocation that mitigates the effect of the consumers' probability distortion on their subjective welfare.

The benevolent government, though, faces an additional incentive. This incentive, stemming from the misalignment of the two agents' preferences, leads the benevolent government to increase the distance in the consumers' subjective welfare across states. Because the government maximizes the consumers' welfare under the true probability model, it recognizes that the decisions of the consumers are based on an incorrect model. Acknowledging this, the benevolent government uses its fiscal policy to reduce this distortion in the consumers' decisions. The government accomplishes this by increasing the distance in the consumers' subjective welfare across state.

Even though the increase in distance raises the consumers' probability distortion, the benevolent government accepts this cost because in doing so, it reduces the consumers' savings distortion. Evidently, the welfare loss due to the inefficient savings profile is larger than the cost of the probability distortion. Reacting to this, the benevolent government implements a fiscal policy that relies more heavily on labor taxes to finance public spending. The goal of this policy is to influence the prices and returns on debt. These prices are manipulated in such a way as to discourage (encourage) the consumers from holding debt that pays off in war (peace). Figure 11 plots the degree to which both governments rely upon tax revenues to finance spending, and Figure 12 plots the implied prices of the state-contingent bonds.

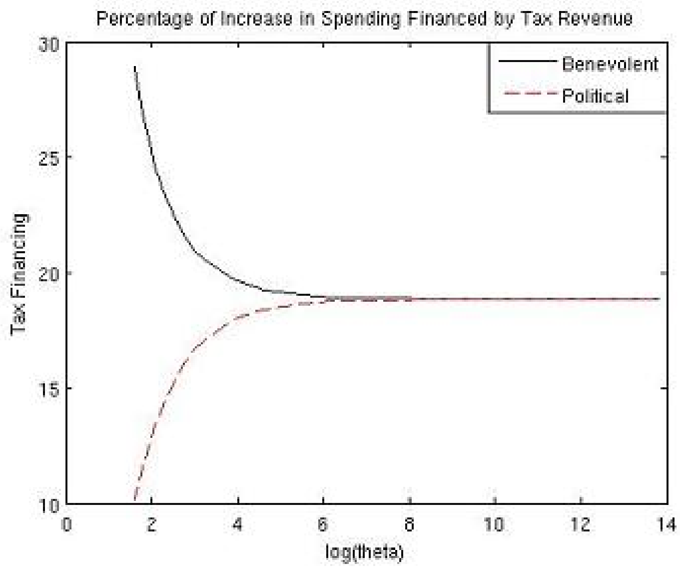


Figure 11: Comparing the degree to which labor taxes finance government spending under the benevolent and political governments

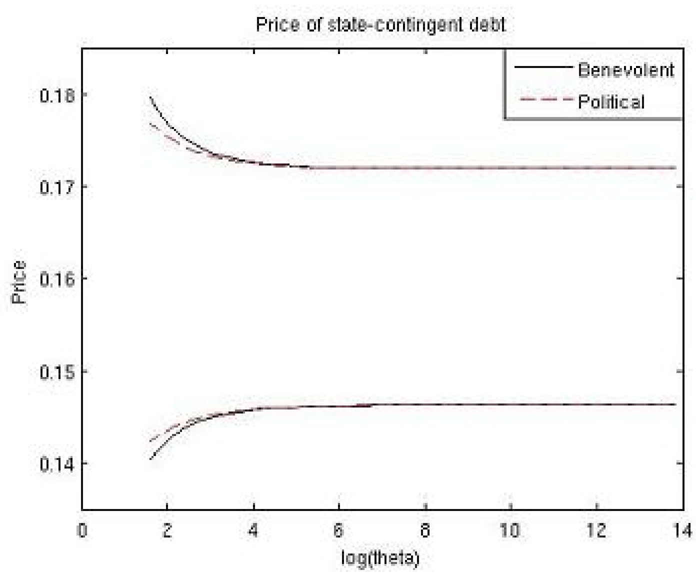


Figure 12: Comparing the price of debt under the benevolent and political governments

Thus, the benevolent government is more concerned than the political government about realigning the consumers' savings behavior with the true model. This leads the benevolent government to induce a higher price on assets that pay off during wars. This higher price encourages the consumers to borrow money from the government, effectively allowing the benevolent government to purchase insurance from them.

For completeness, the fiscal policies of each government are graphed in Figure 13. As indicated above, the political government chooses to set a smooth labor tax across states in order to reduce the distance in the consumers' subjective welfare. The benevolent government, however, sets a more volatile labor tax rate. The purpose of this volatility is to encourage the consumers to provide insurance to the government. These policies imply that the volatility of the public deficit will be higher under the political government than under the benevolent government.

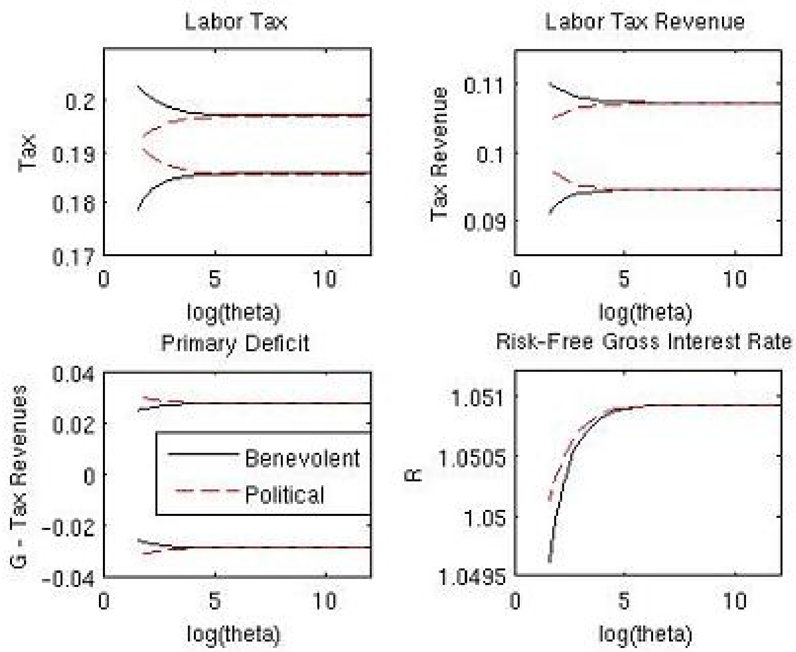


Figure 13: Comparing fiscal policies under the benevolent and political governments

The previous conclusions focus on the volatility of the policies, prices, and allocations. I will now briefly examine the consequences of model uncertainty for persistence. As indicated both theoretically and numerically, both governments implement a persistent fiscal policy when faced with consumer uncertainty. This can be seen in Figure 14, which shows the impulse response of the labor tax to a one period increase in government spending.

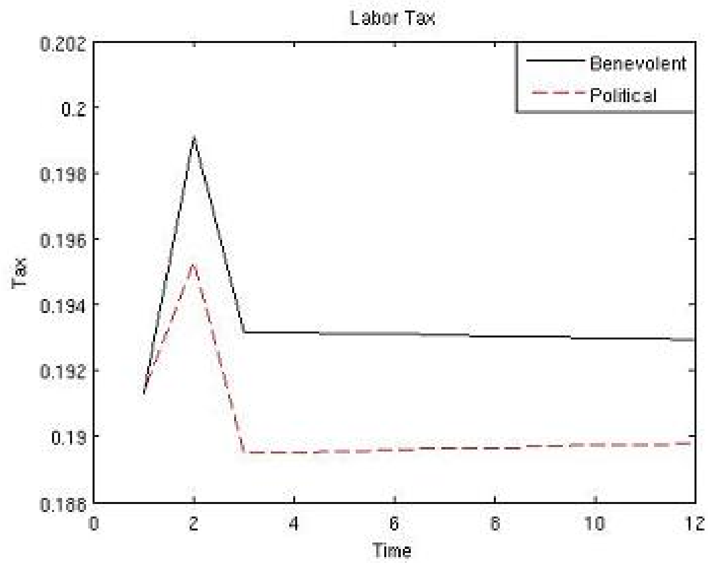


Figure 14: Comparing the impulse response of the labor tax to an increase in government spending under the benevolent and political governments

This graph indicates that the tax rate does not return to its pre-shock level immediately after the war, as would happen if consumers were certain about the government spending process. Instead, the benevolent government maintains a higher tax rate and the political government a lower tax rate for many periods after the shock. The governments induce this additional persistence to counter the persistence in the probability distortion.

**Welfare Comparison:** This section examines the welfare consequences of consumer uncertainty across the benevolent and political governments. Intuitively, the benevolent government's policy should achieve a higher welfare for the consumers than the political government's policy. This is because the spending process evolves according to the model used in the benevolent government's problem. The same is not true for the political government. The question then is the size of the welfare cost associated with political government relative to the benevolent government. To determine this value, I simulate the path of the economy 200 times. For each simulation, the economy runs for 200 periods. In each of those periods, I calculate the consumers' period utility $u(c_t, x_t)$. Discounting these values back to time 0, I obtain the consumers' lifetime utility. I arrive at the average lifetime welfare by averaging this value across all simulations. I compare this number to $W_0$, the benevolent government's value of the value function at time 0, which is equal to the consumers' expected utility under the true probability measure.

The values are listed in Table 2.

| | Benevolent government | Political Government |
|---|---|---|
| Welfare | -89.1818 | -89.2198 |

Table 2: Welfare comparisons under the benevolent and political governments

As predicted, the welfare of the consumers is lower under the political government. Following the Lucas treatment, the percentage increase in consumption that must be given to the consumers living under the political government to make them indifferent to living under the benevolent government is 0.02% per period.

## 6 Conclusion:

This paper compares the fiscal policies chosen by two types of altruistic government when confronted with consumer uncertainty. Even though both governments face the same distortions in the economy, each responds in a dramatically different way. The benevolent government increases the volatility of the labor tax, which smoothes the fluctuations in the primary deficit across states. The political government, on the other hand, decreases the volatility of the labor tax, which amplifies the fluctuations in the primary deficit.

Fundamentally, these stark policy differences hinge upon whether the probability measure used by the government to evaluate the consumers' welfare is aligned with the measure feared by the consumers. In the case of the benevolent government, the two measures do not coincide. This leads the benevolent government to choose a policy that 'corrects' the behavior of the consumers. That is, the planner attempts to re-align the decisions of the consumers with their rational expectations values. The most critical decision that the benevolent government wants to re-align is the consumers' savings decision. To do this, the planner sets policy to raise the price of the war-contingent asset and lower the price of the peace-contingent asset. These movements influence the returns on these assets, reducing the degree to which the consumers' savings profile is different from its rational expectations level.

In the case of the political government, the two probability measures coincide. This means that the government evaluates the consumers' expected utility using their own subjective probability model. Because the two agents' expectations are aligned, the political government does not attempt to correct the consumers' behavior. Rather, the government chooses policy that directly reduces the consumers' probability distortion. This is beneficial because it limits how much additional weight the consumers can place on the bad state, which raises their subjective welfare.

In this model, I have assumed that the actual stochastic process for the government spending shock is the same as the agents' approximating model. As such, the government's confidence in its probability model is well-founded. This assumption also implies that the policy chosen by the benevolent government leads to higher consumer welfare than the policy chosen by the political government. This need not be the case, though. If the actual process does not correspond to the approximating model, then the government's confidence would be misplaced. In this situation, the political government's policy could lead to higher welfare for the consumers, and the paternalism of the benevolent government would be detrimental. Therefore, in determining the welfare and fiscal policy implications of consumer uncertainty, it is crucial to understand the interaction between that uncertainty and the preferences of the planner.

# References


[1] Aiyagari, Rao, Albert Marcet, Thomas Sargent, and Juha Seppala. 2002. "Optimal Taxation without State-Contingent Debt." *Journal of Political Economy.* 110 (6): 1220-1254.

[2] Barro, Robert. 1986. "The Behavior of U.S. Deficits." NBER working paper, w1309.

[3] Bizer, David and Steven Durlauf. 1990. "Testing the Positive Theory of Government Finance." *Journal of Monetary Economics.* 26: 123-141.

[4] Bohn, Henning. 1998. "The Behavior of US Public Debt and Deficits." *The Quarterly Journal of Economics* 113, 949-963.

[5] Camerer, Colin and Roberto Weber. 1992. "Recent Developments in Modeling Preferences: Uncertainty and Ambiguity." *Journal of Risk and Uncertainty.* V, 325-370.

[6] Dennis, Richard. 2007. "Model Uncertainty and Monetary Policy." San Francisco Federal Reserve Bank working paper.

[7] Ellsberg, Daniel. 1961. "Risk, Ambiguity and the Savage Axioms." *Quarterly Journal of Economics.* LXXV, 643-669.

[8] Gilboa, Itzhak and David Schmeidler. 1989. "Maxmin Expected Utility with Non-Unique Prior." *Journal of Mathematical Economics.* 18: 141-153.

[9] Hansen, Lars and Thomas Sargent. 2001. "Robust Control and Model Uncertainty." *American Economics Review.* 91: 20-66.

[10] Hansen, Lars and Thomas Sargent. 2005. "Robust Estimation and Control Under Commitment." *Journal of Economic Theory.* 124: 258-301.

[11] Huang, Chao-Hsi and Kenneth Lin. 1993. "Deficits, Government Expenditures, and Tax Smoothing in the United States: 1929 - 1988." *Journal of Monetary Economics.* 31: 317-339.

[12] Karantounias, Anastasios, Lars Hansen, and Thomas Sargent. 2007. "Ramsey Taxation and Fear of Misspecification." unpublished manuscript.

[13] Lucas, Robert and Nancy Stokey. 1983. "Optimal Fiscal and Monetary Policy in an Economy Without Capital. *Journal of Monetary Economics.* 12: 55-93.

[14] Marcet, Albert and Ramon Marimon. 1998. "Recursive Contracts." unpublished manuscript.

[15] Marcet, Albert and Andrew Scott. 2003. "Optimal Taxation with Endogenously Incomplete Debt Markets." *Journal of Economic Theory.* 127: 36-73.

[16] Sleet, Christopher. 2004. "Optimal Taxation with Private Government Information." *Review of Economic Studies.* 73:1-30.

[17] Sorensen, Bent, Lisa Wu, and Oved Yosha. 2001. "Output Fluctuations and Fiscal Policy." *European Economic Review* 45, 1271-1310.

[18] Woodford, Michael. 2008. "Robustly Optimal Monetary Policy with Near Rational Expectations." NBER working paper.