Addendum to: Importing Skill-Biased Technology

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Abstract

This Addendum derives the results discussed in section 3.3 of our main paper.

1 Alternative quantitative trade models

In this section we embed capital-skill complementarity into an alternative quantitative trade model featuring monopolistic competition and heterogeneous firms. We first analyze the case with restricted entry, in which the mass of active firms in each country is exogenously determined, and firms' productivities are drawn from a Pareto distribution, as in the Chaney (2008) model. We show how to solve for real wages and the skill premium as a function of domestic variables in this setting. We note, however, that the expressions linking changes in the skill premium to changes in domestic expenditure shares differ from those in the Ricardian model developed in the paper, and that changes in real wages now also depend on changes in the domestic trade balance. Finally, we allow for endogenous entry as in Melitz (2003) and show that in this case domestic expenditure shares and the domestic trade balance are no longer sufficient statistics for the impact of trade on the skill premium.

We assume that there is a continuum of producers in each country and sector, each producing a differentiated good. Producers operate the constant returns to scale technology detailed in the paper,

$$y_i(\varphi, j) = A_i(j)\,\varphi b_{c,i},$$

where $b_{c,i} \equiv b_{3,i}^{\zeta} b_{4,i}^{1-\zeta}$ is an input bundle, $b_{3,i}$ and $b_{4,i}$ are defined as in the paper, φ is the producer's idiosyncratic productivity, and we index producers by their productivity and their sector. In addition to production costs, we assume that intermediate producers from country i and sector j must incur an iceberg trade cost $\tau_{in}(j)$ and a fixed cost $f_{in}(j)$ to sell in country n. The fixed cost is denominated in units of the composite production bundle $b_{c,i}$. This implies that producers in sector j must pay a fixed cost $c_i f_{in}(j)$ to sell in country n, where c_i is the unit cost of the composite production bundle and is defined in the paper.

The distribution of productivities is given by $G(\varphi, j)$ in sector j in each country. Productivities are distributed Pareto with shape parameter $\theta(j)$ and support $[1, \infty]$: $G(\varphi, j) = 1 - \varphi^{\theta(j)}$.

1.1 Restricted entry

We will first assume that the mass of firms operating in each sector in each country $M_i(j)$ is determined exogenously.

Firm's problem. The profit maximizing price of a firm with productivity φ, operating in sector j, and selling from country i to country n is

$$p_{in}(\varphi, j) = \frac{\eta_n(j)}{\eta_n(j) - 1} \frac{\tau_{in} c_i}{A_i(j) \varphi}.$$

Variable profits, $\varsigma_{in}(\varphi, j)$, for this firm are

$$\varsigma_{in}(\varphi, j) = \varphi^{\eta_n(j)-1} \frac{\eta_n(j)^{-\eta_n(j)}}{[\eta_n(j) - 1]^{1-\eta_n(j)}} \left[\frac{\tau_{in} c_i}{A_i(j)}\right]^{1-\eta_i(j)} P_n(j)^{\eta_n(j)} Y_n(j).$$

If the firm is active in market n, its demand for the composite bundle is

$$b_{c,in}(\varphi, j) = \frac{\eta_n(j) - 1}{\eta_n(j)} \frac{p_{in}(\varphi, j)\, q_{in}(\varphi, j)}{c_i} + f_{in}(j).$$

The productivity threshold $\varphi^*_{in}(j)$ below which firms decide not to serve the market is given by

$$\varsigma_{in}(\varphi^*_{in}(j), j) = c_i f_{in}(j).$$

Aggregates. Aggregating across firms, countries, and sectors, we obtain total demand of the input bundle to be used in production

$$B^P_{c,in} = \frac{1}{c_i} \sum\nolimits_n \sum\nolimits_j \frac{\eta_n(j) - 1}{\eta_n(j)} \pi_{in}(\varphi, j)\, P_n(j)\, Y_n(j)$$

where $M_i(j)$ is the mass of firms from country i operating in sector j and where $\pi_{in}(\varphi, j) = \left[\int_{\varphi^*_{in}(j)} p_{in}(\varphi, j)\, q_{in}(\varphi, j)\, M_i(j)\, dG(\varphi, j)\right] \Big/ [P_n(j)\, Y_n(j)]$ is the share of country n expenditures in sector j spent on goods produced in country i.

Total demand for the input bundle to cover fixed costs is given by

$$B^{F,c}_i = \sum\nolimits_n \sum\nolimits_j f_{in}(j)\, M_i(j)\, [1 - G(\varphi^*_{in}(j), j)]$$

From the definition of the participation cutoff, we obtain

$$\frac{\pi_{in}(\varphi, j)\, P_n(j)\, Y_n(j)}{c_i} = \frac{\eta_n(j)\, \theta(j)}{\theta(j) + 1 - \eta_n(j)} f_{in}(j)\, M_i(j)\, [1 - G(\varphi^*_{in}(j), j)]$$

Aggregate demand for the input bundle is

$$B^c_i \equiv B^{F,c}_i + B^P_{c,in} = \sum_n \sum_j \left[\frac{\eta_n(j)\, \theta(j) + 1 - \eta_n(j)}{\eta_n(j)\, \theta(j)}\right] \frac{\pi_{in}(j)\, P_n(j)\, Y_n(j)}{c_i}.$$

Note that our modified market structure, relative to the perfectly competitive market structure in our main paper, does not alter how total payments to the input bundle, $c_i B_i^c$, are split among inputs. In addition, note that aggregate profits are:

$$\begin{aligned} \Pi_i &= \sum\nolimits_n \sum\nolimits_j \pi_{in}(j) P_n(j) Y_n(j) - c_i B_i^c \\ &= \sum\nolimits_n \sum\nolimits_j \left[\frac{\eta_n(j) - 1}{\eta_n(j)\theta(j)}\right] \pi_{in}(j) P_n(j) Y_n(j) \end{aligned}$$

Finally, we can write the price index in country i for goods in sector j as

$$P_i(j) = \left[\sum\nolimits_n \int_{\varphi_{ni}^*} \left[\frac{\eta_i(j)}{\eta_i(j) - 1} \frac{\tau_{ni} c_n}{A_n(j)\varphi}\right]^{1-\eta_i(j)} M_n(j)\, dG(\varphi, j)\right]^{\frac{1}{1-\eta_n(j)}},$$

and using the definition of the cutoff, we can write the expenditure share as

$$\pi_{in}(j) = \kappa_i(j) \left[\frac{c_i}{A_i(j)}\right]^{1-\frac{\theta(j)\eta_n(j)}{\eta_n(j)-1}} \left[\frac{f_{in}(j)}{P_n(j) Y_n(j)}\right]^{\frac{\eta_n(j)-1-\theta(j)}{\eta_n(j)-1}} \tau_{in}^{-\theta(j)} P_n(j)^{\theta(j)} M_i(j), \qquad \text{(A1)}$$

with $\kappa_i(j) \equiv \frac{\theta(j)}{\theta(j)-\eta_i(j)+1} \eta_n(j)^{1-\frac{\theta(j)\eta_n(j)}{\eta_n(j)-1}} (\eta_n(j) - 1)^{\theta(j)}$. We now characterize the steady-state equilibrium in the world economy.

1.1.1 Steady-State Equilibrium

We now define and characterize the steady state equilibrium for the world economy. In doing so, we show how aggregate quantities and prices can be determined before solving for product level variables. A steady-state equilibrium for the aggregate variables in the world economy consists of a set of prices $\{v_i, w_i, r_i, s_i\}_{i\in I}$, $\{p_{b_{1,i}}, p_{b_{2,i}}, p_{b_{3,i}}, p_{b_{4,i}}, c_i\}_{i\in I}$, $\{P_i(S), P_i(M), P_i(E)\}_{i\in I}$, aggregate quantities $\{K_i(S), K_i(E), X_i(M), X_i(S)\}_{i\in I}$, $\{C_i(M), C_i(S)\}_{i\in I}$, $\{Y_i(M), Y_i(S), Y_i(E)\}_{i\in I}$, participation cutoffs $\{\varphi_{in}^*(j)\}_{i,n\in I, j\in \mathcal{J}}$ and trade shares $\{\pi_{in}(j)\}_{i,n\in I, j\in\mathcal{J}}$ such that, given factor supplies, $\{H_i, L_i\}_{i\in I}$ and $\{M_i(j)\}_{i,n\in I, j\in\mathcal{J}}$, technologies, $\{A(S)_i, A(M)_i, A(E)_i\}_{i\in I}$, and net exports, $\{nx_i\}_{i\in I}$, in each country, the following are satisfied:

1. Household's maximize utility subject to their budget constraints: The household's optimality conditions in steady state are given by the Euler equations (16) and (17) in the main paper, and the intra-temporal consumption equation (18) in the main paper.

The budget constraint is now written as,

$$\begin{aligned}\left(w_iL_i+s_iH_i+v_iK_i\left(S\right)+r_iK_i\left(E\right)+\Pi_i\right)\left(1+nx_i\right) &= P_i\left(E\right)\delta_i\left(E\right)K_i\left(E\right) \\ &\quad +P_i\left(M\right)C_i\left(M\right)+P_i\left(S\right)\left[C_i\left(S\right)+\delta_i\left(S\right)K_i\left(S\right)\right]\end{aligned} \tag{A2}$$

where aggregate profits are given by: $\Pi_i=\left[\frac{\eta_n(j)-1}{\eta_n(j)\theta(j)}\right]\sum_{n=1}^{I}\pi_{in}\left(\varphi,j\right)P_n\left(j\right)Y_n\left(j\right)$. As in the main paper, nx_i denotes net exports as a share of GDP, which we take as a parameter.

2. Cost minimization by producers of intermediate goods: Cost minimization implies equations $(20)-(24)$ in the main paper.

3. Cost minimization by producers of final goods: The sectorial price indices for final goods are given by

$$P_i\left(j\right)=\left(\sum\nolimits_n\left[\frac{\eta_i\left(j\right)}{\eta_i\left(j\right)-1}\frac{\tau_{ni}c_n}{A_n\left(j\right)\varphi}\right]^{1-\eta_i(j)}\frac{\theta\left(j\right)\varphi_{ni}^{*}\left(j\right)^{\eta_i(j)-1-\theta(j)}M_n\left(j\right)}{\theta\left(j\right)-\eta_i\left(j\right)+1}\right)^{\frac{1}{1-\eta_n(j)}} \tag{A3}$$

and trade shares between any pair of countries are given by equation $(A1)$.

4. Aggregate factor market clearing: Equations $(26)-(31)$ in the main paper are now modified as,

$$\begin{aligned}v_iK_i\left(S\right) &= \zeta_i\alpha_ic_iB_i^c, && \text{(A4)}\\ w_iL_i &= \zeta_i\mu_i\left(1-\alpha_i\right)\left(p_{b_{2,i}}/\,w_i\right)^{\sigma-1}c_iB_i^c, && \text{(A5)}\\ r_iK_i\left(E\right) &= \zeta_i\lambda_i\left(1-\alpha_i\right)\left(1-\mu_i\right)\left(\frac{p_{b_{1,i}}}{r_i}\right)^{\rho-1}\left(\frac{p_{b_{2,i}}}{p_{b_{1,i}}}\right)^{\sigma-1}c_iB_i^c, && \text{(A6)}\\ s_iH_i &= \zeta_i\left(1-\alpha_i\right)\left(1-\mu_i\right)\left(1-\lambda_i\right)\left(\frac{p_{b_{1,i}}}{s_i}\right)^{\rho-1}\left(\frac{p_{b_{2,i}}}{p_{b_{1,i}}}\right)^{\sigma-1}c_iB_i^c, && \text{(A7)}\end{aligned}$$

and,

$$\begin{aligned}P_i\left(S\right)X_i\left(S\right) &= \varepsilon_i\left(1-\zeta_i\right)c_iB_i^c, && \text{(A8)}\\ P_i\left(M\right)X_i\left(M\right) &= \left(1-\varepsilon_i\right)\left(1-\zeta_i\right)c_iB_i^c, && \text{(A9)}\end{aligned}$$

where

$$c_iB_i^c=\sum\nolimits_n\sum\nolimits_j\left[\frac{\eta_n\left(j\right)\theta\left(j\right)+1-\eta_n\left(j\right)}{\eta_n\left(j\right)\theta\left(j\right)}\right]\frac{\pi_{in}P_n\left(j\right)Y_n\left(j\right)}{c_i}. \tag{A10}$$

5. Aggregate goods markets clear in each country: These are given by equations $(33)-(35)$ in the main paper.

6. The participation cutoffs are given by:

$$\varphi_{in}^{*}(j)=\left[\frac{\tau_{in}c_i}{A_i(j)}\right]\left[\frac{c_i f_{in}(j)\left[\eta_n(j)-1\right]^{1-\eta_n(j)}\eta_n(j)^{\eta_n(j)}}{P_n(j)^{\eta_n(j)}Y_n(j)}\right]^{\frac{1}{\eta_n(j)-1}}. \tag{A11}$$

1.1.2 Solving in Terms of Domestic Expenditure Shares

In this section we show how to solve for all aggregate domestic variables as functions of domestic expenditure shares, $\pi_{ii}(j)$, domestic productivities, $A_i(j)$, domestic factor supplies $\{H_i, L_i\}$ and $\{M_i(j)\}$, and net exports nx_i. To better understand the difference with the Ricardian model, we use equation $(A1)$ to write aggregate price indices as functions of domestic expenditure shares,

$$P_i(j)=\left[\frac{\kappa_i(j)}{\pi_{ii}(\varphi,j)}\left(\frac{c_i}{A_i(j)}\right)^{1-\frac{\theta(j)\eta_n(j)}{\eta_n(j)-1}}\left(\frac{f_{ii}(j)}{P_i(j)Y_i(j)}\right)^{\frac{\eta_n(j)-1-\theta(j)}{\eta_n(j)-1}}\right]^{\frac{-1}{\theta(j)}}. \tag{A12}$$

Note that sectoral price indices $P_i(j)$ are functions of sectorial expenditures, $P_i(j)Y_i(j)$, since these determine the participation cutoffs and hence the distribution of prices in each sector. Since sectoral expenditures are not constant across equilibria, we cannot solve for the model's prices independently of the quantities, as we did in the Ricardian model.

Instead, we can use the following algorithm to solve for all aggregate domestic variables as a function of domestic trade shares. Start by guessing expenditures in each sector $P_i(j)Y_i(j)$. From equations $(A5)$ and $(A7)$ we obtain

$$\frac{s_i^{\rho}}{w_i^{\sigma}}\frac{H_i}{L_i}=(1-\lambda_i)\frac{1-\mu_i}{\mu_i}p_{b,i}^{\rho-\sigma}. \tag{A13}$$

The 3 price index equations $(A12)$, together with equation $(A13)$, the Euler equations (16) and (17) and the cost minimization equations $(20)-(24)$ make a system of 11 equations. Together with a choice of numerarie these equations can be used to solve for the 12 domestic prices.

Given prices, we can solve for quantities as follows. First, solve for $K_i(E)$ and $K_i(S)$ using equations $(A4)$, $(A6)$, and $(A7)$. Second, adding equations $(A4)-(A7)$, we solve for $c_i B_i^c$ as

$$\zeta_i c_i B_i^c = v_i K_i(S) + w_i L_i + r_i K_i(E) + s_i H_i.$$

Third, using equations $(A8)$ and $(A9)$, we obtain intermediate inputs $X_i(M)$ and $X_i(S)$. Fourth, from equation (18) in the paper and equation $(A2)$, we can solve for the consumption levels $C_i(S)$ and $C_i(M)$. Finally, from the market clearing equations $(33)-(35)$ we obtain total production in each sector. Use these equations to verify the guess.

1.2 Endogenous Entry

We now examine the case when entry is endogenous. We assume that in every period there is an unbounded mass of potential entrants that can pay a sunk cost $f_{e,i}(j)$ to enter sector j and produce a differentiated good. Producers die each period with probability $\delta_{M(j)} \geq 0$. In an equilibrium with entry, the entry cost must equal discounted expected profits. This implies:

$$c_i f_{e,i}(j) = \frac{\beta}{1 - \beta \left[1 - \delta_{M(j)}\right]} \Pi_i(j) / M_i(j)$$

where

$$\Pi_i(j) = \sum\nolimits_n \left[\frac{\eta_n(j) - 1}{\eta_n(j) \theta(j)}\right] \pi_{in}(j) P_n(j) Y_n(j)$$

are total revenues in sector j. These two equations determine the mass of firms in each sector $M_i(j)$ in this setting. Note that, since $M_i(j)$ depends on revenues accruing from each destination, $\pi_{in}(j) P_n(j) Y_n(j)$, we can no longer solve for the equilibrium as a function of domestic variables only — even if there is trade balance (but not sector-by-sector) for each country.

2 Differences in factor intensities across sectors

In this section we extend the model to allow for heterogeneous production functions across sectors. In particular, we allow for the parameters the parameters $\{\zeta(j), \alpha(j), \mu(j), \lambda(j), \varepsilon(j), \sigma(j), \rho(j)\}_{j \in \mathcal{J}}$ to be sector specific:

$$y_i(\omega, j) = A_i(j) z_i(\omega, j) b_{3,i}^{\zeta(j)} b_{4,i}^{1-\zeta(j)}$$

with

$$\begin{aligned}
b_{1,i}(j) &= \left[\lambda(j)^{1/\rho(j)} k_E^{(\rho(j)-1)/\rho(j)} + (1 - \lambda(j))^{1/\rho(j)} h^{(\rho(j)-1)/\rho(j)}\right]^{\rho(j)/(\rho(j)-1)}, \\
b_{2,i}(j) &= \left[\mu(j)^{1/\sigma(j)} l^{(\sigma(j)-1)/\sigma(j)} + (1 - \mu(j))^{1/\sigma(j)} b_1^{(\sigma(j)-1)/\sigma(j)}\right]^{\sigma(j)/(\sigma(j)-1)}, \\
b_{3,i}(j) &= k_S^{\alpha(j)} b_2^{1-\alpha(j)}, \\
b_{4,i}(j) &= x_S^{\varepsilon(j)} x_M^{1-\varepsilon(j)},
\end{aligned}$$

where we have dropped country-specific subscripts, i, from the production function parameters to facilitate exposition. The unit cost of production—for supplying the domestic market—of a producer with productivity $A_i(j)\, z_i(\omega, j) = 1$ is now sector specific,

$$c_i(j) = p_{b_{3,i}}^{\zeta(j)}(j)\, p_{b_{4,i}}(j)^{1-\zeta(j)}\, \zeta(j)^{-1}\, (1-\zeta(j))^{\zeta(j)-1},$$

where

$$\begin{aligned}
p_{b_{1,i}}(j) &= \left[\lambda(j)\, r_i^{1-\rho(j)} + (1-\lambda(j))\, s_i^{1-\rho(j)}\right]^{\frac{1}{1-\rho(j)}}, \\
p_{b_{2,i}}(j) &= \left[\mu(j)\, w_i^{1-\sigma(j)} + (1-\mu(j))\, p_{b_{1,i}}(j)^{1-\sigma(j)}\right]^{\frac{1}{1-\sigma(j)}}, \\
p_{b_{3,i}}(j) &= v_i^{\alpha(j)} p_{b_{2,i}}(j)^{1-\alpha(j)}\, \alpha(j)^{-1}\, (1-\alpha(j))^{\alpha(j)-1}, \\
p_{b_{4,i}}(j) &= P_i(S)^{\varepsilon(j)}\, P_i(M)^{1-\varepsilon(j)}\, \varepsilon(j)^{-1}\, (1-\varepsilon(j))^{\varepsilon(j)-1}.
\end{aligned}$$

Aggregate factor market clearing conditions are now written as,

$$\begin{aligned}
v_i K_i(S) &= \sum_j \zeta(j)\, \alpha(j)\, \Phi_i(j) \\
w_i L_i &= \sum_j \zeta(j)\, \mu(j)\, (1-\alpha(j)) \left[\frac{p_{b_{2,i}}(j)}{w_i}\right]^{\sigma(j)-1} \Phi_i(j) \\
r_i K_i(E) &= \sum_j \zeta(j)\, \lambda(j)\, (1-\alpha(j))\, (1-\mu(j)) \left[\frac{p_{b_{1,i}}(j)}{r_i}\right]^{\rho(j)-1} \left[\frac{p_{b_{2,i}}(j)}{p_{b_{1,i}}(j)}\right]^{\sigma(j)-1} \Phi_i(j) \\
s_i H_i &= \sum_j \zeta(j)\, (1-\alpha(j))\, (1-\mu(j))\, (1-\lambda(j)) \left[\frac{p_{b_{1,i}}(j)}{s_i}\right]^{\rho(j)-1} \left[\frac{p_{b_{2,i}}(j)}{p_{b_{1,i}(j)}}\right]^{\sigma(j)-1} \Phi_i(j)
\end{aligned}$$

and intermediate input market clearing is written as

$$\begin{aligned}
P_i(S)\, X_i(S) &= \sum_j \varepsilon(j)\, (1-\zeta(j))\, \Phi_i(j) \\
P_i(M)\, X_i(M) &= \sum_j (1-\varepsilon(j))\, (1-\zeta(j))\, \Phi_i(j)
\end{aligned}$$

where $\Phi_i(j) \equiv \sum_n \pi_{in}(j)\, P_n(j)\, Y_n(j)$ denotes total revenue accruing to all country i producers in sector j.

We now express the above conditions in terms of the *factor content of trade*. Define by NX_i^F the units of factor F that are embodied in county i's net exports,

$$NX_i^F = \sum_j F_i(j)\, \omega_i(j), \tag{A14}$$

where $F_i(j)$ denotes the utilization of factor F in country i and sector j, and where $\omega_i(j)$ is the ratio of county i's net exports in sector j to country i's total revenue in sector j,

$$\omega_i(j) = \frac{\sum_n \left[\pi_{in}(j) P_n(j) Y_n(j) - \pi_{ni}(j) P_i(j) Y_i(j)\right]}{\sum_n \pi_{in}(j) P_n(j) Y_n(j)}. \tag{A15}$$

By equations $(A14)$ and $(A15)$, we have

$$p_{Fi} NX_i^F = \alpha_F(j) \sum_{n \neq i} \left[\pi_{in}(j) P_n(j) Y_n(j) - \pi_{ni}(j) P_i(j) Y_i(j)\right], \tag{A16}$$

where $\alpha_F(j)$ denotes the share of sector j revenue paid to factor F. By equation $(A16)$ and $\pi_{ii}(j) = \left[1 - \sum_{n \neq i} \pi_{ni}(j)\right]$, we express equilibrium in the intermediate input markets as

$$P_i(S) = \frac{\sum_j \varepsilon(j)(1-\zeta(j)) P_i(j) Y_i(j)}{X_i(S) - NX_i^{X_i(S)}} \tag{A17}$$

$$P_i(M) = \frac{\sum_j (1-\varepsilon(j))(1-\zeta(j)) P_i(j) Y_i(j)}{X_i(M) - NX_i^{X(M)}} \tag{A18}$$

and in the factor markets as

$$v_i = \sum_j \zeta(j) \alpha(j) P_i(j) Y_i(j) \Big/ \left(K_i(S) - NX_i^{K(S)}\right) \tag{A19}$$

$$w_i = \sum_j \zeta(j) \mu(j)(1-\alpha(j)) \left[\frac{p_{b_{2,i}}(j)}{w_i}\right]^{\sigma(j)-1} P_i(j) Y_i(j) \Big/ \left(L_i - NX_i^L\right) \tag{A20}$$

$$\begin{aligned} r_i = \sum_j \Bigg(& \zeta(j) \lambda(j)(1-\alpha(j))(1-\mu(j)) \left[\frac{p_{b_{1,i}}(j)}{r_i}\right]^{\rho(j)-1} \times \\ & \left[\frac{p_{b_{2,i}}(j)}{p_{b_{1,i}}(j)}\right]^{\sigma(j)-1} P_i(j) Y_i(j) \Bigg) \Big/ \left(K_i(E) - NX_i^{K(E)}\right) \end{aligned} \tag{A21}$$

$$\begin{aligned} s_i = \sum_j \Bigg(& \zeta(j)(1-\alpha(j))(1-\mu(j))(1-\lambda(j)) \left[\frac{p_{b_{1,i}}(j)}{s_i}\right]^{\rho(j)-1} \times \\ & \left[\frac{p_{b_{2,i}}(j)}{p_{b_{1,i}(j)}}\right]^{\sigma(j)-1} P_i(j) Y_i(j) \Bigg) \Big/ \left(H_i - NX_i^H\right) \end{aligned} \tag{A22}$$

Aggregate prices are:

$$P_i(j) = \gamma_i(j)\, c_i(j)\, \pi_{ii}(j)^{\theta(j)} \Big/ A_i(j) \tag{A23}$$

Equations $(16)-(19)$, $(33)-(35)$, and $(A17)-(A23)$ yield a system of equations in $\{w_i, s_i, v_i, r_i\}_{i=1}^{I}$, $\{P_i(j)\}_{i=1, j\in\mathcal{J}}^{I}$, $\{Y_i(j)\}_{i=1, j\in\mathcal{J}}^{I}$, $\{K_i(E), K_i(S)\}_{i=1}^{I}$, $\{C_i(M), C_i(S)\}_{i=1}^{I}$, and $\{X_i(M), X_i(S)\}_{i=1}^{I}$ that characterizes the steady-state equilibrium and that depends only on domestic expenditure shares by sector $\{\pi_{ii}(j)\}_{j\in\mathcal{J}}$, sectoral domestic productivities $\{A_i(j)\}_{j\in\mathcal{J}}$, the factor content of trade for all factors $\left\{NX_i^F\right\}_{F\in\mathcal{F}}$, and the parameters $\{\zeta(j), \alpha(j), \mu(j), \lambda(j), \varepsilon(j), \sigma(j), \rho(j), \theta(j)\}_{j\in\mathcal{J}}$.