Accounting for Changes in Between-Group Inequality*

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Abstract
We offer a quantitative analysis of changes in U.S. between-group inequality from 1984 to 2003. We use an assignment framework with many labor groups, equipment types, and occupations in which changes in inequality are driven by shocks to workforce composition, occupation demand, computerization, and labor productivity. We parameterize our model using detailed data including direct measures of computer usage within labor group-occupation pairs. We separately quantify the impact of each shock for various measures of between-group inequality. We find, for instance, that the combination of computerization and shifts in occupation demand—which are measured without directly using data on changes in wages—account for roughly 80% and that computerization alone accounts for roughly 60% of the rise in the skill premium. We show theoretically how to link the strength of computerization and changes in occupation demand to changes in the extent of international trade.

1 Introduction
The last few decades in the United States have witnessed pronounced changes in relative average wages across groups of workers with different observable characteristics (between-group inequality); see, e.g. Acemoglu and Autor (2011). For example, the wages of more educated relative to less educated workers and of women relative to men have increased substantially.1

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1The relative importance of between- and within-group inequality is an area of active research. Autor (2014) concludes: “In the U.S., for example, about two-thirds of the overall rise of earnings dispersion
A voluminous literature has emerged—following Katz and Murphy (1992)—studying how changes in relative supply and demand for labor groups shape their relative wages. Changes in relative demand across labor groups have been linked prominently to computerization (or a reduction in the price of equipment more generally)—see e.g. Krusell et al. (2000), Acemoglu (2002a), Autor and Dorn (2013), and Beaudry and Lewis (2014)—and changes in relative demand (or productivity) across occupations or sectors, driven by structural transformation, offshoring, and international trade—see e.g. Berman et al. (1994), Autor et al. (2003), Buera et al. (2015), and Galle et al. (2015). Related to the first hypothesis, Table 1 shows that between 1984 and 2003 computer use rose dramatically and that computers are used more intensively by educated workers and women. Related to the second hypothesis, Figure 1 shows that over the same time period education- and female-intensive occupations grew relatively quickly; see Table 10 in Appendix A for details.

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Table 1: Share of hours worked with computers

between 1980 and 2005 is proximately accounted for by the increased premium associated with schooling in general and postsecondary education in particular.” On the other hand, Helpman et al. (2012) conclude: “Residual wage inequality is at least as important as worker observables in explaining the overall level and growth of wage inequality in Brazil from 1986-1995.”

2 We describe our data sources in depth in Section 3.1 and Appendix A.
The goal of our paper is to provide a quantitative evaluation of the sources of changes in between-group inequality in the United States. We base our analysis on an assignment model with many groups of workers and many occupations—building on Eaton and Kortum (2002), Lagakos and Waugh (2013), and Hsieh et al. (2013)—which we extend to incorporate many types of equipment. In order to shed light on the importance of classes of mechanisms through which changes in the economic environment lead to changes in relative wages across labor groups, our model incorporates shocks to (i) the composition of labor supply across groups, (ii) a composite of occupation demand and productivity, which we refer to as “occupation shifters,” (iii) a composite of equipment cost and productivity, which we refer to as “equipment productivity,” and (iv) a residual composite of other factors affecting the relative productivity of labor groups, independent of the equipment they use and occupations in which they are employed, which we refer to as “labor productivity.” The model’s aggregate implications for relative wages nest those of workhorse macro models of between-group inequality, e.g. Katz and Murphy (1992) and Krusell et al. (2000). In spite of its high dimensionality—in our baseline empirics we use 30 education-gender-age groups, 2 types of equipment, and 30 occupations—we parametrize and estimate the model in a transparent manner and perform aggregate counterfactuals to quantify the impact on between-group inequality of these four shocks.

In our model, the impact of changes in the economic environment on between-group inequality is shaped by comparative advantage between labor groups, equipment types, and occupations. Consider, for example, the potential impact of computerization on a labor group—such as educated workers or women—that uses computers intensively. A labor group may use computers intensively for two reasons. First, it may have a comparative advantage with computers, in which case this group would use computers relatively more within occupations, as we show is the case in the data for more educated workers. Computerization increases the relative wage of such groups. Second, a labor group may have a comparative advantage in the occupations in which computers have a comparative advantage, in which case this group would be allocated disproportionately to occupations in which all workers are relatively more likely to use computers, as we show is the case in the data for women. Depending on the value of the elasticity of substitution between occupations, computerization may increase or decrease relative wages of such groups and may increase or decrease employment in computer-intensive occupations.

These factors include, for example, discrimination and the quality of education and health systems; see e.g. Card and Krueger (1992) and Goldin and Katz (2002).

Our model is flexible enough so that computerization may increase the relative wage of workers who are relatively productive using computers and may reduce the relative wage of workers employed in occupations in which computers are particularly productive, as described by, e.g., Autor et al. (1998) and Autor...
Therefore, measuring comparative advantage between labor groups, equipment types, and occupations is a key ingredient in our quantitative analysis.

Comparative advantage can be inferred directly from data on the allocation of workers to equipment type-occupation pairs. Changes in equipment productivity can be inferred from changes in the allocation of workers to computers within labor group-occupation pairs; controlling for labor group-occupation pairs is important because aggregate computer usage rises if labor groups that have a comparative advantage using computers grow or if occupations that have a comparative advantage with computers grow. Changes in occupation shifters can be inferred from changes in the allocation of workers to occupations within labor group-equipment type pairs, changes in labor income shares across occupations, and model parameters. Finally, changes in labor productivity can be inferred as a residual to match changes in observed average wages across labor groups.

Our procedure crucially requires information over time on the allocation of labor groups across equipment type-occupation pairs. We obtain this information for the U.S. from the October Current Population Survey (CPS) Computer Use Supplement, which provides data for five years (1984, 1989, 1993, 1999, and 2003) on whether or not a worker has direct or hands on use of a computer at work—be it a personal computer, laptop, mini computer, or mainframe—in addition to information on worker characteristics, hours worked, and occupation; see, e.g., Krueger (1993) and Autor et al. (1998) for previous studies using the October Supplement. This data is not without limitations: it imposes a narrow view of computerization that does not capture, e.g., automation of assembly lines; it only provides information on the allocation of workers to one type of equipment, computers; it does not detail the share of each worker’s time at work spent using computers; and it does not extend beyond 2003.5

We find that computerization alone accounts for roughly 60% of the forces that increase the skill premium between 1984 and 2003 and plays a similar role in explaining disaggregated measures of between-education inequality (e.g., the relative wage of workers with graduate training relative to high school dropouts). This result from our model is driven by the following three observations in the data. First, within labor group-occupation pairs we observe a large rise in the share of workers using computers, which our model interprets as a large increase in computer productivity (i.e. computerization). Second, more educated workers use computers relatively more within occupations than less educated workers, which—together with computerization—yields a rise in their rel-

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5Our sample period, 1984-2003, accounts for a substantial share of the increase in the skill premium and reduction in the gender gap in the U.S. since the late 1960s.
ative wages according to our model. Third, more educated workers are also dispropor-
tionately employed in occupations in which all workers use computers relatively inten-
sively, which—together with computerization and an estimated elasticity of substitu-
tion between occupations greater than one—also yields a rise in their relative wages accord-
ing to our model. The combination of computerization and occupation shifters account
for roughly 80% of the rise in the skill premium, leaving 20% to be explained by labor
productivity, the shock measured to match changes in relative wages.

We find that computerization, occupation shifters, and labor productivity all play im-
portant roles in accounting for the reduction in the gender gap. Computerization con-
tracts the gender gap in spite of the fact that, unlike educated workers, women do not
have a comparative advantage using computers because, like educated workers, women
are disproportionately employed in occupations in which all workers use computers in-
tensively.

Whereas in our baseline model we treat computerization and occupation shifters as
changes in exogenous parameters, in Section 6 we take a first theoretical step to link these
mechanisms to more primitive shocks. Specifically, we extend our model to incorporate
international trade in equipment and sector output as well as occupation offshoring. We
show that the procedure to quantify the impact on relative wages of moving to autarky
is equivalent to the procedure to calculate changes in relative wages in a closed economy
in which shocks are simple functions of import and export shares in the open economy.
We also provide a simple procedure to quantify the differential effects on wages in a
given country of changes in primitives (i.e. worldwide technologies, labor compositions,
and trade costs) between two time periods relative to the effects of the same changes in
primitives if that country were a closed economy.

We now discuss our contribution relative to a number of related papers in a large lit-
erature. In using an assignment model of the labor market parameterized with a Fréchet
distribution we follow Hsieh et al. (2013) and Lagakos and Waugh (2013). Relative to
Hsieh et al. (2013), we integrate three sources of comparative advantage (between work-
ners, occupations, and equipment) to quantify the role of computerization for between-
group inequality.

In linking the evolution of between-group inequality to observables, rather than ex-
plaining patterns in the skill premium and gender gap through latent skill- or gender-
biasst technological change, our paper’s objective is most similar to Krusell et al. (2000)
and Lee and Wolpin (2010). Krusell et al. (2000) estimate an aggregate production function
which permits capital-skill complementarity and show that changes in aggregate stocks
of equipment, skilled labor, and unskilled labor can account for much of the variation in
the U.S. skill premium. Whereas they identify the degree of capital-skill complementarity using aggregate time series data, we measure comparative advantage using direct measures of computer allocation within labor group-occupation pairs. We also incorporate occupation shifters. Lee and Wolpin (2010) use a model of endogenous human capital accumulation in a dynamic framework to study the evolution of relative wages and labor supply and find that skill-biased technical change (the residual) plays the central role in explaining changes in the skill premium. By allowing for a greater degree of disaggregation (e.g., 30 occupations) and exploiting detailed data on factor allocation we reduce substantially the role of changes in the residual (labor productivity) in shaping changes in the skill premium. On the other hand, in contrast to Lee and Wolpin (2010) we treat labor composition as exogenous, measuring it in each period directly from the data.6

Two important related papers use differential regional exposure to computerization to study the differential effect across regions of technical change on the polarization of U.S. employment and wages, Autor and Dorn (2013), and on the gender gap and skill premium, Beaudry and Lewis (2014). Our approach complements these papers, embedding computerization into a general equilibrium model allowing us to quantify the effect of computerization (as well as other shocks) on changes over time in between-group inequality. Instead of relying on regional variation in the exposure to computerization, we make use of detailed data on computer usage within labor group-occupation pairs.

Our focus on occupation shifters is related to a broad literature using shift-share analyses. For instance, Autor et al. (2003) use a shift-share analysis and find that occupation shifters account for more than 50% of the relative demand shift favoring college labor between 1970 and 1988; we arrive at a similar result using a shift-share analysis between 1984 and 2003 even though occupation shifters account for only 19% of the rise in the relative wage of college labor. Under some assumptions (Cobb-Douglas utility and production functions), shift-share analyses structurally decompose changes in wage bill shares, i.e., changes in labor income for one labor group relative to the sum of labor payments across all labor groups, into within and between occupation shifters. However, changes in wage bill shares can be very different from changes in relative wages (on which we focus), especially when changes in labor composition are large, as they are in the data.7

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6Extending our model using standard assumptions to endogenize education and labor participation would give rise to the same equilibrium equations determining factor allocations and wages, given labor composition. Hence, our measures of shocks—to occupation shifters, equipment productivity, and labor productivity—and our estimates of model parameters would remain unchanged. In our counterfactual exercises, we fix labor composition to isolate the direct effect of individual shocks to occupation shifters, equipment productivity, and labor productivity on labor demand and wages.

7Firpo et al. (2011) uses a statistical model of wage setting to investigate the contribution of changes in the returns to occupational tasks compared to other explanations such as de-unionization and changes in
In modeling international trade in equipment, sectoral output, and occupations (e.g. offshoring) we operationalize the theoretical insights of Costinot and Vogel (2010) and Costinot and Vogel (Forthcoming) regarding the impact of international trade on inequality in a high-dimensional environment. We show, as in concurrent work by Galle et al. (2015), that one can use a similar approach to that introduced by Dekle et al. (2008) in a single-factor trade model—i.e. replacing a large number of unknown parameters with observable allocations in an initial equilibrium—in closed and open-economy many-factor assignment models to quantify the impact of various shocks on relative wages; Burstein et al. (2013), Parro (2013), and Costinot and Rodriguez-Clare (2014) use similar approaches in models with two labor groups.

Our paper is organized as follows. We describe our framework, characterize its equilibrium, and discuss its mechanisms in Section 2. We parameterize the model in Section 3, describe our baseline closed-economy results in Section 4, and consider various robustness exercises and sensitivity analyses in Section 5. Finally, we briefly discuss how we extend the model to incorporate sectors and international trade in Section 6 and conclude in Section 7. Many details and robustness exercises are relegated to the appendix.

2 Model

In this section we introduce the baseline version of our model, characterize the equilibrium, and show how to decompose observed changes in relative average wages between any two periods into four changes in the economic environment: labor composition, equipment productivity, occupation shifters and a residual that we define as labor productivity. Finally, we provide intuition for how each of these changes affects relative wages.

2.1 Environment

At time $t$ there is a continuum of workers indexed by $z \in \mathcal{Z}_t$, each of whom inelastically supplies one unit of labor. We divide workers into a finite number of groups, indexed by $\lambda$. The set of workers in group $\lambda$ is given by $\mathcal{Z}_t (\lambda) \subseteq \mathcal{Z}_t$, which has mass $L_t (\lambda)$. There is a finite number of equipment types, indexed by $\kappa$. Workers and equipment are employed by production units to produce a finite number of occupations, indexed by $\omega$.\footnote{In order to take into consideration the accumulation of occupation-specific human capital as studied in, e.g., Kambourov and Manovskii (2009a) and Kambourov and Manovskii (2009b), empirically we would consider the labor market wide returns to general skills. Our paper complements theirs by incorporating general equilibrium effects and explicitly modeling the endogenous allocation of factors.}
Occupations are used to produce a single final good according to a constant elasticity of substitution (CES) production function

$$Y_t = \left( \sum_{\omega} \mu_t(\omega)^{1/\rho} Y_t(\omega)^{(\rho-1)/\rho} \right)^{\rho/(\rho-1)},$$

(1)

where $\rho > 0$ is the elasticity of substitution across occupations, $Y_t(\omega) \geq 0$ is the endogenous output of occupation $\omega$, and $\mu_t(\omega) \geq 0$ is an exogenous demand shifter for occupation $\omega$.\(^9\) The final good is used to produce consumption, $C_t$, and equipment, $Y_t(\kappa)$,\(^10\) according to the resource constraint

$$Y_t = C_t + \sum_{\kappa} p_t(\kappa) Y_t(\kappa),$$

(2)

where $p_t(\kappa)$ denotes the cost of a unit of equipment $\kappa$ in terms of units of the final good.\(^11\)

Occupation output is produced by perfectly competitive production units. A unit hiring $k$ units of equipment type $\kappa$ and $l$ efficiency units of labor group $\lambda$ produces $ka \left[ T_t(\lambda, \kappa, \omega) l \right]^{1-\alpha}$ units of output, where $\alpha$ denotes the output elasticity of equipment in each occupation and $T_t(\lambda, \kappa, \omega)$ denotes the productivity of an efficiency unit of group $\lambda$'s labor in occupation $\omega$ when using equipment $\kappa$.\(^12\) Comparative advantage between labor and equipment is defined as follows: $\lambda'$ has a comparative advantage (relative to $\lambda$) using equipment $\kappa'$ (relative to $\kappa$) in occupation $\omega$ if $T_t(\lambda', \kappa', \omega)/T_t(\lambda', \kappa, \omega) \geq T_t(\lambda, \kappa', \omega)/T_t(\lambda, \kappa, \omega)$.

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\(^9\)We show in Section 6 that we can disaggregate $\mu_t(\omega)$ further into sector shifters and within-sector occupation shifters. We also show how changes in the extent of international trade/offshoring in sectoral output and occupation output generate endogenous changes in these sector shifters and a within-sector occupation shifters. For now, however, we combine sector and within-sector occupation shifters and treat them as exogenous.

\(^10\)We assume that the sets of equipment types and occupations (as well as labor groups) are disjoint. Hence, the domain of a function such as $Y_t(\cdot)$ may be the union of these sets.

\(^11\)Here we have assumed that equipment fully depreciates every period. Alternatively, we could assume that $Y_t(\kappa)$ denotes investment in capital equipment $\kappa$, which depreciates at a given rate. All our results hold comparing across two balanced growth paths in which the real interest rate and the growth rate of relative productivity are both constant over time. We show in Section 6 how changes in the extent of international trade in equipment generates endogenous changes in $p_t(\kappa)$. For now, however, we treat the cost of producing equipment as exogenous.

\(^12\)We restrict $\alpha$ to be common across $\omega$ because we do not have the data to assign a different value of $\alpha(\omega)$ to each $\omega$. Moreover, without affecting any of our results we can extend the model to incorporate a composite input $s$, which is produced linearly using the final good and which enters the production function multiplicatively as $s^{1-\eta} \left( k^\alpha [ T_t(\lambda, \kappa, \omega) l ]^{1-\alpha} \right)^\eta$. In either case, $\alpha$ is the share of equipment relative to the combination of equipment and labor.
Labor-occupation and equipment-occupation comparative advantage are defined symmetrically.

A worker $z \in Z_t (\lambda)$ supplies $\epsilon (z, \kappa, \omega)$ efficiency units of labor if teamed with equipment $\kappa$ in occupation $\omega$. For each worker $z \in Z_t (\lambda)$, each element of the vector $\epsilon$—which contains one $\epsilon (z, \kappa, \omega)$ for each $(\kappa, \omega)$ pair—is drawn independently from a Fréchet distribution with cumulative distribution function $G (\epsilon) = \exp (\epsilon - \theta)$, where a higher value of $\theta > 1$ decreases the dispersion of efficiency units across $(\kappa, \omega)$ pairs. The assumption that efficiency units are distributed Fréchet is made for analytical tractability; it implies that the average wage of a labor group is a CES function of prices and productivities.

Total output of occupation $\omega$, $Y_t (\omega)$, is the sum of output across all units producing occupation $\omega$ using any labor group $\lambda$ and equipment type $\kappa$. All markets are perfectly competitive and all factors are freely mobile across occupations and equipment types.

Relation to alternative frameworks. Whereas our framework imposes strong restrictions on occupation production functions, its aggregate implications for wages nest those of two frameworks that have been used commonly to study between-group inequality: the canonical model, as named in Acemoglu and Autor (2011), and an extension of the canonical model that incorporates capital-skill complementarity; see e.g. Katz and Murphy (1992) and Krusell et al. (2000), respectively.

2.2 Equilibrium

We characterize the competitive equilibrium, first in partial equilibrium—taking occupation prices as given—and then in general equilibrium.

Partial equilibrium. With perfect competition, equation (2) implies that the price of equipment $\kappa$ relative to the price of the final good (which we normalize to one) is simply $p_t (\kappa)$. An occupation production unit hiring $k$ units of equipment $\kappa$ and $l$ efficiency units of labor $\lambda$ earns revenues $p_t (\omega) k^a [T_t (\lambda, \kappa, \omega)]^{1-a}$ and incurs costs $p_t (\kappa) k + v_t (\lambda, \kappa, \omega) l$, where $v_t (\lambda, \kappa, \omega)$ is the wage per efficiency unit of labor $\lambda$ when teamed with equipment $\kappa$. The wage distribution implied by this assumption is not a poor approximation of the observed distribution of individual wages; see e.g. Saez (2001) and Figure 6 in the Appendix.

The aggregate implications of our model for relative wages are equivalent to those of the canonical model if we assume no equipment (i.e. $a = 0$) and two labor groups, each of which has a positive productivity in only one occupation. The model of capital-skill complementarity is an extension of the canonical model in which there is one type of equipment and the equipment share is positive in one occupation and zero in the other (i.e. $a = 0$ for the latter occupation).
\( \kappa \) in occupation \( \omega \) and where \( p_l(\omega) \) is the price of occupation \( \omega \) output. The profit maximizing choice of equipment quantity and the zero profit condition—due to costless entry of production units—yield

\[
v_l(\lambda, \kappa, \omega) = \bar{a} p_l(\kappa) \frac{a}{1-a} p_l(\omega) \frac{1}{1-\gamma} T_l(\lambda, \kappa, \omega)
\]

if there is positive entry in \((\lambda, \kappa, \omega)\), where \( \bar{a} \equiv (1 - a) \frac{a^h}{1-a} \). Facing the wage profile \( v_l(\lambda, \kappa, \omega) \), each worker \( z \in \mathcal{Z}_l(\lambda) \) chooses the equipment-occupation pair \((\kappa, \omega)\) that maximizes her wage, \( \epsilon_l(z, \kappa, \omega) v_l(\lambda, \kappa, \omega) \).

The assumption that idiosyncratic productivity is distributed Fréchet implies that the probability that a randomly sampled worker, \( z \in \mathcal{Z}_l(\lambda) \), uses equipment \( \kappa \) in occupation \( \omega \) is

\[
\pi_l(\lambda, \kappa, \omega) = \frac{T_l(\lambda, \kappa, \omega) p_l(\kappa) \frac{a}{1-a} p_l(\omega) \frac{1}{1-\gamma}}{\sum_{k', \omega'} T_l(\lambda, k', \omega') p_l(k') \frac{a}{1-a} p_l(\omega') \frac{1}{1-\gamma}} \theta.
\]

The higher is \( \theta \)—i.e. the less dispersed are efficiency units across \((\kappa, \omega)\) pairs—the more responsive are factor allocations to changes in prices or productivities. According to equation (3), comparative advantage shapes factor allocations. As an example, the assignment of workers across equipment types within any given occupation satisfies

\[
\frac{T_l(\lambda', \kappa', \omega)}{T_l(\lambda', \kappa, \omega)} / \frac{T_l(\lambda, \kappa', \omega)}{T_l(\lambda, \kappa, \omega)} = \left( \frac{\pi_l(\lambda', \kappa', \omega)}{\pi_l(\lambda' \kappa, \omega)} / \frac{\pi_l(\lambda, \kappa', \omega)}{\pi_l(\lambda, \kappa, \omega)} \right)^{1/\theta},
\]

so that if \( \lambda' \) workers (relative to \( \lambda \)) have a comparative advantage using \( \kappa' \) (relative to \( \kappa \)) in occupation \( \omega \), then they are relatively more likely to be allocated to \( \kappa' \) in occupation \( \omega \). Similar conditions hold for the allocation of workers to occupations (within an equipment type) and for the allocation of equipment to occupations (within a labor group).

The average wage of workers in group \( \lambda \) teamed with equipment \( \kappa \) in occupation \( \omega \), denoted by \( w_l(\lambda, \kappa, \omega) \), is the integral of \( \epsilon_l(z, \kappa, \omega) v_l(\lambda, \kappa, \omega) \) across workers teamed with \( \kappa \) in occupation \( \omega \), divided by the mass of these workers. Using the distributional assumption on idiosyncratic productivity, we obtain

\[
w_l(\lambda, \kappa, \omega) = \bar{a} \gamma T_l(\lambda, \kappa, \omega) p_l(\kappa) \frac{a}{1-a} p_l(\omega) \frac{1}{1-\gamma} \frac{1}{\pi_l(\lambda, \kappa, \omega)}^{-1/\theta}
\]

where \( \gamma \equiv \Gamma \left(1 - \frac{1}{\theta} \right) \) and \( \Gamma(\cdot) \) is the Gamma function. An increase in productivity or occupation price, \( T_l(\lambda, \kappa, \omega) \) or \( p_l(\omega) \), or a decrease in equipment price, \( p_l(\kappa) \), raises the wages of infra-marginal \( \lambda \) workers allocated to \((\kappa, \omega)\). However, the average wage
across all \( \lambda \) workers in \((\kappa, \omega)\) increases less than proportionately due to self-selection, i.e. \( \pi_t(\lambda, \kappa, \omega) \) increases, which lowers the average efficiency units of \( \lambda \) workers using equipment \( \kappa \) in occupation \( \omega \).

Denoting by \( w_t(\lambda) \) the average wage of workers in group \( \lambda \), the previous expression and equation (3) imply \( w_t(\lambda) = w_t(\lambda, \kappa, \omega) \) for all \((\kappa, \omega)\), where\(^{16}\)

\[
w_t(\lambda) = \bar{\alpha} \gamma \left( \sum_{\kappa, \omega} \left( T_t(\lambda, \kappa, \omega) p_t(\kappa) \frac{1}{1-\bar{\alpha}} p_t(\omega) \right)^{\frac{\alpha}{1-\bar{\alpha}}} \right)^{1/\gamma} . \tag{4}
\]

**General equilibrium.** In any period, occupation prices \( p_t(\omega) \) must be such that total expenditure in occupation \( \omega \) is equal to total revenue earned by all factors employed in occupation \( \omega \),

\[
\mu_t(\omega) p_t(\omega)^{1-\bar{\rho}} E_t = \frac{1}{1-\bar{\alpha}} \zeta_t(\omega)
\]

where \( E_t \equiv \frac{1}{1-\bar{\alpha}} \sum_\lambda w_t(\lambda) L_t(\lambda) \) is total income and \( \zeta_t(\omega) \equiv \sum_{\lambda, \kappa} w_t(\lambda) L_t(\lambda) \pi_t(\lambda, \kappa, \omega) \) is total labor income in occupation \( \omega \). The left-hand side of equation (5) is expenditure on occupation \( \omega \) and the right-hand side is total income earned by factors employed in occupation \( \omega \). In equilibrium, the aggregate quantity of the final good is \( Y_t = E_t \), the aggregate quantity of equipment \( \kappa \) is

\[
Y_t(\kappa) = \frac{1}{p_t(\kappa)^{1-\bar{\rho}}} \bar{\alpha} \sum_{\lambda, \omega} \pi_t(\lambda, \kappa, \omega) w_t(\lambda) L_t(\lambda),
\]

and aggregate consumption is determined by equation (2).

### 2.3 Decomposing changes in relative wages

Our objective in this paper is to quantify the sources of changes in relative wages between labor groups. In what follows, we impose the assumption that \( T_t(\lambda, \kappa, \omega) \) can be expressed as

\[
T_t(\lambda, \kappa, \omega) \equiv T_t(\lambda) T_t(\kappa) T_t(\omega) T(\lambda, \kappa, \omega) . \tag{6}
\]

\(^{16}\)Obviously, the implication that average wage levels are common across occupations within \( \lambda \) (which is inconsistent with the data) implies that the changes in wages will also be common across occupation within \( \lambda \). In Appendix F we decompose changes in average wages for each \( \lambda \) into a between occupation component (which is zero in our model) and a within occupation component and show that the between component is small. Furthermore, we show that by incorporating preference shifters for working in different occupations that generate compensating differentials, similar to Heckman and Sedlacek (1985), our model may be consistent with differences in average wage levels across occupations within a labor group.
Accordingly, whereas we allow labor group, \(T_t(\lambda) \geq 0\), equipment type, \(T_t(\kappa) \geq 0\), and occupation, \(T_t(\omega) \geq 0\), productivity to vary over time, we impose that the interaction between labor group, equipment type, and occupation productivity, \(T(\lambda, \kappa, \omega) \geq 0\), is constant over time. That is, we assume that comparative advantage is fixed over time. This restriction allows us to separate the impact on relative wages of \(\lambda, \kappa,\) and \(\omega\)-specific productivity shocks. In Section 5.3 we relax this restriction.

Given the restriction in equation (6), one could use the model described above to decompose observed changes in relative average wages for any two labor groups into changes in labor composition, \(L_t(\lambda)\); changes in labor productivity, \(T_t(\lambda)\); changes in occupation demand, \(\mu_t(\omega)\); changes in occupation productivity, \(T_t(\omega)\); changes in equipment productivity, \(T_t(\kappa)\); and changes in equipment production costs, \(p_t(\kappa)\). However, given available data, we cannot separately identify occupation demand and productivity; instead, we combine them in a composite occupation shifter, \(a_t(\omega) \equiv \mu_t(\omega)T_t(\omega)^{(1-a)(\rho-1)}\).

Similarly, we cannot separately identify equipment productivity and production cost; instead, we combine them in a composite equipment productivity, \(q_t(\kappa) \equiv p_t(\kappa)^{\frac{1-a}{a}}T_t(\kappa)\). Hence, we consider four shocks: (i) labor composition, (ii) equipment productivity, (iii) occupation shifters, and (iv) labor productivity. \(^{17}\)

Given measures of these shocks and model parameters we will quantify the impact of each shock on relative wages across labor groups. To solve for changes in relative wages it is useful to express the system of equations in changes, denoting changes in any variable \(x\) between any two periods \(t_0\) and \(t_1\) by \(\hat{x} \equiv x_{t_1} / x_{t_0}\). Changes in average wages—using equations (4) and (6)—are given by

\[
\hat{w}(\lambda) = \hat{T}(\lambda) \left( \sum_{\kappa,\omega} (\hat{q}(\omega) \hat{q}(\kappa))^{\theta} \pi_{t_0}(\lambda, \kappa, \omega) \right)^{1/\theta}, \tag{7}
\]

where we have defined transformed occupation prices as \(q_t(\omega) \equiv p_t(\omega)^{1/(1-a)}T_t(\omega)^{1/a}\). \(^{18}\)

\(^{17}\)In order to separate the effects on relative wages of \(T_t(\kappa)\) and \(p_t(\kappa)\) or of \(T_t(\omega)\) and \(\mu_t(\omega)\), one needs to use additional information on changes in equipment or occupation prices, which are hard to measure in practice.

\(^{18}\)Taking a first-order approximation of equation (7) at the period \(t_0\) allocations yields

\[
\log \hat{w}(\lambda) = \log \hat{T}(\lambda) + \sum_{\kappa,\omega} \pi_{t_0}(\lambda, \kappa, \omega) \left( \log \hat{q}(\omega) + \log \hat{q}(\kappa) \right).
\]

Hence, our framework provides a microfoundation for the regression model that Acemoglu and Autor (2011) offer as a stylized example of how their model might be brought to the data. Moreover, changes in average wages in response to given changes in transformed occupation prices and equipment productivities do not depend—to a first-order approximation—on the value of \(\theta\) (or, more generally and using an envelope condition, on the distribution of idiosyncratic draws). However, because the value of \(\theta\) (or, more generally, the shape of the distribution of idiosyncratic draws) determines the extent of factor reallocation in response
Changes in transformed occupation prices—using equations (3), (5), and (6)—are determined by the following system of equations

\[
\hat{\pi}(\lambda, \kappa, \omega) = \frac{(\hat{q}(\omega) \hat{q}(\kappa))^{\theta}}{\sum_{\kappa', \omega'} (\hat{q}(\omega') \hat{q}(\kappa'))^{\theta} \pi_{t_0}(\lambda, \kappa', \omega')},
\]

(8)

\[
\hat{a}(\omega) \hat{q}(\omega)^{(1-a)(1-\rho)} \hat{E} = \frac{1}{\zeta_{t_0}(\omega)} \sum_{\lambda, \kappa} \omega_{t_0}(\lambda) L_{t_0}(\lambda) \pi_{t_0}(\lambda, \kappa, \omega) \hat{w}(\lambda) \hat{L}(\lambda) \hat{\pi}(\lambda, \kappa, \omega).
\]

(9)

In this system, the forcing variables are the four shocks mentioned above: \( \hat{L}(\lambda), \hat{T}(\lambda), \hat{a}(\omega), \) and \( \hat{q}(\kappa) \). Given these variables, equations (7)-(9) yield the model’s implied values of changes in average wages, \( \hat{w}(\lambda) \), allocations, \( \hat{\pi}(\lambda, \kappa, \omega) \), and transformed occupation prices, \( \hat{q}(\omega) \).

2.4 Intuition

In partial equilibrium—i.e. for given changes in occupation prices—changes in wages are proportional to changes in labor productivity, \( \hat{T}(\lambda) \), and are a CES combination of changes in transformed occupation prices and equipment productivities, where the weight given to changes in each of these components depends on factor allocations in the initial period \( t_0, \pi_{t_0}(\lambda, \kappa, \omega) \). An increase in occupation \( \omega \)'s price or equipment \( \kappa \)'s productivity between \( t_0 \) and \( t_1 \) raises the relative average wage of labor groups that disproportionately work in occupation \( \omega \) or use equipment \( \kappa \) in period \( t_0 \). While this partial equilibrium intuition is useful, it is incomplete because all shocks affect occupation prices; indeed, changes in labor composition and occupation shifters affect wages only indirectly through occupation prices.

The impact of shocks on relative wages can be understood as follows. Consider an increase in the occupation shifter \( a_t(\omega) \), i.e. \( \hat{a}(\omega) > 1 \), arising either from an increase in the demand shifter for occupation \( \omega \) or an increase (decrease) in the productivity of all work-

to given changes in transformed occupation prices and equipment productivities, the value of \( \theta \) does affect the first-order response of transformed occupation prices to shocks.

\(^{19}\)Given available data, we can only measure relative shocks to occupation shifters, equipment productivity, and labor productivity. Specifically, rather than measure \( \hat{T}(\lambda), \hat{a}(\omega), \) and \( \hat{q}(\kappa) \), we can only measure \( \hat{T}(\lambda) / \hat{T}(\lambda_1), \hat{a}(\omega) / \hat{a}(\omega_1), \) and \( \hat{q}(\kappa) / \hat{q}(\kappa_1) \) for any arbitrary choice of a benchmark labor group, \( \lambda_1 \), occupation, \( \omega_1 \), and type of equipment, \( \kappa_1 \). However, we can re-express equations (7), (8), and (9)—see Appendix B—so that changes in relative wages, \( \hat{w}(\lambda) / \hat{w}(\lambda_1) \), transformed occupation prices, \( \hat{q}(\omega) / \hat{q}(\omega_1) \), and allocations, \( \hat{\pi}(\lambda, \kappa, \omega) \), depend on relative shocks to labor composition, \( \hat{L}(\lambda) / \hat{L}(\lambda_1) \), occupation shifters, \( \hat{a}(\omega) / \hat{a}(\omega_1) \), equipment productivity, \( \hat{q}(\kappa) / \hat{q}(\kappa_1) \), and labor productivity, \( \hat{T}(\lambda) / \hat{T}(\lambda_1) \).

\(^{20}\)We test and find support for the implication that changes in labor composition affect wages only indirectly through occupation prices in Section 3.3.1.
ers employed in occupation \( \omega \) if \( \rho > 1 \) (\( \rho < 1 \)). This shock raises the transformed price of occupation \( \omega \) (an increase in occupation productivity reduces the primitive occupation price, \( p_t(\omega) \)) and, therefore, the average wages of labor groups that are disproportionately employed in occupation \( \omega \). Similarly, an increase in labor supply \( L_t(\lambda) \) reduces the transformed prices of occupations in which group \( \lambda \) is disproportionately employed. This lowers the relative wage not only of group \( \lambda \), but also of labor groups employed in similar occupations as \( \lambda \). An increase in labor productivity \( T_t(\lambda) \) directly raises the relative wage of group \( \lambda \) and affects all other labor groups through changes in occupation prices similarly to an increase in \( L_t(\lambda) \).\(^{21}\) In all cases, the effect through transformed occupation prices is stronger for lower values of \( \rho \), since occupation prices are more responsive to shocks in this case.

Finally, consider the impact on relative wages of a change in the productivity of equipment \( \kappa \), i.e. \( \hat{q}(\kappa) > 1 \). This shock raises the relative wages of labor groups that use \( \kappa \) intensively. It also reduces the transformed prices of occupations in which \( \kappa \) is used intensively, lowering the relative wages of labor groups that tend to be employed in these occupations. Overall, the impact on relative wages of changes in equipment productivity depend on the value of \( \rho \) and on whether aggregate patterns of labor allocation across equipment types are generated directly by labor-equipment comparative advantage or indirectly by labor-occupation and equipment-occupation comparative advantage. While in practice all sources of comparative advantage are active, it is useful to consider two extreme cases.

If the only form of comparative advantage is between workers and equipment, then an increase in \( q_t(\kappa) \) does not affect relative occupation prices. In this case, for any value of \( \rho \) relative wages are affected only directly through changes in equipment productivity.

On the other hand, if there is no comparative advantage between workers and equipment but there is comparative advantage between workers and occupations and between equipment and occupations, then an increase in \( q_t(\kappa) \) directly increases the relative wage of workers employed in \( \kappa \)-intensive occupations and indirectly, through transformed occupation prices, reduces the relative wage of workers employed in \( \kappa \)-intensive occupations. The relative strength of the direct and indirect channels depends on \( \rho \). The relative wage of workers employed in \( \kappa \)-intensive occupations rises—i.e. the direct effect dominates the indirect occupation price effect—if and only if \( \rho > 1 \). Intuitively, an increase in

\(^{21}\)Costinot and Vogel (2010) provide analytic results on the implications for relative wages of changes in labor composition, \( L_t(\lambda) \), and occupation demand, \( \mu_t(\omega) \), in an environment to which our model limits when there are no differences in efficiency units across workers in the same labor group (i.e. \( \theta = \infty \)), there is no capital equipment (i.e. \( \alpha = 0 \)), and when \( T(\lambda, \omega) \)—i.e. our \( T(\lambda, \kappa, \omega) \) in the absence of equipment—is log-supermodular.
$q_t (\kappa)$ acts like a positive productivity shock to the occupations in which $\kappa$ has a comparative advantage. If $\rho > 1$ this increases employment and the relative wages of labor groups disproportionately employed in the occupations in which $\kappa$ has a comparative advantage.

3 Parameterization

Our quantification of the impact of changes in the economic environment between any two periods $t_0$ and $t_1$ on relative wages using equations (7)-(9) depends on: (i) period $t_0$ measures of factor allocations, $\pi_{t_0} (\lambda, \kappa, \omega)$, average wages, $w_{t_0} (\lambda)$, labor composition, $L_{t_0} (\lambda)$, and labor payments by occupation, $\zeta_{t_0} (\omega)$; (ii) measures of relative shocks to labor composition, $\bar{L} (\lambda) / \bar{L} (\lambda_1)$, occupation shifters, $\bar{a} (\omega) / \bar{a} (\omega_1)$, transformed equipment productivity to the power $\theta$ (henceforth, in an abuse of terminology, we will refer to this as equipment productivity), $\bar{q} (\kappa) / \bar{q} (\kappa_1)$, and labor productivity, $\bar{T} (\lambda) / \bar{T} (\lambda_1)$; and (iii) estimates of the parameters $\alpha, \rho, \text{and } \theta$.

3.1 Data

We use data from the Combined CPS May, Outgoing Rotation Group (MORG CPS) and the October CPS Supplement (October Supplement) in 1984, 1989, 1993, 1997, and 2003. We restrict our sample by dropping workers who are younger than 17 years old, do not report positive paid hours worked, or are self employed. Here we briefly describe our use of these sources; we provide further details in Appendix A. After cleaning, the MORG CPS and October Supplement contain data for roughly 115,000 and 50,000 individuals, respectively, in each year.

We divide workers into 30 labor groups by gender, education (high school dropouts, high school graduates, some college completed, college completed, and graduate training), and age (17-30, 31-43, and 44 and older). We consider two types of equipment: computers and other equipment. We use thirty occupations—which we list, together with summary statistics, in Table 10 in Appendix A—. Although we do not use measures of occupation characteristics in our quantitative exercise, we discuss the relationship between occupation characteristics, constructed using O*NET following Acemoglu and Autor (2011) as detailed in Appendix A, and the growth of occupation labor income in Section 4.

We use the MORG CPS to construct total hours worked and average hourly wages by labor group by year.\textsuperscript{22} We use the October Supplement to construct the share of total

\textsuperscript{22}We measure wages using the MORG CPS rather than the March CPS because the March CPS does not
hours worked by labor group \( \lambda \) that is spent using equipment type \( \kappa \) in occupation \( \omega \) in year \( t \), denoted by \( \pi_t (\lambda, \kappa, \omega) \). In 1984, 1989, 1993, 1997, and 2003, the October Supplement asked respondents whether they “have direct or hands on use of computers at work,” “directly use a computer at work,” or “use a computer at/for his/her/your main job.” Using a computer at work refers only to “direct” or “hands on” use of a computer with typewriter like keyboards, whether a personal computer, laptop, mini computer, or mainframe. We construct \( \pi_t (\lambda, \kappa', \omega) \) as the hours worked in occupation \( \omega \) by \( \lambda \) workers who report that they use a computer \( \kappa' \) at work relative to the total hours worked by labor group \( \lambda \) in year \( t \). Similarly, we construct \( \pi_t (\lambda, \kappa'', \omega) \) as the hours worked in occupation \( \omega \) by \( \lambda \) workers who report that they do not use a computer at work (where \( \kappa'' = \text{other equipment} \)) relative to the total hours worked by labor group \( \lambda \) in year \( t \).23

Constructing factor allocations, \( \pi_t (\lambda, \kappa, \omega) \), as we do introduces four limitations. First, our view of computerization is narrow. Second, at the individual level our computer-use variable takes only two values: zero or one. Third, we are not using any information on the allocation of non-computer equipment. Finally, the computer use question was discontinued after 2003.24

**Factor allocation.** In Table 1 we showed that women and more educated workers use computers more intensively than men and less educated workers, respectively, by aggregating \( \pi_t (\lambda, \kappa, \omega) \) across \( \omega \) and \( \lambda \). To quantify the impact of shocks between \( t_0 \) and \( t_1 \), however, we require disaggregated measures of factor allocations. Here we identify a few key patterns in the \( \pi_t (\lambda, \kappa, \omega) \) data.

To determine the extent to which college educated workers (\( \lambda' \)) compared to workers with high school degrees in the same gender-age group (\( \lambda \)) use computers (\( \kappa' \)) relatively more than non-computer equipment (\( \kappa \)) within occupations (\( \omega \)), the left panel of Figure 2 directly measure hourly wages of workers paid by the hour and, therefore, introduces substantial measurement error in individual wages; see Lemieux (2006). Both datasets imply similar changes in average wages within a labor group.

23We observe \( \pi_t (\lambda, \kappa, \omega) = 0 \) in the data for roughly 27% of \( (\lambda, \kappa, \omega) \) observations in any given year. As a robustness check, in Appendix E.2 we drop age and consider 10 labor groups, reducing the share of zero allocations.

24The German *Qualification and Working Conditions* survey, used in e.g. DiNardo and Pischke (1997), helps mitigate the second and third concerns by providing data on worker usage of multiple types of equipment and, in 2006, the share of hours spent using computers. In Appendix C we show using this more detailed German data similar patterns of comparative advantage—between computers and education groups and between computers and gender—as in the U.S. data described below. We do not conduct our accounting exercise for Germany because wage data is noisy in publicly available datasets, see e.g. Dustmann et al. (2009), and because the German *Qualification and Working Conditions* survey contains many fewer observations than the October Supplement (depending on the year, between roughly 10,700 and 21,150 observations after cleaning).
plots the histogram of
\[
\log \frac{\pi_t(\lambda', \kappa', \omega)}{\pi_t(\lambda', \kappa, \omega)} - \log \frac{\pi_t(\lambda, \kappa', \omega)}{\pi_t(\lambda, \kappa, \omega)}
\]
across all five years, thirty occupations, and six gender-age groups. Clearly, college educated workers are relatively more likely to use computers within occupations compared to high school educated workers. A similar pattern holds comparing across other education groups.

The right panel of Figure 2 plots a similar histogram, where \(\lambda'\) and \(\lambda\) denote female and male workers, respectively, across all five years, thirty occupations, and fifteen education-age groups. This figure shows that on average there is no clear difference in computer usage across genders within occupations (i.e. the histogram is roughly centered around zero). Hence, in order to account for the fact that women use computers more than men at the aggregate level—see Table 1—women must have a comparative advantage in the occupations in which computers have a comparative advantage.

We can similarly study the extent to which labor groups differ in their allocations across occupations conditional on computer usage and the extent to which computers differ in their allocations across occupations conditional on labor groups. For instance, using similar histograms we can show that women are much more likely than men to work in administrative support relative to construction occupations, conditional on the type of equipment used; and that computers are much more likely to be used in administrative support than in construction occupations, conditional on labor group. These comparisons provide an example of a more general relationship: women tend to be employed in occupations in which all labor groups are relatively more likely to use computers.

### 3.2 Measuring shocks

Here we describe our baseline procedure to measure shocks to labor composition, \(\hat{L}(\lambda)/\hat{L}(\lambda_1)\), equipment productivity, \(\hat{q}(\kappa)^{\theta}/\hat{q}(\kappa_1)^{\theta}\), occupation shifters, \(\hat{\alpha}(\omega)/\hat{\alpha}(\omega_1)\), and labor pro-
ductivity, $\hat{T}(\lambda) / \hat{T}(\lambda_1)$. We measure changes in labor composition directly from the MORG CPS. We measure changes in equipment productivity using data only on changes in detailed allocations over time, $\hat{\pi}(\lambda, \kappa, \omega)$. We measure changes in occupation shifters using data on detailed allocations and labor income shares across occupations over time as well as model parameters. Finally, we measure changes in labor productivity as a residual to match changes in relative wages. Details are provided in Appendix B.1 and we provide variations of this baseline procedure in Appendices B.2 and B.3, each of which yields very similar results.

Consider first our measure of changes in equipment productivity. Equations (3) and (6) give

$$\frac{\hat{q}(\kappa)^\theta}{\hat{q}(\kappa_1)^\theta} = \frac{\hat{\pi}(\lambda, \kappa, \omega)}{\hat{\pi}(\lambda, \kappa_1, \omega)}$$

(10)

for any $(\lambda, \omega)$ pair. Hence, if computer productivity rises relative to other equipment between $t_0$ and $t_1$, then the share of $\lambda$ hours spent working with computers relative to other equipment in occupation $\omega$ will increase. It is important to condition on $(\lambda, \omega)$ pairs when identifying changes in equipment productivity because unconditional growth over time in computer usage, shown in Table 1, may also reflect growth in the supply of labor groups who have a comparative advantage using computers and/or changes in occupation shifters that are biased towards occupations in which computers have a comparative advantage. To construct changes in equipment productivity, we average the right-hand side of equation (10) over $(\lambda, \omega)$ pairs, as described in Appendix B.1.

Second, consider our measure of changes in occupation shifters. Equation (9) gives

$$\frac{\hat{a}(\omega)}{\hat{a}(\omega_1)} = \frac{\hat{\xi}(\omega)}{\hat{\xi}(\omega_1)} \left( \frac{\hat{q}(\omega)}{\hat{q}(\omega_1)} \right)^{(1-\alpha)(\beta-1)}$$

(11)

In order to construct occupation shifters, we construct the right-hand side of equation (11) as follows. We use equations (3) and (6) to obtain measures of changes in transformed occupation prices to the power $\theta$ (henceforth, in an abuse of terminology we will refer to these as occupation prices) between $t_0$ and $t_1$,

$$\frac{\hat{q}(\omega)^\theta}{\hat{q}(\omega_1)^\theta} = \frac{\hat{\pi}(\lambda, \kappa, \omega)}{\hat{\pi}(\lambda, \kappa_1, \omega)}$$

(12)

for any $(\lambda, \kappa)$ pair, which we average over $(\lambda, \kappa)$ pairs as described in Appendix B.1.\textsuperscript{25} Hence, if the price of $\omega$ rises relative to $\omega_1$ between $t_0$ and $t_1$, then the share of $\lambda$ hours

\textsuperscript{25}In calculating changes in relative wages in response to a subset of shocks, we solve for counterfactual changes in transformed occupation prices and allocations using equations (8) and (9).
spent working with $\kappa$ in occupation $\omega$ relative to in occupation $\omega_1$ will increase. As above, it is important to condition on $(\lambda, \kappa)$ when identifying changes in occupation prices. Given values of $\alpha$, $\rho$, and $\theta$, we recover $(\hat{q}(\omega)/\hat{q}(\omega_1))^{(1-\alpha)(\rho-1)}$. Next, given values of $\hat{q}(\kappa)\theta/\hat{q}(\kappa_1)\theta$ and $\hat{q}(\omega)\theta/\hat{q}(\omega_1)\theta$, we construct $\hat{\xi}(\omega)/\hat{\xi}(\omega_1)$ using the right-hand side of equation (9). Note that if $\rho = 1$, changes in occupation shifters depend only on changes in the share of labor payments across occupations.

Finally, we measure changes in labor productivity as a residual to match changes in relative wages, expressing equation (7) as

$$\frac{\hat{w}(\lambda)}{\hat{w}(\lambda_1)} = \frac{\hat{T}(\lambda)}{\hat{T}(\lambda_1)} \left( \frac{\hat{s}(\lambda)}{\hat{s}(\lambda_1)} \right)^{1/\theta}. \tag{13}$$

$s(\lambda)$ is a labor-group-specific weighted average of changes in transformed equipment and occupation prices, both raised to the power $\theta$,

$$s(\lambda) = \sum_{\kappa, \omega} \hat{q}(\omega)\theta \hat{q}(\kappa)\theta \hat{q}(\omega_1)\theta \hat{q}(\kappa_1)\theta \pi_{t_0}(\lambda, \kappa, \omega), \tag{14}$$

which we can construct using observed allocations and measured changes in equipment productivity and occupation prices. Clearly, we require a value of $\theta$ to measure changes in labor productivity. Note that changes in relative wages only directly affect our measures of changes in labor productivities. Wage changes have no effect on our measures of changes in labor composition or equipment productivity and they only affect our measures of occupation shifters indirectly through their impact on $\hat{\xi}(\omega)/\hat{\xi}(\omega_1)$.

### 3.3 Parameter estimates

Here we assign values to the parameters $\alpha$, $\theta$, and $\rho$. The parameter $\alpha$ determines payments to all equipment (computers and non-computer equipment) relative to the sum of payments to equipment and labor. We set $\alpha = 0.24$, consistent with estimates in Burstein et al. (2013).

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26 Because our measure of changes in equipment productivity are independent of worker wages, the DiNardo and Pischke (1997) critique of Krueger (1993) does not apply to our approach. More generally, DiNardo and Pischke (1997) argue that measuring the impact of computers on wages requires taking into account two issues: “(1) computers are only productive in conjunction with a specific set of skills (e.g., programming); and (2) computers are of value only in certain jobs (e.g., for empirical economists but not for ballet dancers).” Our framework is designed precisely to address these issues. By allowing for comparative advantage between computers and labor groups we address issue (1) and by allowing for comparative advantage between computers and occupations we address issue (2).
3.3.1 Estimating $\theta$

Equation (13) can be expressed as

$$\log \hat{\omega}(\lambda) = \xi_\theta(t) + \beta_\theta \log \hat{s}(\lambda) + \iota_\theta(\lambda).$$

We directly observe $\log \hat{\omega}(\lambda)$, $\xi_\theta(t) \equiv \log \hat{q}(\omega_1) \hat{q}(k_1)$ is a time effect that is common across $\lambda$, $\beta_\theta \equiv 1/\theta$ is the coefficient of interest, we construct $\log \hat{s}(\lambda)$ using our previous estimates, and $\iota_\theta(\lambda) \equiv \log \hat{T}(\lambda)$ captures unobserved changes in labor group $\lambda$ productivity (measuring changes in labor productivity requires a value of $\theta$ according to equation (13)).

Regressing $\log \hat{\omega}(\lambda)$ on $\log \hat{s}(\lambda)$ would generate biased estimates of $\beta_\theta$, and therefore $\theta$, because changes in equilibrium occupation prices, $\hat{q}(\omega)$, depend on changes in unobserved labor productivity, $\iota_\theta(\lambda)$. Specifically, we expect the error term $\iota_\theta(\lambda)$ and the covariate $\log \hat{s}(\lambda)$ to be negatively correlated: the higher the growth in the productivity of a particular labor group, the lower the growth in the price of those occupations that use that type of labor more intensively. Therefore, we expect the OLS estimate of $\beta_\theta$ to be biased downwards and the estimate of $\theta$ to be biased upwards. To address the endogeneity of the covariate $\log \hat{s}(\lambda)$, we construct the following instrument for $\log \hat{s}(\lambda)$,

$$\chi_\theta(\lambda) \equiv \log \sum_{\kappa} \frac{\hat{q}(\kappa)^\theta}{\hat{q}(k_1)^\theta} \sum_{\omega} \pi_{1984}(\lambda, \kappa, \omega),$$

which is a labor-group-specific average of the observed changes in equipment productivity $\hat{q}(\kappa)^\theta / \hat{q}(k_1)^\theta$. An increase in the relative productivity of $\kappa$ between $t_0$ and $t_1$ raises the wage of group $\lambda$ relatively more if a larger share of $\lambda$ workers use equipment $\kappa$ in period $t_0$. While in our model we have not imposed assumptions on the stochastic structure of the shocks, a sufficient condition for our estimator to be consistent is that, in any two periods $t_0$ and $t_1$, the change in unobserved labor productivity, $\log \hat{T}(\lambda)$, and the weighted changes in equipment productivity are uncorrelated across labor groups. In constructing the instrument for $\log \hat{s}(\lambda)$ between any two periods $t_0$ and $t_1$, we could have used the observed labor allocations at period $t_0$. However, in order to minimize the correlation between a possibly serially correlated $\iota_\theta(\lambda)$ and the instrument, we construct our instrument for $\log \hat{s}(\lambda)$ between any two sample periods $t_0$ and $t_1$ using allocations in the initial sample year, 1984.

We estimate $\theta$ (and $\rho$ below) using four time periods: 1984-1989, 1989-1993, 1993-1997, and 1997-2003. The resulting IV estimate is $\beta_\theta = 0.56$, with a robust standard error of 0.08. The F-stat in the first stage is 157 and the coefficient on the instrument has the expected
positive sign; see Table 2 for details. This estimate implies $\theta = 1.78$.\footnote{If we had estimated $\beta_\theta$ using an OLS estimator then we would have obtained an estimate of $\beta_\theta = 0.38$, which would imply $\theta = 2.61$. The fact that the OLS estimate of $\beta_\theta$ is lower than its IV estimate is consistent with the prediction of our model that the error term $\iota_\theta(\lambda)$ should be negatively correlated with the covariate $\log \hat{s}(\lambda)$.}

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<th>Estimate</th>
<th>(SE)</th>
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<th>First stage (F-stat)</th>
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</table>

Table 2: Parameter estimates
Notes: Estimate refers to $\beta_\theta$ in the first four rows and $\beta_\rho$ in the last four rows. We report Huber-White standard errors (SE). First stage reports the coefficient on the instrument in the first stage regression and the final column reports the Kleibergen-Paap Wald rk F statistic (F-stat).

**Alternative estimations of $\theta$.** Here we show that our baseline estimate of $\theta$ is robust to a number of alternative estimation approaches. We use these alternative estimates in our sensitivity analyses in Section 5.1.

First, Acemoglu (2002b) raises a concern that the presence of common trends in unobserved labor-group-specific productivity, explanatory variables, and instruments may bias the estimates of wage elasticities. Here we show that our estimate of $\theta$ is robust accounting for labor-group-specific time trends in equation (15). Specifically, we express $\hat{T}(\lambda)$ as following a $\lambda$-specific time trend with deviations around this trend, $\log \hat{T}(\lambda) = \beta_\theta(\lambda) \times (t_1 - t_0) + \iota_\theta(\lambda)$, which is the same assumption imposed in Katz and Murphy (1992) and in the estimation of the canonical model more generally. We then estimate

$$\log \hat{\varphi}(\lambda) = \zeta_\theta(t) + \beta_\theta(\lambda) \times (t_1 - t_0) + \beta_\theta \log \hat{s}(\lambda) + \iota_\theta(\lambda)$$

instead of equation (15) using the same instrumental variable for $\log \hat{s}(\lambda)$ as in our baseline approach. Here we identify our parameter of interest, $\beta_\theta$, using variation in deviations from trend in wage changes and predicted changes in labor-group-specific averages of the product of equipment productivity and occupation prices, $\log \hat{s}(\lambda)$, instrumented by the corresponding variation in deviations from trend in the variable $\chi_\theta(\lambda)$. A sufficient condition for our estimator to be consistent is that, conditional on a linear time
trend, in any two periods $t_0$ and $t_1$, the change in unobserved labor productivity and the weighted changes in equipment productivity are uncorrelated across labor groups. This approach implies $\theta = 1.13$, as reported in Table 2.

Second, whereas it is (slightly) simpler to take our framework to the data using time differences, in Appendix B.4 we show how to estimate $\theta$ using a version of equation (15) in levels and expanding the right-hand side variables with $\lambda$-specific fixed effects. This approach implies $\theta = 1.48$, as reported in Table 2. Third, we can combine both of the previous approaches, including a labor-group-specific time trend and using levels with $\lambda$-specific fixed effects. This approach implies $\theta = 2.04$, as reported in Table 2.

Fourth, according to our theory, labor composition only affects wages indirectly through occupation prices. We test this prediction by including changes in labor supply as an additional explanatory variable in equation (15). This yields an estimate of $\beta_\theta = 0.54$ that is not statistically different from our baseline; this estimate implies $\theta = 1.84$. Moreover, we cannot reject at the 10% significance level the null hypothesis that the effect of changes in labor supply on changes in wages, conditional on the composite term $\log \hat{s}(\lambda)$, is equal to zero.

Finally, in Appendix D we take an alternative approach—based on Lagakos and Waugh (2013) and Hsieh et al. (2013)—in which we use the empirical distribution of wages within each $\lambda$ to estimate $\theta$ using maximum likelihood. This approach yields $\theta = 2.62$.28

3.3.2 Estimating $\rho$

Equation (9) can be expressed as

$$\log \hat{z}(\omega) = \zeta_\rho(t) + \beta_\rho \log \frac{\hat{q}(\omega)^\theta}{\hat{q}(\omega_1)^\theta} + \eta_\rho(\omega).$$

We directly observe $\log \hat{z}(\omega)$ in the MORG CPS, $\zeta_\rho(t) \equiv \log \hat{E} + (1 - \alpha)(1 - \rho) \log \hat{q}(\omega_1)$ is a time effect that is common across $\omega$, $\beta_\rho \equiv (1 - \alpha)(1 - \rho)/\theta$ is the coefficient of interest, we measure $\log \hat{q}(\omega)^\theta/\hat{q}(\omega_1)^\theta$ following the procedure indicated in Section 3.2, and $\eta_\rho(\omega) \equiv \log \hat{\alpha}(\omega)$ captures unobserved changes in occupation shifters (measuring changes in occupation shifters requires a value of $\rho$ according to equation (11)).

According to our model, regressing $\log \hat{z}(\omega)$ on $\log \hat{q}(\omega)^\theta/\hat{q}(\omega_1)^\theta$ would generate biased estimates of $\beta_\rho$, and therefore $\rho$ because changes in equilibrium occupation prices, $\hat{q}(\omega)$, depend on changes in unobserved occupation shifters, $\eta_\rho(\omega)$. Specifically, we expect the error term $\eta_\rho(\omega)$ and the covariate $\log \hat{q}(\omega)^\theta/\hat{q}(\omega_1)^\theta$ to be positively correlated:

28In Appendix D we also conduct our analysis allowing for $\theta$ to vary across $\lambda$. 

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the higher the growth in the occupation shifter of a particular occupation, the higher the
growth in the occupation prices. Therefore, we expect the OLS estimate of $\beta_p$ to be biased
upwards. To address the endogeneity of the covariate $\log \hat{q}(\omega)^\theta / \hat{q}(\omega_1)^\theta$, we construct
the following Bartik-style instrument for $\log \hat{q}(\omega)^\theta / \hat{q}(\omega_1)^\theta$,

$$
\chi_p(\omega) \equiv \log \sum_\kappa \hat{q}(\kappa)^\theta \sum_\lambda L_{1984}(\lambda) \pi_{1984}(\lambda, \kappa, \omega) \sum_{\lambda', \kappa'} L_{1984}(\lambda') \pi_{1984}(\lambda', \kappa', \omega),
$$

which is an occupation-specific average of the observed changes in equipment productivity, $\hat{q}(\kappa)^\theta / \hat{q}(\kappa_1)^\theta$. An increase in the relative productivity of $\kappa$ raises occupation $\omega$’s output—and, therefore, reduces its price—relatively more if a larger share of workers
employed in occupation $\omega$ use equipment $\kappa$ in period $t_0$. While in our model we have
not imposed assumptions on the stochastic structure of the shocks, a sufficient condition
for our estimator to be consistent is that the change in the (unobserved) occupation
shifter and the weighted changes in equipment productivity are uncorrelated across occupations.\(^{29}\) In order to minimize the correlation between a possibly serially correlated
$i_r(\omega)$ and the instrument, we construct our instrument using allocations in 1984, where
$L_{1984}(\lambda) \pi_{1984}(\lambda, \kappa, \omega)$ is the number of $\lambda$ workers using equipment $\kappa$ employed in occupation $\omega$ in 1984 whereas the denominator is total employment in $\omega$ in 1984.

The resulting IV estimate is $\beta_p = -0.33$, with a robust standard error of 0.11. Together
with our baseline values of $\alpha$ and $\theta$, this implies $\rho = 1.78$, as reported in Table 2. The F-stat in the first stage is 45 and the coefficient on the instrument has the expected negative sign.\(^{30}\)

**Alternative estimations of $\rho$.** Here we show that our baseline estimate of $\rho$ is robust to
a number of alternative approaches to the estimation of $\beta_p$. We use these alternative estimates in our sensitivity analyses in Section 5.1. In each case we use our baseline estimates
of $\alpha$ and $\theta$ to construct $\rho$ from our estimate of $\beta_p$.

We obtain similar estimates of $\rho$ if we incorporate an occupation-specific time trend in
equation (16). In this case, we implicitly decompose $\hat{a}(\omega)$ into an $\omega$-specific time trend
and deviations around this trend, $\log \hat{a}(\omega) = \beta_p(\omega) \times (t_1 - t_0) + \iota_{p1}(\omega)$. We then esti-

\(^{29}\)Theoretically, if the U.S. specializes in traded sectors that employ a large share of computer-intensive occupations, then a rise in trade between 1984 and 2003 might generate a demand shift towards computer-intensive occupations within traded sectors, inducing a potential correlation between occupation shifters and weighted changes in equipment productivity. In practice, however, we find that computer-intensive occupations do not grow faster relative to non-computer-intensive occupations in manufacturing than non-manufacturing, suggesting that this theoretical concern is not a problem in practice.

\(^{30}\)If we had estimated $\beta_p$ using an OLS estimator then we would have obtained an estimate of $\beta_p = 0.23$, which would imply $\rho = 0.46$. The fact that the OLS estimate of $\beta_p$ is higher than its IV estimate is consistent with the prediction of our model that the error term $i_r(\omega)$ should be negatively correlated with the covariate $\log \hat{q}(\omega)^\theta / \hat{q}(\omega_1)^\theta$. 

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mate
\[ \log \hat{\xi}(\omega) = \xi \rho(t) + \beta \rho(\omega)(t_1 - t_0) + \beta \log \frac{\hat{q}(\omega)}{\hat{q}(\omega_1)} + \tau(\omega), \]

instead of equation (16) using the same instrument as in our baseline approach. A sufficient condition for this estimator to be consistent is that, for any interval \(t_0\) and \(t_1\), the deviations from linear time trends in the changes in the unobserved occupation shifters and the weighted changes of equipment productivity are uncorrelated across occupations. This approach implies \(\rho = 2.59\), as reported in Table 2.

In Appendix B.4 we show how to estimate \(\rho\) using levels with \(\omega\)-specific fixed effects rather than time differences. This approach implies \(\rho = 2.9\) whether or not we incorporate occupation-specific time trends, although in this case the instrument is weak when we include occupation-specific time trends, as reported in Table 2.

4 Results

In this section we summarize our baseline results, quantifying the implications for relative wages of changes in labor composition, occupation shifters, equipment productivity, and labor productivity. We construct various measures of changes in between-group inequality that aggregate wage changes across our thirty labor groups (e.g., the skill premium). As is standard, when doing so, both in the model and in the data, we construct composition-adjusted wage changes; that is, for each aggregated measure we average wage changes across the corresponding labor groups using constant weights over time, as described in detail in Appendix A. For each measure of inequality we report its cumulative log change between 1984 and 2003, calculated as the sum of the log change over all sub-periods in our data.\(^{31}\) We also report the log change over each sub-period in our data.

Skill premium. We begin by decomposing changes in the skill premium over the full sample period and over each sub-period, displayed in Table 3. The first column reports the change in the data, which is also the change predicted by the model when all changes—in labor composition, occupation shifters, equipment productivity, and labor productivity—are simultaneously considered. Between 1984 and 2003 the skill premium increased by 15.1 log points, with the largest increases occurring between 1984 and 1993. The subsequent four columns summarize the counterfactual change in the skill premium predicted by the model if only one component is changed (i.e. holding the other components at their \(t_0\) level).

\(^{31}\)For cumulative changes in log relative wages we obtain very similar results if we set \(t_0 = 1984\) and \(t_1 = 2003\) instead of adding changes in log relative wages over all sub-periods.
Table 3: Decomposing changes in the log skill premium

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1984 - 1989</td>
<td>0.057</td>
<td>-0.031</td>
<td>0.026</td>
<td>0.052</td>
</tr>
<tr>
<td>1989 - 1993</td>
<td>0.064</td>
<td>-0.017</td>
<td>-0.009</td>
<td>0.045</td>
</tr>
<tr>
<td>1993 - 1997</td>
<td>0.037</td>
<td>-0.023</td>
<td>0.044</td>
<td>0.021</td>
</tr>
<tr>
<td>1997 - 2003</td>
<td>-0.007</td>
<td>-0.043</td>
<td>-0.011</td>
<td>0.042</td>
</tr>
<tr>
<td>1984 - 2003</td>
<td>0.151</td>
<td>-0.114</td>
<td>0.049</td>
<td>0.159</td>
</tr>
</tbody>
</table>

Changes in labor composition decrease the skill premium over each sub-period in response to the large increase in the share of hours of more educated workers. The increase in hours worked by those with college degrees relative to those without of 47.4 log points between 1984 and 2003 decreases the skill premium by 11.4 log points. Changes in relative demand across labor groups must, therefore, compensate for the impact of changes in labor composition in order to generate the observed rise of the skill premium in the data.

Changes in equipment productivity, i.e. computerization, account for roughly 60% of the sum of the demand-side forces pushing the skill premium upwards: \(0.60 \approx \frac{0.159}{(0.049 + 0.159 + 0.056)}\). Over sub-periods, changes in equipment productivity are particularly important in generating increases in the skill premium over the years in which the skill premium rose most dramatically: 1984-1989 and 1989-1993. These are precisely the years in which the overall share of workers using computers rose most rapidly; see Table 1. We measure large growth in computer productivity, consistent with ample direct evidence showing a rapid decline in the price of computers relative to all other equipment types and structures, which we do not directly use in our estimation procedure.\(^{32}\)

Computerization raises the skill premium for two reasons, as described in detail in Section 2.4. First, educated workers have a direct comparative advantage using computers within occupations, as shown in Figure 2. Second, educated workers have a comparative advantage in occupations in which computers have a comparative advantage and \(\rho > 1\).

Changes in occupation shifters account for roughly 19% of the sum of the forces push-
ing the skill premium upwards over the full sample. This result is intuitively related to the expansion of education-intensive occupations—including, for example, executive, administrative, managerial as well as health assessment and treating—in the data, as documented in Figure 1 as well as in Table 10 in Appendix A.

As discussed in, e.g., Autor et al. (2003), these changes have been systematically related to occupation characteristics; for example, there has been an expansion of occupations intensive in non-routine cognitive analytical, non-routine cognitive interpersonal, and socially perceptive tasks and a corresponding contraction in occupations intensive in routine manual and non-routine manual physical tasks, as defined using O*NET constructed task measures following Acemoglu and Autor (2011) as described in Appendix E.1. In our model, occupation shifters generate the largest movements in income shares across occupations, and these movements have been biased systematically towards occupations intensive in certain characteristics, e.g. social perceptiveness, as we show in Appendix E.1. However, with \( \rho \neq 1 \) all shocks affect income shares across occupations. We find that computerization and labor composition play significant roles in explaining the observed relationship between occupation characteristics and growth. For example, they both increase the size of occupations intensive in non-routine cognitive analytical and non-routine cognitive interpersonal tasks as well as decrease the size of occupations intensive in routine manual and non-routine manual physical tasks, as we show in Appendix E.1.

Finally, labor productivity, the residual to match observed changes in relative wages, accounts for roughly 21% of the sum of the effects of the three demand-side mechanisms. **Gender gap.** The average wage of men relative to women, the gender gap, declined by 13.3 log points between 1984 and 2003. Table 4 decomposes changes in the gender gap over the full sample and over each sub-period. The increase in hours worked by women relative to men of roughly 12.6 log points between 1984 and 2003 increased the gender gap by 4.2 log points. Changes in relative demand across labor groups must, therefore, compensate for the impact of changes in labor composition in order to generate the observed fall of the gender gap in the data.

Changes in equipment productivity, i.e. computerization, account for roughly 27% of the sum of the forces decreasing the gender gap over the full sample in spite of the fact that women do not have a comparative advantage using computers. This results from the finding that women have a comparative advantage in the occupations in which computers have a comparative advantage, which together with \( \rho > 1 \) implies that computerization raises the wages of labor groups disproportionately employed in computer-intensive occupations.
Table 4: Decomposing changes in the log gender gap

Changes in occupation shifters account for roughly 38% of the sum of the forces decreasing the gender gap over the full sample. This is driven in part by the fact that a number of male-intensive occupations—including, for example, mechanics/repairers as well as machine operators/assemblers/inspectors—contracted substantially between 1984 and 2003; see Table 10 in the Appendix for details.

Unlike the skill premium, changes in labor productivity account for a sizable share, roughly 35%, of the impact of the demand-side forces affecting the gender gap and plays a central role in each sub-period except for 1997-2003. This suggests that factors such as changes in gender discrimination—if they affect labor productivity irrespective of \((\kappa, \omega)\)—may have played a substantial role in reducing the gender gap, especially early in our sample (in the 1980s and early 1990s); see e.g. Hsieh et al. (2013).

**Five education groups.** Table 5 decomposes changes in between-education-group wage inequality at a higher level of worker disaggregation, comparing changes in average wages across the five education groups considered in our analysis over the full sample, 1984-2003. The 15.1 log point change in the skill premium aggregates across heterogeneous changes in relative wages between more disaggregated education groups.

Table 5: Decomposing changes in log relative wages across education groups between 1984 and 2003

The results reported in Table 3 are robust: computerization is the central force driving changes in between-education group inequality whereas labor productivity plays a relatively minor role.
Thirty disaggregated labor groups. One of the advantages of our framework is that we can solve for wage changes across a large number of labor groups. Whereas above we quantify the impact of shocks on measures of between-group inequality that aggregate across a number of labor groups, here we show the model’s implications for a subset of shocks across all of our disaggregated labor groups. For all labor groups we construct changes in wages over the full sample predicted by changes in each of the three demand-side shocks—occupation shifters, equipment productivity, and labor productivity—both separately and together relative to a weighted average across all labor groups; i.e. we calculate the log of the change between 1984 and 2003 in the predicted wage relative to the average change across all groups for each shock. We plot these in Figure 3, ordering the thirty labor groups by the change in the wage predicted by the combination of all three shocks. As can be seen in Figure 3, changes in wages predicted by changes in equipment productivity are most closely related to changes in wages predicted by all three shocks together. More formally, the average (across labor groups) squared difference between the wage change predicted by our model when we simultaneously allow for changes in equipment productivity, occupation shifters and labor productivity and that predicted by our model when we only allow for each of these three shocks one at a time is minimized when we use the wages predicted by changes in equipment productivity.

Figure 3: Changes in log relative wages predicted separately and together by changes in equipment productivity, occupation shifters, and labor productivity between 1984-2003
5 Robustness and sensitivity analyses

In this section we consider three types of sensitivity exercises. First, we perform sensitivity to different values of \( \rho \) and \( \theta \). Second, we illustrate the importance of integrating all three forms of comparative advantage by eliminating some of them from the model. Finally, we allow for changes in comparative advantage over time.

5.1 Alternative parameter values

We first consider the sensitivity of our results for the skill premium and gender gap over the period 1984-2003 by varying \( \theta \) and \( \rho \) using alternative values reported in Table 2. For each alternative approach to estimating \( \theta \) or \( \rho \) we fully re-parameterize the model. Recall that whereas our estimated value of \( \theta \) is independent of our approach to estimate \( \rho \), our estimated value of \( \rho \) does depend on our approach to estimate \( \theta \). Hence, when varying \( \theta \) we also re-calculate \( \rho \) using our baseline estimation approach.

Alternative values of \( \theta \). In the left and right panels of Table 6 we decompose changes in the skill premium and gender gap, respectively, between 1984 and 2003 using the highest and lowest alternative values of \( \theta \) reported in Table 2. The first row reports our lowest value of \( \theta = 1.13 \), estimated using time differences and including a time trend. The second row reports our baseline value of \( \theta = 1.78 \), estimated using time differences but without time trend. The final row reports our highest value of \( \theta = 1.90 \), estimated using levels and including a time trend.

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>Skill premium</th>
<th>Gender gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.13</td>
<td>-0.159</td>
<td>0.066</td>
</tr>
<tr>
<td>1.78</td>
<td>-0.114</td>
<td>0.049</td>
</tr>
<tr>
<td>1.90</td>
<td>-0.109</td>
<td>0.047</td>
</tr>
</tbody>
</table>

Table 6: Decomposing changes in the log skill premium and gender gap between 1984 and 2003 for alternative values of \( \theta \).

A higher value of \( \theta \) implies a larger role for changes in labor productivity, and a smaller role for the other shocks in accounting for changes in the skill premium and the gender gap. The intuition is straightforward. According to equation (13), the elasticity of changes in average wages of workers in labor group \( \lambda \), \( \hat{\omega} (\lambda) \), to changes in the labor-group-specific average of equipment productivities and occupation prices, \( \hat{s} (\lambda) \), is \( 1/\theta \). Because our measure of \( \hat{s} (\lambda) \) is independent of \( \theta \), a higher value of \( \theta \) reduces the impact
on wages of changes in the labor-group-specific average of equipment productivities and 
occupation prices and, therefore, increases the impact of changes in labor productivity, 
identified as a residual to match observed changes in average wages.

Our main results are robust for all alternative values of \( \theta \) that we estimate. First, 
computerization is the most important force accounting for the rise in between-education-
group inequality between 1984 and 2003. Alone, it accounts for between roughly 59% 
and 72% of the demand-side forces raising the skill premium. Second, the role of the 
residual in accounting for the rise in between-education-group inequality is relatively 
small, between roughly 7% and 23% of the demand-side forces raising the skill premium. 
Third, changes in each of the demand-side forces play an important role in accounting 
for the reduction in the gender gap between 1984 and 2003. Specifically, changes in labor 
productivity play a larger role in accounting for the reduction in the gender gap than the 
rise in the skill premium. Alone, they account for between roughly 16% and 37% of the 
demand-side forces reducing the gender gap.

**Alternative values of \( \rho \).** In the left and right panels of Table 7 we decompose observed 
changes in the skill premium and gender gap, respectively, between 1984 and 2003 using 
the highest alternative value of \( \rho \) reported in Table 2; our baseline value is the lowest of 
our estimated values. The first row reports our baseline value of \( \rho = 1.78 \). The second 
row reports the value of \( \rho = 2.9 \), estimated using differences either including a time trend 
or not.

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>1.78</td>
<td>-0.114</td>
<td>0.049</td>
<td>0.159</td>
<td>0.056</td>
<td>0.042</td>
<td>-0.067</td>
<td>-0.047</td>
<td>-0.061</td>
</tr>
<tr>
<td>2.90</td>
<td>-0.080</td>
<td>-0.029</td>
<td>0.199</td>
<td>0.060</td>
<td>0.030</td>
<td>-0.030</td>
<td>-0.071</td>
<td>-0.063</td>
</tr>
</tbody>
</table>

Table 7: Decomposing changes in the log skill premium and gender gap between 1984 
and 2003 for alternative values of \( \rho \).

The value of \( \rho \) may potentially affect the contribution of each shock to relative wages 
through two channels: by affecting the measured shock itself and by affecting the elastic-
ty of occupation prices to these measured shocks. As shown in Section 3.2, \( \rho \) does 
not affect our measurement of either the labor composition or equipment productivity 
shock; hence, \( \rho \) affects the importance of these shocks for relative wages only through 
the elasticity of occupation prices. Because labor composition only affects relative wages 
through occupation prices, a higher value of \( \rho \) must reduce the impact of labor compo-
sition on relative wages. As described in Section 2.4, computerization has two effects.
First, it raises the relative wages of labor groups that disproportionately use computers. Second, by lowering the prices of occupations in which computers are disproportionately used, it lowers the wages of labor groups that are disproportionately employed in these occupations. A higher value of $\rho$ mitigates the second effect and, therefore, strengthens the impact of computerization on the skill premium and gender gap.

On the other hand, the value of $\rho$ impacts occupation shifters both through the magnitude of the measured shocks, see equation (11), and through the elasticity of occupation prices to these measured shocks. In practice, a higher value of $\rho$ yields measured occupation shifters that are less biased towards educated workers; in fact, occupation shifters reduce the skill premium for sufficiently high values of $\rho$. In practice, a higher value of $\rho$ tends to reduce the effect of occupation shifters on the gender gap by reducing the elasticity of occupation prices to shocks.

Our main results are robust for all alternative values of $\rho$ that we estimate. First, whereas computerization accounts for roughly 60% (27%) of the impact of the demand side forces on the skill premium (gender gap) in our baseline, it accounts for 87% (43%) under our alternative value of $\rho$. Only when $\rho \leq 1.2$ are occupation shifters more important for the skill premium than computerization. Second, the role of labor productivity is stable as we vary $\rho$. Whereas it accounts for roughly 21% (35%) of the impact of the demand side forces on the skill premium (gender gap) in our baseline, it accounts for 26% (38%) under our alternative values of $\rho$.

### 5.2 Sources of comparative advantage

To demonstrate the importance of including each of the three forms of comparative advantage, we perform two exercises. We first assume there is no comparative advantage related to occupations and then we redo the decomposition under the assumption that there is no comparative advantage related to equipment. In all cases, we hold the values of $\alpha$, $\rho$, and $\theta$ fixed.

Table 8 reports our baseline decomposition between 1984-2003 both for the skill premium (in the left panel) and the gender gap (in the right panel) as well as decompositions under the restriction that there is comparative advantage only between labor and equipment or only between labor and occupations.

Abstracting from any comparative advantage at the level of occupations (i.e. assuming away worker-occupation and equipment-occupation comparative advantage) has two effects. First—because changes in labor composition and occupation shifters affect relative wages only through occupation prices—it implies that the labor composition and occu-
Table 8: Decomposing changes in the log skill premium and log gender gap between 1984 and 2003 under different assumptions on the evolution of comparative advantage

<table>
<thead>
<tr>
<th></th>
<th>Skill premium</th>
<th>Gender gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>-0.114</td>
<td>0.049</td>
</tr>
<tr>
<td>Only labor-equip. CA</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Only labor-occ. CA</td>
<td>-0.114</td>
<td>0.116</td>
</tr>
</tbody>
</table>

Table 8 shows that if we were to abstract from any comparative advantage at the level of occupations, we would incorrectly conclude that all of the rise in the skill premium has been driven by changes in relative equipment productivities. Similarly, because we would infer that women have a strong comparative advantage with computers, we would incorrectly conclude that changes in equipment productivity account for almost all of the fall in the gender gap.

Similarly, assuming there is no comparative advantage at the level of equipment implies that the equipment productivity component of our decomposition is zero and that the only force giving rise to the allocation of labor groups to occupations is worker-occupation comparative advantage. Table 8 shows that abstracting from any comparative advantage at the level of equipment magnifies the importance of labor productivity in explaining the rise of the skill premium and the fall in the gender gap. The impact of occupation shifters on the gender gap does not change significantly, suggesting that the results in Hsieh et al. (2013) are robust to the inclusion of equipment.

In summary, abstracting from comparative advantage at the level of either occupations or equipment has a large impact on the decomposition of changes in between-group inequality. It does so by forcing changes in labor productivity to absorb the impact of the missing component(s) and by changing the inferred importance of the remaining source of comparative advantage.
5.3 Evolving comparative advantage

In our baseline model we imposed that the only time-varying components of productivity are multiplicatively separable between labor, equipment, and occupation components. In practice, over time some labor groups may have become relatively more productive in some occupations or using some types of equipment, perhaps caused by differential changes in discrimination of labor groups across occupations, by changes in occupation characteristics that differentially affect labor groups (e.g. job flexibility, which women may value relatively more, see e.g. Goldin 2014), or by changes in the characteristics of equipment.

In the most general case, we could allow \( T_t(\lambda, \kappa, \omega) \) to vary freely over time in which case we would match \( \hat{p}(\lambda, \kappa, \omega) \) exactly in each time period. The impact of labor composition would be exactly the same as in our baseline. However, we would only be able to report the joint effects of the combination of all \( \lambda-, \kappa-, \) and \( \omega-\)specific shocks on relative wages. Instead, here we generalize our baseline model to incorporate changes over time in comparative advantage in a restricted manner.

Specifically, we consider separately three extensions of our baseline model:

\[
T_t(\lambda, \kappa, \omega) = \begin{cases} 
T_t(\lambda) T_t(\lambda, \omega) T(\lambda, \kappa, \omega) & \text{case 1} \\
T_t(\omega) T_t(\lambda, \kappa) T(\lambda, \kappa, \omega) & \text{case 2} \\
T_t(\lambda) T_t(\kappa, \omega) T(\lambda, \kappa, \omega) & \text{case 3}
\end{cases}
\]

We allow for changes over time in comparative advantage between workers and occupations in case 1, workers and equipment in case 2, and equipment and occupations in case 3. In Appendix G we show how to measure the relevant shocks and how to decompose changes in between-group inequality into labor composition, occupation shifter, and labor-equipment components in case 2. Details for cases 1 and 3 are similar. Table 9 reports our results from decomposing changes in the skill premium between 1984 and 2003 in our baseline exercise as well as in cases 1, 2, and 3. In all cases, we hold the values of \( a, \rho, \) and \( \theta \) fixed.

The intuition for why our results are largely unchanged is straightforward in cases 1 and 2. In each case our measures of initial factor allocations and changes in labor composition as well as the system of equations that determines the impact of changes in labor composition on relative wages (i.e., abstracting from all other shocks) are exactly the same as in our baseline model. Hence, the labor composition component of our baseline decomposition is unchanged if we incorporate time-varying comparative advantage.
Similarly, in case 1 our measure of changes in equipment productivity as well as the system of equations that determines their impact are exactly the same as in our baseline model. Hence, the equipment productivity component is unchanged from the baseline in case 1. In case 2, whereas our measure of changes in transformed occupation prices is exactly the same as in our baseline model, our measure of changes in occupation labor payment shares—and, therefore, our measure of occupation shifters—differs slightly from our baseline, since predicted allocations in period $t_1$ differ slightly. However, since these differences aren’t large and since the system of equations determining the impact of occupation shifters is the same, our results on occupation shifters are very similar to those in the baseline in case 2. Finally, since (when fed in one at a time) the sum of all four components of our decomposition in the baseline model match the change in relative wages in the data well and the sum of all three components of our decomposition in the extensions considered here match the data reasonably well (in each case they match wage changes perfectly when fed in together), the change in wages resulting from the sum of the labor productivity and occupation productivity components in our baseline (when fed in one at a time) must closely match the change in wages from the labor-occupation component in case 1; similarly, the sum of the labor productivity and equipment productivity components in our baseline must closely match the labor-equipment component in case 2.

### Table 9: Decomposing changes in the log skill premium between 1984 and 2003 allowing comparative advantage to evolve over time

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>None (baseline)</td>
<td>-0.114</td>
<td>0.049</td>
<td>0.159</td>
<td>0.056</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Worker-occ. (case 1)</td>
<td>-0.114</td>
<td>-</td>
<td>0.159</td>
<td>-</td>
<td>0.069</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Worker-equip. (case 2)</td>
<td>-0.114</td>
<td>0.046</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.223</td>
<td>-</td>
</tr>
<tr>
<td>Equip.-occ. (case 3)</td>
<td>-0.114</td>
<td>-</td>
<td>-</td>
<td>0.013</td>
<td>-</td>
<td>-</td>
<td>0.251</td>
</tr>
</tbody>
</table>

6 International trade

Our exercise is intended to shed light on classes of mechanisms through which changes in the economic environment lead to changes in between-group inequality. Here we extend our model to incorporate international trade in equipment and sector output as well as occupation offshoring and show theoretically how the degree of openness is reflected in what we had treated—in our closed-economy model—as exogenous primitive shocks to
(i) the cost of producing equipment, (ii) sector shifters, and (iii) occupation shifters.33 As a preliminary step, we first introduce sectors in the closed-economy model and show that we can decompose the effects of changes in occupation shifters into changes in (i) sector shifters and (ii) within-sector occupation shifters. Here we briefly sketch the main elements of the extended models. Details are provided in Appendices H and I.

**Sectors in a closed economy.** Whereas in our baseline model the final good was produced using a CES combination of occupational output, here we assume that the final good is produced using a CES combination of sectoral output with elasticity $\rho_s$ and that sectors are produced using a CES combination of occupational output with elasticity $\rho$. Occupations are produced exactly as in our baseline specification: a worker’s productivity depends only on her occupation $\omega$, and not on her sector of employment, $s$.34

In Appendix H we show how we can use this extension to further decompose the effects of changes in occupation shifters into changes in (i) sector shifters, $a_t(\sigma)$, and (ii) within-sector occupation shifters, $a_t(\omega,\sigma)$. The system of equations to solve for changes in wages is very similar to equations (7)-(9). All shocks excluding sector shifters and within-sector occupation shifters are measured as in our baseline model. Moreover, given a value of the elasticity of substitution across sectors, $\rho_s$, constructing sector shifters and within-sector occupation shifters is straightforward using readily available data on the share of labor payments across sectors and across occupations within sectors.

Finally, in the special case in which the elasticity of substitution across occupations equals the elasticity of substitution across sectors, $\rho = \rho_s$, the model that incorporates sectors is equivalent to our baseline model where occupation shifters in our baseline model, $a_t(\omega)$, are equal to a simple combination of sector shifters and within-sector occupation shifters in our extended model, $a_t(\omega) \equiv \sum_{\sigma} a_t(\omega,\sigma) a_t(\sigma)$. Therefore, the quantitative results from our decomposition in the baseline and extended models are exactly the same in this case.

**International trade in occupations, sectors, and equipment.** Whereas in our baseline model we assumed a closed economy in which output equaled absorption within each

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33See Burstein et al. (2013) and Parro (2013) for quantitative analyses of the impact of trade in capital equipment on the skill premium and Stokey (1996) for a theoretical treatment of trade in skill complementary capital. See e.g. Grossman and Rossi-Hansberg (2008) for a theoretical analysis of occupation trade and inequality and Feenstra and Hanson (1999) for an empirical treatment of offshoring and relative wages. See Galle et al. (2015) for an analysis of the impact of sectoral trade on between-group inequality. Our framework, which includes multiple types of equipment and comparative advantage between labor groups and equipment types rationalizes the findings in Caselli and Wilson (2004), that countries with different distributions of education import different mixes of equipment.

34It is straightforward to assume, alternatively, that worker productivity depends both on occupation and sector of employment.
market, here we assume a global economy in which countries trade occupational, sectoral, and equipment output subject to iceberg costs. Each country’s absorption of occupation, sector, and equipment goods is itself a CES aggregate of these goods sourced from all countries in the world. In Appendix I we provide the details of this extension and show that the system of equations for quantifying the impact on relative wages if a given country were to move to autarky at time $t_0$ is the same as the system in our closed-economy model, where shocks that we had treated as exogenous in our closed-economy model are now simple functions of import and export shares at time $t_0$.

The mapping between import and export shares at time $t_0$ and the corresponding closed-economy shocks is intuitive. First, if the import share of equipment type $\kappa$ is relatively high and trade elasticities are common across equipment goods, then moving to autarky is equivalent to decreasing equipment $\kappa$ productivity in the closed economy. Second, if sector $\sigma$ has a relatively low export share and/or a high import share relative to sector $\sigma'$ then, under mild parametric restrictions, moving to autarky is equivalent to increasing the sector shifter for $\sigma$ relative to $\sigma'$ in the closed economy. Finally, if occupation $\omega$ has a relatively low export share and/or a high import share relative to occupation $\omega'$ then, under mild parametric restrictions, moving to autarky is equivalent to increasing the within-sector occupation shifter for $\omega$ relative to $\omega'$ in the closed economy.\footnote{T35}{Our extended model does not capture some of the mechanisms that have been studied in the literature linking international trade to between-group inequality. For example, as studied in Yeaple (2005), Bustos (2011), and Burstein and Vogel (2012), trade liberalization increases the measured skill bias of technology by reallocating resources from less to more skill-intensive firms within industries and/or inducing firms to increase their skill intensity. Extending the model to capture these mechanisms and mapping them into the components of our decomposition is a promising area for future work.}

Finally, we also show that the same approach can be used to provide a quantitative answer to the following question about the impact of international trade on relative wages between any two non-autarkic time periods. What are the differential effects on wages in a given country of changes in primitives (i.e. worldwide technologies, labor compositions, and trade costs) between two time periods relative to the effects of the same changes in primitives if that country were a closed economy? To answer this question we simply calculate wage changes caused by moving to autarky in each period and take the difference in these wage changes across periods.

7 Conclusions

In this paper we have developed a simple extension to a standard assignment model in which changes in workforce composition, occupation shifters, computerization, and labor

36
productivity shape the evolution of between-group inequality across many labor groups. We have parameterized and estimated the model to match observed factor allocations and wages in the United States between 1984 and 2003. We have shown that computerization alone accounts for the majority of the observed rise in between-education-group inequality over this period (e.g. 60% of the rise in the skill premium). The combination of observables—computerization and occupation shifters—explain roughly 80% of the rise in the skill premium, almost all of the rise in inequality across more disaggregated education groups, and the majority of the fall in the gender gap.

Our framework remains tractable in spite of its high dimensionality—using strong parametric assumptions including Fréchet, which would be interesting to relax in future work—lending itself to a variety of extensions and applications. We have extended our model to incorporate international trade in equipment and sector output as well as offshoring of occupations and have shown that changes in import and export shares in each of these markets shape what we treated, in our baseline closed-economy model, as exogenous primitive shocks. It would be interesting to bring this extended model to the data. One challenge in implementing such an application is the lack of available data on trade in occupations. Another interesting extension would be to model inter- and intra-national trade—and, potentially frictional labor mobility as in, e.g., Redding (2012)—and use the information from regional analyses—see e.g. Edmonds et al. (2010), Autor et al. (2013), Kovak (2013), and Dix-Carneiro and Kovak (2015)—to estimate the parameters that shape the outcomes of our aggregate counterfactual analyses.

Finally, the focus of this paper has been on the distribution of labor income between-groups of workers with different observable characteristics. A fruitful avenue for future research is to extend our framework to address the changing distribution of income accruing to labor and capital, as analyzed in e.g. Karabarbounis and Neiman (2014) and Oberfield and Raval (2014), as well as the changing distribution of income across workers within groups, as analyzed in e.g. Huggett et al. (2011), Hornstein et al. (2011), and Helpman et al. (2012).

References


_ and David Autor, Skills, Tasks and Technologies: Implications for Employment and Earnings, Vol. 4 of Handbook of Labor Economics, Elsevier,


Dix-Carneiro, Rafael and Brian Kovak, “Trade Reform and Regional Dynamics: Evidence from 25 Years of Brazilian Matched Employer-Employee Data,” mimeo, 2015.


A Data details

Throughout, we restrict our sample by dropping workers who are younger than 17 years old, do not report positive paid hours worked, are self-employed, or are in the military.

**MORG.** We use the MORG CPS to form a sample—for each labor group—of hours worked and income. We use the “hour wage sample” from Acemoglu and Autor (2011). Hourly wages are equal to the reported hourly earnings for those paid by the hour and the usual weekly earnings divided by hours worked last week for non-hourly workers. Top-coded earnings are multiplied by 1.5. Workers earning below $1.675/hour in 1982 dollars are dropped, as are workers whose hourly wages exceed 1/35th the top-coded value of weekly earnings (i.e., workers paid by the hour whose wages are sufficiently high so that their weekly income would be top-coded if they worked at least 35 hours and were not paid by the hour). Observations with allocated earnings are excluded in all years. Our measure of labor composition, $L_t(\lambda)$, is hours worked within each labor group $\lambda$ (weighted by sample weights).

**October Supplement.** In 1984, 1989, 1993, 1997, and 2003, the October Supplement asked respondents whether they “have direct or hands on use of computers at work,” “directly use a computer at work,” or “use a computer at/for his/her/your main job.” Using a computer at work refers only to “direct” or “hands on” use of a computer with typewriter like keyboards, whether a personal computer, laptop, mini computer, or mainframe.

**Occupations.** The occupations we include are listed in Table 10, where we also list the share of hours worked in each occupation by college educated workers and by women as well as the occupation share of labor payments in 1984 and in 2003. Our concordance of occupations across time is based on the concordance developed in Autor and Dorn (2013).

**Composition-adjusted wages.** When we construct measures of changes in relative wages between broader groups that aggregate across our labor groups—e.g. the average wage of college educated workers combines ten of our thirty labor groups—we composition adjust wages by holding constant the relative employment shares of our thirty labor groups—defined by the intersection of five education, two gender, and three age categories—across all years of the sample. Specifically, after calculating mean log wages within each labor group (either from the model or the data), we construct mean wages for broader groups as fixed-weighted averages of the relevant labor group means, using an average
<table>
<thead>
<tr>
<th>Occupations</th>
<th>College intensity</th>
<th>Female intensity</th>
<th>Income share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Executive, administrative, managerial</td>
<td>0.48</td>
<td>0.58</td>
<td>0.32</td>
</tr>
<tr>
<td>Management related</td>
<td>0.53</td>
<td>0.61</td>
<td>0.43</td>
</tr>
<tr>
<td>Architect</td>
<td>0.86</td>
<td>0.88</td>
<td>0.15</td>
</tr>
<tr>
<td>Engineer</td>
<td>0.71</td>
<td>0.79</td>
<td>0.06</td>
</tr>
<tr>
<td>Life, physical, and social science</td>
<td>0.65</td>
<td>0.55</td>
<td>0.30</td>
</tr>
<tr>
<td>Computer and mathematical</td>
<td>0.86</td>
<td>0.91</td>
<td>0.31</td>
</tr>
<tr>
<td>Community and social services</td>
<td>0.76</td>
<td>0.73</td>
<td>0.46</td>
</tr>
<tr>
<td>Lawyers</td>
<td>0.98</td>
<td>0.98</td>
<td>0.24</td>
</tr>
<tr>
<td>Education, training, etc...*</td>
<td>0.90</td>
<td>0.87</td>
<td>0.63</td>
</tr>
<tr>
<td>Arts, design, entertainment, sports, media</td>
<td>0.49</td>
<td>0.57</td>
<td>0.39</td>
</tr>
<tr>
<td>Health diagnosing</td>
<td>0.96</td>
<td>0.98</td>
<td>0.20</td>
</tr>
<tr>
<td>Health assessment and treating</td>
<td>0.51</td>
<td>0.64</td>
<td>0.85</td>
</tr>
<tr>
<td>Technicians and related support</td>
<td>0.30</td>
<td>0.43</td>
<td>0.46</td>
</tr>
<tr>
<td>Financial sales and related</td>
<td>0.31</td>
<td>0.33</td>
<td>0.31</td>
</tr>
<tr>
<td>Retail sales</td>
<td>0.17</td>
<td>0.24</td>
<td>0.54</td>
</tr>
<tr>
<td>Administrative support</td>
<td>0.12</td>
<td>0.16</td>
<td>0.78</td>
</tr>
<tr>
<td>Housekeeping, cleaning, laundry</td>
<td>0.01</td>
<td>0.03</td>
<td>0.83</td>
</tr>
<tr>
<td>Protective service</td>
<td>0.16</td>
<td>0.21</td>
<td>0.11</td>
</tr>
<tr>
<td>Food preparation and service</td>
<td>0.05</td>
<td>0.06</td>
<td>0.61</td>
</tr>
<tr>
<td>Health service</td>
<td>0.04</td>
<td>0.08</td>
<td>0.90</td>
</tr>
<tr>
<td>Building, grounds cleaning, maintenance</td>
<td>0.04</td>
<td>0.05</td>
<td>0.20</td>
</tr>
<tr>
<td>Miscellaneous**</td>
<td>0.12</td>
<td>0.17</td>
<td>0.67</td>
</tr>
<tr>
<td>Child care</td>
<td>0.11</td>
<td>0.12</td>
<td>0.91</td>
</tr>
<tr>
<td>Agriculture and mining</td>
<td>0.05</td>
<td>0.06</td>
<td>0.10</td>
</tr>
<tr>
<td>Mechanics and repairers</td>
<td>0.04</td>
<td>0.07</td>
<td>0.03</td>
</tr>
<tr>
<td>Construction</td>
<td>0.04</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>Precision production</td>
<td>0.07</td>
<td>0.08</td>
<td>0.15</td>
</tr>
<tr>
<td>Machine operators, assemblers, inspectors</td>
<td>0.03</td>
<td>0.06</td>
<td>0.40</td>
</tr>
<tr>
<td>Transportation and material moving</td>
<td>0.03</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>Handlers, equip. cleaners, helpers, laborers</td>
<td>0.03</td>
<td>0.04</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 10: Thirty occupations, their college and female intensities, and the occupational share of labor payments

*Education, training, etc... also includes library, legal support/assistants/paralegals

**Miscellaneous includes personal appearance, misc. personal care and service, recreation and hospitality

College intensity (Female intensity) indicates hours worked in the occupation by those with college degrees (females) relative to total hours worked in the occupation. Income share denotes labor payments in the occupation relative to total labor payments. Each is calculated using the MORG CPS.
share of total hours worked by each labor group over 1984 to 2003 as weights. This adjustment ensures that changes in average wages across broader groups are not driven by shifts in the education × age × gender composition within these broader groups.

**O*NET.** We follow Acemoglu and Autor (2011) in our use of O*NET and construct six composite measures of O*NET Work Activities and Work Context Importance scales: (i) non-routine cognitive analytical, (ii) non-routine cognitive interpersonal, (iii) routine cognitive, (iv) routine manual, (v) non-routine manual physical, and (vi) social perceptive. We simply aggregate their measures up to our thirty occupations and standardize each to have mean zero and standard deviation one.

### B Measurement

Equations (7), (8), and (9) can be written so that changes in relative wages, $\hat{w}(\lambda) / \hat{w}(\lambda_1)$, relative transformed occupation price changes, $\hat{q}(\omega) / \hat{q}(\omega_1)$, and allocations, $\hat{\pi}(\lambda, \kappa, \omega)$, depend on relative shocks to labor composition, $\hat{L}(\lambda) / \hat{L}(\lambda_1)$, occupation shifters, $\hat{a}(\omega) / \hat{a}(\omega_1)$, equipment productivity, $\hat{q}(\kappa) / \hat{q}(\kappa_1)$, and labor productivity, $\hat{T}(\lambda) / \hat{T}(\lambda_1)$:

$$\frac{\hat{w}(\lambda)}{\hat{w}(\lambda_1)} = \frac{\hat{T}(\lambda)}{\hat{T}(\lambda_1)} \left[ \sum_{\kappa, \omega} \left( \frac{\hat{q}(\omega)}{\hat{q}(\omega_1)} \right)^\theta \tau_{t_0}(\lambda, \kappa, \omega) \right]^{1/\theta}$$

$$\hat{a}(\omega) / \hat{a}(\omega_1) = \frac{\hat{q}(\kappa) / \hat{q}(\kappa_1)}{\sum_{\kappa', \omega'} (\hat{q}(\kappa') / \hat{q}(\kappa_1))^{\theta}}$$

and

$$\frac{\hat{a}(\omega)}{\hat{a}(\omega_1)} \left( \frac{\hat{q}(\omega)}{\hat{q}(\omega_1)} \right)^{(1-\alpha)(1-\rho)} = \frac{\sum_{\lambda, \kappa} w_{t_0}(\lambda) \pi_{t_0}(\lambda, \kappa, \omega) \hat{w}(\lambda) \hat{L}(\lambda) / \hat{w}(\lambda_1) \hat{L}(\lambda_1) \hat{T}(\lambda, \kappa, \omega)}{\sum_{\lambda', \kappa'} w_{t_0}(\lambda') \pi_{t_0}(\lambda', \kappa', \omega) \hat{w}(\lambda') \hat{L}(\lambda') / \hat{w}(\lambda_1) \hat{L}(\lambda_1) \hat{T}(\lambda', \kappa', \omega)}$$

### B.1 Baseline

Here we describe in detail the steps that we follow to obtain our measures of changes in labor composition, occupation shifters, equipment productivity and labor productivity. The relative shocks to labor composition $\hat{L}(\lambda) / \hat{L}(\lambda_1)$ are directly observed in the data.

First, we measure changes in equipment productivity (to the power $\theta$) using equation
\[ \frac{q(\kappa_2)^\theta}{q(\kappa_1)^\theta} = \exp \left( \frac{1}{N(\kappa_1, \kappa_2)} \sum_{\lambda, \omega} \log \frac{\hat{\pi}(\lambda, \kappa_2, \omega)}{\hat{\pi}(\lambda, \kappa_1, \omega)} \right), \]

dropping all \((\lambda, \omega)\) pairs for which \(\pi_t(\lambda, \kappa_1, \omega) = 0\) or \(\pi_t(\lambda, \kappa_2, \omega) = 0\) in either period \(t_0\) or \(t_1\). \(N(\kappa_1, \kappa_2)\) is the number of \((\lambda, \omega)\) pairs over which we average; in the absence of any zeros in allocations we have \(N(\kappa_1, \kappa_2) = 900\), which is the number of labor groups multiplied by the number of occupations.

Second, we measure changes in transformed occupation prices relative to occupation \(\omega_0\) (to the power \(\theta\)) using equation (12) as

\[ \frac{q(\omega)^\theta}{q(\omega_0)^\theta} = \exp \left( \frac{1}{N(\omega, \omega_0)} \sum_{\lambda, \kappa} \log \frac{\hat{\pi}(\lambda, \kappa, \omega)}{\hat{\pi}(\lambda, \kappa, \omega_0)} \right), \]

dropping all \((\lambda, \kappa)\) pairs for which \(\pi_t(\lambda, \kappa, \omega_0) = 0\) or \(\pi_t(\lambda, \kappa, \omega) = 0\) in either period \(t_0\) or \(t_1\). \(N(\omega, \omega_0)\) is the number of \((\lambda, \kappa)\) pairs over which we average; in the absence of any zeros in allocations we have \(N(\omega, \omega_0) = 60\), which is the number of labor groups multiplied by the number of equipment types. In our model, the estimates of the relative occupation shifters for any two occupations \(\omega_A\) and \(\omega_B\) should not depend on the choice of the reference category \(\omega_0\). However, even if this prediction of the model is right, measurement error in changes in allocations implies that the estimate of changes in relative transformed occupation prices varies with the choice of \(\omega_0\). In order to avoid this sensitivity to the choice of \(\omega_0\), we compute changes in relative transformed occupation prices using the following geometric average

\[ \frac{q(\omega)^\theta}{q(\omega_1)^\theta} = \exp \left( \frac{1}{30} \sum_{\omega_0} \left( \log \frac{q(\omega)^\theta}{q(\omega_0)^\theta} - \log \frac{q(\omega_1)^\theta}{q(\omega_0)^\theta} \right) \right), \]

where \(\frac{q(\omega)^\theta}{q(\omega_0)^\theta}\) for each \(\omega\) and \(\omega_0\) is calculated as described above. This expression yields estimates that do not depend on the choice of \(\omega_1\). Furthermore, as Section C.2. shows, this approach yields measures of relative changes in occupation shifters that are very similar to those that arise from projecting changes in allocations on a set of fixed effects.

Third, given our measures of changes in equipment and transformed occupation prices (both to the power \(\theta\)), we construct \(\hat{s}(\lambda)\) using equation (14). Given \(\hat{s}(\lambda)\), we estimate \(\theta\) using equation (15) as described in Section 3.3.1.

Fourth, given the transformed measures of changes in equipment productivity and
occupation prices in equations (10) and (12), the estimate of $\theta$, and observed values both of
the initial allocation $\pi_{t_0} (\lambda, \kappa, \omega)$ and changes in relative wages $\hat{w} (\lambda) / \hat{w} (\lambda_1)$, we measure
changes in labor productivity $\hat{T} (\lambda) / \hat{T}(\lambda_1)$ using equation (13).

Fifth, using data on changes in payments to occupations, $\hat{z} (\omega)$, the transformed measures
of changes in equipment productivity and occupation prices in equations (10) and
(12), and an estimate of $\theta$, we estimate $\rho$ as described in Section 3.3.2.

Finally, we measure changes in occupation shifters, $\hat{a} (\omega) / \hat{a} (\omega_1)$, using equation (11).
A variable in this equation is the relative changes in total payments to occupations $\omega$
relative to those in a benchmark occupation $\omega_1$, $\hat{z} (\omega) / \hat{z}(\omega_1)$. We construct this variable
as follows. The initial levels, $\xi_{t_0} (\omega) / \xi_{t_0} (\omega_1)$, are calculated directly using the observed
values of $\pi_{t_0} (\lambda, \kappa, \omega)$, $w_{t_0} (\lambda)$, and $L_{t_0} (\lambda)$. The terminal levels, $\xi_{t_1} (\omega) / \xi_{t_1} (\omega_1)$, are con-
structed as

\[
\frac{\xi_{t_1} (\omega)}{\xi_{t_1} (\omega_1)} = \frac{\sum_{\lambda, \kappa} w_{t_0} (\lambda) L_{t_0} (\lambda) \pi_{t_0} (\lambda, \kappa, \omega) \frac{\hat{w}(\lambda)}{\hat{w}(\lambda_1)} \hat{L} (\lambda) \hat{\pi} (\lambda, \kappa, \omega)}{\sum_{\lambda', \kappa'} w_{t_0} (\lambda') L_{t_0} (\lambda') \pi_{t_0} (\lambda', \kappa', \omega_1) \frac{\hat{w}(\lambda')}{\hat{w}(\lambda_1)} \hat{L} (\lambda') \hat{\pi} (\lambda', \kappa', \omega_1)},
\]

where $\hat{\pi} (\lambda, \kappa, \omega)$ are those constructed by the model given the measures of $\hat{q} (\omega)^\theta / \hat{q} (\omega_1)^\theta$
and $\hat{q} (\kappa)^\theta / \hat{q} (\kappa_1)^\theta$. The correlation between $\log(\hat{z}(\omega) / \hat{z}(\omega_1))$ implied by the model and
in the data is 0.77 between 1984 and 2003; the correlation between $\log(\hat{z}(\omega) / \hat{z}(\omega_1))$
implied by the model using the alternative approach in Appendix B.3 and in the data is 1
and the quantitative results we obtain from these approaches are very similar.

**B.2 Alternative approach 1: Regression based**

Instead of using the expressions in equations (10) and (12), we can measure $\hat{q}(\omega)^\theta / \hat{q}(\omega_1)^\theta$
and $\hat{q}(\kappa)^\theta / \hat{q}(\kappa_1)^\theta$ using the coefficients of a regression of the observed changes in factor
allocations on labor group, occupation, and equipment type fixed effects. Specifically, we
can express equation (8) as

\[
\hat{\pi} (\lambda, \kappa, \omega) = \hat{q} (\lambda) \hat{q} (\omega)^\theta \hat{q} (\kappa)^\theta
\]

where we define

\[
\hat{q} (\lambda) \equiv \sum_{\lambda', \omega'} \hat{q} (\omega')^\theta \hat{q} (\kappa')^\theta \pi_{t_0} (\lambda, \kappa', \omega')
\]
Hence, in the presence of multiplicative measurement error in the observed changes in allocations, \( \pi(\lambda, \kappa, \omega), \tau_l (\lambda, \kappa, \omega) \), we have

\[
\log \hat{\pi} (\lambda, \kappa, \omega) = \log \hat{q} (\lambda) + \log \hat{q} (\omega) + \log \hat{q} (\kappa) + \tau_l (\lambda, \kappa, \omega).
\]

Using this equation, we regress observed values of \( \log \hat{\pi} (\lambda, \kappa, \omega) \) on labor group, equipment, and occupation effects. Exponentiating the resulting occupation and equipment fixed effects, we obtain estimates of \( \hat{q} (\omega)^\theta / \hat{q} (\omega_1)^\theta \) and \( \hat{q} (\kappa)^\theta / \hat{q} (\kappa_1)^\theta \). Using these estimates instead of those derived from equations (10) and (12), we can recover measures of occupation shifters, \( \hat{\alpha} (\omega) / \hat{\alpha}(\omega_1) \), and labor productivity, \( \hat{T} (\lambda) / \hat{T}(\lambda_1) \), as well as estimate \( \rho \) and \( \theta \), following the same steps outlined in Appendix B.1.

Our alternative and baseline approaches are identical in the absence of zeros in the allocation data. In practice, the correlation between the measures obtained using these two approaches is above 0.99 for both equipment productivity and occupation prices. We use the procedure described in Appendix B.1 as our baseline approach simply because, in our opinion, it more clearly highlights what variation in the data is being used to identify the changes in occupation prices and equipment productivity (to the power \( \theta \)).

### B.3 Alternative approach 2: Matching income shares

Our baseline approach does not exactly match changes in total labor income by occupation, \( \zeta_t (\omega) \equiv \sum_{\lambda, \omega} w_l (\lambda) L_t (\lambda) \pi_l (\lambda, \kappa, \omega) \), and total labor income by equipment type \( \zeta_t (\kappa) \equiv \sum_{\lambda, \omega} w_l (\lambda) L_t (\lambda) \pi_l (\lambda, \kappa, \omega) \). In this alternative approach we calibrate \( \frac{\partial(\omega)^\theta}{\partial(\omega_1)^\theta} \) and \( \frac{\partial(\kappa)^\theta}{\partial(\kappa_1)^\theta} \) to match \( \frac{\hat{\zeta}(\omega)}{\hat{\zeta}(\omega_1)} \) and \( \frac{\hat{\zeta}(\kappa)}{\hat{\zeta}(\kappa_1)} \) exactly.

For each time period we solve simultaneously for \( \frac{\partial(\omega)^\theta}{\partial(\omega_1)^\theta} \) and \( \frac{\partial(\kappa)^\theta}{\partial(\kappa_1)^\theta} \) to match observed values of \( \frac{\hat{\zeta}(\omega)}{\hat{\zeta}(\omega_1)} \) and \( \frac{\hat{\zeta}(\kappa)}{\hat{\zeta}(\kappa_1)} \), whereas in our baseline procedure we measure \( \frac{\partial(\omega)^\theta}{\partial(\omega_1)^\theta} \) and \( \frac{\partial(\kappa)^\theta}{\partial(\kappa_1)^\theta} \) independently to match changes in allocations. Specifically, for every \( t_0 \), \( \frac{\partial(\omega)^\theta}{\partial(\omega_1)^\theta} \) and \( \frac{\partial(\kappa)^\theta}{\partial(\kappa_1)^\theta} \) is the solution to the following non-linear system of equations:

\[
\frac{\hat{\zeta}(\omega)}{\hat{\zeta}(\omega_1)} = \frac{\zeta_{t_0} (\omega_1) \sum_{\lambda, \omega} \omega_{t_0} (\lambda) L_{t_0} (\lambda) \pi_{t_0} (\lambda, \kappa, \omega) \hat{\omega} (\lambda) \hat{L} (\lambda) \hat{\pi} (\lambda, \kappa, \omega)}{\zeta_{t_0} (\omega) \sum_{\lambda, \omega} \omega_{t_0} (\lambda) L_{t_0} (\lambda) \pi_{t_0} (\lambda, \kappa, \omega_1) \hat{\omega} (\lambda) \hat{L} (\lambda) \hat{\pi} (\lambda, \kappa, \omega_1)}
\]

\[
\frac{\hat{\zeta}(\kappa)}{\hat{\zeta}(\kappa_1)} = \frac{\zeta_{t_0} (\kappa_1) \sum_{\lambda, \omega} \omega_{t_0} (\lambda) L_{t_0} (\lambda) \pi_{t_0} (\lambda, \kappa, \omega) \hat{\omega} (\lambda) \hat{L} (\lambda) \hat{\pi} (\lambda, \kappa, \omega)}{\zeta_{t_0} (\kappa) \sum_{\lambda, \omega} \omega_{t_0} (\lambda) L_{t_0} (\lambda) \pi_{t_0} (\lambda, \kappa_1, \omega) \hat{\omega} (\lambda) \hat{L} (\lambda) \hat{\pi} (\lambda, \kappa_1, \omega)}
\]

where \( \hat{\omega} (\lambda) \hat{L} (\lambda), \frac{\hat{\zeta}(\omega)}{\hat{\zeta}(\omega_1)} \) and \( \frac{\hat{\zeta}(\kappa)}{\hat{\zeta}(\kappa_1)} \) are given by data, and \( \hat{\pi} (\lambda, \kappa, \omega) = \frac{(\hat{q}(\omega)^\theta \hat{q}(\kappa)^\theta)}{\sum_{\lambda', \omega'} (\hat{q}(\omega')^\theta \hat{q}(\kappa')^\theta) \pi_{t_0} (\lambda, \kappa', \omega')} \)
is constructed given \( \frac{\dot{q}(\omega)^\theta}{\dot{q}(\omega_1)^\theta} \) and \( \frac{\dot{q}(\kappa)^\theta}{\dot{q}(\kappa_1)^\theta} \). After solving for \( \frac{\dot{q}(\omega)^\theta}{\dot{q}(\omega_1)^\theta} \) and \( \frac{\dot{q}(\kappa)^\theta}{\dot{q}(\kappa_1)^\theta} \) the remaining shocks and parameters are chosen exactly as in our baseline procedure. We also consider a variation in which we first measure \( \frac{\dot{q}(\kappa)^\theta}{\dot{q}(\kappa_1)^\theta} \) using our baseline procedure, and then \( \frac{\dot{q}(\omega)^\theta}{\dot{q}(\omega_1)^\theta} \) in order to match \( \frac{\xi(\omega)}{\xi(\omega_1)} \) in the data. Results using these alternative approaches are very similar to our baseline results.

**B.4 Using levels rather than changes**

In Sections 3.3.1 and 3.3.2 we describe alternative regression approaches to estimate \( \theta \) and \( \rho \) using levels rather than time differences. Here we show how we measure the required inputs into these level regressions—\( q_t (\omega)^\theta / q_t (\omega_1)^\theta, q_t (\kappa)^\theta / q_t (\kappa_1)^\theta \) where \( q_t (\kappa) \equiv p_t (\kappa) \bar{T}_t (\kappa) \), and \( T (\lambda, \kappa, \omega) \)—and provide details on the estimation of \( \theta \) and \( \rho \). Equation (3) can be expressed as

\[
\log \pi_t (\lambda, \kappa, \omega) = \log q_t (\kappa)^\theta + \log q_t (\omega)^\theta + \log q_t (\lambda) + l_t^L (\lambda, \kappa, \omega) \tag{17}
\]

where

\[
q_t (\lambda) \equiv \left( \sum_{k',\omega'} T (\lambda, \kappa', \omega') q_t (k')^\theta q_t (\omega')^\theta \right)^{-1}
\]

and

\[
l_t^L (\lambda, \kappa, \omega) = \log T (\lambda, \kappa, \omega)^\theta.
\]

Hence, regressing observed values of \( \log \pi_t (\lambda, \kappa, \omega) \) on labor group, equipment, and occupation effects and exponentiating the resulting occupation and equipment fixed effects, we obtain estimates of \( q_t (\omega)^\theta / q_t (\omega_1)^\theta \) and \( q_t (\kappa)^\theta / q_t (\kappa_1)^\theta \). We then construct \( T (\lambda, \kappa, \omega)^\theta \) as

\[
T (\lambda, \kappa, \omega)^\theta = \exp \left( l_t (\lambda, \kappa, \omega) \right)
\]

where \( l_t (\lambda, \kappa, \omega) \) is the average of \( l_t^L (\lambda, \kappa, \omega) \) across time. Given \( q_t (\omega)^\theta / q_t (\omega_1)^\theta, q_t (\kappa)^\theta / q_t (\kappa_1)^\theta, \) and \( T (\lambda, \kappa, \omega)^\theta \) we can estimate \( \theta \) and \( \rho \) using levels rather than time differences, as described in Sections 3.3.1 and 3.3.2.

**Estimating \( \theta \).** In levels, we can express wages as

\[
w_t (\lambda) = \tilde{a} \gamma \times T_t (\lambda) \times S_t (\lambda)^{1/\theta} \tag{18}
\]

where \( S_t (\lambda) \) is a labor-group-specific average of equipment productivities and occupa-
tion prices, both to the power $\theta$,

$$ S_t(\lambda) \equiv \sum_{k,\omega} \left( T(\lambda, \kappa, \omega) q_{t}(\kappa) q_{t}(\omega) \right)^{\theta}. \quad (19) $$

To estimate $\theta$ using levels rather than changes, we decompose $\log T_t(\lambda)$ into a labor group effect, a time effect, and labor-group-time-specific deviations and express equation (18) as

$$ \log w_t(\lambda) = \zeta_{\theta2}(t) + \beta_{\theta2}(\lambda) + \beta_{\theta} \log s_t(\lambda) + \nu_{\theta2}(\lambda), $$

where $s_t(\lambda) \equiv S_t(\lambda) q_t(\omega_1)^{-\theta} q_t(\kappa_1)^{-\theta}$ is a transformation of $S_t(\lambda)$. We require an instrument for $\log s_t(\lambda)$ for the same reason we require an instrument for $\hat{s}(\lambda)$ in our baseline approach. We use a similar instrument,

$$ \chi_{\theta2}(\lambda) \equiv \log \sum_{\kappa} \frac{q_t(\kappa)^{\theta}}{q_t(\kappa_1)^{\theta}} \sum_{\omega} \pi_{1984}(\lambda, \kappa, \omega), $$

which is a labor-group-specific productivity shifter generated by the level rather than change in equipment productivity, $q_t(\kappa)^{\theta}/q_t(\kappa_1)^{\theta}$. A higher value of equipment $\kappa$ productivity in period $t$ raises the wage of group $\lambda$ relatively more if a larger share of $\lambda$ workers use equipment $\kappa$. We can construct both $\log s_t(\lambda)$ and $\chi_{\theta2}(\lambda)$ using the estimates described above.

Finally, including a labor-group-specific time trend and using levels is straightforward.

**Estimating $\rho$.** We decompose $\log a_t(\omega)$ into a labor group effect, a time effect, and a deviation and express equation (5) as

$$ \log \zeta_t(\omega) = \zeta_{\rho2}(t) + \beta_{\rho2}(\omega) + \beta_{\rho2} \log \frac{q_t(\omega)^{\theta}}{q_t(\omega_1)^{\theta}} + \nu_{\rho2}(\omega). $$

We require an instrument for $q_t(\omega)^{\theta}/q_t(\omega_1)^{\theta}$ for the same reason we require an instrument for $\hat{q}(\omega)^{\theta}/\hat{q}(\omega_1)^{\theta}$ in our baseline approach. We use a similar instrument

$$ \chi_{\rho2}(\omega) \equiv \log \sum_{\lambda,\kappa} \frac{q_t(\kappa)^{\theta}}{q_t(\kappa_1)^{\theta}} \frac{L_{1984}(\lambda) \pi_{1984}(\lambda, \kappa, \omega)}{\sum_{\lambda',\kappa'} L_{1984}(\lambda') \pi_{1984}(\lambda', \kappa', \omega)}, $$

which is a labor-group-specific productivity shifter generated by the level, rather than change, in equipment productivity, $q_t(\kappa)^{\theta}/q_t(\kappa_1)^{\theta}$. A higher value of equipment $\kappa$ productivity in period $t$ lowers the price of occupation $\omega$ relatively more if a larger share of
workers use equipment $\kappa$ in occupation $\omega$. We can construct both $q_t(\omega)^{\theta} / q_t(\omega_1)^{\theta}$ and $\chi_{\rho_2}(\omega)$ using the estimates described above.

Finally, including occupation-specific time trends and using levels is straightforward.

C Factor allocation in Germany

As discussed above, constructing factor allocations, $\pi_t(\lambda, \kappa, \omega)$, as we do using U.S. data from the October Supplement introduces certain limitations. For example, (i) our view of computerization is narrow, (ii) at the individual level our computer-use variable is zero-one, (iii) we are not using any information on the allocation of non-computer equipment, and (iv) the computer use question was discontinued after 2003. Here, we use data on the allocation of German workers to address possible concerns raised by limitations (ii) and (iii).

We use the 1986, 1992, 1999, and 2006 waves of the German Qualification and Working Conditions survey, which asks detailed questions about usage of different types of equipment (i.e. tools) at work. Specifically, respondents are asked which tool, out of many, they use most frequently at work. In 1986, 1992, and 1999 respondents are also asked whether or not they use each tool, regardless of whether it is the tool they use most frequently whereas in 2006 respondents are asked about the share of time they spend using computers. The list of tools changes over time (discussed below) and is extensive. For instance, workers are asked if they use simple transportation tools such as wheelbarrows or fork lifts, computers, and writing implements such as pencils. After cleaning there are between 10,700 and 21,150 observations, depending on the year.

We group workers into twelve labor groups using three education groups (low education workers who do not have post-secondary education or an apprenticeship degree, medium education workers who have either post-secondary education or an apprenticeship degree, and high education workers who have a university degree), two age groups (20-39 and 40-up), and two genders. We consider twelve occupations. We drop workers who do not report using any tool most frequently. Because the list of tools changes over time, we allocate workers to computer usage (using the question about most used equipment type or the questions about whether a worker uses a type of equipment at all) as follows. In 1986 and 1992, we allocate a worker to computer usage if she reports using a computer terminal, computer-controlled medical instrument, electronic lists or forms, personal computer, computer, screen operated system, or CAD graphics systems. In 1999 we allocate a worker to computer usage if she reports using computerized control or measure tools, personal computers, computers with connection to intranet or internet,
laptops, computers to control machines, or other computers. In 2006 we allocate a worker to computer usage if she reports using computerized control or measure tools, computers, personal computers, laptops, peripheries, or computers to control machines. We have considered a range of alternative allocations and obtained similar results.

According to our baseline definition, the share of workers for whom computers are the most-used tool rose from roughly 5% to 50% between 1986 and 2006. Clearly, no other equipment type reported in the data either grew or shrank at a similar pace.

**Computer-education and computer-gender comparative advantage.** We first use the question about the most used equipment type to study comparative advantage in Germany. This question helps address limitation (iii) in the U.S. data, since here we are using information on the allocation of non-computer equipment, both by dropping workers who do not report a most used equipment type and by including in the group of computer users only those workers who report that they use computers more than any other type of equipment. Specifically, we construct histograms in Figure 4 for Germany—analogous to Figure 2 for the U.S.—detailing education × computer and gender × computer comparative advantage. The left panel of Figure 4 shows that German workers with a high level of education have a strong comparative advantage using computers relative to workers with a low level of education, the middle panel shows that German workers with a high level of education have a mild comparative advantage using computers relative to workers with a medium level of education, and the right panel shows that women have no discernible comparative advantage with computers relative to men, where each of these patterns is identified within occupation and holding all other worker characteristics fixed. These patterns in Germany resemble the patterns we document in the U.S. in Figure 2.

![Figure 4: Computer relative to non-computer usage](image)

Second, we study the extent to which allocating workers to computers using the most-used or the used-at-all-question matters for measuring comparative advantage, since we
only have access to the second type of question in the U.S. In the three years with available data (1986, 1992, and 1999) we construct allocations separately using these two questions. We then construct the share of hours worked with computers within each \((\lambda, \omega)\), i.e. 
\[
\pi_t^{Comp}(\lambda, \omega) \equiv \frac{\pi_t(\lambda, \omega_{Comp})}{\pi_t(\lambda, \omega_{Comp}) + \pi_t(\lambda, \omega_{Non-comp})},
\]
separately using each type of question. The correlation of \(\pi_t^{Comp}(\lambda, \omega)\) constructed using the two different questions is high: 0.86, 0.78, and 0.55 in 1999, 1992, and 1986, respectively. To further understand the similarities and differences in measures of comparative advantage constructed using these questions, in Figure 5 we replicate the histograms in Figure 4 using the question on whether computers are used at all. The patterns of comparative advantage of education groups with computers, the left and middle panels of Figure 5, replicate the patterns in Figure 4: high education German workers have a strong and mild comparative advantage using computers relative to low and medium education German workers, respectively. However, constructing allocations using the used-at-all-question we measure a mild comparative advantage between men and computers in the right panel in Figure 5, unlike what we observe in Figure 4 in Germany or in Figure 2 in the U.S.

![Histograms](image)

Figure 5: Computer relative to non-computer usage (constructed using questions about whether computers are used at all) for high relative to low education, high relative to medium education, and female relative to male workers, respectively, in Germany. Outliers have been truncated.

Finally, we use the question asked only in 2006 about the share of a worker’s time spent using a computer. When constructing allocations using this question, we allocate the share of each worker’s hours to computer or non-computer accordingly, whereas in our baseline approach we must allocate all of each worker’s hours either to computers or non-computer equipment. Hence, this question helps address limitation \((ii)\) in the U.S. data. Since figures like 2, 4, and 5 are noisy when constructed using a single year of data (using any question to determine allocations), here we focus instead on the correlation in the share of hours worked with computers within each \((\lambda, \omega)\), \(\pi_t^{Comp}(\lambda, \omega)\), constructed using the most-used equipment type question and the share of hours worked with computers question. This correlation is above 0.9.
D Multivariate Fréchet

In this section we generalize our baseline assumption that each worker \( z \in \mathcal{Z}_t (\lambda) \) draws idiosyncratic efficiency units from a Fréchet distribution with CDF \( G (\varepsilon) = \exp (\varepsilon^{-\theta}) \). Instead, we allow for correlation across draws, following the approach of, e.g., Ramondo and Rodríguez-Clare (2013) and Hsieh et al. (2013); see Lagakos and Waugh (2013) for a related approach. For each worker \( z \in \mathcal{Z}_t (\lambda) \), the vector \( \varepsilon (z) \) is drawn from a multivariate Fréchet distribution,

\[
G (\varepsilon (z) ; \lambda) = \exp \left( - \left( \sum_{\kappa, \omega} \varepsilon (z, \kappa, \omega)^{-\tilde{\theta}(\lambda) / 1 - \nu(\lambda)} \right)^{1-\nu(\lambda)} \right).
\]

The parameter \( \tilde{\theta}(\lambda) > 1 \) governs the \( \lambda \)--specific dispersion of efficiency units across \((\kappa, \omega)\) pairs; a higher value of \( \tilde{\theta}(\lambda) \) decreases this dispersion. The parameter \( 0 \leq \nu(\lambda) \leq 1 \) governs the \( \lambda \)--specific correlation of each worker’s efficiency units across \((\kappa, \omega)\) pairs; a higher value of \( \nu(\lambda) \) increases this correlation. We define \( \theta(\lambda) \equiv \tilde{\theta}(\lambda) / (1 - \nu(\lambda)) \).

**Imposing a common \( \theta(\lambda) \) across \( \lambda \).** It is straightforward to show that our baseline equations, parameterization strategy, and results hold exactly in the case in which \( \theta(\lambda) \) is constant across \( \lambda \). Hence, given \( \theta = \theta(\lambda) \) for all \( \lambda \), all of our results are independent of the values of \( \tilde{\theta}(\lambda) \) and \( \nu(\lambda) \). Our baseline assumption that \( \nu(\lambda) = 0 \) is, therefore, without loss of generality under the common assumption, see e.g. Hsieh et al. (2013), that \( \theta(\lambda) \) is constant across \( \lambda \).

**Allowing \( \theta(\lambda) \) to vary across \( \lambda \).** In what follows, we describe the generalized model, parameterization, and results allowing \( \theta(\lambda) \) to vary across \( \lambda \).

Our baseline equilibrium equations in levels—(3), (4), and (5)—and in changes—(7), (8), and (9)—are unchanged except \( \theta(\lambda) \) replaces \( \theta \). The key distinction between our baseline and extended model is the parameterization.

When \( \theta(\lambda) \) varies across \( \lambda \), we parameterize \( \theta(\lambda) \) first, following the approaches in Lagakos and Waugh (2013) and Hsieh et al. (2013). Our assumption on the distribution of idiosyncratic productivity implies that the distribution of wages within labor group \( \lambda \) is Fréchet with shape parameter \( \tilde{\theta}(\lambda) \), where \( \theta(\lambda) \equiv \tilde{\theta}(\lambda) / (1 - \nu(\lambda)) \). We, therefore, use the empirical distribution of wages within each \( \lambda \) to estimate \( \tilde{\theta}(\lambda) \), separately for each labor group \( \lambda \), using maximum likelihood. Specifically, we jointly estimate the shape and scale parameter for each \( \lambda \) in each year \( t \) using maximum likelihood (MLE). Figure 6 plots the empirical and predicted wage distributions for all middle-aged workers in 2003. We average across years our estimates of the shape parameter to obtain \( \tilde{\theta}(\lambda) \).
Figure 6: Empirical and predicted (Fréchet distribution estimated using maximum likelihood) wage distributions for all middle-aged labor groups in 2003
Finally, we obtain an estimate of $\theta(\lambda)$ from $\hat{\theta}(\lambda)$ using Hsieh et al.’s (2013) implied estimate of $\nu \equiv \nu(\lambda) \approx 0.1$. Consistent with the observation that higher earning labor groups have more within-group wage dispersion, see e.g. Lemieux (2006), we find that $\theta(\lambda)$ is lower for more educated groups than less educated groups—averaging within each of the five education groups across age and gender, we obtain estimates that fall monotonically from 3.46 amongst high school dropouts to 2.21 amongst those with graduate training—for men than women—averaging within each gender across age and education, we obtain an estimate of 2.41 for men and 2.82 for women—and for older than younger workers—averaging within each age group across education and gender we obtain estimates that fall monotonically with age from 2.96 to 2.37. The average across $\lambda$ varies non-monotonically across years from a low of 2.56 to a high of 2.69. Finally, averaging across all groups and years yields an estimates of $\theta = 2.62$.

Given values of $\theta(\lambda)$, we measure changes in equipment productivity and transformed occupation prices, not to the power $\theta(\lambda)$, using the following variants of equations (10) and (12)

$$\frac{\hat{q}(\kappa)}{\hat{q}(\kappa_1)} = \left( \frac{\hat{p}(\lambda, \kappa, \omega)}{\hat{p}(\lambda, \kappa_1, \omega)} \right)^{1/\theta(\lambda)}$$

and

$$\frac{\hat{q}(\omega)}{\hat{q}(\omega_1)} = \left( \frac{\hat{p}(\lambda, \kappa, \omega)}{\hat{p}(\lambda, \kappa, \omega_1)} \right)^{1/\theta(\lambda)}.$$ 

Given changes in transformed occupation prices, we measure changes in occupation shifters using equation (11). Finally, we could also estimate $\rho$ using the following variant of equation (16)

$$\log \hat{\xi}(\omega) = \beta_1^\rho (t) + \beta_1^\rho \log \frac{\hat{q}(\omega)}{\hat{q}(\omega_1)} + \rho^\prime(\omega),$$

where $\beta_1^\rho \equiv (1 - \alpha)(1 - \rho)$ is the coefficient of interest, using the same instrument as in our baseline approach.\textsuperscript{36}

Table 11 reports the results of our decomposition of the skill premium and the gender gap over the period 1984-2003 under three alternative specifications. The first row reports our baseline results in which $\theta$ is constant across all groups and estimated as described in Section 3.3.1. The second row reports results in which $\theta(\lambda)$ is estimated separately for each $\lambda$, but we use the average value $\theta$ for each $\lambda$. Finally the final row reports results using distinct values of $\theta(\lambda)$ across each $\lambda$. The key message of Table 11 is that our results are robust. This is particularly true comparing between the second and the third rows of Table 11, in which the average value of $\theta(\lambda)$ is constant by construction.

\textsuperscript{36}In practice, we will impose the same $\rho$ as in our baseline in our exercises below.
<table>
<thead>
<tr>
<th></th>
<th>Skill premium</th>
<th></th>
<th>Gender gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>-0.114</td>
<td>0.049</td>
<td>0.159</td>
</tr>
<tr>
<td>$\theta = 2.62$</td>
<td>-0.094</td>
<td>0.058</td>
<td>0.108</td>
</tr>
<tr>
<td>$\theta (\lambda)$</td>
<td>-0.098</td>
<td>0.046</td>
<td>0.110</td>
</tr>
</tbody>
</table>

Table 11: Decomposing changes in the log skill premium and gender gap between 1984 and 2003 allowing $\theta (\lambda)$ to vary with $\theta$

### E Additional details

#### E.1 Occupation characteristics

Here we use the standardized characteristics of our thirty occupations, derived from O*NET following Acemoglu and Autor (2011) to have mean zero and standard deviation one as described in Appendix A, to understand how each shock shapes the observed evolution of labor income shares across occupations. The first row of Table 12 shows that patterns identified by a large literature hold when we consider our thirty occupations. Specifically, we regress the change in the share of labor income earned in each occupation between 1984 and 2003, measured using the MORG CPS, separately on six occupation characteristics and find a systematic contraction in occupations that are intensive in routine manual as well as non-routine manual physical tasks and a systematic expansion of occupations that are intensive in non-routine cognitive analytical, non-routine cognitive interpersonal, and socially perceptive tasks. Rows two through five replicate this exercise, but instead of using the change in labor income shares across occupations from the data, we use the change predicted by our model in response to each shock separately. Because $\rho \neq 1$, shocks other than occupation shifters generate changes in occupation income shares. We find that these other shocks play a significant role in accounting for the observed systematic evolution of occupation income shares over the years 1984-2003.

#### E.2 Worker aggregation

In theory we could incorporate as many labor groups, equipment types, and occupations as the data permits without complicating our measurement of shocks or our estimation of
parameters. In practice, we are constrained by data. Specifically, as we increase the number of labor groups, equipment types, or occupations we increase the share of \((l, k, w)\) triplets for which \(\pi_t(l, k, w) = 0\) and measurement error in factor allocations in general.

Our objective here is to understand the extent to which our particular disaggregation may be driving our results. To do so, we decrease the number of labor groups from 30 to 10 by dropping age as a characteristic. In this case, the share of \((l, k, w)\) observations for which \(\pi_t(l, k, w) = 0\) falls from (roughly) 27% to 12%. Because in our baseline we composition adjust the skill premium and the gender gap using gender, education, and age, whereas here we only use gender and education, we find slightly different changes in the skill premium, 16.1 instead of 15.1 log points, and the gender gap, -13.2 instead of -13.3 log points, between 1984 and 2003.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.173**</td>
<td>0.257**</td>
<td>-0.035</td>
<td>-0.273***</td>
<td>-0.214***</td>
<td>0.244***</td>
</tr>
<tr>
<td>Labor composition</td>
<td>0.038***</td>
<td>0.052***</td>
<td>0.010</td>
<td>-0.042***</td>
<td>-0.042***</td>
<td>0.032***</td>
</tr>
<tr>
<td>Occupation shifters</td>
<td>-0.044</td>
<td>0.061</td>
<td>-0.072</td>
<td>-0.093</td>
<td>-0.030</td>
<td>0.147***</td>
</tr>
<tr>
<td>Equipment prod.</td>
<td>0.068***</td>
<td>0.059***</td>
<td>0.042**</td>
<td>-0.063***</td>
<td>-0.073***</td>
<td>0.026</td>
</tr>
<tr>
<td>Labor productivity</td>
<td>0.002**</td>
<td>0.004***</td>
<td>-0.001</td>
<td>-0.004***</td>
<td>-0.005***</td>
<td>0.004***</td>
</tr>
</tbody>
</table>

Table 12: The evolution of labor income shares across occupations in the data and predicted separately by each shock. Each cell represents the coefficient estimated from a separate OLS regression across thirty occupations of the change in the income share between 1984 and 2003—either in the data or predicted in the model by each shock—on a constant and a single occupation characteristic derived from O*NET.

Non-routine cogn. anlyt. refers to Non-routine cognitive analytical; Non-routine cogn. inter. refers to Non-routine cognitive interpersonal; Routine cogn. refers to Routine cognitive; Non-routine man. phys. refers to Non-routine manual physical; and Social perc. refers to Social perceptiveness

*** Significant at the 1 percent level, ** Significant at the 5 percent level, * Significant at the 10 percent level

We conduct our decomposition with 10 labor groups using two different approaches. In both approaches we re-measure all shocks. However, in one approach we use our
baseline values of $\theta$ and $\rho$ estimated with 30 labor groups, $\theta = 1.78$ and $\rho = 1.78$, whereas in the other approach we re-estimate these parameters with 10 labor groups using our baseline estimation approach, yielding $\theta = 1.39$ and $\rho = 1.63$. We report results for both approaches and our baseline in Table 13.

Our baseline results are robust to decreasing the number of labor groups, suggesting that measurement error introduced from disaggregating into 30 labor groups is not driving our results. If anything, our key results seem stronger with 10 than with 30 labor groups. For example, the relative importance of labor productivity, the residual, in accounting for the rise of the skill premium and the fall of the gender gap falls relative to our baseline.

F Average wage variation within a labor group

Our baseline model implies that the average wage of workers in group $\lambda$ is the same across all equipment-occupation pairs. This implication is rejected by the data. In Section F.1 we argue that these differences in average wages across $(\kappa, \omega)$ do not drive our results. In Section F.2 we show that incorporating preference shifters for working in different occupations generates differences in average wages across occupations within a labor group and we show how to conduct a decomposition in this case.

F.1 Between-within decomposition

Here, we conduct a between and within decomposition of changes in the average wage of group $\lambda$, $w_t(\lambda)/w_t$, where $w_t$ is the composition-adjusted average wage across all labor groups. We consider variation in average wages within a labor group across occupations but not across equipment types, $w_t(\lambda, \omega)$, because the MORG CPS contains wage data for only a subset of observations (those respondents in the Outgoing Rotation Group).

The following accounting identity must hold at each $t$,

$$
\frac{w_t(\lambda)}{w_t} = \sum_{\omega} \frac{w_t(\lambda, \omega)}{w_t} \pi_t(\lambda, \omega).
$$

The previous expression implies

$$
\Delta \frac{w_t(\lambda)}{w_t} = \sum_{\omega} \Delta \frac{w_t(\lambda, \omega)}{w_t} \pi_t(\lambda, \omega) + \sum_{\omega} \frac{w_t(\lambda, \omega)}{w_t} \Delta \pi_t(\lambda, \omega)
$$

(20)

where $\Delta x_t = x_t - x_{t_0}$ and $\bar{x}_t = (x_t + x_{t_0})/2$. The first term on the right-hand side
of the equation (20) is the within component whereas the second term is the between component. According to the model, the contribution to changes in wages of the within component should be 100\% for each labor group.\footnote{Of course, this does not mean that changes in occupation shifters do not drive changes in wages in our model.} We conduct this decomposition using the MORG CPS data between 1984 and 2003 for each of 30 labor groups and find that the median contribution across labor groups of the within component is above 86\%. Hence, while in practice there are large differences in average wages for a labor group across occupations, these differences do not appear to be first order in explaining changes in labor group wages over time.

F.2 Compensating differentials

Here we extend our model to incorporate preference heterogeneity in addition to productivity heterogeneity. This simple extension implies that the average wage of workers in group $\lambda$ varies across equipment-occupation pairs. We show how to use data on average wages across equipment-occupation pairs to identify the parameters of the extended model.

**Environment and equilibrium.** The indirect utility function of a worker $z \in \mathcal{Z}$ earning income $I_t(z)$ and employed in occupation $\omega$ with equipment $\kappa$ is

$$U(z, \kappa, \omega) = I_t(z) u_t(\lambda, \kappa, \omega)$$

(21)

where $u_t(\lambda, \kappa, \omega) > 0$ is a time-varying preference shifter.\footnote{In this extended environment it is straightforward to allow for $u_t(\lambda, \kappa, \omega) = 0$, in which case no workers in group $\lambda$ would choose $(\kappa, \omega)$ in period $t$.} We have normalized the price index to one. We normalize $u_t(\lambda, \kappa_1, \omega_1) = 1$ for all $\lambda$ and $t$. This model limits to our baseline model when $u_t(\lambda, \omega, \kappa) = 1$ for all $t$ and $(\lambda, \kappa, \omega)$.

A occupation production unit hiring $k$ units of equipment $\kappa$ and $l$ efficiency units of labor $\lambda$ earns profits $p_t(\omega) k^a \left[T_t(\lambda, \kappa, \omega) l\right]^{1-a} - p_t(\kappa) k - v_t(\lambda, \kappa, \omega) l$. The profit maximizing choice of equipment quantity and the zero profit condition yield

$$v_t(\lambda, \kappa, \omega) = \tilde{a} p_t(\kappa) \tilde{t}^{-\frac{a}{1-a}} p_t(\omega) l^{-\frac{1}{1-a}} T_t(\lambda, \kappa, \omega)$$

if there is positive entry in $(\lambda, \kappa, \omega)$. Facing the wage profile $v_t(\lambda, \kappa, \omega)$, each worker $z \in \mathcal{Z}$ chooses $(\kappa, \omega)$ to maximize her indirect utility, $v_t(z, \kappa, \omega) u_t(\lambda, \kappa, \omega) v_t(\lambda, \kappa, \omega)$. In our extended model, preference parameters $u_t(\lambda, \kappa, \omega)$ and productivity parameters, $T_t(\lambda, \kappa, \omega)$, affect worker utility in the same way, as shown by the previous expres-
sion. Hence, they also affect worker allocation in the same way: the probability that a randomly sampled worker, \( z \in Z(\lambda) \), uses equipment \( \kappa \) in occupation \( \omega \) is

\[
\pi_t(\lambda, \kappa, \omega) = \frac{\sum_{\kappa', \omega'} \gamma(\lambda)}{u_t(\lambda, \kappa, \omega)} \left[ \sum_{\kappa', \omega'} \gamma(\lambda) \right]^{\theta(\lambda)}.
\] (22)

On the other hand, preferences and productivities affect wages differently. The average wage of workers \( z \in Z(\lambda) \) teemed with equipment \( k \) in occupation \( w \) is now given by

\[
w_t(\lambda, \kappa, \omega) = \frac{\gamma(\lambda)}{u_t(\lambda, \kappa, \omega)} \left( \sum_{\kappa', \omega'} \gamma(\lambda) \right)^{\theta(\lambda)}.
\] (23)

If \( u_t(\lambda, \kappa, \omega) > u_t(\lambda, \kappa', \omega') \), then the average wage of group \( \lambda \) is lower in \( (\kappa, \omega) \) than in \( (\kappa', \omega') \) in period \( t \).

The general equilibrium condition is identical to our baseline model and is given by equation (5), although total labor income in occupation \( \omega \) is now given by

\[
\zeta_t(\omega) \equiv \sum_{\lambda, \kappa} w_t(\lambda, \kappa, \omega) L_t(\lambda) \pi_t(\lambda, \kappa, \omega).
\]

**Parameterization.** Here, we focus on measuring preference shifters and shocks under the restriction that \( \theta(\lambda) = \theta \) for all \( \lambda \), taking \( \theta \) as given.

From equation (23), we have

\[
\frac{w_t(\lambda, \kappa, \omega)}{w_t(\lambda, \kappa_1, \omega_1)} = \frac{1}{u_t(\lambda, \kappa, \omega)}.
\] (24)

Hence, we measure preference shifters directly from average wages.

Equations (6) and (22) give us,

\[
\frac{\pi_t(\lambda, \kappa_2, \omega)}{\pi_t(\lambda, \kappa_1, \omega)} = \frac{u_t(\lambda, \kappa_2, \omega)^{\theta}}{u_t(\lambda, \kappa_1, \omega)^{\theta}} \frac{q_t(\kappa_2)^{\theta}}{q_t(\kappa_1)^{\theta}}
\]

which, together with equation (24), gives us

\[
\frac{q_t(\kappa_2)^{\theta}}{q_t(\kappa_1)^{\theta}} = \frac{\pi_t(\lambda, \kappa_2, \omega)}{\pi_t(\lambda, \kappa_1, \omega)} \frac{w_t(\lambda, \kappa_2, \omega)^{\theta}}{w_t(\lambda, \kappa_1, \omega)^{\theta}}
\]
Hence, we obtain

\[
\log \frac{\hat{q}(k_2)}{\hat{q}_k(k_1)}^\theta = \log \frac{\pi_{t_1}(\lambda, k_2, \omega)}{\pi_{t_1}(\lambda, k_1, \omega)} - \log \frac{\pi_{t_0}(\lambda, k_2, \omega)}{\pi_{t_0}(\lambda, k_1, \omega)} + \theta \log \frac{w_{t_1}(\lambda, k_2, \omega)}{w_{t_1}(\lambda, k_1, \omega)} - \theta \log \frac{w_{t_0}(\lambda, k_2, \omega)}{w_{t_0}(\lambda, k_1, \omega)}
\]

We can then average over all \((\lambda, \omega)\) and then exponentiate, exactly as in our baseline, to obtain a measure of changes in equipment productivity (to the power \(\theta\)). We obtain a measure of changes in transformed occupation prices to the power \(\theta\) similarly and use this to measure changes in occupation shifters using equation (11), as in our baseline approach. Finally, we have

\[
w_t(\lambda, k_1, \omega_1) = T_t(\lambda) \gamma(\lambda) \left( \sum_{k, \omega} \left[ u_t(\lambda, k, \omega) q_t(k) q_t(\omega) \right]^{\theta} \right)^{1/\theta}
\]

so that

\[
\bar{w}(\lambda, k_1, \omega_1) = \bar{T}(\lambda) \left( \sum_{k, \omega} \left( \hat{u}(\lambda, k, \omega) \hat{q}(k) \hat{q}(\omega) \right)^\theta \pi_{t_0}(\lambda, k, \omega) \right)^{1/\theta}
\]

Hence, given measures of changes in transformed occupation prices (to the power \(\theta\)), changes in equipment productivity (to the power \(\theta\)), and changes in preference shifters (obtained above) as well as observed changes in wages and observed allocations in period \(t_0\), we can measure changes in relative labor productivities using the previous expression for group \(\lambda\) relative to group \(\lambda_1\).

### G  Evolving comparative advantage: Details

Here we study case 2 described in Section 5.3, where

\[
T_t(\lambda, \kappa, \omega) \equiv T_t(\omega) T_t(\lambda, \kappa) T(\lambda, \kappa, \omega) .
\]

(25)

Cases 1 and 3 are similar and available upon request.

The equilibrium conditions are unchanged: equations (3), (4), and (5) hold as in our baseline model. However, we can re-express the system in changes as follows. Defining

\[
q_t(\lambda, \kappa) \equiv T_t(\lambda, \kappa) p_t(\kappa)^{\frac{\kappa}{\gamma}} ,
\]
equations (7) and (8) become
\[
\hat{\omega} (\lambda) = \left( \sum_{k,\omega} \pi_{t_0} (\lambda, \kappa, \omega) \left( \hat{q} (\lambda, \kappa) \hat{q} (\omega) \right)^{\theta (\lambda)} \right)^{1/\theta (\lambda)} \tag{26}
\]
\[
\hat{\pi} (\lambda, \kappa, \omega) = \frac{\left( \hat{q} (\omega) \hat{q} (\lambda, \kappa) \right)^{\theta (\lambda)}}{\sum_{k',\omega'} \left( \hat{q} (\omega') \hat{q} (\lambda, \kappa') \right)^{\theta (\lambda)} \pi_{t_0} (\lambda, \kappa', \omega')}, \tag{27}
\]
whereas equation (9) remains unchanged. Expressing equation (26) in relative terms yields
\[
\frac{\hat{\omega} (\lambda)}{\hat{\omega} (\lambda_1)} = \frac{\hat{q} (\lambda, \kappa_1)}{\hat{q} (\lambda_1, \kappa_1)} \left( \frac{\sum_{k,\omega} \pi_{t_0} (\lambda, \kappa, \omega) \left( \hat{q} (\lambda, \kappa) \hat{q} (\omega) \right)^{\theta (\lambda)}}{\sum_{k',\omega'} \left( \hat{q} (\omega') \hat{q} (\lambda, \kappa') \right)^{\theta (\lambda)} \pi_{t_0} (\lambda, \kappa', \omega')} \right)^{1/\theta (\lambda)} \tag{28}
\]
Hence, the decomposition requires that we measure \( \hat{q} (\lambda, \kappa) / \hat{q} (\lambda, \kappa_1) \) for each \( \lambda, \kappa \) as well as \( \hat{q} (\lambda, \kappa_1) / \hat{q} (\lambda_1, \kappa_1) \) for each \( \lambda \).

Here we provide an overview—similar in structure to that provided in Section 3.2—of how we measure shocks taking as given the parameters \( \alpha, \rho, \) and \( \theta \). Equations (3) and (25) give us
\[
\frac{\hat{q} (\lambda, \kappa_1)^{\theta}}{\hat{q} (\lambda, \kappa_2)^{\theta}} = \frac{\hat{\pi} (\lambda, \kappa_1, \omega)}{\hat{\pi} (\lambda, \kappa_2, \omega)}
\]
for each \( \lambda \) and \( \omega \). Hence, we can measure \( \hat{q} (\lambda, \kappa_1)^{\theta} / \hat{q} (\lambda, \kappa_2)^{\theta} \) for each \( \lambda \) as the exponential of the average across \( \omega \) of the log of the right-hand side of the previous expression. We can recover changes in transformed occupation prices to the power \( \theta \) and use these to measure changes in occupation shifters exactly as in our baseline. Finally, given these measures, we can recover \( \hat{q} (\lambda, \kappa_1)^{\theta} / \hat{q} (\lambda_1, \kappa_1)^{\theta} \) to match changes in relative wages using equation (28).

H Model with sectors: Details

Here we provide additional details on the closed economy extension with sectors. Sectors are indexed by \( \sigma \). The final good combines sectoral output, \( Y_t (\sigma) \), according to a CES production function,
\[
Y_t = \left( \sum_{\sigma} h_t (\sigma)^{1/\rho_s} Y_t (\sigma) (\rho_s - 1)/\rho_s \right)^{\rho_s/(\rho_s - 1)} \tag{29}
\]
where $\rho_s > 0$ is the elasticity of substitution across sectors and $\mu_t(\sigma) \geq 0$ is an exogenous demand shifter for sector $\sigma$. Sectoral output is itself a CES combination of the output of different occupations,

$$Y_t(\sigma) = \left( \sum_\omega \mu_t(\omega, \sigma)^{1/\rho} Y_t(\omega, \sigma)^{(\rho-1)/\rho} \right)^{\rho/(\rho-1)} \tag{30}$$

where $Y_t(\omega, \sigma) \geq 0$ denotes the absorption of occupation $\omega$ in the production of sector $\sigma$, $\mu_t(\omega, \sigma) \geq 0$ is an exogenous demand shifter for occupation $\omega$ in sector $\sigma$, and $\rho > 0$ is the elasticity of substitution across occupations within each sector.

Occupations are produced exactly as in our baseline specification: a worker’s productivity depends only on her occupation $\omega$, and not on her sector of employment $\sigma$.\(^{39}\)

Production and absorption must satisfy the resource constraint $Y_t(\omega) = \sum_\sigma Y_t(\omega, \sigma)$.

Because the partial equilibrium is the same as in our baseline model, the equations in levels determining the the allocations $\pi_t(\lambda, \kappa, \omega)$ and the average wage $w_t(\lambda)$ are the same as in the baseline model and are given by (3) and (4), respectively. The only change to the equilibrium equations (in levels) is to the occupation market clearing condition, which becomes

$$\sum_\sigma E_t(\omega, \sigma) = \frac{1}{1-\alpha} \zeta_t(\omega)$$

where $E_t(\omega, \sigma)$ denotes income (or expenditure) on occupation $\omega$ in sector $\sigma$,

$$E_t(\omega, \sigma) = \mu_t(\sigma) \mu_t(\omega, \sigma) p_t(\omega)^{1-\rho} p_t(\sigma)^{\rho-\rho_s} E_t$$

and $p_t(\sigma)$ denotes the price index of sector $\sigma$

$$p_t(\sigma) = \left( \sum_\omega \mu_t(\omega, \sigma) p_t(\omega)^{1-\rho} \right)^{1/1-\rho}$$

We now provide the system of equations in changes, analogous to equations (7)-(9) with which to calculate wage changes that result from changes in the primitives between periods $t_0$ and $t_1$. The expressions for changes in wages and in allocations are given, as in the baseline model, by (7) and (8), respectively. The right hand side of the occupation market

\(^{39}\)Alternatively, we could assume that worker productivity depends both on occupation and sector of employment, $T_t(\lambda, \kappa, \omega, \sigma) \in (z, \kappa, \omega, \sigma)$. Our estimation approach extends directly to this alternative assumption; however, in practice, the data may become sparse, in the sense that there might be many $(\lambda, \kappa, \omega, \sigma, t)$ for which $\pi_t(\lambda, \kappa, \omega, \sigma) = 0$. 

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clearing condition in changes is the same as in the baseline model,

\[ \hat{\xi} (\omega) = \frac{1}{\xi t0 (\omega)} \sum_{\lambda, \kappa} \hat{w} t0 (\lambda) L t0 (\lambda) \pi t0 (\lambda, \kappa, \omega) \hat{w} (\lambda) \hat{L} (\lambda) \hat{\alpha} (\lambda, \kappa, \omega). \quad (31) \]

The left hand side of the occupation market clearing condition in changes and the change in the sectoral price index are, respectively,

\[ \hat{p} (\omega)^{1-\rho} \hat{E} \sum_{\sigma} v t0 (\sigma | \omega) \hat{\mu} (\sigma) \hat{\mu} (\omega, \sigma) \hat{p} (\sigma)^{\rho - \rho e} = \hat{\xi} (\omega) \quad (32) \]

and

\[ \hat{p} (\sigma) = \left[ \sum_{\omega} \hat{v} t0 (\omega | \sigma) \hat{\mu} (\omega, \sigma) \hat{p} (\omega)^{1-\rho} \right]^{1/1-\rho} \quad (33) \]

Here, \( v t (\sigma | \omega) \equiv \frac{E t (\omega, \sigma)}{\sum_{\sigma} E t (\omega, \sigma)} \) denotes the share of expenditure on occupation \( \omega \) across all sectors that occurs within sector \( \sigma \) and \( v t (\omega | \sigma) = \frac{E t (\omega, \sigma)}{\sum_{\omega} E t (\omega, \sigma)} \) denotes the share of expenditure on occupation \( \omega \) across all occupations employed within sector \( \sigma \). Defining, as in the baseline model, changes in transformed within-sector occupation shifters \( \hat{a} (\omega, \sigma) = \hat{\mu} (\omega) \hat{T} (\omega)^{(1-\alpha)(\rho - 1)} \) and changes in transformed occupation prices \( \hat{q} (\omega) = \hat{p} (\omega)^{1/(1-\alpha)} \hat{T} (\omega) \) we can write (32) and (33) as

\[ \hat{q} (\omega)^{(1-\alpha)(1-\rho)} \hat{E} \sum_{\sigma} v t0 (\sigma | \omega) \hat{\mu} (\sigma) \hat{\mu} (\omega, \sigma) \hat{a} (\omega, \sigma) \hat{a} (\omega_1, \sigma) \hat{p} (\sigma)^{\rho - \rho e} = \hat{\xi} (\omega) \]

and

\[ \hat{p} (\sigma) = \hat{a} (\omega_1, \sigma)^{1/1-\rho} \left[ \sum_{\omega} \hat{v} t0 (\omega | \sigma) \hat{\mu} (\omega, \sigma) \hat{a} (\omega, \sigma) \hat{a} (\omega_1, \sigma) \hat{q} (\omega)^{(1-\rho)(1-\alpha)} \right]^{1/1-\rho} \]

Furthermore, defining changes in transformed sector prices \( \hat{q} (\sigma) = \hat{p} (\sigma) \hat{a} (\omega_1, \sigma)^{1-1/1-\rho} \) and changes in transformed sector shifters \( \hat{a} (\sigma) = \hat{a} (\omega_1, \sigma)^{1-1/1-\rho} \hat{\mu} (\sigma) \) we can re-write these two equations as

\[ \hat{q} (\omega)^{(1-\alpha)(1-\rho)} \hat{E} \sum_{\sigma} v t0 (\sigma | \omega) \hat{a} (\sigma) \hat{a} (\omega, \sigma) \hat{a} (\omega_1, \sigma) \hat{q} (\sigma)^{\rho - \rho e} = \hat{\xi} (\omega) \quad (34) \]

\[ \hat{q} (\sigma) = \left[ \sum_{\omega} v t0 (\omega | \sigma) \hat{a} (\omega, \sigma) \hat{a} (\omega_1, \sigma) \hat{q} (\omega)^{(1-\rho)(1-\alpha)} \right]^{1/1-\rho} \quad (35) \]

Therefore we can solve for changes in relative wages \( \hat{w} (\lambda) / \hat{w} (\lambda_1) \), relative occupation
We use equation (\( \hat{\eta} (\omega) \)) and relative sectoral prices \( \hat{\eta} (\sigma) \) using equations (7), (8), (31), (34) and (35), given shocks \( \hat{L} (\lambda) / \hat{L} (\lambda_1), \hat{T} (\lambda) / \hat{T} (\lambda_1) \), \( \hat{q} (\kappa) / \hat{q} (\kappa_1), \hat{a} (\omega, \sigma) / \hat{a} (\omega_1, \sigma) \) and \( \hat{a} (\sigma) / \hat{a} (\sigma_1) \). Note that when \( \rho = \rho_\sigma, \hat{q} (\sigma) \) drops out from equation (34) and the extension of our baseline model that accounts for sectors is equivalent to our baseline model where the occupation shifter in our baseline model, \( \hat{a} (\omega) \), is replaced by \( \sum_\sigma v_{\lambda_0} (\sigma | \omega) \hat{a} (\sigma) \hat{a} (\omega, \sigma) / \hat{a} (\omega_1, \sigma) \).

We measure \( \hat{T} (\lambda) / \hat{T} (\lambda_1), \hat{q} (\kappa) / \hat{q} (\kappa_1) \) and \( \hat{q} (\omega) / \hat{q} (\omega_1) \) between any two time periods using the same procedure and data as in our baseline model. To measure changes in transformed within-sector occupation shifters and transformed sector shifters, and to construct \( v_1 (\sigma | \omega) \) and \( v_1 (\omega | \sigma) \) we need data on \( E_t (\omega, \sigma) \) in \( t_0 \) and \( t_1 \). To measure within-sector occupation shifters, \( \hat{a} (\omega, \sigma) / \hat{a} (\omega_1, \sigma) \), we start from the equilibrium relationship

\[
\frac{\hat{E} (\omega, \sigma)}{\hat{E} (\omega_1, \sigma)} = \frac{\hat{a} (\omega, \sigma)}{\hat{a} (\omega_1, \sigma)} \left( \frac{\hat{p} (\sigma)}{\hat{p} (\omega)} \right)^{1-\rho}
\]

which can be re-expressed in terms of transformed shifters as

\[
\frac{\hat{E} (\omega, \sigma)}{\hat{E} (\omega_1, \sigma)} = \frac{\hat{a} (\omega, \sigma)}{\hat{a} (\omega_1, \sigma)} \left( \frac{\hat{p} (\sigma)}{\hat{p} (\omega_1)} \right)^{(1-\rho)(1-\rho)}.
\]

We use equation (36) to back-out \( \hat{a} (\omega, \sigma) / \hat{a} (\omega_1, \sigma) \). To estimate \( \hat{a} (\omega, \sigma) / \hat{a} (\omega_1, \sigma) \), we start from the equilibrium relationship

\[
\frac{\hat{E} (\sigma)}{\hat{E} (\sigma_1)} = \frac{\hat{\mu} (\sigma)}{\hat{\mu} (\sigma_1)} \left( \frac{\hat{p} (\sigma)}{\hat{p} (\sigma_1)} \right)^{1-\rho_\sigma}
\]

or in terms of transformed variables

\[
\frac{\hat{E} (\sigma)}{\hat{E} (\sigma_1)} = \frac{\hat{a} (\sigma)}{\hat{a} (\sigma_1)} \left( \frac{\hat{q} (\sigma)}{\hat{q} (\sigma_1)} \right)^{1-\rho_\sigma}.
\]

The previous expression and equation (35) yield

\[
\frac{\hat{E} (\sigma)}{\hat{E} (\sigma_1)} = \frac{\hat{\mu} (\sigma)}{\hat{\mu} (\sigma_1)} \left( \frac{\sum_\omega v_{\lambda_0} (\omega | \sigma) \hat{a} (\omega, \sigma) / \hat{a} (\omega_1, \sigma)}{\sum_\omega v_{\lambda_0} (\omega' | \sigma_1) \hat{a} (\omega', \sigma_1) / \hat{a} (\omega_1, \sigma_1)} \right)^{1-\rho_\sigma} \left( \frac{\hat{a} (\omega_1, \sigma)}{\hat{a} (\omega_1, \sigma_1)} \right)^{1-\rho_\sigma}.
\]

We use equation (38) to back-out \( \hat{a} (\sigma) / \hat{a} (\sigma_1) \).
I Model with international trade: Details

We first describe the full model with international trade, then provide details on the two types of counterfactuals we can conduct without solving for the full world general equilibrium or estimating parameters in any country other than the U.S., and finally show how to conduct these counterfactuals.

Setup. All variables are indexed by country, $n$, and we omit time subscripts for simplicity. We use $Y$ to indicate output and $D$ to indicate absorption; this distinction is required in the open economy but not in the closed economy. We assume that labor is internationally immobile.

The final good and sector production functions in country $n$, the open economy counterparts of equations (29) and (30), are given by

\[
Y_n = \left( \sum \mu_n (\sigma)^{1/\rho} D_n (\sigma)^{(\rho-1)/\rho} \right)^{\rho/(\rho-1)}
\]

\[
Y_n (\sigma) = \left( \sum \mu_n (\omega, \sigma)^{1/\rho} D_n (\omega, \sigma)^{(\rho-1)/\rho} \right)^{\rho/(\rho-1)}
\]

Country $n$ produces occupation, sector, and equipment output: $Y_n (\omega)$, $Y_n (\sigma)$, and $Y_n (\kappa)$, respectively. Its absorption of occupation, sector, and equipment goods is itself a CES aggregate of these goods sourced from all countries in the world. For example, absorption of occupation $\omega$ in country $n$ is

\[
D_n (\omega) = \left( \sum_i D_{in} (\omega)^{\eta(\omega)-1/\eta(\omega)} \right)^{\eta(\omega)/(\eta(\omega)-1)}
\]

where $D_{in} (\omega)$ is absorption in country $n$ of occupation $\omega$ sourced from country $i$, and $\eta (\omega) > 1$ is the elasticity of substitution across source countries for occupation $\omega$.\(^{40}\) Trade is subject to iceberg transportation costs, where $d_{ni} (x) \geq 1$ denotes units of $x$ output that must be shipped from origin country $n$ in order for one unit to arrive in destination country $i$. For example, output of occupation $\omega$ in country $n$ satisfies

\[
Y_n (\omega) = \sum_i d_{ni} (\omega) D_{ni} (\omega).
\]

\(^{40}\)We assume an Armington trade model only for expositional simplicity. Our results would also hold in a Ricardian model as in Eaton and Kortum (2002).
Without loss of generality for our results on relative wages, we abstract from trade in the consumption good. Production and absorption must also satisfy $Y_n = C_n + \sum_\kappa q_n(\kappa) Y_n(\kappa)$ and $D_n(\omega) = \sum_\sigma D_n(\omega, \sigma)$. As in our baseline model, $Y_n(\omega)$ is the sum of output across all workers employed in $\omega$. For the exercises we consider below, we do not need to specify conditions on trade balance in each country.

**Counterfactual exercises considered.** We consider two counterfactual exercises quantifying the impact of international trade on country $n$ that do not require solving the full world general equilibrium or assigning parameters in any country other than $n$. The first counterfactual quantifies the impact on wages in country $n$ at time $t$ if it were to move to autarky, holding all of country $n$’s parameters fixed. The second counterfactual answers the following question: what are the differential effects of changes in primitives (i.e. worldwide technologies, labor compositions, and trade costs) between periods $t_0$ and $t_1$ on wages in country $n$, relative to the effects of the same changes in primitives if country $n$ were a closed economy?

To understand these counterfactuals, define $w_n(\lambda; \Phi_t, \Phi_t^*, d_t)$ to be the average wage of labor group $\lambda$ in country $n$ given that country $n$ fundamentals are $\Phi_t$, fundamentals in the rest of the world are $\Phi_t^*$, and the full matrix of world trade costs (for equipment, sectors, and occupations) are $d_t$. Define $d_{n,t}^A$ to be an alternative matrix of world trade costs in which country $n$’s trade costs are infinite ($d_{n,t} = \infty$ for all $i \neq n$). Our first counterfactual calculates $\hat{w}_{n,t}(\lambda) \equiv w_n(\lambda; \Phi_t, \Phi_t^*, d_{n,t}^A) / w_n(\lambda; \Phi_t, \Phi_t^*, d_n, t)$. Our second counterfactual calculates

$$\frac{w_n(\lambda; \Phi_{t_1}, \Phi_{t_1}^*, d_{t_1})}{w_n(\lambda; \Phi_{t_0}, \Phi_{t_0}^*, d_{t_0})} \frac{w_n(\lambda; \Phi_t, \Phi_t^*, d_{n,t}^A)}{w_n(\lambda; \Phi_{t_1}, \Phi_{t_1}^*, d_{t_1}^A)}$$

which is simply $\hat{w}_{n,t_0}(\lambda) / \hat{w}_{n,t_1}(\lambda)$. Because the second counterfactual amounts to conducting the first counterfactual twice, at different points in time, we only explain how to solve the first counterfactual.

Here we derive the system of equations (analogous to equations (7), (8), and (9) in the closed economy model without sectors) that can be used to calculate changes in relative wages in some country $n$ at time $t_0$ when this country moves to autarky ($d_{n_1}(x)$ becomes infinite for all $i \neq n$ and all other primitives remain constant). We show that, moving to autarky, the equilibrium system of equations in an open economy is equivalent to the system that characterizes a closed economy with sectors as detailed in Appendix H. Here, however, changes in equipment productivity, sector shifters, and within-sector occupation shifters are induced by moving to autarky.
Conducting the counterfactuals. Variables with the superscript $A$ refer to counterfactual autarky values (holding all other parameters fixed such as productivities, primitive occupation and sector shifters, and labor composition at their time $t_0$ levels) and variables without the superscript $A$ refer to factual values in period $t_0$. Variables with hats denote the ratio of the value of this variable in autarky relative to the value of this variable at time $t_0$: $\hat{y} = y^A_{t_0} / y_{t_0}$. For simplicity, in this section we omit time indices.

In an open economy, we must distinguish between production prices and absorption prices. For example, we denote by $p_{in}(\omega)$ the price of country $i$’s output of occupation $\omega$ in country $n$ (inclusive of trade costs) and by $p_n(\omega)$ the absorption price of occupation $\omega$ in country $n$, given by

$$p_{in}(\omega) = \left[ \sum_i p_{in}(\omega)^{1-\eta(\omega)} \right]^{1/(1-\eta(\omega))};$$

output prices $p_{in}(\sigma), p_{in}(\kappa)$, and absorption prices $p_n(\sigma)$, and $p_n(\kappa)$ are defined analogously.

Changes in relative wages and in allocations depend on changes in absorption prices for equipment (since it is an input in production) and production prices for occupations (since occupations are produced in each country). Since productivities are assumed constant when moving to autarky, we set $\hat{q}(\kappa) = \hat{p}_n(\kappa)^{\frac{1}{1-\sigma}}$ and $\hat{q}(\omega) = \hat{p}_{nn}(\omega)^{\frac{1}{1-\kappa}}$, and equations (7) and (8) (moving to autarky) become

$$\hat{w}_{n}(\lambda) = \left\{ \sum_{i,\omega} \left[ \hat{p}_{nn}(\omega)^{\frac{1}{1-\sigma}} \hat{p}_n(\kappa)^{\frac{1}{1-\kappa}} \right]^{\theta(\lambda)} \pi_n(\lambda, \kappa, \omega) \right\}^{1/\theta(\lambda)} \quad (39)$$

$$\hat{\pi}_n(\lambda, \kappa, \omega) = \frac{\left[ \hat{p}_{nn}(\omega)^{\frac{1}{1-\sigma}} \hat{p}_n(\kappa)^{\frac{1}{1-\kappa}} \right]^{\theta(\lambda)}}{\sum_{\omega',\kappa'} \left[ \hat{p}_{nn}(\omega')^{\frac{1}{1-\sigma}} \hat{p}_n(\kappa')^{\frac{1}{1-\kappa}} \right]^{\theta(\lambda)} \pi_n(\lambda, \kappa', \omega')} \quad (40)$$

The remaining equations are the open-economy versions of the occupation-market clearing condition (9) (the analog of equation (32)) and the sectoral price index (the analog of equation (33)).

The right-hand side of equation (9) remains unchanged, so we focus on the left-hand side only. The level of worldwide absorption expenditure on country $n$’s produced occupation $\omega$ is $\sum_i E_{ni}(\omega)$, where $E_{ni}(\omega)$ denotes country $i$’s absorption expenditure of occupation $\omega$ from country $n$,

$$E_{ni}(\omega) = p_{ni}(\omega) D_{ni}(\omega) = \left( \frac{p_{ni}(\omega)}{p_i(\omega)} \right)^{1-\eta(\omega)} E_i(\omega)$$
In autarky, \( E_{ni}(\omega) = 0 \) for \( i \neq n \). Hence, the ratio of \( \sum_i E_{ni}(\omega) \) between autarky and \( t_0 \) (the left hand side of equation (9)) is

\[
\frac{\sum_i E_{ni}(\omega)}{\sum_i E_{ni}(\omega)} = \frac{E_{nn}(\omega)}{E_{nn}(\omega)} \frac{E_{nn}(\omega)}{E_{nn}(\omega)} = f_{nn}(\omega) \left( \frac{\hat{p}_{nn}(\omega)}{\bar{p}_n(\omega)} \right)^{1-\eta(\omega)} \hat{E}_n(\omega)
\]

where \( f_{nn}(\omega) = \frac{E_{nn}(\omega)}{\sum_i E_{ni}(\omega)} \) denotes the share of domestic sales of occupation \( \omega \) relative to its total sales (one minus the export share).

We now calculate an expression for \( \hat{E}_n(\omega) \), the change in total expenditure on absorption of occupation \( \omega \) in country \( n \). The level of \( E_n(\omega) \) is given by

\[
E_n(\omega) = \sum_\sigma E_n(\omega,\sigma)
\]

(41)

where \( E_n(\omega,\sigma) \) denotes country \( n \)'s absorption expenditures on occupation \( \omega \) in sector \( \sigma \) and is given by

\[
E_n(\omega,\sigma) = \mu_n(\omega,\sigma) \left( \frac{p_n(\omega)}{p_{nn}(\sigma)} \right)^{1-\rho} Y_n(\sigma) p_{nn}(\sigma)
\]

(42)

where \( p_{nn}(\sigma) = \left( \sum_\sigma \mu_n(\omega,\sigma) p_n(\omega)^{1-\rho} \right)^{1/(1-\rho)} \). The value of sector \( \sigma \) production in country \( n \) is

\[
Y_n(\sigma) p_{nn}(\sigma) = \sum_i Y_{ni}(\omega) d_{in}(\sigma) p_{nn}(\sigma) = \sum_i E_{ni}(\sigma)
\]

(43)

where \( E_{ni}(\sigma) \) denotes expenditures on absorption in country \( i \) of country \( n \)'s sector \( \sigma \) output, given by

\[
E_{ni}(\sigma) = \mu_i(\sigma) \left( \frac{p_{ni}(\sigma)}{p_i(\sigma)} \right)^{1-\eta(\sigma)} \left( \frac{p_i(\sigma)}{p_i} \right)^{1-\rho_e} E_i
\]

(44)

and \( E_i \) denotes total expenditures on the final good in country \( i \). Combining equations (42), (43), and (44) yields

\[
E_n(\omega,\sigma) = \mu_n(\omega,\sigma) \left( \frac{p_n(\omega)}{p_{nn}(\sigma)} \right)^{1-\rho} \sum_i \mu_i(\sigma) p_{ni}(\sigma)^{1-\eta(\sigma)} p_i(\sigma)^{\eta(\sigma)-\rho_e} p_i^{1-\rho_e} E_i.
\]

The ratio of \( E_n(\omega,\sigma) \) in autarky relative to its level at time \( t_0 \) is then

\[
\hat{E}_n(\omega,\sigma) = \hat{p}_n(\omega)^{1-\rho} \hat{p}_{nn}(\sigma)^{\rho_e} \left( \frac{\hat{p}_n(\sigma)}{\hat{p}_{nn}(\sigma)} \right)^{\eta(\sigma)-\rho_e} f_{nn}(\sigma) \hat{E}_n
\]
where $f_{nn}(\sigma)$ denotes the share of domestic sales of sector $\sigma$ relative to its total sales, the defined analogously to $f_{nn}(\omega)$. Equation (41) therefore yields

$$\hat{E}_n(\omega) = \sum_\sigma v_n(\sigma|\omega) \hat{E}_n(\omega, \sigma) = \sum_\sigma v_n(\sigma|\omega) \hat{p}_n(\omega)^{1-\rho} \hat{p}_{nn}(\sigma)^{\rho-\rho_e} \left( \frac{\hat{p}_n(\sigma)}{\hat{p}_{nn}(\sigma)} \right)^{\eta(\sigma) - \rho_e} f_{nn}(\sigma) \hat{E}_n.$$  

Combining these results, we have

$$\frac{\sum_i E_{ni}(\omega)}{\sum_i E_{ni}(\omega)} = f_{nn}(\omega) \hat{p}_n(\omega)^{1-\rho} \left( \frac{\hat{p}_{nn}(\omega)}{\hat{p}_n(\omega)} \right)^{1-\eta(\omega)} \sum_\sigma v_n(\sigma|\omega) \hat{p}_{nn}(\sigma)^{\rho-\rho_e} \left( \frac{\hat{p}_n(\sigma)}{\hat{p}_{nn}(\sigma)} \right)^{\eta(\sigma) - \rho_e} f_{nn}(\sigma) \hat{E}_n$$  

where the change in the production sectoral price index is

$$\hat{p}_n(\sigma) = \left( \sum_\omega v_n(\omega|\sigma) \hat{p}_n(\omega)^{1-\rho} \right)^{1/(1-\rho)}.$$  

Finally, we calculate the differential change in absorption and production prices, $\hat{p}_n(\omega) / \hat{p}_{nn}(\omega)$, $\hat{p}_n(\kappa) / \hat{p}_{nn}(\kappa)$, and $\hat{p}_n(\sigma) / \hat{p}_{nn}(\sigma)$. When moving to autarky at time $t_0$, the change in import prices is infinite. The change in the absorption price of occupation $\omega$, for example, is

$$\hat{p}_n(\omega) = \frac{p_{nn}(\omega) / p_{nn}(\omega)}{\left( \sum_i (p_{\text{in}}(\omega) / p_{nn}(\omega))^{1-\eta(\omega)} \right)^{1/(1-\eta(\omega))}} = \frac{\hat{p}_{nn}(\omega)}{s_{nn}(\omega)^{1/(\eta(\omega) - 1)}}$$  

where $s_{nn}(\omega)$ denotes expenditure on domestic occupation $\omega$ relative to total expenditure on occupation $\omega$ in country $n$ (one minus the import share),

$$s_{nn}(\omega) = \frac{p_{nn}(\omega) D_{nn}(\omega)}{\sum_i p_{\text{in}}(\omega) D_{\text{in}}(\omega)}.$$  

The second equality in (47) uses the following relationship between the prices of domestic and imported goods

$$(p_{\text{in}}(\omega) / p_{nn}(\omega))^{1-\eta(\omega)} = (p_{\text{in}}(\omega) D_{\text{in}}(\omega)) / (p_{nn}(\omega) D_{nn}(\omega)).$$

Similarly, changes in absorption prices of sector $\sigma$ are

$$\hat{p}_n(\sigma) = \frac{\hat{p}_{nn}(\sigma)}{s_{nn}(\sigma)^{1/(\eta(\sigma) - 1)}}.$$
where \( s_{nn} (\sigma) \) is defined analogously to \( s_{nn} (\omega) \). The change in the absorption price of \( \kappa \) is simply

\[
\hat{\rho}_n (\kappa) = s_{nn} (\kappa)^{\frac{1}{\eta(\kappa)}},
\]  

(49)

where \( s_{nn} (\kappa) \) is defined analogously to \( s_{nn} (\omega) \) and where have used the fact that \( \hat{\rho}_{nn} (\kappa) = 1 \) given our choice of numeraire.

We can substitute equation (49) directly into equations (39) and (40). Similarly, substituting (47) and (48) into (45) and (46) we have

\[
\frac{\sum_i E_{ni}^A (\omega)}{\sum_i E_{ni} (\omega)} = \hat{\rho}_{nn} (\omega)^{1-\rho} \frac{f_{nn} (\omega)}{s_{nn} (\omega)^{\frac{1}{\eta(\omega)} - 1}} \sum_{\sigma} v_n (\sigma|\omega) \frac{f_{nn} (\sigma)}{s_{nn} (\sigma)^{\frac{1}{\eta(\sigma)} - 1}} \hat{\rho}_{nn} (\sigma)^{\rho - \rho_o} \hat{E}_n
\]

and

\[
\hat{\rho}_{nn} (\sigma) = \left( \sum_{\omega} v_n (\omega|\sigma) s_{nn} (\omega)^{\frac{1}{\eta(\omega)} - 1} \hat{\rho}_{nn} (\omega)^{1-\rho} \right)^{1/(1-\rho)}.
\]  

(50)

In sum, the system of equations to solve for changes in factor allocations and relative prices when moving to autarky is given by

\[
\hat{\pi}_n (\lambda, \kappa, \omega) = \frac{\left[ \left( \hat{\rho}_{nn} (\omega) \right)^{\frac{1}{\eta(\omega)}} \frac{1}{\eta(\omega)} s_{nn} (\kappa)^{\frac{1}{\eta(\omega)} - 1} \right]^{\theta(\lambda)}}{\sum_{\omega', \kappa'} \left[ \left( \hat{\rho}_{nn} (\omega') \right)^{\frac{1}{\eta(\omega')}} \frac{1}{\eta(\omega')} s_{nn} (\kappa')^{\frac{1}{\eta(\omega')} - 1} \right]^{\theta(\lambda)} \pi_{nt0} (\lambda, \kappa', \omega')}
\]

\[
\hat{\omega}_n (\lambda) = \left\{ \sum_{\kappa, \omega} \left[ \left( \hat{\rho}_{nn} (\omega) \right)^{\frac{1}{\eta(\omega)}} \frac{1}{\eta(\omega)} s_{nn} (\kappa)^{\frac{1}{\eta(\omega)} - 1} \right]^{\theta(\lambda)} \pi_{nt0} (\lambda, \kappa, \omega) \right\}^{1/\theta(\lambda)}
\]

\[
\hat{\rho}_{nn} (\sigma) = \left( \sum_{\omega} v_{nt0} (\omega|\sigma) s_{nt0} (\omega)^{\frac{1}{\eta(\omega)} - 1} \left( \hat{\rho}_{nn} (\omega) \right)^{1-\rho} \right)^{1/(1-\rho)}
\]

and

\[
\left( \hat{\rho}_{nn} (\omega) \right)^{1-\rho} \frac{f_{nt0} (\omega)}{s_{nt0} (\omega)^{\frac{1}{\eta(\omega)} - 1}} \sum_{\sigma} v_n (\sigma|\omega) \frac{f_{nt0} (\sigma)}{s_{nt0} (\sigma)^{\frac{1}{\eta(\sigma)} - 1}} \hat{\rho}_{nn} (\sigma)^{\rho - \rho_o} \hat{E}
\]

\[
= \frac{1}{\hat{\xi}_{t0} (\omega)} \sum_{\lambda, \kappa} \omega_{t0} (\lambda) L_{t0} (\lambda) \pi_{t0} (\lambda, \kappa, \omega) \hat{\omega} (\lambda) \hat{L} (\lambda) \hat{\pi} (\lambda, \kappa, \omega)
\]

All variables in the previous four equations that are indexed by \( t_0 \) represent either their observed level or are constructed based on estimates. Note that this system of equations corresponds to the system of equations in the closed economy version of the model with sectors, where within-sector occupation shifters, \( \hat{a}_n (\omega, \sigma) \), and between-sector shifters,
\( \hat{a}_n (\sigma) \), are equal to

\[
\hat{a}_n (\omega, \sigma) \equiv f_{nt0} (\omega) s_{nt0} (\omega)^{\frac{\rho - \eta (\omega)}{\eta (\omega) - 1}} \\
\hat{a}_n (\sigma) \equiv f_{nt0} (\sigma) s_{nt0} (\sigma)^{\frac{\rho - \eta (\sigma)}{\eta (\sigma) - 1}} ,
\]

and changes in equipment costs are equal to

\[
\hat{q}_n (\kappa) = s_{nt0} (\kappa) \frac{1}{1 - \eta (\kappa)} \frac{-\alpha}{1 - \alpha} .
\]