Addendum to: International Trade, Technology, and the Skill Premium

Ariel Burstein
UCLA and NBER

Jonathan Vogel
Columbia and NBER

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Abstract

In this Addendum we set up a perfectly competitive version of the model and conduct a range of comparative static exercises under simplifying assumptions.

Compared to the model described in Section 2 of our main paper, we assume that there is a large number of firms in each variety (ω, j) in each country and that within a country all firms have access to the same productivity z. This implies that firms will price at marginal cost, exactly as in Eaton and Kortum (2002), henceforth EK.

Throughout this Addendum we maintain the following simplifying assumptions:

1. There are two countries, $I = \{1, 2\}$

2. Trade costs are symmetric, $\tau = \tau_{12} = \tau_{21}$

3. The elasticity of substitution between sectors is one, $\sigma = 1$

4. Trade is balanced, which implies $Y_i = P_i Q_i$

5. All sectors are tradeable, $\gamma_i = 1$ for all i and $J_M = J$

In what follows we only write the equations that differ between the model presented in Section 2 and the perfectly competitive model presented here.

Denote by $c_{in}(\omega, j)$ the unit cost of all country i variety (ω, j) firms supplying country n. Under perfect competition, the price of variety (ω, j) in country n is

$$p_n(\omega, j) = \min_i \{c_{in}(\omega, j)\},$$

exactly as in EK. Profits in each variety are zero with perfect competition and constant returns to scale. Hence, with balanced trade, the budget constraint in each country n satisfies

$$P_i Q_i = s_i H_i + w_i L_i.$$

Hicks-Neutral Technology

In this section we study a special version of the model: a standard H-O model extended to incorporate within- and across-sector productivity heterogeneity. We first show that our framework captures the key mechanisms of the H-O model: comparative advantage is shaped by cross-country differences in endowment ratios, a country is a net exporter in its comparative advantage sector, and trade raises the relative wage of a country's abundant factor. Unlike the standard H-O model, in which countries share identical technologies, comparative advantage is also shaped by productivity heterogeneity in our model. Second, we show how the extent of productivity heterogeneity shapes the response of relative wages to trade liberalization. Burstein and Vogel (2011) derive these results in a version of the model with monopolistically competitive firms similar to the one studied in Bernard, Redding, and Schott (2007).

In addition to assumptions 1-5 made above, we also make the following assumptions:

6. There are two sectors $J = \{x, y\}$ with $\alpha_y < \alpha_x$

7. Production functions are Cobb Douglas, $\rho = 1$

8. $\phi = 1/2$

Throughout this section we will also assume that country 1 has a comparative advantage in sector x. This is without loss of generality, as the condition on parameters (provided below) under which this is true must be satisfied for one of the two countries. Before presenting a set of results, we show how the model simplifies under these assumptions. With either $\rho = 1$ or $\phi = 1/2$, technology is Hicks-neutral, $\varphi = 0$. Skilled labor's share of revenue in sector j is equal to α_j. $c_{in}(\omega, j)$ simplifies to

$$c_{in}(\omega, j) = \frac{\tau_{in}}{z} v_i(j),$$

where

$$v_i(j) = \frac{1}{\alpha_j^{\alpha_j}(1-\alpha_j)^{(1-\alpha_j)}} \frac{1}{A_i(j)} s_i^{\alpha_j} w_i^{1-\alpha_j}$$

represents the unit cost of production for a producer with productivity $z = 1$. As in EK, we obtain simple analytic expressions that characterize the probability that country i supplies country n with an arbitrary variety (ω, j). This probability, denoted by $\pi_{in}(j)$, is

$$\pi_{in}(j) = \frac{(\tau_{in} v_i(j))^{-1/\theta}}{(\tau_{in} v_i(j))^{-1/\theta} + (\tau_{-in} v_{-i}(j))^{-1/\theta}}. \tag{1}$$

EK show that $\pi_{in}(j)$ is also equal to the share of country n's expenditure in sector j that is allocated to goods purchased from country i,

$$\pi_{in}(j) = X_{in}(j) \Big/ \sum_{k=1}^{2} X_{kn}(j),$$

where $X_{in}(j)$ are the sales of country i varieties of sector j in country n. This implies that the amount of unskilled (and skilled) labor used in country i sector j to supply country n can be written as a simple function of factor prices, aggregate prices, and aggregate quantities. Labor market clearing conditions for unskilled and skilled labor are simply

$$w_i L_i = \frac{1}{2} \sum_{n=1}^{2} \sum_{j=\{x,y\}} (1-\alpha_j) \pi_{in}(j) Q_n P_n \tag{2}$$

$$s_i H_i = \frac{1}{2} \sum_{n=1}^{2} \sum_{j=\{x,y\}} \alpha_j \pi_{in}(j) Q_n P_n, \tag{3}$$

where we have used the fact that $X_{in}(j) = \pi_{in}(j) P_n Q_n$.

In this specification of the model, wages can be expressed in terms of what is called the factor content of trade, which we define below. Denote by $NX_i(L)$ and $NX_i(H)$ the units of unskilled and skilled labor, respectively, embodied in country i's net exports. That is,

$$\begin{aligned} NX_i(L) &= \sum\nolimits_j L_i(j)\,\omega_i(j) \\ NX_i(H) &= \sum\nolimits_j H_i(j)\,\omega_i(j), \end{aligned}$$

where $L_i(j)$ and $H_i(j)$ denote the employment of unskilled and skilled labor in country i sector j, respectively, and where $\omega_i(j)$ equals the ratio of country i's net exports in sector j to country i's total revenue in sector j,

$$\omega_i(j) = \frac{X_{i-i}(j) - X_{-ii}(j)}{X_{ii}(j) + X_{i-i}(j)}.$$

Equations (2) and (3) and the definitions of $NX_i(L)$ and $NX_i(H)$ imply

$$\begin{aligned} w_i &= \frac{1}{L_i - NX_i(L)}\frac{1}{2}\sum\nolimits_j \alpha_j Q_i P_i \\ s_i &= \frac{1}{H_i - NX_i(H)}\frac{1}{2}\sum\nolimits_j (1-\alpha_j)\, Q_i P_i. \end{aligned}$$

Hence, we obtain

$$\frac{s_i}{w_i} = \frac{L_i - NX_i(L)}{H_i - NX_i(H)}\frac{\sum_j (1-\alpha_j)}{\sum_j \alpha_j}. \tag{4}$$

Burstein and Vogel (2011) derive a generalized version of equation (4) under much less restrictive conditions. Equation (4) gives us a simple relationship between the skill premium in countries 1 and 2,

$$\frac{s_1}{w_1}\Big/\frac{s_2}{w_2} = \frac{L_1 - NX_1(L)}{H_1 - NX_1(H)}\Big/\frac{L_2 - NX_2(L)}{H_2 - NX_2(H)}. \tag{5}$$

Finally, in order to study the effects of trade on the skill premium under the assumptions imposed in this section, we introduce the concept of comparative advantage. We say that country 1 has a comparative advantage in sector x if, in autarky and for a common z, country 1's unit cost of production in sector x relative to sector y is relatively lower than country 2's: i.e., $\frac{v_1(x)}{v_1(y)} \le \frac{v_2(x)}{v_2(y)}$. In general, we have

$$\frac{v_1(x)}{v_1(y)} \le \frac{v_2(x)}{v_2(y)} \Leftrightarrow a\left(\frac{s_2}{w_2}\Big/\frac{s_1}{w_1}\right)^{\alpha_x - \alpha_y} \ge 1 \tag{6}$$

where the equivalence follows from the definition of $v_i(j)$ and the definition

$$a = \frac{A_1(x)\,A_2(y)}{A_1(y)\,A_2(x)},$$

where a indexes country 1's Ricardian comparative advantage (if $a > 1$) or disadvantage (if $a < 1$) in sector x. Note that in autarky $NX_i(L) = NX_i(H) = 0$. Hence, according to equation (5) and inequality (6), country 1 has a comparative advantage in sector x if and only if

$$a\left[(H_1/L_1)/(H_2/L_2)\right]^{\alpha_x-\alpha_y} \geq 1. \tag{7}$$

Note that Condition (7) is a strict generalization of comparative advantage in the Ricardian and H-O models. If $a = 1$, so that there is no Ricardian comparative advantage, then country 1 has a comparative advantage in sector x if and only if $H_1/L_1 \geq H_2/L_2$, exactly as in the Heckscher-Ohlin model. If endowment ratios are the same across countries, $H_1/L_1 = H_2/L_2$, so that there is no H–O-based comparative advantage, then country 1 has a comparative advantage in sector x if and only if $a \geq 1$, exactly as in the Ricardian model. We impose Condition (7) throughout this section; this is obviously without loss of generality.

While comparative advantage is defined as a condition on relative costs in autarky, it is straightforward to show that if country 1 has a comparative advantage in sector x, then $v_1(x)/v_1(y) \leq v_2(x)/v_2(y)$ also holds in any trade equilibrium.

Lemma 1 *Under the assumptions of this section, $v_1(x)/v_1(y) \leq v_2(x)/v_2(y)$.*

Proof. The proof requires one preliminary step.

Preliminary Step. *Country i has positive net exports in sector x if and only if $NX_i(H) > 0 > NX_i(L)$.*

We can re-express $NX_i(L)$ as

$$w_i NX_i(L) = \sum\nolimits_j \frac{w_i L_i(j)}{X_{ii}(j) + X_{i-i}(j)}\left[X_{i-i}(j) - X_{-ii}(j)\right]. \tag{8}$$

Together with balanced trade, which implies

$$X_{i-i}(x) - X_{-ii}(x) = X_{-ii}(y) - X_{i-i}(y),$$

and Cobb-Douglas production functions, which imply

$$1 - \alpha_j = \frac{w_i L_i(j)}{X_{ii}(j) + X_{i-i}(j)},$$

equation (8) yields

$$w_i NX_i(L) = (\alpha_y - \alpha_x)\left[X_{i-i}(x) - X_{-ii}(x)\right].$$

Since $\alpha_y < \alpha_x$, we obtain the result that $NX_i(L) < 0$ if and only if country i is a net exporter in sector x (i.e. if and only if $X_{i-i}(x) > X_{-ii}(x)$). Similarly,

$$s_i NX_i(H) = (\alpha_x - \alpha_y)\left[X_{i-i}(x) - X_{-ii}(x)\right],$$

so that $NX_i(H) > 0$ if and only if country i is a net exporter in sector x. This concludes the proof of the Preliminary Step.

We now proceed by contradiction. Suppose that $v_1(x)/v_1(y) > v_2(x)/v_2(y)$. This is equivalent to $\pi_{1n}(x) < \pi_{1n}(y)$ and $\pi_{2n}(x) > \pi_{2n}(y)$ for $n = 1, 2$ according to equation (1). By balanced trade, this implies $\omega_1(x), \omega_2(y) < 0$ and $\omega_1(y), \omega_2(x) > 0$. Hence, we have $X_{12}(x) < X_{21}(x)$ and $X_{12}(y) > X_{21}(y)$. By the Preliminary Step we therefore have $NX_1(L) > 0 > NX_1(H)$ and $NX_2(L) < 0 < NX_2(H)$. Together with equation (5), this implies

$$\frac{s_1}{w_1}\Big/\frac{s_2}{w_2} < \frac{L_1}{H_1}\Big/\frac{L_2}{H_2}.$$

The previous inequality gives us

$$\frac{v_1(x)}{v_1(y)}\Big/\frac{v_2(x)}{v_2(y)} = \frac{1}{a}\left(\frac{s_1}{w_1}\Big/\frac{s_2}{w_2}\right)^{\alpha_x-\alpha_y} < \frac{1}{a}\left(\frac{L_1}{H_1}\Big/\frac{L_2}{H_2}\right)^{\alpha_x-\alpha_y} \leq 1,$$

a contradiction. **QED.** ■

We are now equipped to study the effects of trade liberalization on the skill premium. Starting in autarky, a reduction in trade costs leads factors to reallocate towards the skill-intensive x sector in country 1 and towards the unskill-intensive y sector in country 2. This increases the relative demand and, therefore, the relative price of skilled labor in country 1 and unskilled labor in country 2. We refer to this force as the H-O effect. This result is summarized in the following Proposition.

Proposition 1 *Under the assumptions of this section, moving from autarky to any positive level of trade increases s_1/w_1 and decreases s_2/w_2.*

Proof. This proposition follows directly from the Lemma 1. According to the Lemma 1, $v_1(x)/v_1(y) \leq v_2(x)/v_2(y)$ in any equilibrium. $v_1(x)/v_1(y) \leq v_2(x)/v_2(y)$ implies $\pi_{1n}(x) \geq \pi_{1n}(y)$ and $\pi_{2n}(x) \leq \pi_{2n}(y)$ for $n = 1, 2$ according to equation (1). Hence, country 1 (2) has weakly positive net exports in sector x (sector y). According to the Preliminary Step in the proof of Lemma 1, this implies $NX_1(H) \geq 0 \geq NX_1(L)$ and $NX_2(H) \leq 0 \leq NX_2(L)$. Combined with equation (4), this implies

$$\frac{s_1}{w_1} > \frac{L_1}{H_1}\frac{\sum_j(1-\alpha_j)}{\sum_j \alpha_j} \text{ and } \frac{s_2}{w_2} < \frac{L_2}{H_2}\frac{\sum_j(1-\alpha_j)}{\sum_j \alpha_j},$$

where $\frac{L_1}{H_1}\frac{\sum_j(1-\alpha_j)}{\sum_j \alpha_j}$ and $\frac{L_2}{H_2}\frac{\sum_j(1-\alpha_j)}{\sum_j \alpha_j}$ equal the skill premia in countries 1 and 2 in autarky, respectively. This concludes the proof of Proposition 1. **QED.** ■

When there are no productivity differences between sectors and between varieties within sectors, our model is similar to the Heckscher-Ohlin model, in which the location of production of each variety is determined solely by trade costs and factor endowments. In this case, Proposition 1 captures what is often called the Stolper-Samuelson effect. However, in our model a given variety's location of production is determined not only by trade costs and factor endowments, but also by sectoral productivities, $A_i(j)$s, and variety-specific idiosyncratic productivities, zs. A higher dispersion of productivities across varieties within a sector (a higher θ) increases the relative importance of the idiosyncratic component of production costs. Intuitively, if θ is very high, then in any variety (ω, j) one country is likely to have a much higher productivity than the other, and this country is likely to export (ω, j) even if it has a comparative disadvantage in sector j.

On the other hand, a higher value of a increases the relative importance of the systematic Ricardian component of comparative advantage (given that country 1 has a comparative advantage in the x sector). Intuitively, if a is very high, then country 1's comparative advantage in the x sector is likely to be sufficiently strong to overcome even large idiosyncratic productivity disadvantages in a given vareity (ω, x) within sector x, so that country 1 is likely to export this variety. The following proposition confirms this intuition.[1]

Proposition 2 *Under the assumptions of this section, if τ and T_1/T_2 are chosen to match fixed values of trade shares, then the increase in the s_1/w_1 and the decrease in s_2/w_2 caused by moving from autarky to these trade shares is decreasing in θ and increasing in a.*

Proof. After setting out the necessary notation we provide two preliminary steps before proving the proposition. Denote by $\Delta_i = \frac{1}{2}\left[\pi_{-ii}(x) + \pi_{-ii}(y)\right]$ for $i = 1, 2$ country i's expenditure share of trade and by $\Delta_3 = \pi_{21}(y) - \pi_{21}(x)$ the difference in import shares between sector x and sector y in country 1.

Step 1. *If τ, τ', a, a', θ, and θ' are chosen such that $\Delta_3 > \Delta_3' \geq 0$, $\Delta_1 = \Delta_1' > 0$, and $\Delta_2 = \Delta_2' > 0$, then the following conditions are satisfied: (i) $\pi_{12}(x) > \pi_{12}'(x)$, (ii) $\pi_{12}(y) < \pi_{12}'(y)$, (iii) $\pi_{21}(x) < \pi_{21}'(x)$, (iv) $\pi_{21}(y) > \pi_{21}'(y)$, (v) $s_1'/w_1' < s_1/w_1$, and (vi) $s_2'/w_2' > s_2/w_2$.*

[1] In Proposition 2 we hold trade shares constant, rather than holding trade costs constant, while varying θ and a for the following reason. As we increase θ holding trade costs constant, the impact on the the skill premium is ambiguous because trade shares rise and greater volumes of trade tend to strengthen the H-O effect, all else equal.

We first show that $\Delta_3 > \Delta_3' \geq 0$, $\Delta_1 = \Delta_1' > 0$, and $\Delta_2 = \Delta_2' > 0$ imply conditions $(i) - (iv)$. Conditions (iii) and (iv) follow directly from

$$\pi_{21}(x) = \Delta_1 - \frac{1}{2}\Delta_3 < \Delta_1 - \frac{1}{2}\Delta_3' = \pi_{21}'(x) \tag{9}$$

and

$$\pi_{21}(y) = \Delta_1 + \frac{1}{2}\Delta_3 > \Delta_1 + \frac{1}{2}\Delta_3' = \pi_{21}'(y). \tag{10}$$

respectively. Equations (1), (9), and (10) yield

$$\left(\frac{v_2(x)}{v_1(x)}\right)^{\frac{-1}{\theta}} \left(\frac{v_1'(x)}{v_2'(x)}\right)^{\frac{-1}{\theta'}} < \frac{(\tau')^{-1/\theta'}}{\tau^{-1/\theta}} < \left(\frac{v_2(y)}{v_1(y)}\right)^{\frac{-1}{\theta}} \left(\frac{v_1'(y)}{v_2'(y)}\right)^{\frac{-1}{\theta'}}.$$

Equation (1) and $\Delta_2 = \Delta_2'$ give us

$$\gamma_1 \left[1 - \frac{(\tau')^{\frac{-1}{\theta'}}}{\tau^{\frac{-1}{\theta}}} \left(\frac{v_2(x)}{v_1(x)}\right)^{\frac{-1}{\theta}} \left(\frac{v_1'(x)}{v_2'(x)}\right)^{\frac{-1}{\theta'}}\right] = \gamma_2 \left[\frac{(\tau')^{\frac{-1}{\theta'}}}{\tau^{\frac{-1}{\theta}}} \left(\frac{v_2(y)}{v_1(y)}\right)^{\frac{-1}{\theta}} \left(\frac{v_1'(y)}{v_2'(y)}\right)^{\frac{-1}{\theta'}} - 1\right]$$

where $\gamma_1 > 0$ and $\gamma_2 > 0$ are functions of τ, τ', and $v_i(j)$ for $i = 1, 2$ and $j = x, y$. The two previous equations yield

$$\gamma_1 \left[1 - \frac{(\tau')^{\frac{-1}{\theta'}}}{\tau^{\frac{-1}{\theta}}} \left(\frac{v_2(y)}{v_1(y)}\right)^{\frac{-1}{\theta}} \left(\frac{v_1'(y)}{v_2'(y)}\right)^{\frac{-1}{\theta'}}\right] < -\gamma_2 \left[1 - \frac{(\tau')^{\frac{-1}{\theta'}}}{\tau^{\frac{-1}{\theta}}} \left(\frac{v_2(y)}{v_1(y)}\right)^{\frac{-1}{\theta}} \left(\frac{v_1'(y)}{v_2'(y)}\right)^{\frac{-1}{\theta'}}\right]$$

which gives us

$$\frac{(\tau')^{\frac{-1}{\theta'}}}{\tau^{\frac{-1}{\theta}}} > \left(\frac{v_1(y)}{v_2(y)}\right)^{\frac{-1}{\theta}} \left(\frac{v_2'(y)}{v_1'(y)}\right)^{\frac{-1}{\theta'}}. \tag{11}$$

Equation (1) and inequality (11) yield condition (ii), which, in turn, implies condition (i). To conclude Step 1, note that the skill premium in country 1 can be expressed as

$$\frac{s_1}{w_1} = \frac{L_1}{H_1} \frac{\alpha_x R_1(x) + \alpha_y R_1(y)}{(1-\alpha_x) R_1(x) + (1-\alpha_y) R_1(y)},$$

where $R_i(j) = X_{ii}(j) + X_{i-i}(j)$ denotes country i's revenue in sector j. Hence, $s_1/w_1 > s_1'/w_1'$ if and only if $R_1(x) R_1'(y) > R_1'(x) R_1(y)$. $\Delta_1 = \Delta_1'$ and $\Delta_2 = \Delta_2'$ imply $Q_1P_1/Q_2P_2 = Q_1'P_1'/Q_2'P_2' = \Delta_2/\Delta_1$ for $i = 1, 2$. Therefore, $R_1(j) = Q_2P_2[\pi_{12}(j) + \pi_{11}(j)\Delta_2/\Delta_1]$ and $R_1'(j) = Q_2'P_2'[\pi_{12}'(j) + \pi_{11}'(j)\Delta_2/\Delta_1]$. Together with the definition of $R_1(j)$ and $R_1'(j)$, Conditions $(i) - (iv)$ imply $R_1(x) R_1'(y) > R_1'(x) R_1(y)$, concluding the proof of condition (v). The proof of condition (vi) is identical. This concludes the proof of Step 1.

Step 2. $\Delta_3 > \Delta_3' \geq 0$, $\Delta_1 = \Delta_1' > 0$, *and* $\Delta_2 = \Delta_2' > 0$ *imply*

$$\left(\frac{1}{a'} \times \left(\frac{s_1'/w_1'}{s_2'/w_2'}\right)^{\alpha_x - \alpha_y}\right)^{1/\theta'} > \left(\frac{1}{a} \times \left(\frac{s_1/w_1}{s_2/w_2}\right)^{\alpha_x - \alpha_y}\right)^{1/\theta}. \tag{12}$$

Condition (i) in Step 1, together with equation (1), implies

$$(\tau')^{-1/\theta'} v_2(x)^{-1/\theta} v_1'(x)^{-1/\theta'} < \tau^{-1/\theta} v_2'(x)^{-1/\theta'} v_1(x)^{-1/\theta}$$

while condition (ii) in Step 1, together with equation (1), implies

$$(\tau')^{-1/\theta'} v_2(y)^{-1/\theta} v_1'(y)^{-1/\theta'} > \tau^{-1/\theta} v_2'(y)^{-1/\theta'} v_1(y)^{-1/\theta}.$$

The two previous inequalities give us

$$\frac{v_2(x)^{-1/\theta} v_1'(x)^{-1/\theta'}}{v_2(y)^{-1/\theta} v_1'(y)^{-1/\theta'}} < \frac{v_2'(x)^{-1/\theta'} v_1(x)^{-1/\theta}}{v_2'(y)^{-1/\theta'} v_1(y)^{-1/\theta}},$$

which is equivalent to inequality (12), concluding the proof of Step 2.

We now prove Proposition 2. We first prove the comparative static result for θ. We proceed by contradiction. Suppose that $\theta > \theta'$, $a = a'$, and that $\Delta_3 > \Delta_3' > 0$. Then

$$\left(\frac{1}{a'}\left(\frac{s_1'/w_1'}{s_2'/w_2'}\right)^{(\alpha_x - \alpha_y)}\right)^{\theta/\theta'} > \frac{1}{a'}\left(\frac{s_1/w_1}{s_2/w_2}\right)^{(\alpha_x - \alpha_y)} \geq \frac{1}{a'}\left(\frac{s_1'/w_1'}{s_2'/w_2'}\right)^{(\alpha_x - \alpha_y)} \tag{13}$$

where the first inequality follows from inequality (12) and $a = a'$ while the second weak inequality follows from parts (v) and (vi) of Step 1. Inequality (13) and $\theta > \theta'$ contradicts $\Delta_3' \geq 0$, which implies

$$\frac{1}{a'} \times \left(\frac{s_1'/w_1'}{s_2'/w_2'}\right)^{\alpha_x - \alpha_y} \leq 1.$$

Thus, if $a = a'$ and $\theta > \theta'$, then $\Delta_3 \leq \Delta_3'$. Combined with conditions (v) and (vi) in Step 1, this yields the desired comparative static results for θ.

Next, we prove the comparative static result for a. We proceed by contradiction. Suppose that $\theta = \theta'$, $a \leq a'$, and $\Delta_3 > \Delta_3' > 0$. Then inequality (12) yields

$$\frac{1}{a'} \times \left(\frac{s_1'/w_1'}{s_2'/w_2'}\right)^{(\alpha_x - \alpha_y)} > \frac{1}{a} \times \left(\frac{s_1/w_1}{s_2/w_2}\right)^{(\alpha_x - \alpha_y)}. \tag{14}$$

With $a \leq a'$, inequality (14) requires $\frac{s_1'/w_1'}{s_2'/w_2'} > \frac{s_1/w_1}{s_2/w_2}$, which contradicts conditions (v) and (vi) in Step 1. Thus, if $\theta = \theta'$ and $a \leq a'$, then $\Delta_3 \leq \Delta_3'$. Combined with conditions (v) and

(vi) in Step 1, this yields the desired comparative static results for a. **QED.** ■

Skill-Biased Technology

In this section we focus on a one-sector, symmetric country model in which technology is skill biased. Under certain assumptions, we show that a reduction in trade raises the skill premium. Specifically, in addition to assumptions 1-5 presented above, we also make the following assumptions:

6. There is one sector, $J = 1$

7. The sector-level aggregator is Cobb Douglas, $\eta = 1$

8. Technology is skilled biased, $\varphi > 0$

9. Countries are symmetric: $H = H_n$ and $L = L_n$ for $n = 1, 2$ and $A_1(j) = A_2(j) = 1$.

The assumptions that countries are symmetric and that there is a single sector allow us to abstract from the H-O effect and isolate the skill-biased technology effect; it also implies that $s_i = s$ and $w_i = w$ for $i = 1, 2$. The assumption that $\eta = 1$ simplifies the algebra: a consequence of $\eta = 1$ is that, in the factor demand equations, the direct effect of a reduction in trade costs—less labor is required to sell a given quantity of output in the foreign market—and the indirect effect—falling export prices increase the quantity sold in export markets—exactly offset each other.

Under these assumptions, we can write the unit cost of country i firms supplying country n as

$$c_{in}(\omega; \tau) = \tau_{in} w z^{2(\phi-1)} \left[\alpha z^{2(2\phi-1)(\rho-1)} \left(\frac{s}{w}\right)^{1-\rho} + 1 - \alpha \right]^{\frac{1}{1-\rho}},$$

which we have written explicitly as a function of τ. Denote by $\chi_{in}(z; \tau) / \int_0^\infty \chi_{in}(k; \tau)\, dk$ the density of country i varieties supplying country n using productivity z, written explicitly as a function of trade costs. Let $\chi(z; \tau) = \chi_{ii}(z; \tau) + \chi_{i-i}(z; \tau)$, where $\chi(z; \tau)$ is a density because—with symmetric countries—we have $\chi_{i-i}(k; \tau) = \chi_{-ii}(k; \tau)$, so that $\int_0^\infty \left[\chi_{ii}(k; \tau)\, dk + \chi_{i-i}(k; \tau)\right] dk = 1$. With this notation, we can express the factor market clearing conditions as

$$\begin{aligned} wL &= \textstyle\int_0^\infty l(z, w, s/w)\, \chi(z; \tau)\, dz \\ sH &= \textstyle\int_0^\infty h(z, s, s/w)\, \chi(z; \tau)\, dz \end{aligned} \tag{15}$$

where

$$l(z, w, s/w) = \frac{1}{w} \left[1 + \frac{\alpha}{1-\alpha} z^{\varphi} \left(\frac{s}{w}\right)^{1-\rho} \right]^{-1} PQ \tag{16}$$

$$h(z, s, s/w) = \frac{1}{s} \left[1 + \left(\frac{1-\alpha}{\alpha}\right) z^{-\varphi} \left(\frac{s}{w}\right)^{\rho-1} \right]^{-1} PQ. \tag{17}$$

Note that, as discussed above, conditional on supplying a country, the amount of skilled and unskilled labor employed is independent of trade costs because $\eta = 1$. With skill-biased technology, we cannot solve explicitly for $\pi_{in}(j)$, unlike in the Hicks-neutral section above. However, we are able to obtain analytic comparative static results without this explicit solution.

If countries are symmetric and technology is Hicks-neutral, $\varphi = 0$, then reductions in the cost of trade do not affect the skill premium. On the other hand, if technology is skill biased, $\varphi > 0$, then reductions in the cost of trade increase the skill premium. The intuition behind this result is as follows. As in standard models with heterogeneous productivities (Ricardian or heterogeneous firm models), reductions in trade costs induce a reallocation of factors of production within sectors towards relatively productive firms. With skill-biased technology, relatively productive firms are also relatively skill intensive. Hence, trade liberalization increases the relative demand for skill and the skill premium. This result is summarized in Proposition 3.

Proposition 3 *Under the assumptions in this section, s/w is strictly decreasing in τ.*

Proof. We normalize $wL + sH = 1$ and we denote by $\Omega_{in}(\tau)$ the set of varieties in which country n is supplied by production in country i as a function of trade costs. Before proving the proposition, we first outline our proof strategy.

Proof outline: We prove the proposition by contradiction. In the first three steps of the proof, we prove that if trade costs rise from τ to $\tau' > \tau$ and the skill premium rises from s/w to $s'/w' \geq s/w$, then the new distribution of productivities $\chi(z;\tau')$ is first-order stochastically dominated by the original distribution of productivities $\chi(z;\tau)$. Given this result, we show that $\tau' > \tau$ and $s'/w' \geq s/w$ leads to a contradiction. Specifically, if $\chi(z;\tau)$ first-order stochastically dominates $\chi(z;\tau')$ and $s/w \leq s'/w'$, then the demand for unskilled labor must strictly rise from the original equilibrium to the new equilibrium. However, the supply of unskilled labor is fixed between equilibria, which yields our contradiction and proves our result.

Step 1: *If $\tau < \tau'$, $s'/w' \geq s/w$, $\omega \in \Omega_{ni}(\tau)$, and $\omega \in \Omega_{n'i}(\tau')$ for $n' \neq n$, then $z_n(\omega) \geq z_{n'}(\omega)$.*

There are two cases to consider: $\omega \in \Omega_{ii}(\tau)$ and $\omega \in \Omega_{-ii}(\tau)$. First consider $\omega \in \Omega_{ii}(\tau)$. In this case, we must have $\omega \in \Omega_{ii}(\tau')$. This is obvious if $z_i(\omega) \geq z_{-i}(\omega)$, since in this case $c_{ii}(\omega;\tau') < c_{-ii}(\omega;\tau')$ for any $\tau' > 1$. It is also true if $z_i(\omega) < z_{-i}(\omega)$, since

$$1 \geq \frac{c_{ii}(\omega;\tau)}{c_{-ii}(\omega;\tau)} > \frac{c_{ii}(\omega;\tau')}{c_{-ii}(\omega;\tau')},$$

where the first weak inequality follows from $\omega \in \Omega_{ii}(\tau)$ and the second strict inequality follows from $\tau' > \tau$, $s'/w' \geq s/w$, and $z_i(\omega) < z_{-i}(\omega)$. Since $c_{ii}(\omega;\tau') < c_{-ii}(\omega;\tau')$, we obtain $\omega \in \Omega_{ii}(\tau')$. Hence, if $\tau < \tau'$, $s'/w' \geq s/w$, and $\omega \in \Omega_{ii}(\tau)$, then $\omega \in \Omega_{ii}(\tau')$, in which case Step 1 holds vacuously. Second, consider $\omega \in \Omega_{-ii}(\tau)$. In this case, we must have $z_{-i}(\omega) \geq z_i(\omega)$; otherwise $c_{ii}(\omega;\tau) \leq c_{-ii}(\omega;\tau)$ for any $\tau \geq 1$, in which case $\omega \in \Omega_{ii}(\tau)$. Hence, if $\tau < \tau'$, $s'/w' \geq s/w$, and the firms that supply country i with variety ω switch from n to n' as τ rises to τ' (which, as we have just shown can only happen if $n = -i$ and $n' = i$) then we must have $z_n(\omega) \geq z_{n'}(\omega)$, which concludes the proof of Step 1.

Step 2: *If $\tau < \tau'$ and $s'/w' \geq s/w$, then there is a positive measure of ω satisfying $\omega \in \Omega_{-ii}(\tau)$, $\omega \in \Omega_{ii}(\tau')$, and $z_{-i}(\omega) > z_i(\omega)$.*

By Step 1, if $\omega \in \Omega_{-ii}(\tau)$ and $\omega \in \Omega_{ii}(\tau')$, then $z_{-i}(\omega) \geq z_i(\omega)$. Since $z_i(\omega)$ and $z_{-i}(\omega)$ have full support on $[1,\infty)$ and τ is finite, there exists a positive measure of $\omega \in \Omega_{-ii}(\tau)$. If $\tau' > \tau$ and $s'/w' \geq s/w$, then we can show that

$$\frac{c_{ii}(\omega;\tau')}{c_{-ii}(\omega;\tau')} < \frac{c_{ii}(\omega;\tau)}{c_{-ii}(\omega;\tau)}.$$

Given that $z_i(\omega)$ and $z_{-i}(\omega)$ are *i.i.d.* and have full support on $[1,\infty)$, for any $\tau' > \tau$ there exists a positive measure of ω for which

$$\frac{c_{ii}(\omega;\tau')}{c_{-ii}(\omega;\tau')} < 1 < \frac{c_{ii}(\omega;\tau)}{c_{-ii}(\omega;\tau)}.$$

Hence, there is a positive measure of ω satisfying $\omega \in \Omega_{-ii}(\tau)$, $\omega \in \Omega_{ii}(\tau')$, and $z_{-i}(\omega) \geq z_i(\omega)$. Finally, since $z_i(\omega)$ and $z_{-i}(\omega)$ are *i.i.d.* and atomless, there is a measure zero of ω satisfying $z_{-i}(\omega) = z_i(\omega)$. Hence, there is a positive measure of ω satisfying $\omega \in \Omega_{-ii}(\tau)$, $\omega \in \Omega_{ii}(\tau')$, and $z_{-i}(\omega) > z_i(\omega)$, concluding the proof of Step 2.

Step 3: *If $\tau < \tau'$ and $s'/w' \geq s/w$, then $\int_0^z \chi(v;\tau)\,dv \leq \int_0^z \chi(v;\tau')\,dv$ for all $z > 0$ and $\int_0^z \chi(v;\tau)\,dv < \int_0^z \chi(v;\tau')\,dv$ for z sufficiently large.*

We can write $\int_0^z \chi(v;\tau)\,dv$ as

$$\int_0^z \chi(v;\tau)\,dv = \Pr[z_i(\omega) < z \;\&\; \omega \in \Omega_{ii}(\tau)] + \Pr[z_{-i}(\omega) < z \;\&\; \omega \in \Omega_{-ii}(\tau)] \tag{18}$$

where we used the fact that $\Pr[z_i(\omega) < z \;\&\; \omega \in \Omega_{i-i}(\tau)] = \Pr[z_{-i}(\omega) < z \;\&\; \omega \in \Omega_{-ii}(\tau)]$ by symmetry across countries. For any ω, we have either $\omega \in \Omega_{ii}(\tau)$ or $\omega \in \Omega_{-ii}(\tau)$. Hence, by Step 1 and equation (18) we have $\int_0^z \chi(v;\tau)\,dv \leq \int_0^z \chi_i(v;\tau')\,dv$ for all $z > 0$. By Step 2 we therefore have $\int_0^z \chi(v;\tau)\,dv < \int_0^z \chi(v;\tau')\,dv$ for z sufficiently large, which concludes the proof of Step 3.

We now use Step 3 to prove the proposition. We proceed by contradiction. Suppose that $\tau < \tau'$ and $s'/w' \geq s/w$. According to the balanced trade condition and our normalization

$wL + sH = 1$, we have $w \geq w'$ and $s \leq s'$. Equation (15) and $w \geq w'$ yield

$$\int_0^\infty l\left(z, w, s/w\right) \chi\left(z; \tau\right) dz = wL \geq w'L = \int_0^\infty l\left(z, w', s'/w'\right) \chi\left(z; \tau'\right) dz,$$

while $\frac{d}{dw} l\left(z, w, s/w\right) < 0$ and $\frac{d}{d(s/w)} l\left(z, w, s/w\right) > 0$ together with $w \geq w'$ and $s/w \leq s'/w'$ yield

$$\int_0^\infty l\left(z, w', s'/w'\right) \chi\left(z; \tau'\right) dz \geq \int_0^\infty l\left(z, w, s/w\right) \chi\left(z; \tau'\right) dz.$$

The previous two inequalities give us

$$\int_0^\infty l\left(z, w, s/w\right) \chi\left(z; \tau\right) dz \geq \int_0^\infty l\left(z, w, s/w\right) \chi\left(z; \tau'\right) dz. \tag{19}$$

Finally, $\frac{d}{dz} l\left(z, w, s/w\right) < 0$ with $\varphi > 0$, and Step 3 imply[2]

$$\int_0^\infty l\left(z, w, s/w\right) \chi\left(z; \tau\right) dz < \int_0^\infty l\left(z, w, s/w\right) \chi\left(z; \tau'\right) dz,$$

which contradicts inequality (19). Therefore, $\tau < \tau'$ implies $s'/w' < s/w$. **QED.** ■

References

Bernard, Andrew B., Stephen J. Redding, and Peter K. Schott. 2007. "Comparative Advantage and Heterogeneous Firms." *Review of Economic Studies*, 74(1): 31-66.

Burstein, Ariel and Jonathan Vogel. 2011. "Factor Prices and International Trade: A Unifying Perspective." Mimeo. Columbia University.

[2]This follows from the fact that if $\int_0^z h\left(v\right) dv \leq \int_0^z g\left(v\right) dv$ for all $z > 0$, $\int_0^z h\left(v\right) dv < \int_0^z g\left(v\right) dv$ for z sufficiently large, and $f'\left(z\right) < 0$, then $\int_0^\infty h\left(v\right) f\left(v\right) dv < \int_0^\infty g\left(v\right) f\left(v\right) dv$.