

# Factor Prices and International Trade: A Unifying Perspective\*

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## Abstract

How do trade liberalizations affect relative factor prices and to what extent do they cause factors to reallocate across sectors? We first present a general accounting framework that nests a wide range of models that have been used to study the link between globalization and factor prices and from which we obtain two sufficient statistics that determine factor prices. Under some restrictions, changes in the "factor content of trade" (FCT) fully determine the impact of trade on relative factor prices.

We then study the determination of the FCT in a specific version of our general framework that unifies traditional models of trade and factor prices featuring sectoral productivity and factor endowment differences and new models featuring imperfect competition and heterogeneous producers. We show how heterogeneous firms' decisions shape the FCT, and, therefore, the impact of trade liberalization on relative factor prices and between-sector factor allocation.

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# 1 Introduction

How do trade liberalizations affect the skill premium, or relative factor prices more generally, and to what extent do they cause factors to reallocate between sectors and across producers within sectors? This paper offers a unifying perspective on the fundamental forces that shape factor prices and factor allocation in a global economy. In the first part of the paper we present a general accounting framework from which we obtain sufficient statistics that determine factor prices. In the second part of the paper we study the impact of trade on these sufficient statistics (and, hence, on factor prices and factor allocation) in a specific model, nested by our general framework, that unifies traditional models of trade and factor prices featuring sectoral productivity and factor endowment differences and new models featuring imperfect competition and heterogeneous producers.

The general accounting framework that we present in the first part of the paper nests a wide range of international trade models such as the traditional Heckscher-Ohlin model, which emphasizes differences in factor intensities across sectors and factor endowments across countries. It also nests other models—emphasizing, e.g., differences in skill intensities between exporters and non-exporters within sectors, differences in the tradeability of skill-intensive and unskill-intensive goods, and complementarities between skilled labor and traded goods such as capital<sup>1</sup>—that have been used to study the link between international trade and relative factor prices. While we focus on factor prices and between-factor inequality, the framework also covers recent models featuring unemployment, within-factor heterogeneity, and within-group inequality.<sup>2</sup>

We show that within this framework, each factor price can be expressed as the product of two components. The first component is the inverse of the trade-adjusted factor supply, which is the domestic employment of that factor less the factor content of trade (FCT). The FCT of a given factor is the quantity of that factor embodied in the country’s net exports. A decrease in the trade-adjusted factor supply increases the factor’s price, just like a decrease in its domestic supply. The second component is the factor payments for domestic absorption, which is the counterfactual payments to that factor if domestic sectoral absorption were produced domestically; this component depends on domestic sectoral expenditure shares and factor shares in sectoral revenues. An increase in the average revenue share of a factor increases the price of this factor.

We use this decomposition to show how various specific models and mechanisms operate

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<sup>1</sup>See e.g. Yeaple (2005), Matsuyama (2007), and Burstein and Vogel (2010) for the first mechanism, Epifani and Gancia (2006) for the second, and Burstein, Cravino, and Vogel (2010) and Parro (2010) for the third.

<sup>2</sup>See e.g. Helpman, Itskhoki, and Redding (2010).

through these two components. More generally, this decomposition provides a single lens to view and compare a wide range of models and mechanisms linking trade and technology to factor prices.

Under some restrictions, the decomposition simplifies further. In particular, we provide conditions under which the ratio of factor payments for domestic absorption between any two factors is constant across equilibria, so that changes in relative factor prices depend only on changes in trade-adjusted factor supplies. Hence, in any model satisfying these restrictions, if the domestic supplies of two factors are fixed, then changes in the FCT for these two factors are sufficient statistics for the impact of trade on the relative price of those two factors: changes in the economic environment—such as trade costs, foreign productivities, foreign factor supplies, foreign production functions, domestic productivities, or domestic supplies of other factors—affect domestic relative factor prices only through changes in the FCT. Moreover, we show that under these restrictions the FCT can be constructed using industry-level trade and production data.<sup>3</sup> A similar result has been obtained previously by Deardorff and Staiger (1988) and Deardorff (2000) in perfectly competitive environments with constant returns to scale and common production technologies across producers within sectors. Deardorff and Staiger (1988) motivated a substantial number of empirical studies; see e.g. Katz and Murphy (1992) and Krugman (1995). We show that this empirical approach remains valid in significantly more general economic environments featuring, for instance, imperfect competition, increasing returns to scale, heterogeneous producers, and/or fixed costs of exporting.<sup>4</sup>

While our general framework provides a single lens to view and compare a wide range of models and mechanisms linking trade and technology to factor prices, and makes a clear link between the FCT and factor prices, it takes the FCT as given. Without any further structure, this accounting framework does not provide insights into how changes in the economic environment, such as changes in trade costs, affect the FCT and relative factor prices.

In the second part of the paper we study the determination of the FCT and the impact of trade liberalization on factor prices and factor allocation in an environment that combines a number of key elements of an important class of workhorse models in international trade. To maintain analytic tractability, we specialize the general framework above to an environ-

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<sup>3</sup>Note that even in cases in which the FCT can be constructed using industry-level data, measuring the FCT requires detailed information on factor employment and trade across highly disaggregated industries, which may be unavailable in practice; see e.g. Feenstra and Hanson (2000).

<sup>4</sup>A large literature studies Vanek's (1968) prediction that each country is a net exporter of the services of its abundant factors, using the FCT; see e.g. Treffer (1993 and 1995) and Davis and Weinstein (2001). Our results are related to Helpman and Krugman (1985) and Treffer and Zhu (2010), who show that Vanek's prediction holds across a wide range of models.

ment with two-countries, two-factors (skilled and unskilled labor), and two-sectors, as in the Heckscher-Ohlin model. We allow for sectoral productivity differences across countries, as in the Ricardian model, and we introduce monopolistic competition and heterogeneous firms, as in Melitz (2003) and Bernard, Redding, and Schott (2007). While this model is significantly less general than our accounting framework, it provides a unified environment to study analytically how the FCT, factor prices, the extent of between-sector factor reallocation, and the extent of between-sector trade are all shaped by cross-country differences in factor endowments and sectoral productivities, as in the standard Heckscher-Ohlin and Ricardian models, and by firms' decisions to enter and to operate in each market, as introduced in the recently developed models of monopolistic competition and heterogeneous producers.

In this model, a reduction in trade costs induces countries to expand production and exports in their comparative advantage sector and contract production elsewhere, as in the standard Heckscher-Ohlin and Ricardian models. This between-sector reallocation lowers the trade-adjusted supply of the factor used intensively in the comparative advantage sector (by raising its FCT) and hence raises its relative price, as in the Heckscher-Ohlin model. This effect, which is often referred to as the Stolper-Samuelson effect, is the only channel through which trade affects factor prices (in contrast to the general framework) because the ratio of factor payments for domestic absorption between any two factors is constant in this environment. Moreover, in this model changes in the FCT fully determine not only changes in the skill premium, but also changes in the extent of between-sector factor reallocation and between-sector trade.

Our main objective in this second part of the paper is to study how the impact of trade liberalization on the FCT, the skill premium, the extent of between-sector trade, and the extent of factor reallocation is shaped by the new margins of heterogeneous-firm trade models that are absent in standard Heckscher-Ohlin and Ricardian models; e.g., the extent of productivity heterogeneity and heterogeneous firms' decisions to enter and operate in each market. To provide intuition for these comparative static exercises, we find it useful to relate the implications of these new margins to the well-known effects of changes in exogenous Ricardian comparative advantage. In particular, exploiting the model's simple expressions for sectoral trade shares, we show that an increase in the mass of country 1 firms that sell in a given destination market in a given sector is equivalent—in terms of its impact on the FCT, the skill premium, between-sector factor reallocation, and between-sector trade—to an increase in country 1's exogenous Ricardian productivity in that sector. The mass of firms selling to a given destination increases either because of an increase in the mass of entering firms or because of an increase in the fraction of entrants that operate in the destination. Moreover, the extent to which changes in the mass of firms selling in each destination affects

the FCT depends on the degree of within-sector productivity heterogeneity. In this sense, the strength of exogenous sources of comparative advantage (resulting from sectoral productivity differences and factor endowment differences) is shaped by the decisions of firms to enter and to sell in each destination market.

We use this logic to obtain the following results on the impact of trade liberalization on the FCT, the skill premium, the extent of between-sector factor reallocation, and the extent of between-sector trade. We first show that greater within-sector productivity heterogeneity reduces the magnitudes of the changes in the FCT, the skill premium, the extent of between-sector trade, and the extent of factor reallocation induced by a given change in trade shares. This is because greater technological heterogeneity mitigates the relative importance of exogenous Ricardian technological differences and factor price differences in shaping sectoral trade patterns. Given the extensive evidence of large productivity differences within narrowly-defined sectors, this comparative static exercise provides a rationale for empirical results suggesting that the FCT is not very large for many countries like the US, and that the extent of between-sector factor reallocation induced by trade and its impact on the skill premium are small in practice; see e.g. Goldberg and Pavcnik (2007).

We next show that endogenous entry and endogenous selection of firms into markets each increases the magnitudes of the changes in the FCT, the skill premium, the extent of between-sector trade, and the extent of factor reallocation induced by a given change in trade shares. This is because the mass of entrants from country 1 relative to the mass of entrants from country 2 and the fraction of these entrants from country 1 relative to the fraction of these entrants from country 2 that choose to sell in any given market is relatively larger in country 1's comparative advantage sector than in country 2's; and an increase in the mass of firms that supply a given market is equivalent to an increase in that sector's exogenous Ricardian productivity. These results imply that measures of sectoral productivity and endowment differences across countries would underestimate the impact of trade liberalization on the skill premium and between-sector factor reallocation if firm entry and selection decisions are not taken into account. Note, however, that given our earlier results, the extent of within-sector productivity heterogeneity, endogenous entry, and selection of firms into markets have no effect whatsoever on changes in factor prices, between-sector factor allocation, or between-sector trade, for given changes in the FCT.

Our results are related to recent papers in international trade identifying robust insights for welfare analysis across different models; see e.g., Arkolakis, Costinot, and Rodriguez-Clare (Forthcoming) and Atkeson and Burstein (2010). Whereas these papers focus on the welfare implications of international trade, we focus on the distributional implications of international trade. We show that across a wide range of workhorse models, the effects of

international trade on the skill premium can be summarized by changes in the FCT.

The second part of our paper is most closely related to Bernard, Redding, and Schott (2007), henceforth BRS, which consider a very similar framework. As in other models with factor-endowment differences across countries and factor-intensity differences across sectors, we both obtain standard Heckscher-Ohlin-like results: trade generates factor reallocation and a Stolper-Samuelson-type effect. Our contribution relative to BRS is as follows. First, we show that changes in the FCT are sufficient statistics for the impact of international trade on the skill premium and between-sector factor allocation. Second, we demonstrate analytically how the new margins in heterogeneous-firm models (the extent of within-sector productivity heterogeneity, endogenous entry, and selection of firms into markets) affect the impact of trade on the skill premium, between-sector trade, and factor reallocation. In particular, while propositions in BRS state that the new margins in heterogeneous firm models do affect the aggregate implications of trade liberalization, none of their analytic results establish *how* any of these margins affects the skill premium, the extent of factor reallocation, or the extent of between sector trade, whereas this is the focus of the second part of our paper. Third, we revisit their finding that differences in factor endowments induce what BRS call "endogenous Ricardian productivity differences" at the industry level.<sup>5</sup>

## 2 Factor Prices: A Unifying Framework

In this section we present a general accounting framework to examine the link between factor prices and trade. We first derive a simple expression relating equilibrium factor prices to two components: the *trade-adjusted factor supply* and *factor payments for domestic absorption*. We then show how changes in relative factor prices within a range of workhorse models of trade can be mapped into these two components. Finally, we describe a set of assumptions that are standard in the literature under which changes in the FCT are sufficient statistics for the impact of trade on relative factor prices.

### 2.1 General Framework

There are  $N$  countries, indexed by  $n = 1, \dots, N$ , and  $J$  sectors, indexed by  $j = 1, \dots, J$ . Production requires inputs, and inputs are grouped into disjoint sets of factors, indexed by  $k = 1, \dots, K$ . Factors could be highly aggregated, e.g. capital and labor, or highly disaggregated; e.g. workers with a given number of years of education. Inputs within a given

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<sup>5</sup>Ho (2010) uses a similar framework to study the implications of idiosyncratic distortions on between-sector factor allocation, the skill premium, and welfare, while Lu (2010) uses it to study how export market participation decisions of Chinese firms vary across sectors.

factor set  $k$  can be heterogeneous in all respects; e.g. the efficiency units provided by each input of factor  $k$  can vary within firms in a sector and across sectors.

In a given equilibrium and at a given point in time, we denote by  $L_{k,i}$  the mass of inputs of factor  $k$  employed in country  $i$ . Note that in the presence of idle inputs (e.g. labor unemployment),  $L_{k,i}$  can be strictly less than the total supply of factor  $k$  in country  $i$ . Let  $L_{k,in}(j)$  denote the mass of inputs of factor  $k$  in country  $i$  that are employed in supplying destination market  $n$  in sector  $j$ . The mass of inputs of factor  $k$  used in country  $i$  in sector  $j$  is  $L_{k,i}(j) = \sum_n L_{k,in}(j)$ , and the sum of  $L_{k,i}(j)$  across industries must equal total employment of factor  $k$ ,  $L_{k,i} = \sum_j L_{k,i}(j)$ .

At this point,  $L_{k,in}(j)$  is an accounting variable describing how factor usage is distributed across destination markets. Below we discuss how  $L_{k,in}(j)$  can be constructed in a range of specific models. In some cases it may not be straightforward to allocate sectoral employment,  $L_{k,i}(j)$ , across destination markets,  $L_{k,in}(j)$ . As discussed in detail below, this can be the case, for example, if firms must incur fixed costs that do not depend on the set of destination markets they supply (or if marginal costs are not constant). We show, however, that in some of these cases we do not need to construct  $L_{k,in}(j)$  to apply our results that follow.

We denote by  $E_i(j) \geq 0$  country  $i$ 's total expenditure on sector  $j$ , and by  $\Lambda_{in}(j) \in [0, 1]$  the share of country  $n$ 's total expenditure in sector  $j$  that is allocated to goods from country  $i$ , with  $\sum_i \Lambda_{in}(j) = 1$ . Then total revenues by producers in country  $i$  are  $\sum_n \sum_j \Lambda_{in}(j) E_n(j)$ .

**Factor payments:** Let  $w_{k,in}(j)$  denote the equilibrium average price paid in country  $i$  to inputs in factor group  $k$  employed in sector  $j$  in the production of goods bound for country  $n$ . This average price is equal to total earnings of these inputs divided by the mass of these inputs  $L_{k,in}(j)$ . The average price paid in country  $i$  to inputs in factor group  $k$  employed in sector  $j$  can then be expressed as

$$w_{k,i}(j) = \frac{1}{L_{k,i}(j)} \sum_n L_{k,in}(j) w_{k,in}(j).$$

Similarly, the average price paid in country  $i$  to inputs in factor group  $k$ ,  $w_{k,i}$ , can be expressed as

$$w_{k,i} = \frac{1}{L_{k,i}} \sum_j L_{k,i}(j) w_{k,i}(j).$$

Note that we do not impose the restriction that two identical inputs within a factor set (e.g. two identical workers) are paid the same price within or across plants, firms, or sectors.

Denote by  $\lambda_{in}(j)$  the share of country  $i$  revenues from sales in country  $n$  in sector  $j$  that

is paid to all factors,

$$\lambda_{in}(j) = \frac{\sum_k w_{k,in}(j) L_{k,in}(j)}{\Lambda_{in}(j) E_n(j)}.$$

Denote by  $\alpha_{k,in}(j)$  the share of factor payments that are paid to inputs in factor set  $k$ ,

$$\alpha_{k,in}(j) = \frac{w_{k,in}(j) L_{k,in}(j)}{\sum_{k'} w_{k',in}(j) L_{k',in}(j)},$$

where  $\sum_k \alpha_{k,in}(j) = 1$  for all  $n$  and  $j$ . If  $\lambda_{in}(j) < 1$ , then any remaining revenues are profits or rents paid to an input that is not included in any set  $k = 1, \dots, K$ . Given these definitions, it follows that

$$w_{k,i} L_{k,i} = \sum_j \sum_n \alpha_{k,in}(j) \lambda_{in}(j) \Lambda_{in}(j) E_n(j). \quad (1)$$

Equation (1) states that the total earnings of employed inputs within a factor set must equal the payments to these inputs in the production of goods bound for all destination markets and across all sectors.

**Factor content of trade:** Denote by  $FCT_{k,i}$  the factor content of trade for factor  $k$  in country  $i$ , defined as

$$FCT_{k,i} = \sum_j \sum_n \left[ L_{k,in}(j) - L_{k,ii}(j) \frac{\Lambda_{ni}(j) w_{k,ii}(j)}{\Lambda_{ii}(j) w_{k,i}} \right].$$

To better understand the definition of the FCT, we use our definitions of  $\lambda_{in}(j)$  and  $\alpha_{k,in}(j)$  to express the payments for the FCT,  $w_{k,i} FCT_{k,i}$ , as

$$w_{k,i} FCT_{k,i} = \sum_j \sum_{n \neq i} [\alpha_{k,in}(j) \lambda_{in}(j) \Lambda_{in}(j) E_n(j) - \alpha_{k,ii}(j) \lambda_{ii}(j) \Lambda_{ni}(j) E_i(j)]. \quad (2)$$

The first term in the summation in equation (2),  $\alpha_{k,in}(j) \lambda_{in}(j) \Lambda_{in}(j) E_n(j)$ , represents the payments to factor  $k$  embodied in country  $i$ 's exports to destination market  $n$ . The second term in the summation,  $\alpha_{k,ii}(j) \lambda_{ii}(j) \Lambda_{ni}(j) E_i(j)$ , represents the counterfactual payments to factor  $k$  in country  $i$ , had country  $i$  produced for itself the value of goods that it imported from country  $n$ . Therefore, the FCT corresponds to the net exports of factor  $k$  embodied in country  $i$ 's trade.

Note that constructing the FCT in the data requires input usage and average factor prices by destination country, which may be difficult to observe in practice. In Section 2.2, we discuss a range of models in which the construction of  $FCT_{k,i}$  is simplified significantly.

**Factor prices:** To show how  $FCT_{k,i}$  is related to  $w_{k,i}$ , we proceed as follows. By equations (1) and (2), and using  $\Lambda_{ii}(j) = 1 - \sum_{n \neq i} \Lambda_{ni}(j)$ , we decompose payments to factor  $k$  into



two components:

$$w_{k,i}L_{k,i} = w_{k,i}FCT_{k,i} + \Phi_{k,i}. \quad (3)$$

The first component is the payments for the FCT defined in equation (2). The second component is the factor payments for domestic absorption (FPD),

$$\Phi_{k,i} = \sum_j \lambda_{ii}(j) \alpha_{k,ii}(j) E_i(j),$$

which is the counterfactual payments to factor  $k$  if domestic absorption were produced domestically. By equation (3), factor  $k$ 's average price is

$$w_{k,i} = \Phi_{k,i} / \mathcal{L}_{k,i}, \quad (4)$$

where

$$\mathcal{L}_{k,i} = L_{k,i} - FCT_{k,i}$$

denotes the trade-adjusted supply of factor  $k$ .<sup>6</sup>

By comparing equations (1) and (4), it is apparent that for given values of  $\lambda_{ii}(j)$ ,  $\alpha_{k,ii}(j)$ , and  $E_i(j)$ , the average price paid to factor  $k$  in a trade equilibrium is equal to the average price that would have been paid to factor  $k$  in autarky had country  $i$ 's employment of factor  $k$  been  $\mathcal{L}_{k,i}$  rather than  $L_{k,i}$ . If a country is a net exporter of factor  $k$ , then its average factor price is determined as if it has a smaller stock of this factor. In this sense, we can think of  $\mathcal{L}_{k,i}$  as the counterfactual employment of factor  $k$  available in economy  $i$  in the presence of international trade.

Using equation (4), we express the average price of factor  $k_1$  relative to factor  $k_2$  as

$$w_{k_1,i}/w_{k_2,i} = (\mathcal{L}_{k_2,i}/\mathcal{L}_{k_1,i}) \times (\Phi_{k_1,i}/\Phi_{k_2,i}), \quad (5)$$

for any  $k_1, k_2 \leq K$ . Equation (5) decomposes the relative price of factor  $k_1$  to factor  $k_2$  into two terms: (i) the trade-adjusted employment of  $k_2$  relative to  $k_1$  and (ii) the FPD of  $k_1$  relative to  $k_2$ . An increase in  $\mathcal{L}_{k_2,i}/\mathcal{L}_{k_1,i}$ , either through a decrease in the relative employment of factor  $k_1$  or an increase in the FCT of  $k_1$ , increases the relative average price of  $k_1$ . Similarly, an increase in  $\Phi_{k_1,i}/\Phi_{k_2,i}$ , either through an increase in expenditure shares in sectors intensive in factor  $k_1$  or an increase in the average revenue share of factor  $k_1$  across sectors, increases the relative price of  $k_1$ .

We summarize these results in the following proposition, which provides an equation for

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<sup>6</sup>While we use the term "trade-adjusted factor supply," it should be clear from the definition of  $L_{k,i}$  that in an environment in which some inputs remain idle,  $\mathcal{L}_{k,i}$  is really the trade-adjusted factor employment.

the change in the relative price of factor  $k_1$  to factor  $k_2$  between any two equilibria.

**Proposition 1** *If  $w'_{k_1,i}/w'_{k_2,i}$ ,  $\mathcal{L}'_{k,i}$ , and  $\Phi'_{k,i}$  denote the relative average price of factor  $k_1$  to factor  $k_2$ , the trade-adjusted supply of factor  $k$ , and the factor payments for domestic absorption of factor  $k$  in a counterfactual equilibrium, then*

$$\frac{w'_{k_1,i}/w'_{k_2,i}}{w_{k_1,i}/w_{k_2,i}} = \left[ \frac{\mathcal{L}'_{k_2,i}}{\mathcal{L}'_{k_1,i}} \bigg/ \frac{\mathcal{L}_{k_2,i}}{\mathcal{L}_{k_1,i}} \right] \times \left[ \frac{\Phi'_{k_1,i}}{\Phi'_{k_2,i}} \bigg/ \frac{\Phi_{k_1,i}}{\Phi_{k_2,i}} \right]. \quad (6)$$

Of course, both  $\mathcal{L}_{k,i}$  and  $\Phi_{k,i}$  are endogenous, and their equilibrium determination—and therefore, how they are affected by trade liberalization—is outside the scope of this accounting framework. In Section 3, we specialize our general framework to study the determination of these variables.

## 2.2 Mapping Specific Models into Framework

In this section we discuss how a variety of models of international trade, technological change, and relative factor prices can be mapped into the general framework above. We also describe a range of model assumptions under which expression (6) and the calculation of the FCT can be simplified significantly. In the examples we consider, we follow the literature and assume that all inputs within a factor set are homogeneous. Moreover, because the labor market is perfectly competitive in the examples below, all inputs within a factor set receive a common price,  $w_{k,i}$ .

**Heckscher-Ohlin-like perfectly competitive models:** Here we focus on perfectly competitive models with constant returns to scale in which all producers within a sector share a common factor intensity that does not depend on the destination in which output is sold. These assumptions are satisfied in the Heckscher-Ohlin model—see, e.g., Stolper and Samuelson (1941)—and its multi-sector and multi-factor extensions—see, e.g., Ethier (1984), Jones and Scheinkman (1977), and Costinot and Vogel (2010).

In these models,  $L_{k,in}(j)$  can be constructed easily, as the product of sector  $j$ 's employment of factor  $k$ ,  $L_{k,i}(j)$ , and the ratio of country  $i$  sector  $j$  revenues earned in market  $n$  to total revenues earned in that sector,  $\Lambda_{in}(j) E_n(j) / (\sum_{n'} \Lambda_{in'}(j) E_{n'}(j))$ . Hence, the share of factor payments accruing to factor  $k$  in sector  $j$  production—i.e. the factor  $k$  intensity of production in sector  $j$ —is the same across destination markets,  $\alpha_{k,in}(j) = \alpha_{k,i}(j)$  for all  $i$ ,  $n$ ,  $k$ , and  $j$ , where

$$\alpha_{k,i}(j) = \frac{w_{k,i} L_{k,i}(j)}{\sum_{k'} w_{k',i} L_{k',i}(j)}. \quad (7)$$

Hence, constructing the  $\alpha_{k,in}(j)$  terms requires only sectoral employment and factor prices. Moreover, with constant returns to scale and perfect competition, firm profits are zero, so  $\lambda_{in}(j) = 1$  for all  $i$ ,  $n$ , and  $j$ .

In any setting in which  $\alpha_{k,in}(j)$  and  $\lambda_{in}(j)$  are common across destination markets, we can simplify significantly the construction of net exports of factor  $k$ . In particular, we have

$$FCT_{k,i} = \sum_j L_{k,i}(j) \omega_i(j), \quad (8)$$

where

$$\omega_i(j) = \frac{\sum_{n \neq i} [\Lambda_{in}(j) E_n(j) - \Lambda_{ni}(j) E_i(j)]}{\sum_n \Lambda_{in}(j) E_n(j)}$$

denotes the ratio of country  $i$ 's net exports in sector  $j$  to country  $i$ 's total revenue in sector  $j$ . The variables  $L_{k,i}(j)$  and  $\omega_i(j)$ , and hence the factor  $k$  content of trade, can be measured in principle using sectoral production and trade data.

In this environment, the expression in Proposition 1 is simplified only because  $\alpha_{k,in}(j) = \alpha_{k,i}(j)$  and  $\lambda_{in}(j) = 1$ . However, we can further simplify this expression under a few additional assumptions. If preferences and production functions are Cobb-Douglas and the Cobb-Douglas share parameters are unchanged across equilibria, then equation (6) simplifies to

$$\frac{w'_{k_1,i}}{w'_{k_2,i}} \bigg/ \frac{w_{k_1,i}}{w_{k_2,i}} = \frac{\mathcal{L}'_{k_2,i}}{\mathcal{L}'_{k_1,i}} \bigg/ \frac{\mathcal{L}_{k_2,i}}{\mathcal{L}_{k_1,i}}. \quad (9)$$

In this special case, relative factor prices change only due to changes in trade-adjusted factor supplies. For fixed domestic supplies of factors  $k_1$  and  $k_2$ , any change in the economic environment—such as trade costs, foreign productivities, foreign factor supplies, foreign production functions, domestic productivities, or domestic supplies of factors other than  $k_1$  and  $k_2$ —affects domestic relative factor prices only through changes in the FCT.

Expression (9) was also obtained in Deardorff and Staiger (1988) and Deardorff (2000) in a perfectly competitive environment with constant returns to scale and common productivities across producers within each sector. Our result allows for heterogeneous productivities within sectors, as in a multi-sector and multi-factor version of Eaton and Kortum (2002).<sup>7</sup>

**Common factor intensities across sectors:** A particular class of models nested by the perfectly competitive, constant returns to scale models above are those in which factor intensity is identical across producers, sectors, and destination markets,  $\alpha_{k,in}(j) = \alpha_{k,i}$ . These

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<sup>7</sup>An alternative assumption that simplifies equation (6) is that countries are symmetric. In this case, trade is balanced sector-by-sector, so that  $\omega_i(j) = 0$  for all  $i$  and  $j$ , and  $FCT_{k,i} = 0$  for all  $k$  and  $i$ . Hence, changes in relative factor prices depend only on changes in factor supplies and relative payments for domestic absorption.

assumptions are satisfied in, e.g., Katz and Murphy (1992), Krusell, Ohanian, Rios-Rul, and Violante (2000), and Burstein, Cravino, and Vogel (2010). Under these assumptions, equation (5) simplifies to

$$w_{k_1,i}/w_{k_2,i} = (\mathcal{L}_{k_2,i}/\mathcal{L}_{k_1,i}) \times (\alpha_{k_1,i}/\alpha_{k_2,i})$$

Moreover, because  $FCT_{k,i}$  equals  $\alpha_{k,i}/w_{k,i}$  times country  $i$ 's net aggregate exports ( $i$ 's trade balance), we have  $\mathcal{L}_{k_2,i}/\mathcal{L}_{k_1,i} = L_{k_2,i}/L_{k_1,i}$ . Hence, equation (6) becomes

$$\frac{w'_{k_1,i}/w_{k_1,i}}{w'_{k_2,i}/w_{k_2,i}} = \left[ \frac{L'_{k_2,i}/L_{k_2,i}}{L'_{k_1,i}/L_{k_1,i}} \right] \times \left[ \frac{\alpha'_{k_1,i}/\alpha_{k_1,i}}{\alpha'_{k_2,i}/\alpha_{k_2,i}} \right].$$

In this class of models, changes in relative factor prices across two points in time are driven entirely by changes in relative factor supplies and by changes in relative factor intensities. Changes in relative factor intensities can be driven by technological change (see e.g. Katz and Murphy 1992),<sup>8</sup> capital accumulation (see e.g. Krusell, Ohanian, Rios-Rul, and Violante 2000), and capital accumulation and international trade (see e.g. Burstein, Cravino, and Vogel 2010).<sup>9</sup>

**Factor intensity varies by destination market:** In Matsuyama (2007) and Burstein and Vogel (2010), markets are perfectly competitive, production is constant returns to scale, and average factor intensities vary depending on destination market. In Matsuyama (2007) producers are homogeneous within a sector, and trade costs are assumed to be skill intensive relative to production. In Burstein and Vogel (2010), for a given producer, skill intensity is independent of destination market, but the most productive producers tend to export and to be more skill intensive. Hence in these models,  $\lambda_i(j) = 1$  but  $\alpha_{k,in}(j)$  tends not to equal  $\alpha_{k,ii}(j)$  for  $n \neq i$ .

With constant returns to scale it is straightforward to allocate aggregate sectoral factor employment,  $L_{k,i}(j)$ , to each destination market,  $L_{k,in}(j)$ . Hence, these models fit into the general framework presented above. However, equation (6) simplifies only because  $\lambda_{ii}(j) = 1$ . In general, changes in trade costs will affect relative factor prices through both trade-adjusted factor supplies and the factor payments for domestic absorption.

**Heckscher-Ohlin-like imperfectly competitive models:** In Section 3 we consider a range of models featuring imperfect competition, heterogeneous firms, and increasing returns

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<sup>8</sup>While some papers treat technological change as exogenous, there is a large literature on endogenous factor-biased technological change; see, e.g. Acemoglu (2002). The effect of such changes on relative factor wages operate through changes in relative factor intensities (changes in  $\alpha$ 's).

<sup>9</sup>Parro (2010) considers a model similar to Burstein, Cravino, and Vogel (2010) in which factor intensities vary across sectors.

to scale, as in, e.g., Romalis (2004) and BRS (2007). With imperfect competition, firms may earn profits, so  $\lambda_{in}(j)$  is not generally equal to one. Moreover, in some cases it is not straightforward to allocate sectoral employment,  $L_{k,i}(j)$ , across destination markets,  $L_{k,in}(j)$ . This can be the case, for example, if a firm must incur fixed costs that do not depend on the set of destination markets it supplies. However, we show that in the model of Section 3, Proposition 1 holds with the FCT being constructed using equation (8), and that equation (6) simplifies to equation (9).

It is straightforward to show that the same results hold in a two-factor version of Bernard, Eaton, Jensen, and Kortum (2003), which is an extension of Eaton and Kortum (2002) with Bertrand instead of perfect competition. Since there are constant returns to scale (and no fixed costs), allocating factors across destination markets is straightforward. With Frechet distributed productivities and CES demand,  $\lambda_{in}(j)$  is constant and equal across destination markets.<sup>10</sup>

### 3 The FCT in a Heterogeneous Firm Model

While the framework in Section 2 provides a single lens to view and compare a wide range of models and mechanisms linking trade and technology to factor prices, and links the FCT to relative factor prices, it takes the FCT as given. Without any further structure, this accounting framework does not provide insights into how changes in the economic environment, such as changes in trade costs, affect the FCT and relative factor prices.

We now focus on understanding the determination of the FCT and the impact of trade liberalization on factor prices and factor allocation in a specific environment, covered by our accounting framework above, that combines key elements of an important class of workhorse models in international trade. While this model is significantly less general than our accounting framework, it provides a unified environment to study analytically how the FCT, factor prices, the extent of between-sector factor reallocation, and the extent of between-sector trade are all shaped by cross-country differences in factor endowments and sectoral productivities, as in the standard Heckscher-Ohlin and Ricardian models, and by firms' decisions to enter and to operate in each market, as in models of monopolistic competition and heterogeneous firms.

We use this model to obtain three sets of results. First, in Section 3, we show that in

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<sup>10</sup>Epifani and Gancia (2008) consider an alternative model of international trade and monopolistic competition. In their model, changes in relative factor wages are driven by changes in the FCT and sectoral expenditures. Trade raises expenditures,  $E_i(j)$ , in the skill-intensive sector relative to the unskill-intensive sector in all countries, which tends to increase the skill premium in all countries, as is evident in equation (5).

this environment, Proposition 1 holds, the FCT is given by equation (8), and the calculation of the FCT in equation (6) simplifies to equation (9). Second, in Section 4, we demonstrate that the FCT and factor endowments fully determine not only the relative price of skilled to unskilled labor (the skill premium), but also the extent of between-sector factor reallocation and between-sector trade. Finally, in Section 5, we show how the extent of productivity heterogeneity between and within sectors, and heterogeneous firms' decisions to enter and operate in each market shape the impact of trade liberalization on the FCT, and, therefore, on factor allocation and the skill premium.

### 3.1 Model

Our model economy features two countries,  $i = 1, 2$ ; two factors, which we refer to as skilled labor and unskilled labor; and two sectors,  $j = x, y$ , where  $x$  is skill intensive.<sup>11</sup> Here we assume, without loss of generality for our results, that within each factor, all inputs are homogeneous. While factors are perfectly mobile across producers within a country, they are internationally immobile. The exogenous and fixed endowments of skilled and unskilled labor in country  $i$  are denoted by  $L_{s,i}$  and  $L_{u,i}$ , respectively. Each country produces a final non-tradeable good using output of both sectors. Output in each sector is produced using a continuum of differentiated intermediate goods, which are produced by firms using skilled and unskilled labor. To focus on cases in which changes in the FCT fully determine the impact of trade on factor prices, we assume that intermediate goods production functions and the final non-tradeable good aggregator are Cobb Douglas. International trade of intermediate goods is subject to variable and fixed costs. Factors are perfectly mobile across firms and sectors but are immobile across countries.

**Preferences:** The representative consumer's utility is defined over a non-tradeable final good,  $Q_i$ , that (for expositional purposes) places equal weight on the output of each sector

$$Q_i = Q_i(x)^{1/2} Q_i(y)^{1/2},$$

where  $Q_i(j)$  denotes the output of sector  $j$ . The aggregate price index is  $P_i = \frac{1}{2}P_i(x)^{1/2} P_i(y)^{1/2}$ , where  $P_i(j)$  is the price of sector  $j$ . Demand for the sector  $j$  good is  $Q_i(j) = \frac{E_i}{2P_i(j)}$ , where  $E_i = Q_i P_i$  denotes total expenditure in country  $i$ .

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<sup>11</sup>We impose that there are two countries and two sectors for analytic tractability. We believe that the intuition for our results extend to more general environments. For an analysis of Stolper-Samuelson-like results in environments with many factors and sectors, see Costinot and Vogel (2010).

**Sectoral aggregates:** Sector  $j$ 's output,  $Q_i(j)$ , is a CES aggregate of varieties

$$Q_i(j) = \left( \int_{\omega \in \Omega_j} q_i(\omega, j)^{(\eta-1)/\eta} d\omega \right)^{\eta/(\eta-1)}.$$

Here,  $q_i(\omega, j)$  denotes country  $i$  consumption of variety  $(\omega, j)$ , and  $\eta > 1$  is the elasticity of substitution between varieties. The price index in sector  $j$  is  $P_i(j) = \left[ \int_{\omega \in \Omega_j} p_i(\omega, j)^{1-\eta} d\omega \right]^{1/(1-\eta)}$ , where  $p_i(\omega, j)$  denotes the price of good  $(\omega, j)$  in country  $i$ . Demand for variety  $(\omega, j)$  is  $q_i(\omega, j) = \left( \frac{p_i(\omega, j)}{P_i(j)} \right)^{-\eta} Q_i(j)$ .

**Intermediate good technologies:** There are a continuum of firms, each producing a unique variety  $(\omega, j)$ . Firms face variable costs of production, fixed (market access) costs of selling in each country, and iceberg costs of international trade. Both fixed and variable costs use skilled and unskilled labor, where the factor intensity of production varies across sectors but is constant across firms within a sector and across fixed and variable costs within a firm.

A sector  $j$  firm from country  $i$  with Hicks-neutral productivity  $z \geq 1$  that hires  $l_s$  units of skilled labor and  $l_u$  units of unskilled labor in variable production activities produces  $y = z A_i(j) l_s^{\alpha_s(j)} l_u^{\alpha_u(j)}$  units of output, where  $\alpha_s(j) + \alpha_u(j) = 1$ . Here,  $\alpha_k(j)$  denotes the share of skilled ( $k = s$ ) and unskilled ( $k = u$ ) labor in production of all country  $i$  firms in sector  $j$ , where we omit the dependence of  $\alpha_k(j)$  on  $i$  since factor intensities are equal in both countries. Because  $x$  is skill intensive, we have  $\alpha_s(x) > \alpha_s(y)$ .  $A_i(j) > 0$  denotes country  $i$ 's exogenous total factor productivity in sector  $j$ .

To facilitate exposition in our results below, we decompose  $A_i(j)$  into two components—national TFP,  $T_i$ , and sectoral TFP,  $T_i(j)$ —so that  $A_i(j) = T_i \times T_i(j)$ . We normalize  $T_1 = 1$ . We define  $a = A_1(x) A_2(y) / A_1(y) A_2(x)$  to be a measure of country 1's relative productivity advantage (if  $a > 1$ ) or disadvantage (if  $a < 1$ ) in sector  $x$ .

Firms from country  $i$  must ship  $\tau_{in} q$  units of output in order for  $q$  units to arrive in country  $n$ , with  $\tau_{ii} = 1$  and  $\tau_{in} = \tau_{ni} = \tau \geq 1$ . We refer to  $\tau$  as the iceberg transportation cost. Additionally, in order to supply a positive amount of goods to country  $n$ , a country  $i$  firm incurs a fixed market access cost of  $f_{in} \geq 0$  units of the sectoral composite input bundle in country  $i$ ; we assume that these fixed costs are produced using the same input bundle as the production of intermediate goods in that sector. For simplicity, but without loss of generality for our results, we assume that variable and fixed trade costs are common across sectors. We denote by  $f = f_{12}/f_{11} = f_{21}/f_{22}$  the relative fixed costs of international versus intra-national trade in all sectors and countries.

Under these assumptions on technology, a sector  $j$  firm with productivity  $z$  from country

$i$  incurs a cost

$$C_{in}(q) = v_i(j) \left[ \frac{q\tau_{in}}{z} + f_{in} \right]$$

to supply  $q > 0$  units of goods to country  $n$ . We refer to  $v_i(j)$  as the cost of the sector  $j$  composite input bundle in country  $i$ , where

$$v_i(j) = \frac{1}{A_i(j)} \left[ \frac{w_{s,i}}{\alpha_s(j)} \right]^{\alpha_s(j)} \left[ \frac{w_{u,i}}{\alpha_u(j)} \right]^{\alpha_u(j)}. \quad (10)$$

and where country  $i$ 's wages for unskilled and skilled labor are  $w_{s,i}$  and  $w_{u,i}$ , respectively. We denote by  $c_{in}(z, j) = v_i(j) \tau_{in}/z$  the marginal cost of a firm with productivity  $z$ , sector  $j$ , in country  $i$  to supply a good to country  $n$ .

Conditional on a country  $i$  firm paying the fixed cost to access market  $n$ , profit maximization implies that it charges a constant markup over its marginal cost,  $p_{in}(z, j) = \frac{\eta}{\eta-1} c_{in}(z, j)$ . In this case, a firm's market-specific revenue is proportional to its marginal cost,

$$r_{in}(z, j) = \frac{E_n}{2P_n(j)^{1-\eta}} \left[ \frac{\eta}{\eta-1} c_{in}(z, j) \right]^{1-\eta}, \quad (11)$$

and its market-specific variable profit is proportional to its revenue  $\pi_{in}(z, j) = r_{in}(z, j)/\eta$ .

**Selection of firms into markets:** A country  $i$  firm chooses to supply market  $n$  if the variable profit it earns there covers its fixed market access cost,  $\pi_{in}(z, j) \geq v_i(j) f_{in}(j)$ . Denote by  $z_{in}^*(j)$  the productivity threshold at which the least productive sector  $j$  firm from country  $i$  sells in country  $n$ :

$$z_{in}^*(j) = \max \left\{ \frac{\tau_{in}}{P_n(j)} \left[ \frac{2\eta^\eta f_{in}}{(\eta-1)^{1-\eta} E_n} \right]^{\frac{1}{\eta-1}} v_i(j)^{\frac{\eta}{\eta-1}}, 1 \right\}. \quad (12)$$

In order to understand the implications of endogenous selection for trade patterns and relative factor rewards, we consider specifications in which endogenous selection into markets is and is not active. In the specification in which endogenous selection is not active, we assume that  $f_{in} = 0$  for all  $i, n \in I$ , so that every entrant sells to each market:  $z_{in}^*(j) = 1$  for all  $i, n \in I$  and  $j \in J$ .<sup>12</sup> We refer to this as the case "without selection." This case corresponds to a multi-factor extension of Krugman (1980), as in Helpman and Krugman (1985) and in Romalis (2004).

In the specification in which endogenous selection is active, we assume that  $f_{in}$  is sufficiently large for all  $i, n \in I$  and  $j \in J$  such that there is selection into every market, i.e.

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<sup>12</sup>Under this specification, our results remain unchanged if market access costs are strictly greater than zero and all firms sell in all markets.



$z_{in}^*(j) > 1$  for all  $i, n \in I$  and  $j \in J$ . We refer to this as the case "with selection." This case corresponds to a multi-factor extension of Melitz (2003)—as in BRS—or of Chaney (2008). Note that the two cases we consider are not exhaustive. There are parameter values for which there exist country-pairs and sectors such that  $z_{in}^*(j) = 1$  and  $z_{kl}^*(j') > 1$ .

**Entry:** In order to understand the implications of endogenous entry for trade patterns and relative factor prices, we consider two alternative specifications on the determination of the mass of entering firms in each sector,  $M_i(j)$ ; we refer to these specifications as exogenous and endogenous entry. The difference between the two specifications is the timing regarding when entrepreneurs (potential entrants) realize their productivities.

In the specification with exogenous entry, we assume that entrepreneurs know their productivities ex-ante. In this case, the mass of entrepreneurs is fixed at  $M_i(j)$ —since if it were unbounded then only the most productive would enter—but the number of operating firms in each sector is endogenous because firms must pay a fixed cost to sell in each market. Firms in each sector/country draw their productivity  $z$  from a Pareto distribution with shape parameter  $\gamma$  and location parameter one:  $G(z) = \Pr(Z \leq z) = 1 - z^{-\gamma}$ . This case corresponds to, e.g., Chaney (2008), Arkolakis (Forthcoming), and Eaton et. al. (Forthcoming). For simplicity and without loss of generality, we assume in the exogenous entry case that  $M_i(j) = M_i$ .<sup>13</sup>

In the specification with endogenous entry, we assume that entrepreneurs are identical ex-ante. In this case, in each country/sector there is an unbounded mass of ex-ante identical potential entrants. To enter, an entrepreneur incurs a fixed entry cost of  $f^e > 0$  units of the sectoral composite input bundle (in the exogenous entry case, we assume that  $f^e = 0$  for all  $j$ ). That is, sector  $j$  startup costs in country  $i$  are  $f^e v_i(j)$ . Upon entry, firms draw their productivity  $z$  from the same distribution  $G(z)$  defined above and subsequently chose whether or not to pay a fixed cost to sell in each market. This case corresponds to a version of Melitz (2003) and BRS (2007) with Pareto distributed productivities. The free entry condition, for all  $j$ , is given by

$$\sum_{n=1}^I \int_{z_{in}^*(j)}^{\infty} [\pi_{in}(z, j) - v_i(j) f_{in}] dG(z) \leq v_i(j) f^e \quad \text{with equality if } M_i(j) > 0.$$

Finally, in all that follows we focus exclusively on cases with incomplete specialization; i.e. in which  $M_i(j) > 0$  for all  $i \in I$  and  $j \in J$ .

**Trade balance:** We assume trade balance in both countries. This implies that total expen-

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<sup>13</sup>This assumption implies that there are only two independent sources of comparative advantage: relative endowments and sectoral productivities. Alternatively, we could combine differences in  $M_i(j)$  and  $A_i(j)$  into a single parameter,  $\tilde{A}_i(j)$ . The parameter  $a$  in this case would be defined using the  $\tilde{A}_i(j)$ s.

diture equals total income (wages and profits) in each country,

$$E_i = \sum_{k=s,u} w_{k,i} L_{k,i} + \sum_j \sum_n M_i(j) \left\{ \int_{z_{in}^*(j)}^{\infty} [\pi_{in}(z,j) - v_i(j) f_{in}] dG(z) - v_i(j) f^e \right\}$$

### 3.2 Equilibrium Characterization

In this section we derive the equations that we use to solve for equilibrium factor prices and trade patterns. We consider specifications (i) with endogenous or exogenous entry and (ii) with or without selection.

**International trade:** Denote by  $\Lambda_{in}(j)$  the sector  $j$  expenditure share in country  $n$  on goods from country  $i$ . By definition, we have

$$\Lambda_{in}(j) = \frac{M_i(j) \int_{z_{in}^*(j)}^{\infty} r_{in}(z,j) dG(z)}{\sum_{k=1}^I M_k(j) \int_{z_{kn}^*(j)}^{\infty} r_{kn}(z,j) dG(z)}.$$

Substituting in for  $G(z)$  and  $r_{in}(z,j)$  yields

$$\Lambda_{in}(j) = \frac{M_i(j) v_i(j)^{1-\eta} z_{in}^*(j)^{\eta-\gamma-1} \tau_{in}^{1-\eta}}{\sum_{k=1}^I M_k(j) v_k(j)^{1-\eta} z_{kn}^*(j)^{\eta-\gamma-1} \tau_{kn}^{1-\eta}}. \quad (13)$$

In the specification without selection, in which  $z_{in}^*(j) = 1$  for all  $i, n \in I$  and  $j \in J$ , Equation (13) implies

$$\Lambda_{in}(j) = \frac{M_i(j) v_i(j)^{1-\eta} \tau_{in}^{1-\eta}}{\sum_{k=1}^I M_k(j) v_k(j)^{1-\eta} \tau_{kn}^{1-\eta}}. \quad (14)$$

In the specification with selection, in which  $z_{in}^*(j) > 1$  for all  $i, n \in I$  and  $j \in J$ , Equation (13) implies

$$\Lambda_{in}(j) = \frac{M_i(j) [v_i(j)]^{\frac{\gamma\eta-\eta+1}{1-\eta}} f_{in}^{\frac{\gamma-\eta+1}{1-\eta}} \tau_{in}^{-\gamma}}{\sum_{k=1}^I M_k(j) [v_k(j)]^{\frac{\gamma\eta-\eta+1}{1-\eta}} f_{kn}^{\frac{\eta-\gamma-1}{\eta-1}} \tau_{kn}^{-\gamma}}. \quad (15)$$

We define  $t$  to be the relative size of international versus intra-national trade costs,

$$t = \begin{cases} \tau^{\eta-1} & \text{without selection} \\ \tau^\gamma f^{\frac{\gamma+1-\eta}{\eta-1}} & \text{with selection.} \end{cases}$$

It is apparent from inspection of equations (14) and (15) that it is this relative cost  $t$  that matters, rather than  $\tau$  and  $f$  separately. We assume that relative costs of international trade are strictly greater than those of intra-national trade, so that  $t > 1$ .

We denote by  $\Delta_i = \frac{1}{2} [\Lambda_{ni}(x) + \Lambda_{ni}(y)]$ , for  $n \neq i$ , country  $i$ 's trade share. Note that

$\Delta_i$  is the share of country  $i$ 's expenditure allocated to imports from country  $n \neq i$ . We also denote by  $\Theta_i = \Lambda_{ni}(y) - \Lambda_{ni}(x)$ , for  $n \neq i$ , the share of country  $i$ 's expenditure allocated to imports in sector  $y$  minus the share of expenditures allocated to imports in sector  $x$ . The greater in absolute value is  $\Theta_i$ , the greater is the difference between net imports in the  $x$  and  $y$  sectors. Hence, for a given trade share  $\Delta_i$ ,  $\Theta_i$  indicates the importance of between sector trade relative to within sector trade.

**Labor market clearing:** In Appendix A we show that the labor market clearing conditions—when entry is endogenous or exogenous and with or without selection—are given by

$$w_{k,i}L_{k,i} = \sum_j \sum_n \lambda \alpha_k(j) \Lambda_{in}(j) \left( \frac{E_n}{2} \right), \quad (16)$$

where  $\lambda$  is the share of revenues paid to all factors in both sectors,

$$\lambda E_i = \sum_{k=s,u} w_{k,i}L_{k,i}, \quad (17)$$

and is given by

$$\lambda = \begin{cases} 1 & \text{with endogenous entry} \\ \frac{\gamma\eta - \eta + 1}{\gamma\eta} & \text{with exogenous entry and with selection} \\ \frac{\eta - 1}{\eta} & \text{with exogenous entry and without selection} \end{cases} \quad (18)$$

in the different specifications of the model.

**Equilibrium firm entry:** In Appendix A we show that with endogenous entry, the mass of entering firms in each sector is given by

$$M_i(j) v_i(j) f^e = \tilde{\lambda} \sum_n \Lambda_{in}(j) \left( \frac{E_n}{2} \right) \quad (19)$$

where  $\tilde{\lambda} = 1/\eta$  without selection and  $\tilde{\lambda} = (\eta - 1) / (\gamma\eta)$  with selection.

**Solving for an equilibrium:** Equilibrium factor prices, total expenditures  $E_i$ , expenditure shares  $\Lambda_{in}(j)$ , and entrants  $M_i(j)$  can be solved for using factor market clearing as given by equation (16) (note that, by Walras' law, one equation is redundant), equation (17), expenditure shares  $\Lambda_{in}(j)$  as given by Equation (14) without selection and by Equation (15) with selection, and the free-entry conditions (with endogenous entry) as given by Equation (19).<sup>14</sup>

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<sup>14</sup>After solving for an equilibrium assuming that the model is either with selection or without selection,

We compute production and consumption of the final non-tradeable good,  $Q_i$ , as follows. Given factor prices, nominal expenditures, and entry levels, the solution for sectoral price indices is provided in Appendix B. Using sectoral price indices and the definition of the aggregate price level,  $P_i$ , above we obtain  $Q_i$ . Our model and this solution procedure can be extended to any number of factors, sectors, and countries.

In some comparative static exercises, in Section 5, we simplify the model solution by assuming that *countries and sectors are mirror symmetric*:  $A_1(x) = A_2(y)$ ,  $A_1(y) = A_2(x)$ ,  $L_{s1} = L_{u2}$ ,  $L_{u1} = L_{s2}$ , and  $\alpha_x = 1 - \alpha_y$ . Mirror symmetry makes the model more tractable because  $w_{s1} = w_{u2}$ ,  $w_{u1} = w_{s2}$ , and  $E_1 = E_2$ .

### 3.3 Mapping to General Framework

The model clearly fits into the general framework presented in Section 2. In the specification with exogenous entry, constructing  $L_{k,in}(j)$  is straightforward. It is the sum of factor  $k$  employment in variable production and market access costs for supplying destination market  $n$ . With CES sectoral aggregators, the share of variable costs in total sectoral revenue is constant. With CES sectoral aggregators and Pareto-distributed productivity, the share of market access costs in total sectoral revenue is also constant. Hence, with common  $\eta$  and  $\gamma$  across sectors and countries,  $\lambda_{in}(j) = \lambda$  for all destination markets and in each sector, where  $\lambda = (\gamma\eta - \eta + 1) / (\gamma\eta)$  with selection and  $\lambda = (\eta - 1) / \eta$  without selection. Since factor intensity is common across fixed and variable costs as well as across source and destination markets, we have  $\alpha_{k,in}(j) = \alpha_k(j)$ . Hence, equation (1) from the general framework of Section 2 is simplified to equation (16) in our specialized model.

In the specification with endogenous entry, constructing  $L_{k,in}(j)$  is more subtle because there are multiple ways of allocating entry costs,  $f^e$ , across destination markets. However, in this specification we do not need to construct  $L_{k,in}(j)$  to use the results in Section 2. This is because for *any* construction of  $L_{k,in}(j)$  consistent with equilibrium sectoral factor allocation (i.e.,  $L_{k,i}(j) = \sum_n L_{k,in}(j)$ ), we can simplify equations (1) from the general framework of Section 2 to equation (16) in our model. To obtain equation (16), we make use of two results: (i) free entry implies that revenues are equal to total costs (including entry, market access, and variable costs) in each sector, and (ii) fixed and variable costs have a common factor intensity in each sector. Note that to obtain this result in the specification with endogenous entry, we do not make use of Pareto distributed productivity or CES aggregators.

Given that factor market clearing conditions are given by equation (16), it follows that we can express the FCT using equation (8) in all specifications of our model. Finally,

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one must verify that all cutoffs are either greater than one or equal to one, respectively, using equation (12).

with Cobb-Douglas preferences and production functions and unchanged share parameters ( $\alpha_k(j) = \alpha'_k(j)$ ), equation (6) from the general framework simplifies to equation (9), so that the change in the skill premium across two equilibria is given by

$$\frac{w'_{s,i} / w_{s,i}}{w'_{u,i} / w_{u,i}} = \frac{\mathcal{L}'_{u,i} / \mathcal{L}_{u,i}}{\mathcal{L}'_{s,i} / \mathcal{L}_{s,i}}.$$

Hence, in all specifications of our model, changes in the skill premium are fully determined by changes in trade-adjusted factor supplies. Moreover, since we impose that factor supplies are fixed parameters ( $L_{k,i} = L'_{k,i}$ ), changes in the FCT are sufficient statistics for the impact of trade on the skill premium: changes in trade costs or in productivities affect the skill premium only through changes in the FCT.

## 4 The Skill Premium, Factor Allocation, and Trade

We now investigate the impact of trade liberalizations on the skill premium, factor allocation, and trade patterns in our model. We first show that if country 1 has a comparative advantage in the skill intensive good, then the trade-adjusted relative supply of skill,  $\mathcal{L}_{s,i}/\mathcal{L}_{u,i}$ , falls in country 1 and rises in country 2 when countries open to trade. We then show that changes in  $\mathcal{L}_{s,i}/\mathcal{L}_{u,i}$  fully determine the impact of trade liberalization not only on the skill premium, as shown in the previous section, but also on between-sector factor allocation and between-sector trade. Through these results, we obtain a generalized version of what is often referred to as the Stolper-Samuelson effect. The Stolper-Samuelson effect relates changes in factor prices to exogenous changes in goods prices, whereas we relate changes in factor prices, factor allocation, and trade patterns to changes in trade costs, via changes in trade shares.

We say that country 1 has a comparative advantage in sector  $x$  if the cost of the composite input bundle in sector  $x$  relative to sector  $y$  is relatively lower in country 1 than in country 2 in autarky:  $v_1(x)/v_1(y) < v_2(x)/v_2(y)$  in autarky. According to this definition, country 1 has a comparative advantage in the skill-intensive sector if and only if

$$a \left( \frac{H_1/L_1}{H_2/L_2} \right)^{\alpha_x - \alpha_y} > 1. \quad (\text{CA})$$

Condition CA follows from the definition of  $v_i(j)$  in equation (10), from the factor-market clearing condition in equation (16), and from the observation that  $\Lambda_{12}(j) = \Lambda_{21}(j) = 0$  in autarky. Without loss of generality, we impose Condition CA throughout the remainder of the paper.

To understand Condition CA, consider two special cases that are standard in the liter-

ature. First, if  $a = 1$  so that there is no Ricardian comparative advantage, then country 1 has a comparative advantage in sector  $x$  if and only if  $H_1/L_1 > H_2/L_2$ , exactly as in the Heckscher-Ohlin model. Second, if endowment ratios are the same across countries so that there is no Heckscher-Ohlin-based comparative advantage, then country 1 has a comparative advantage in sector  $x$  if and only if  $a > 1$ , exactly as in the Ricardian model.

The consequences of moving from autarky ( $\Delta_1, \Delta_2 = 0$ ) to positive trade shares ( $\Delta'_1, \Delta'_2 > 0$ ) on the trade-adjusted relative supply of skill in country 1 are stated in the following proposition.

**Proposition 2** *If  $\Delta_1, \Delta_2 = 0$  and  $\Delta'_1, \Delta'_2 > 0$ , then  $\mathcal{L}'_{s,1}/\mathcal{L}'_{u,1} < \mathcal{L}_{s,1}/\mathcal{L}_{u,1} = L_{s,1}/L_{u,1}$ .*

Country 1 is a net exporter in the sector in which it has a comparative advantage, sector  $x$ :  $\Theta_1 = \Lambda_{21}(y) - \Lambda_{21}(x) > 0$  if  $\Delta_1 > 0$ . Because the  $x$  sector is skill intensive, country 1's net exports embody a positive amount of skilled labor,  $FCT_{s,1} > 0$ , and a negative amount of unskilled labor,  $FCT_{u,1} < 0$ , if  $\Delta_1 > 0$ . Hence, moving from autarky to any positive trade shares reduces the trade-adjusted relative supply of skill in country 1.

For given trade shares  $\Delta_1$  and  $\Delta_2$ , the level of the trade-adjusted relative supply of skill in either country,  $\mathcal{L}_{s,i}/\mathcal{L}_{u,i}$ , determines important economic outcomes in both countries: the skill premium  $w_{s,i}/w_{u,i}$ ; between-sector factor allocation  $L_{k,i}(j)$ ; and between-sector trade (the absolute value of  $\Theta_i$ ). The following proposition states specifically how these economic outcomes vary across two equilibria with equal trade shares but different trade-adjusted factor supplies.

**Proposition 3** *In any two trade equilibria with equal trade shares  $\Delta'_1 = \Delta_1 > 0$  and  $\Delta'_2 = \Delta_2 > 0$ , the following eight statements are equivalent:*

- |       |   |        |   |
|-------|---|--------|---|
| (i)   | $\mathcal{L}'_{s,1}/\mathcal{L}'_{u,1} < \mathcal{L}_{s,1}/\mathcal{L}_{u,1}$ | (ii)   | $\mathcal{L}'_{s,2}/\mathcal{L}'_{u,2} > \mathcal{L}_{s,2}/\mathcal{L}_{u,2}$ |
| (iii) | $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$   | (iv)   | $w'_{s,2}/w'_{u,2} < w_{s,2}/w_{u,2}$   |
| (v)   | $L'_{k,1}(x) > L_{k,1}(x)$ for $k = s, u$                                     | (vi)   | $L'_{k,2}(x) < L_{k,2}(x)$ for $k = s, u$                                     |
| (vii) | $\Theta'_1 > \Theta_1$  | (viii) | $\Theta'_2 < \Theta_2$  |

The broad intuition behind Proposition 3 can be understood as follows. A lower trade-adjusted relative supply of skill in country 1 (statement *i*) increases the skill premium in country 1 (statement *iii*), as stated in Proposition 1. For fixed factor supplies, a lower trade-adjusted relative supply of skill requires a higher absolute value of the FCT of skilled and unskilled labor in country 1, which requires that the extent of between-sector trade be greater (statement *vii*, since  $\Theta_1 > 0$  from Condition CA). More between-sector trade requires that a greater share of factors be allocated to country 1's comparative advantage

sector (statement *v*). With trade balance and fixed trade shares, more between-sector trade in country 1 (statement *vii*) requires more between-sector trade in country 2 (statement *viii*, since  $\Theta_2 < 0$ ), a greater share of factors allocated to country 2's CA sector (statement *vi*), and a greater absolute value of the FCT of skilled and unskilled labor, which is associated with both a higher trade-adjusted relative supply of skill (statement *ii*) and a lower skill premium (statement *iv*).

Combining Propositions 2 and 3, we establish the following corollary.

**Corollary 1** *Reducing trade costs so that countries move from autarky ( $\Delta_1, \Delta_2 = 0$ ) to any positive level of trade ( $\Delta'_1, \Delta'_2 > 0$ ) raises the skill premium in country 1 and reduces it in country 2, reallocates factors towards the  $x$  sector in country 1 and towards the  $y$  sector in country 2, and generates positive net exports in the  $x$  sector in country 1 and in the  $y$  sector in country 2.*

Intuitively, starting in autarky, a reduction in trade costs increases each country's net exports in its comparative advantage sector. This requires factors to reallocate towards that sector, which increases the relative demand and, therefore, the relative price of the factor that is used intensively in the comparative advantage sector.

## 5 Technology, Selection, and Entry

Changes in the trade-adjusted relative supply of skill are determined by changes in the FCT, which are endogenous. Our next goal is to study how key margins in our model—the extent of productivity heterogeneity between and within sectors, and heterogeneous firms' decisions to enter and operate in each market—shape the impact of trade liberalization on the FCT, and, therefore, on factor allocation and the skill premium.

These margins matter for equilibrium outcomes only through their impacts on expenditure shares  $\Lambda_{in}(j)$ . This follows from Proposition 3, which shows that for given factor supplies and trade shares, changes in trade-adjusted relative factor supplies in each country are fully determined by changes in the extent of between-sector trade  $\Theta_1$ , which is itself determined by changes in  $\Lambda_{in}(j)$ .

Equation (13) illustrates the various exogenous and endogenous determinants of these expenditure shares. First, composite input costs,  $v_i(j)$ , have a direct effect on expenditure shares through the prices charged by active firms. All else equal, lowering  $v_i(j)$  increases  $\Lambda_{in}(j)$  for all  $n$ . From equation (10), composite input costs can be decomposed into two components: (i) factor prices and intensities,  $w_{k,i}^{\alpha_k(j)}$ , as in the Heckscher-Ohlin model, and (ii) exogenous sectoral technologies,  $A_i(j)$ , as in the Ricardian model.

Second, the mass of operating firms from each country shapes expenditure shares: an increase in the mass of country  $i$  firms operating in country  $n$  increases  $\Lambda_{in}(j)$ , all else equal. This mass of firms can be decomposed into two components: (i) the mass of entering firms in country  $i$ , given by  $M_i(j)$ , and (ii) the fraction of country  $i$  entrants that operate in country  $n$ , which is negatively related to  $z_{in}^*(j)$ . All else equal, an increase in the mass of operating firms, either through an increase in  $M_i(j)$  or a decrease in  $z_{in}^*(j)$ , is equivalent, in terms of expenditure shares, to an increase in sectoral productivity  $A_i(j)$ .

Third, the extent of productivity heterogeneity affects the elasticity of expenditure shares to a change in the productivity cutoff,  $z_{in}^*(j)$ . In particular, a greater dispersion of productivity, a lower  $\gamma$ , decreases the concentration of firms around the cutoff. This implies a smaller decrease in the mass of operating firms for a given increase in the productivity cutoff.

In what follows, we study how each of these margins affects the impact of trade liberalization on trade-adjusted relative supplies of skill, and therefore on the skill premium and the extent of both between-sector factor reallocation and trade. In order to isolate the effects of these margins in our comparative static exercises, we choose trade costs,  $t$ , and relative country productivities,  $T_1/T_2$ , so that trade shares,  $\Delta_1$  and  $\Delta_2$ , remain fixed.<sup>15</sup> When comparing across equilibria under different parameter values, we always impose that factor supplies, factor shares, and the elasticity of substitution between varieties within sectors remain fixed:  $L_{k,i} = L'_{k,i}$ ,  $\alpha_k(j) = \alpha'_k(j)$ , and  $\eta = \eta'$ .

## 5.1 Productivity heterogeneity

Proposition 4 summarizes our findings about how productivity heterogeneity affects the impact of trade liberalization on trade-adjusted relative supplies of skill, and therefore on the skill premium and the extent of both between-sector factor reallocation and trade.<sup>16</sup>

**Proposition 4** *In the specification of the model with selection, the decline in the trade-adjusted relative supply of skill in country 1 caused by moving from autarky to trade shares  $\Delta_1, \Delta_2 > 0$  is greater the higher is  $a$  or the lower is  $\gamma$  if either (i) entry is exogenous, or (ii) entry is endogenous and countries and sectors are mirror symmetric.*

<sup>15</sup>If we were to hold trade costs rather than trade shares fixed, then some of our comparative static results would be ambiguous. For example, an increase in technological dispersion, i.e. a reduction in  $\gamma$ , would increase total trade. This could offset the direct effect of  $\gamma$  discussed in Proposition 4 below. Note also that for given trade shares, the partial elasticity of trade flows with respect to variable trade costs does not fully determine the implications of trade liberalization for the skill premium and factor allocation.

<sup>16</sup>Based on economic intuition and many numerical examples, we believe that in the case of endogenous entry Propositions 4 and 6 continue to hold even if we do not impose mirror symmetry. However, we have not yet been successful proving this more general result.



Consider first the intuition for Proposition 4 in the exogenous entry case. Increasing  $a$  (i.e., increasing country 1's relative productivity advantage in sector  $x$ ) reduces country 1's cost in the  $x$  sector relative to its cost in the  $y$  sector, relative to that in country 2. These changes in relative costs reinforce country 1's comparative advantage in sector  $x$ , inducing country 1 to specialize further in sector  $x$ . Hence,  $\mathcal{L}_{s,1}/\mathcal{L}_{u,1}$  falls because the  $x$  sector is skill intensive.<sup>17</sup>

Consider second the role of  $\gamma$  in the exogenous entry case. As discussed above, the elasticity of  $\Lambda_{in}(j)$  to a change in the cutoff productivity  $z_{in}^*(j)$  is increasing in  $\gamma$ . To understand how this elasticity matters for economic outcomes, consider a change in  $v_i(j)$ , the composite input cost in sector  $j$ . The direct effect of such a change on  $z_{in}^*(j)$  is independent of  $\gamma$ . However, a given change in  $z_{in}^*(j)$  has a larger effect on sectoral expenditures the less dispersed are productivities, i.e., the higher is  $\gamma$ . Hence, higher values of  $\gamma$  increase the responsiveness of expenditure shares to a change in the cost of the composite input bundle. This implies that factor endowment differences and sectoral productivity differences, which affect the relative cost of the composite input bundle across countries, play a larger role in shaping expenditure shares, and therefore trade-adjusted relative supplies of skill, when  $\gamma$  is larger.

In the endogenous entry case, changes in  $a$  and  $\gamma$  have indirect effects on expenditure shares through  $M_i(j)$ , in addition to the direct effects we discuss above in the exogenous entry case. An increase in  $a$  increases relative entry in the  $x$  sector in country 1 relative to country 2, which reinforces the direct effect. That is, endogenous entry magnifies exogenous comparative advantage. To understand the impact of an increase in  $\gamma$  on entry, consider the following thought experiment: Starting in autarky, consider a move to trade first holding both  $z_{in}^*(j)$  and  $M_i(j)$  fixed. International trade increases market-specific relative profits in a country's comparative advantage sector. Note that the impact on profits does not depend directly on  $\gamma$  for a fixed  $z_{in}^*(j)$ . Allowing now for changes in  $z_{in}^*(j)$  while still holding entry fixed, the previous discussion implies that changes in  $z_{in}^*(j)$  are also independent of  $\gamma$ . However, the less dispersed are productivities (i.e. the greater is  $\gamma$ ), the greater is the change in a potential entrant's expected market-specific profit given equal-sized changes in  $z_{in}^*(j)$ . Hence, given  $M_i(j)$ , opening up to trade induces larger changes in the expected value of firms at entry, the higher is  $\gamma$ . Hence, we should anticipate a rise in relative entry in the comparative advantage sector, and this rise should be greater the higher is  $\gamma$ . Thus, the indirect effect of a change in  $\gamma$  on entry reinforces the direct effect of  $\gamma$ .

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<sup>17</sup>Had we not imposed  $M_i(j) = M_i$ , then increasing the relative mass of entrants in a sector would have identical implications as increasing the relative sectoral productivity in that sector.

While Proposition 4 focuses on the specification of the model with selection, we obtain similar results in the specification without selection. In this specification, the parameter  $a$  has the same effect as in Proposition 4. On the other hand, since the partial elasticity of expenditure shares with respect to composite input costs is  $1 - \eta$  instead of  $(\gamma\eta + 1 - \eta) / (1 - \eta)$ , in this case Proposition 4 holds when  $\gamma$  is replaced by  $\eta$ . Without selection, within-sector productivity heterogeneity does not matter for the trade-adjusted relative supply of skill.

## 5.2 Entry

Proposition 5 summarizes our findings about how the extent of endogenous entry affects the impact of trade liberalization on trade-adjusted relative supplies of skill, and therefore on the skill premium and the extent of both between-sector factor reallocation and trade.

**Proposition 5** *The decline in the trade-adjusted relative supply of skill in country 1 caused by moving from autarky to trade shares  $\Delta_1, \Delta_2 > 0$  is greater in the specification with endogenous entry than in the specification with exogenous entry.*

In the specification with endogenous entry, trade liberalization increases entry in a country's comparative advantage sector relative to its comparative disadvantage sector. Recall that a larger mass of entrants in a given sector is equivalent—in terms of its implications for the skill premium and the extent of both between-sector factor reallocation and trade—to an increase in that sector's exogenous Ricardian productivity. Hence, endogenous entry decreases  $\mathcal{L}_{s,1}/\mathcal{L}_{u,1}$ , just as an increase in exogenous Ricardian comparative advantage  $a$ .

## 5.3 Selection

Proposition 6 summarizes our findings about how the extent of selection affects the impact of trade liberalization on trade-adjusted relative supplies of skill, and therefore on the skill premium and the extent of both between-sector factor reallocation and trade.

**Proposition 6** *The decline in the trade-adjusted relative supply of skill in country 1 caused by moving from autarky to trade shares  $\Delta_1, \Delta_2 > 0$ , is greater in the specification with selection than in the specification without selection if either (i) entry is exogenous, or (ii) entry is endogenous and countries and sectors are mirror symmetric.*

This result follows directly from the following two observations; hence, we omit a formal proof of this proposition. First, trade patterns and factor prices obtained using the equations in the specification with selection limit to those obtained using the equations in the

specification without selection, as  $\gamma$  converges to  $\eta - 1$ , when all parameters are the same across specifications (obviously with the exception of market access costs, which are assumed to be zero without selection). This is because, as  $\gamma$  converges to  $\eta - 1$ , almost all production occurs within an arbitrarily small mass of very productive firms. Hence, in this limiting case  $\mathcal{L}_{s,1}/\mathcal{L}_{u,1}$  is equivalent in the specification with selection and the specification without selection. Second, in the specification with selection, the decline in  $\mathcal{L}_{s,1}/\mathcal{L}_{u,1}$  (holding trade shares fixed) is greater the higher is  $\gamma$ , as shown in Proposition 4.

Intuitively, with endogenous selection the fraction of country 1 entrants, relative to country 2 entrants, that choose to sell in any given market is relatively larger in country 1's comparative advantage sector because country 1 has a relatively lower composite input cost in this sector:<sup>18</sup>

$$\frac{z_{1n}^*(x)}{z_{2n}^*(x)} < \frac{z_{1n}^*(y)}{z_{2n}^*(y)} \text{ for all } n. \quad (20)$$

Recall that a larger fraction of firms that supply a given market is equivalent—in terms of its implications for the skill premium and the extent of both between-sector factor reallocation and trade—to a larger exogenous sectoral productivity. Hence, endogenous selection reinforces ex-ante comparative advantage. Note that when  $a = 1$  this implies that the average productivity of country 1 firms supplying a given country is relatively *lower* in country 1's comparative advantage sector, relative to country 2.

**Relation of Proposition 6 to BRS:** Proposition 6 and Condition (20) are reminiscent of a result in BRS that, with selection and endogenous entry, differences in endowments across countries lead to stronger selection for domestic production in a country's comparative advantage sector:

$$\frac{z_{11}^*(x)}{z_{11}^*(y)} > \frac{z_{22}^*(x)}{z_{22}^*(y)}. \quad (21)$$

That is, endogenous selection implies that the average productivity of firms that choose to produce for the domestic market is relatively greater in country 1's comparative advantage sector, compared to country 2. This leads to their interpretation that differences in endowments across countries induce what they call "endogenous Ricardian productivity differences" at the industry level, which magnify Heckscher-Ohlin-based comparative advantage. This interpretation may appear similar to our result in Proposition 6, but it is not. BRS do not show what implications, if any, Condition (21) has for the skill premium or for the extent of either between-sector factor reallocation or trade.

Moreover, Condition (20), which plays a central role in Proposition 6, differs from Condition (21) in three important respects. First, Condition (20) depends on a comparison of

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<sup>18</sup>With endogenous entry, the relative mass of entrants also plays a role in this result.

cutoffs in a *common destination market*, whereas Condition (21) depends on a comparison of cutoffs in *each country's domestic market*. Second, Condition (20) emphasizes that a country is *less* selective in any given destination in its comparative advantage sector, relative to the other country, whereas Condition (21) emphasizes that a country is *more* selective in its domestic market in its comparative advantage sector, relative to the other country. Third, while Condition (20) is satisfied with either endogenous or exogenous entry, Condition (21) is reversed in the specification with exogenous entry. That is, while endogenous selection magnifies the effect of trade on the skill premium and the extent of both between-sector factor reallocation and trade in both the specification with endogenous entry and the specification without, Condition (21) holds in one specification but is reversed in the other. The following Proposition shows that whether the average productivity of domestic firms is relatively higher or lower in a country's comparative advantage sector compared to another country depends on whether entry is endogenous or exogenous.<sup>19</sup>

**Proposition 7** *Consider the specification of our model with selection and suppose that  $A_i(j) = 1$  and, if entry is exogenous, that  $M_i(j) = 1$ . If entry is endogenous and trade shares are positive, then  $z_{11}^*(x)/z_{11}^*(y) > z_{22}^*(x)/z_{22}^*(y)$ . If entry is exogenous and there is no factor price equalization, then  $z_{11}^*(x)/z_{11}^*(y) < z_{22}^*(x)/z_{22}^*(y)$ .*

Why does the relationship between the average productivity of domestic firms across sectors depend on whether entry is endogenous or exogenous? In our model, countries specialize in their comparative advantage sector. Recall from equation (13) that an expansion of the comparative advantage sector can occur along three margins: (i) firms of equal productivity can be larger, (ii) the productivity cutoff can be lower, and (iii) entry can be greater. With exogenous entry, only margins (i) and (ii) are active. Equally productive firms are larger in the comparative advantage sector and, in order to have a larger mass of operating firms, the comparative advantage sector must be relatively less selective. With endogenous entry, all three margins are active. Moreover, margins (ii) and (iii) are not independent. When entry is endogenous, a relatively higher entry level in the comparative advantage sector makes survival relatively more difficult in the domestic market. Hence, this sector is larger while also being more selective.<sup>20</sup>

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<sup>19</sup>In Lemma 7, we impose  $M_i(j) = 1$  with exogenous entry and  $A_i(j) = 1$  so that factor endowment differences are the unique source of exogenous comparative advantage, as in BRS.

<sup>20</sup>Another result in BRS—that the export cutoff relative to the domestic cutoff is relatively lower in each country's comparative advantage sector, e.g.  $z_{12}^*(x)/z_{11}^*(x) < z_{12}^*(y)/z_{11}^*(y)$ —holds both with endogenous entry (as considered in BRS) and exogenous entry.

## 6 Conclusions

In this paper we have provided a unifying framework to study how factor prices and factor allocation respond to trade liberalizations. We derived a simple expression relating equilibrium factor prices to two components: *trade-adjusted factor supplies* and *relative factor payments for domestic absorption*. We showed how changes in relative factor prices within a range of workhorse models of trade can be mapped into these two components and described a set of standard assumptions under which changes in the FCT are sufficient statistics for the impact of trade on relative factor prices.

We then specialized the general framework to an environment that combines the key elements of the Heckscher-Ohlin model, the Ricardian model, and the Melitz model. Changes in the FCT fully determine not only relative factor prices, but also the extent of between-sector factor reallocation and between-sector trade. We used this model to examine how the FCT is shaped by heterogeneous firms' decisions to enter and to operate in each market. Endogenous entry and endogenous selection of firms into markets magnify the impact of trade on the FCT and hence the change in the skill premium and the extent of between-sector trade and factor reallocation, while greater within-sector productivity heterogeneity weakens these effects. Given the extensive evidence of large productivity differences within narrowly-defined sectors, our prediction about the implications of within-sector productivity heterogeneity provides a rationale for empirical results suggesting that the FCT, the extent of between-sector factor reallocation induced by trade, and the impact of trade on the skill premium are small in practice.

## Appendix A: Additional Derivations

### Labor Market Clearing with Exogenous Entry

**Variable input costs:** With Cobb-Douglas production functions, payments to skilled and unskilled labor hired as a variable input in the production of a variety of sector  $j$  in country  $i$  that is bound for country  $n$ , denoted by  $l_{s,in}(z, j)$  and  $l_{u,in}(z, j)$ , are proportional to market-specific revenues

$$w_{k,i} l_{k,in}(z, j) = \frac{\eta - 1}{\eta} \alpha_k(j) r_{in}(z, j). \quad (22)$$

Equation (22) implies that total payments to country  $i$  labor employed in variable production in sector  $j$  are  $\frac{1}{2} \sum_n \frac{\eta - 1}{\eta} \Lambda_{in}(j) E_n$ , of which a share  $\alpha_k(j)$  is paid to factor  $k$ .

**Market access input costs:** Country  $i$ 's total market access fixed costs associated with selling

sector  $j$  goods in country  $n$  are given by

$$F_{in}(j) = M_i(j) f_{in} v_i(j) z_{in}^*(j)^{-\gamma}.$$

Equation (11) implies that total sector  $j$  revenue in country  $i$  from goods shipped to country  $n$  is

$$\Lambda_{in}(j) Q_n(j) P_n(j) = M_i(j) \frac{E_n}{2P_n(j)^{1-\eta}} \left( \frac{\eta}{\eta-1} \right)^{1-\eta} [v_i(j) \tau_{in}]^{1-\eta} \frac{\gamma}{\gamma+1-\eta} z_{in}^*(j)^{\eta-\gamma-1}$$

so that in general

$$F_{in}(j) = \frac{\eta^\eta}{(1-\eta)^{1-\eta}} \frac{\gamma+1-\eta}{\gamma\eta} P_n(j)^{1-\eta} v_i(j)^\eta f_{in} \tau_{in}^{\eta-1} \Lambda_{in}(j) z_{in}^*(j)^{1-\eta}.$$

In the case with no selection  $F_{in}(j) = 0$ . In the case with selection

$$F_{in}(j) = \frac{\gamma+1-\eta}{\gamma\eta} \frac{1}{2} \Lambda_{in}(j) E_n, \quad (23)$$

of which a share  $\alpha_k(j)$  is paid to factor  $k$ .

**Total factor payments without selection:** With no selection into any market, variable labor costs represent a share  $(\eta-1)/\eta$  of total revenues and market access payments are zero. Therefore, total labor payments equal  $\frac{1}{2} \sum_n \left( \frac{\eta-1}{\eta} \right) \Lambda_{in}(j) E_n$ , of which a share  $\alpha_k(j)$  is paid to factor  $k$ . Factor market clearing implies

$$w_{k,i} L_{k,i} = \sum_j \sum_n \frac{\eta-1}{\eta} \alpha_k(j) \Lambda_{in}(j) \frac{E_n}{2} \quad (24)$$

In the exogenous entry case, Equation (24) and balanced trade imply  $E_n = \frac{\eta}{\eta-1} (w_{s,n} L_{s,n} + w_{u,n} L_{u,n})$ . Hence, equation (24) is equivalent to equation (16), where  $\lambda = (\eta-1)/\eta$ .

**Total factor payments with selection:** Total payments to sector  $j$  labor in country  $i$  are the sum of variable input payments and market access fixed cost payments. With selection, these payments equal  $\frac{1}{2} \sum_n \left( \frac{\gamma\eta-\eta+1}{\gamma\eta} \right) \Lambda_{in}(j) E_n$ , of which a share  $\alpha_k(j)$  is paid to factor  $k$ . Hence, factor market clearing implies

$$w_{k,i} L_{k,i} = \sum_j \sum_n \left( \frac{\gamma\eta-\eta+1}{\gamma\eta} \right) \alpha_k(j) \Lambda_{in}(j) \frac{E_n}{2} \quad (25)$$

Equation (25) and balanced trade imply  $E_n = \frac{\gamma\eta}{\gamma\eta-\eta+1} (w_{s,n} L_{s,n} + w_{u,n} L_{u,n})$ . Hence, equation (25) is equivalent to equation (16), where  $\lambda = (\gamma\eta-\eta+1)/\gamma\eta$ .

## Labor Market Clearing and Entry with Endogenous Entry

**Labor market clearing:** With free entry, total revenue equals total factor payments, sector by sector,

$$\sum_k w_{k,i} L_{k,i}(j) = \sum_n \Lambda_{in}(j) \left( \frac{E_n}{2} \right).$$

Moreover, the share of factor payments that accrue to factor  $k$  is  $\alpha_k(j)$  in sector  $j$ ,

$$w_{k,i} L_{k,i}(j) = \alpha_k(j) \sum_n \Lambda_{in}(j) \left( \frac{E_n}{2} \right). \quad (26)$$

By summing equation (26) across sectors, we obtain equation (16), where  $\lambda = 1$ .

**Entry:** Total entry costs in sector  $j$  are  $M_i(j) v_i(j) f^e$ . From the free entry condition, total entry costs,  $M_i(j) v_i(j) f^e$ , are equal to total revenues,  $\sum_n \Lambda_{in}(j) \left( \frac{E_n}{2} \right)$ , minus variable production costs,  $\sum_n \frac{\eta-1}{\eta} \Lambda_{in}(j) \left( \frac{E_n}{2} \right)$ , and market access costs,  $F_{in}(j)$ . Together with  $F_{in}(j) = 0$ , without selection, and with equation (23), with selection, we obtain equation (19) both with and without selection.

## Price Indices

The sector  $j$  price level in country  $n$  equals

$$P_n(j) = \left( \frac{\gamma}{\gamma + 1 - \eta} \right)^{\frac{1}{1-\eta}} \frac{\eta}{\eta - 1} \left[ \sum_i M_i(j) [\tau_{in} v_i(j)]^{1-\eta} z_{in}^*(j)^{\eta-\gamma-1} \right]^{1/(1-\eta)}. \quad (27)$$

Without selection Equation (27) is equivalent to

$$P_n(j)^{\eta-1} = \frac{\lambda_2}{\sum_i M_i(j) \tau_{in}^{1-\eta} v_i(j)^{1-\eta}} \quad (28)$$

where  $\lambda_2 = \frac{\gamma+1-\eta}{\gamma} \left( \frac{\eta}{\eta-1} \right)^{\eta-1}$ . With selection, Equation (27) is equivalent to

$$P_n(j)^\gamma = \frac{\lambda_1 (Q_n P_n)^{\frac{\eta-\gamma-1}{\eta-1}}}{\sum_i M_i(j) v_i(j)^{\frac{\eta\gamma+1-\eta}{1-\eta}} f_{in}^{\frac{\gamma+1-\eta}{1-\eta}} \tau_{in}^{-\gamma}} \quad (29)$$

where  $\lambda_1 = \frac{\gamma+1-\eta}{\gamma} \left( \frac{\eta}{\eta-1} \right)^{\eta-1} \left( \frac{1}{J\eta^\eta(\eta-1)^{1-\eta}} \right)^{\frac{\eta-\gamma-1}{\eta-1}}$ .

## Appendix B: Proofs

**Proof of Proposition 2.** We proceed by contradiction. Suppose  $\Lambda_{21}(y) \leq \Lambda_{21}(x)$ . By equation (14) or (15),  $\Lambda_{21}(y) < (=)\Lambda_{21}(x)$  is equivalent to

$$\left[ \frac{v_2(x) v_1(y)}{v_2(y) v_1(x)} \right]^\zeta < (=) \frac{M_1(y) M_2(x)}{M_1(x) M_2(y)},$$

where  $\zeta = \eta - 1 > 0$  without selection and  $\zeta = (\gamma\eta - \eta + 1) / (\eta - 1) > 0$  with selection, and where

$$\frac{v_2(x) v_1(y)}{v_2(y) v_1(x)} = a \frac{w_{s,2}/w_{u,2}}{w_{s,1}/w_{u,1}}.$$

With exogenous entry,  $\frac{M_1(y) M_2(x)}{M_1(x) M_2(y)} = 1$ .<sup>21</sup> With endogenous entry, equation (19) implies

$$\frac{M_1(y) M_2(x)}{M_1(x) M_2(y)} = \frac{v_1(x) v_2(y)}{v_1(y) v_2(x)} \Gamma,$$

where

$$\Gamma = \frac{\sum_n \Lambda_{1n}(y) E_n \sum_n \Lambda_{2n}(x) E_n}{\sum_n \Lambda_{1n}(x) E_n \sum_n \Lambda_{2n}(y) E_n}.$$

Therefore,  $\Lambda_{21}(y) \leq \Lambda_{21}(x)$  is equivalent to

$$\left[ \frac{v_2(x) v_1(y)}{v_2(y) v_1(x)} \right]^{\zeta + \Xi} \leq \Gamma^\Xi, \quad (30)$$

where  $\Xi = 0$  with exogenous entry and  $\Xi = 1$  with endogenous entry. In autarky, Inequality (30) is violated, since the left-hand-side is strictly greater than one under Condition CA and since  $\Gamma = 1$  simply because  $\Lambda_{ii}(j) = 1$  and  $\Lambda_{in}(j) = 0$  for all  $i \neq n$ . Note that for arbitrarily small trade shares, Inequality (30) remains violated because  $\Gamma$  and  $\frac{v_2(x)v_1(y)}{v_2(y)v_1(x)}$  depend on trade costs only through the  $\Lambda_{in}(j)$ 's, are continuous in the  $\Lambda_{in}(j)$ 's, and for arbitrarily small trade shares  $\Lambda_{ii}(j)$  and  $\Lambda_{in}(j)$  are arbitrarily close to their autarky values.

Since both the left- and right-hand sides of Inequality (30) are continuous in the  $\Lambda_{in}(j)$ 's, a necessary condition for Inequality (30) to be satisfied is that there exist trade costs such that  $\Delta_1, \Delta_2 > 0$  and Inequality (30) is satisfied with equality; i.e.  $\Lambda_{21}(y) = \Lambda_{21}(x)$ . Equation (14) or (15), and  $\Lambda_{21}(y) = \Lambda_{21}(x)$  imply  $\Lambda_{in}(x) = \Lambda_{in}(y)$  for all  $i, n$ . Hence,  $\Lambda_{21}(y) = \Lambda_{21}(x)$  implies  $\Gamma = 1$ . Equation (16) and  $\Lambda_{21}(y) = \Lambda_{21}(x)$  also imply that  $\frac{v_2(x)v_1(y)}{v_2(y)v_1(x)}$  equals its autarky value, which by Condition CA is strictly greater than one. Hence, Inequality (30) can never be satisfied with equality. By continuity, Inequality (30) can never be satisfied. **QED.** ■

**Proof of Proposition 3.** We decompose the proof of Proposition 3 into four parts. First, we prove the equivalence of (i) and (iii). Second, we prove the equivalence of (iii) and (vii). Third, we prove the equivalence of (iii) and (v). The proofs for the equivalence of (ii), (iv), (vi), and (vii) are identical, and therefore omitted. Fourth, we prove the equivalence of (vii) and (viii). Throughout the proof, we impose  $L'_{k,i} = L_{k,i}$ ,  $\alpha'_k(j) = \alpha_k(j)$ ,  $\Theta'_1, \Theta_1 \geq 0$ ,  $\Delta'_1 = \Delta_1$ , and  $\Delta'_2 = \Delta_2$ .

**Part I:** (i)  $\mathcal{L}'_{s,1}/\mathcal{L}'_{u,1} < \mathcal{L}_{s,1}/\mathcal{L}_{u,1}$  if and only if (iii)  $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$ .

Part I follows directly from equation (9).

**Part II:** (iii)  $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$  if and only if (vii)  $\Theta'_1 > \Theta_1$ .

The proof of Part II proceeds in 3 steps.

**Step 1:** The following inequalities are equivalent (a)  $\Lambda'_{11}(x) > \Lambda_{11}(x)$ , (b)  $\Lambda'_{11}(y) < \Lambda_{11}(y)$ , (c)  $\Lambda'_{12}(y) < \Lambda_{12}(y)$ , (d)  $\Lambda'_{12}(x) > \Lambda_{12}(x)$ , and (e)  $\Theta'_1 > \Theta_1$ .

<sup>21</sup>Of course, if  $M_i(x) \neq M_i(y)$  in the case of exogenous entry, then we can always include the  $M$ 's into the  $a$  term, as described in Section 3 and the proof remains unchanged.



$\Delta_1 = \Delta'_1$  and  $\Delta_2 = \Delta'_2$ , together with the identities  $\Lambda_{12}(j) = 1 - \Lambda_{22}(j)$  and  $\Lambda_{21}(j) = 1 - \Lambda_{11}(j)$ , directly imply that Inequalities (a) and (b) are equivalent, as are Inequalities (c) and (d). In what follows, we first show that Inequality (a) is equivalent to Inequality (e) and we conclude by showing that Inequality (c) is equivalent to Inequality (e).

We have  $\Lambda_{21}(x) = \Delta_1 - \frac{1}{2}\Theta_1$ . Hence,  $\Theta'_1 > \Theta_1$  if and only if  $\Lambda'_{21}(x) < \Lambda_{21}(x)$ , since  $\Delta'_1 = \Delta_1$ . Moreover,  $\Lambda'_{21}(x) < \Lambda_{21}(x)$  is equivalent to  $\Lambda'_{11}(x) > \Lambda_{11}(x)$ , since  $\Lambda'_{11}(x) = 1 - \Lambda'_{21}(x)$ . Thus, Inequality (e) is equivalent to Inequality (a).

We conclude by showing that Inequality (c) is equivalent to Inequality (e). To show that Inequality (e) implies Inequality (c), we proceed by contradiction. Suppose that  $\Theta'_1 > \Theta_1$  and  $\Lambda'_{12}(y) \geq \Lambda_{12}(y)$ .  $\Lambda'_{12}(y) \geq \Lambda_{12}(y)$  is equivalent to  $\Lambda'_{12}(x) \leq \Lambda_{12}(x)$  while  $\Theta'_1 > \Theta_1$  is equivalent to both  $\Lambda'_{21}(y) > \Lambda_{21}(y)$  and  $\Lambda'_{21}(x) < \Lambda_{21}(x)$ . In the specification without or with selection, equation (14) or (15),  $\Lambda'_{12}(y) \geq \Lambda_{12}(y)$  and  $\Lambda'_{21}(y) > \Lambda_{21}(y)$  imply  $t < t'$ ; while equation (14) or (15),  $\Lambda'_{12}(x) \leq \Lambda_{12}(x)$ , and  $\Lambda'_{21}(x) < \Lambda_{21}(x)$  imply  $t > t'$ , a contradiction. Hence, in the specifications with and without selection,  $\Theta'_1 > \Theta_1$  implies Inequality (c). Finally, to show that Inequality (c) implies Inequality (e), we proceed by contradiction. Suppose that  $\Lambda'_{12}(y) < \Lambda_{12}(y)$  and  $\Theta'_1 \leq \Theta_1$ .  $\Lambda'_{12}(y) < \Lambda_{12}(y)$  implies  $\Lambda'_{12}(x) > \Lambda_{12}(x)$  while  $\Theta'_1 \leq \Theta_1$  implies both  $\Lambda'_{21}(y) \leq \Lambda_{21}(y)$  and  $\Lambda'_{21}(x) \geq \Lambda_{21}(x)$ . In the specification without selection (or with selection), equation (14) (or equation (15)),  $\Lambda'_{12}(y) < \Lambda_{12}(y)$  and  $\Lambda'_{21}(y) \leq \Lambda_{21}(y)$  imply  $t > t'$ ; while equation (14) (or equation (15)),  $\Lambda'_{12}(x) > \Lambda_{12}(x)$ , and  $\Lambda'_{21}(x) \geq \Lambda_{21}(x)$  imply  $t < t'$ , a contradiction. Hence, Inequality (c) is equivalent to Inequality (e).

**Step 2:** If  $\Delta_1 = \Delta'_1 > 0$  and  $\Delta_2 = \Delta'_2 > 0$ , then  $E_1/E_2 = E'_1/E'_2$ .

$\Delta_i = \Delta'_i$  for  $i = 1, 2$  is equivalent to

$$\Lambda_{ni}(x) + \Lambda_{ni}(y) = \Lambda'_{ni}(x) + \Lambda'_{ni}(y), \text{ for } i = 1, 2 \text{ and } n \neq i \quad (31)$$

and trade balance implies

$$\left[ \Lambda_{21}^{(i)}(x) + \Lambda_{21}^{(i)}(y) \right] E_1^{(i)} = \left[ \Lambda_{12}^{(i)}(x) + \Lambda_{12}^{(i)}(y) \right] E_2^{(i)} \quad (32)$$

in both the original equilibrium (without  $'$ ) and the new equilibrium (with  $'$ ). Equations (31) and (32) yields  $E_1/E_2 = E'_1/E'_2$ .

**Step 3:** (iii)  $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$  if and only if (vii)  $\Theta'_1 > \Theta_1$ .

By equation (16), we have

$$\frac{w_{s,1}}{w_{u,1}} = \frac{L_{u,1} \sum_j \sum_n \alpha_s(j) \Lambda_{1n}(j) E_n}{L_{s,1} \sum_j \sum_n \alpha_u(j) \Lambda_{1n}(j) E_n}. \quad (33)$$

With  $L_{k,i} = L'_{k,i}$  and  $\alpha_k(j) = \alpha'_k(j)$ , we have  $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$  if and only if

$$\frac{\Lambda_{11}(y)E_1 + \Lambda_{12}(y)E_2}{\Lambda'_{11}(y)E'_1 + \Lambda'_{12}(y)E'_2} > \frac{\Lambda_{11}(x)E_1 + \Lambda_{12}(x)E_2}{\Lambda'_{11}(x)E'_1 + \Lambda'_{12}(x)E'_2}.$$

By Step 2, the previous inequality is equivalent to

$$\frac{\Lambda_{11}(y)E_1 + \Lambda_{12}(y)E_2}{\Lambda'_{11}(y)E_1 + \Lambda'_{12}(y)E_2} > \frac{\Lambda_{11}(x)E_1 + \Lambda_{12}(x)E_2}{\Lambda'_{11}(x)E_1 + \Lambda'_{12}(x)E_2}. \quad (34)$$

By Step 1,  $\Theta'_1 > \Theta_1$  is equivalent to Inequalities (a) – (d). Inequalities (a) – (d) imply equation (34). Therefore  $\Theta'_1 > \Theta_1$  implies  $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$ .

Now suppose that  $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$ . This is equivalent to equation (34). To prove that equation (34) implies  $\Theta'_1 > \Theta_1$ , we proceed by contradiction. Suppose that  $\Theta'_1 \leq \Theta_1$ . By Step 1, this implies  $\Lambda'_{11}(x) \leq \Delta_{11}(x)$ ,  $\Lambda'_{11}(y) \geq \Lambda_{11}(y)$ ,  $\Lambda'_{12}(y) \geq \Lambda_{12}(y)$ , and  $\Lambda'_{12}(x) \leq \Lambda_{12}(x)$ . These four inequalities contradict equation (34). This concludes the proof of Step 3, and Part II follows directly.

**Part III:** (iii)  $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$  if and only if (v)  $L'_{k,1}(x) > L_{k,1}(x)$  for  $k = s, u$ .

In the proof of Part III, we normalize  $E_1 = E'_1 = 1$ . By Step 2 of the proof of Part II, we have  $E'_2 = E_2$ . Moreover, with  $E_1 = E'_1$ , we have  $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$  if and only if

$$w'_{u,1} < w_{u,1}. \quad (35)$$

The proof of Part III proceeds in two steps.

**Step 1:**  $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$  implies  $L'_{k,1}(x) > L_{k,1}(x)$  for  $k = s, u$ .

From equation (16), we have

$$w_{k,i}L_{k,i}(x) = \lambda\alpha_k(x) \sum_n \left[ \Lambda_{in}(x) \left( \frac{E_n}{2} \right) \right]. \quad (36)$$

By Part II,  $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$  implies  $\Lambda'_{1n}(x) > \Lambda_{1n}(x)$  for  $n = 1, 2$ . By equation (36),  $E'_n = E_n$ , and  $\Lambda'_{1n}(x) > \Lambda_{1n}(x)$  for  $n = 1, 2$ , we have

$$w'_{u,1}L'_{u,1}(x) = \lambda\alpha_u(x) \sum_n \left[ \Lambda'_{1n}(x) \left( \frac{E'_n}{2} \right) \right] > \lambda\alpha_u(x) \sum_n \left[ \Lambda_{1n}(x) \left( \frac{E_n}{2} \right) \right] = w_{u,1}L_{u,1}(x). \quad (37)$$

Equations (35) and (37) imply  $L'_{u,1}(x) > L_{u,1}(x)$ . We similarly have

$$w'_{s,1}L'_{s,1}(y) = \lambda\alpha_s(y) \sum_n \left[ \Lambda'_{1n}(y) \left( \frac{E'_n}{1} \right) \right] < \lambda\alpha_s(y) \sum_n \left[ \Lambda_{1n}(y) \left( \frac{E_n}{1} \right) \right] = w_{s,1}L_{s,1}(y)$$

and

$$w'_{s,1} > w_{s,1},$$

which imply  $L'_{s,1}(y) < L_{s,1}(y)$ . Since  $L_{s,1}(y) + L_{s,1}(x) = L_{s,1}$ , we therefore have  $L'_{s,1}(x) > L_{s,1}(x)$ . Hence,  $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$  implies  $L'_{k,1}(x) > L_{k,1}(x)$  for  $k = s, u$ .

**Step 2:**  $L'_{k,1}(x) > L_{k,1}(x)$  for  $k = s, u$  implies  $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$ .

We proceed by contradiction. Suppose that  $L'_{k,1}(x) > L_{k,1}(x)$  for  $k = s, u$  and  $w'_{s,1}/w'_{u,1} \leq w_{s,1}/w_{u,1}$ . By Part II,  $w'_{s,1}/w'_{u,1} \leq w_{s,1}/w_{u,1}$  is equivalent to  $\Theta'_1 \leq \Theta_1$ , which, by Step 1 in the proof of Part II, implies  $\Lambda'_{1n}(x) \leq \Lambda_{1n}(x)$  for  $n = 1, 2$ . Therefore,  $w'_{s,1}/w'_{u,1} \leq w_{s,1}/w_{u,1}$  implies

$$\lambda\alpha_u(x) \sum_n \left[ \Lambda'_{1n}(x) \left( \frac{E'_n}{2} \right) \right] \leq \lambda\alpha_u(x) \sum_n \left[ \Lambda_{1n}(x) \left( \frac{E_n}{2} \right) \right]. \quad (38)$$

By equations (36) and (38), we have  $w'_{u,1}L'_{u,1}(x) \leq w_{u,1}L_{u,1}(x)$ . By  $w'_{s,1}/w'_{u,1} \leq w_{s,1}/w_{u,1}$  and  $E'_1 = E_1$ , we also have  $w'_{u,1} \geq w_{u,1}$ , so that  $L'_{u,1}(x) \leq L_{u,1}(x)$ , a contradiction. Hence,  $L'_{k,1}(x) > L_{k,1}(x)$  for  $k = s, u$  implies  $w'_{s,1}/w'_{u,1} > w_{s,1}/w_{u,1}$ .

**Part IV:** (vii)  $\Lambda'_{21}(y) - \Lambda'_{21}(x) > \Lambda_{21}(y) - \Lambda_{21}(x)$  if and only if (viii)  $\Lambda'_{12}(x) - \Lambda'_{12}(y) > \Lambda_{12}(x) - \Lambda_{12}(y)$ .

From Step 1 of Part II, we have  $\Lambda'_{21}(y) - \Lambda'_{21}(x) > \Lambda_{21}(y) - \Lambda_{21}(x)$  if and only if  $\Lambda'_{12}(y) < \Lambda_{12}(y)$  and  $\Lambda'_{12}(x) > \Lambda_{12}(x)$ . These inequalities imply  $\Lambda'_{12}(x) - \Lambda'_{12}(y) > \Lambda_{12}(x) - \Lambda_{12}(y)$ . The proof that  $\Lambda'_{12}(x) - \Lambda'_{12}(y) > \Lambda_{12}(x) - \Lambda_{12}(y)$  implies  $\Lambda'_{21}(y) - \Lambda'_{21}(x) > \Lambda_{21}(y) - \Lambda_{21}(x)$  is identical and omitted.

The proof of Proposition 3 follows directly from Parts I-IV and the similar, but omitted, proofs that (ii), (iv), (vi), and (viii) are equivalent. **QED.** ■

**Proof of Proposition 4 Part 1.** First,  $\Theta_1 \geq \Theta'_1$  if and only if

$$\left[ \frac{1}{a'} \left( \frac{w'_{s,1}/w'_{u,1}}{w'_{s,2}/w'_{u,2}} \right)^{(\alpha_x - \alpha_y)} \right]^{\frac{\eta\gamma' - \eta + 1}{\eta - 1}} \geq \left[ \frac{1}{a} \left( \frac{w_{s,1}/w_{u,1}}{w_{s,2}/w_{u,2}} \right)^{(\alpha_x - \alpha_y)} \right]^{\frac{\eta\gamma - \eta + 1}{\eta - 1}} \quad (39)$$

Second,  $\Theta'_1 > 0$  implies

$$1 > \left[ \frac{1}{a'} \left( \frac{w'_{s,1}/w'_{u,1}}{w'_{s,2}/w'_{u,2}} \right)^{(\alpha_x - \alpha_y)} \right]^{\frac{\eta\gamma' - \eta + 1}{\eta - 1}}. \quad (40)$$

Third, Proposition 3 and  $\Theta_1 \geq \Theta'_1 > 0$  imply

$$w_{s,1}/w_{u,1} \geq w'_{s,1}/w'_{u,1} \quad (41)$$

$$w_{s,2}/w_{u,2} \leq w'_{s,2}/w'_{u,2}. \quad (42)$$

We now prove the comparative static result for  $\gamma$ . We proceed by contradiction. Suppose that  $\gamma' > \gamma$  and that  $\Theta_1 \geq \Theta'_1 > 0$ . Then

$$\left[ \frac{1}{a'} \left( \frac{w'_{s,1}/w'_{u,1}}{w'_{s,2}/w'_{u,2}} \right)^{(\alpha_x - \alpha_y)} \right]^{\frac{\eta\gamma' - \eta + 1}{\eta\gamma' - \eta + 1}} \geq \frac{1}{a'} \left( \frac{w_{s,1}/w_{u,1}}{w_{s,2}/w_{u,2}} \right)^{(\alpha_x - \alpha_y)} \geq \frac{1}{a'} \left( \frac{w'_{s,1}/w'_{u,1}}{w'_{s,2}/w'_{u,2}} \right)^{(\alpha_x - \alpha_y)} \quad (43)$$

where the first weak inequality follows from Condition (39),  $a = a'$ , and  $(\eta\gamma - \eta + 1) / (\eta - 1) > 0$  while the second weak inequality follows from Conditions (41) and (42). Condition (43) and  $\gamma'/\gamma > 1$  contradict Condition (40). Thus, if  $t$  and  $T_1/T_2$  are chosen to match fixed values of  $\Delta_1, \Delta_2 > 0$ , and if  $\gamma' > \gamma$ ,  $\Theta_1, \Theta'_1 > 0$ , then  $\Theta'_1 > \Theta_1$ . Combined with Proposition 3, this yields the desired comparative static result for  $\gamma$ .

We now prove the comparative static result for  $a$ . We proceed by contradiction. Suppose that  $a' > a$  and that  $\Theta_1 \geq \Theta'_1 > 0$ . Then Condition (39) implies

$$\frac{1}{a'} \times \left( \frac{w'_{s,1}/w'_{u,1}}{w'_{s,2}/w'_{u,2}} \right)^{(\alpha_x - \alpha_y)} \geq \frac{1}{a} \times \left( \frac{w_{s,1}/w_{u,1}}{w_{s,2}/w_{u,2}} \right)^{(\alpha_x - \alpha_y)}$$

which, contradicts Conditions (41) and (42). Thus, if  $t$  and  $T_1/T_2$  are chosen to match fixed values of  $\Delta_1, \Delta_2 > 0$ , and if  $a' > a$ , then  $\Theta'_1 > \Theta_1$ . Combined with Proposition 3, this yields the desired comparative static result for  $a$ . **QED. ■**

**Proof of Proposition 4 Part 2.** The proof requires two preliminary steps and uses the following notation:  $\zeta = \frac{M_2(x)}{M_1(x)} \left( \frac{v_2(x)}{v_1(x)} \right)^{1 - \frac{\eta\gamma}{\eta - 1}}$ .

**Step 1.**  $\frac{d}{d\gamma} \left( \frac{w_{s,1}}{w_{u,1}} \right) \leq 0 \Leftrightarrow \frac{d}{d\gamma} \zeta \geq 0$ .

With endogenous entry  $w_{u,n}L_{u,n} + w_{s,n}L_{s,n} = Q_nP_n$  and with mirror-symmetry  $Q_1P_1 = Q_2P_2$ ,  $\alpha_y = 1 - \alpha_x$ ,  $\Lambda_{12}(y) = 1 - \Lambda_{11}(x)$ ,  $\Lambda_{11}(y) = 1 - \Lambda_{12}(x)$ , and  $\Delta_2 = \frac{1}{2} [\Lambda_{12}(x) + 1 - \Lambda_{11}(x)]$ , in which case equation (16) is equivalent to

$$w_{s,1}L_{s,1} = \frac{Q_1P_1}{2} \{ (2\alpha_x - 1) [2\Delta_2 - 1 + 2\Lambda_{11}(x)] + 2(1 - \alpha_x) \}$$

Choosing  $Q_1P_1$  as the numeraire, implies  $\frac{d}{d\gamma} \left( \frac{w_{s,1}}{w_{u,1}} \right) \leq 0$  if and only if  $\frac{d}{d\gamma} \Lambda_{11}(x) \leq 0$ . Differentiating  $\Delta_2$  with respect to  $\gamma$  and setting  $\frac{d}{d\gamma} \Delta_2 = 0$  yields  $\frac{d}{d\gamma} \Lambda_{12}(x) = \frac{d}{d\gamma} \Lambda_{11}(x)$ . Equation (15) implies

$$\frac{d}{d\gamma} \Lambda_{12}(x) = -\Lambda_{12}(x)^2 \left\{ \zeta \frac{dt}{d\gamma} + t \frac{d\zeta}{d\gamma} \right\}$$

and

$$\frac{d}{d\gamma} \Lambda_{11}(x) = -\Lambda_{11}(x)^2 \left\{ -t^{-2} \zeta \frac{dt}{d\gamma} + t^{-1} \frac{d\zeta}{d\gamma} \right\}. \quad (44)$$

Hence,  $\frac{d}{d\gamma}\Lambda_{12}(x) = \frac{d}{d\gamma}\Lambda_{11}(x)$  if and only if

$$\frac{dt}{d\gamma} = \frac{\Lambda_{11}(x)^2 - t^2\Lambda_{12}(x)^2}{\Lambda_{11}(x)^2 + t^2\Lambda_{12}(x)^2} \left(\frac{t}{\zeta}\right) \left(\frac{d\zeta}{d\gamma}\right). \quad (45)$$

Equations (44) and (45) imply

$$\begin{aligned} \frac{d}{d\gamma}\Lambda_{11}(x) &= -t^{-1}\Lambda_{11}(x)^2 \left\{ -\frac{\Lambda_{11}(x)^2 - t^2\Lambda_{12}(x)^2}{\Lambda_{11}(x)^2 + t^2\Lambda_{12}(x)^2} + 1 \right\} \left(\frac{d\zeta}{d\gamma}\right) \\ &= -\frac{2t\Lambda_{11}(x)^2\Lambda_{12}(x)^2}{\Lambda_{11}(x)^2 + t^2\Lambda_{12}(x)^2} \left(\frac{d\zeta}{d\gamma}\right) \end{aligned}$$

Hence,  $\frac{d}{d\gamma}\Lambda_{11}(x)$  has the opposite sign as  $\frac{d\zeta}{d\gamma}$ . Hence,  $\frac{d}{d\gamma}\left(\frac{w_{s,1}}{w_{u,1}}\right) \leq 0 \Leftrightarrow \frac{d}{d\gamma}\Lambda_{11}(x) \leq 0 \Leftrightarrow \frac{d}{d\gamma}\zeta \geq 0$ .

**Step 2.** If  $\frac{d}{d\gamma}\zeta > 0$  then  $\frac{d}{d\gamma}t < 0$ .

Equation (45) and  $\frac{d}{d\gamma}\zeta > 0$  imply  $\frac{d}{d\gamma}t < 0$  if and only if  $\Lambda_{11}(x) < t\Lambda_{12}(x)$ . Equation (15) implies both (i)  $\Lambda_{11}(x) < t\Lambda_{12}(x)$  is equivalent to  $\zeta < 1$  and (ii)  $\Lambda_{12}(x) > \Lambda_{12}(y)$  is equivalent to  $\zeta < 1$ . Hence,  $\Theta_1 > 0$ —which implies  $\Lambda_{12}(x) > \Lambda_{12}(y)$ —implies  $\Lambda_{11}(x) < t\Lambda_{12}(x)$ , which itself implies  $\frac{d}{d\gamma}t < 0$ .

We now use Steps 1 and 2 to prove Proposition 4 Part 2. With mirror-symmetry

$$\zeta = \frac{(t^2 + 1) \left[ \frac{v_1(x)}{v_1(y)} \right]^{\frac{\eta\gamma}{\eta-1}} - 2t}{(t^2 + 1) - 2t \left[ \frac{v_1(x)}{v_1(y)} \right]^{\frac{\eta\gamma}{\eta-1}}}$$

To obtain a contradiction, suppose that  $\frac{d}{d\gamma}\zeta > 0$ .  $\frac{d}{d\gamma}\zeta > 0$  if and only if

$$\begin{aligned} \left\{ t^2 + 1 - 2t \left[ \frac{v_1(x)}{v_1(y)} \right]^{\frac{\eta\gamma}{\eta-1}} \right\} \left[ 2 \left( t \left[ \frac{v_1(x)}{v_1(y)} \right]^{\frac{\eta\gamma}{\eta-1}} - 1 \right) \frac{dt}{d\gamma} + (t^2 + 1) \frac{d \left[ \frac{v_1(x)}{v_1(y)} \right]^{\frac{\eta\gamma}{\eta-1}}}{d\gamma} \right] > \\ 2 \left( (t^2 + 1) \left[ \frac{v_1(x)}{v_1(y)} \right]^{\frac{\eta\gamma}{\eta-1}} - 2t \right) \left[ \left( t - \left[ \frac{v_1(x)}{v_1(y)} \right]^{\frac{\eta\gamma}{\eta-1}} \right) \frac{dt}{d\gamma} - t \frac{d \left[ \frac{v_1(x)}{v_1(y)} \right]^{\frac{\eta\gamma}{\eta-1}}}{d\gamma} \right]. \end{aligned}$$

Since  $t > 1$ ,  $\frac{d}{d\gamma}\zeta > 0$  if and only if

$$(1 - t^2) \frac{d}{d\gamma} \left[ \frac{v_1(x)}{v_1(y)} \right]^{\frac{\eta\gamma}{\eta-1}} + 2 \left( \left[ \frac{v_1(x)}{v_1(y)} \right]^{\frac{2\eta\gamma}{\eta-1}} - 1 \right) \frac{dt}{d\gamma} < 0 \quad (46)$$

We consider the first and second terms of Condition (46) separately. According to Step 1,  $\frac{d}{d\gamma}\zeta > 0$  implies  $\frac{d}{d\gamma}\left(\frac{w_{s,1}}{w_{u,1}}\right) < 0$ . Hence,  $\frac{d}{d\gamma}\zeta > 0$  implies  $\frac{d}{d\gamma}\left[ \frac{v_1(x)}{v_1(y)} \right]^{\frac{\eta\gamma}{\eta-1}} < 0$ . The first term in Condition (46)

is, therefore, positive. In the second term we know that  $\left[\frac{v_1(x)}{v_1(y)}\right]^{\frac{2\eta\gamma}{\eta-1}} < 1$ , which follows from the fact that  $\Theta_1 > 0$  is equivalent to  $\zeta < 1$ , and  $\zeta < 1$  implies  $v_1(x) < v_1(y)$ . In the second term we also have  $\frac{dt}{d\gamma} < 0$ , which follows from Step 2. Hence, the second term in Condition (46) is positive. This contradicts  $\frac{d}{d\gamma}\zeta > 0$ . **QED. ■**

**Proof of Proposition 5.** The proof of Proposition 5 requires additional notation and two preliminary steps. Let  $\vartheta_1 = \frac{\Lambda_{11}(x)\Delta_2/\Delta_1+\Lambda_{12}(x)}{\Lambda_{11}(y)\Delta_2/\Delta_1+\Lambda_{12}(y)}$  denote the ratio of country 1's revenue in the  $x$  sector to country 1's revenue in the  $y$  sector. Let  $k_i(\vartheta_i) = \frac{M_i^T(x)/M_i^T(y)}{M_i^A(x)/M_i^A(y)}$ , where  $M_i^T(j)$  and  $M_i^A(j)$  denote the mass of sector  $j$  entrants in country  $i$  in a trade equilibrium for a given  $\vartheta_i$  and in the autarky equilibrium ( $\vartheta_i^A = 1$ ), respectively.

**Step 1.**  $k_1$  is increasing in  $\vartheta_1$ .

Equations (10) and (19) imply

$$\frac{M_i(x)}{M_i(y)} = \frac{\alpha_x^{\alpha_x} (1 - \alpha_x)^{1-\alpha_x} A_i(x)}{\alpha_y^{\alpha_y} (1 - \alpha_y)^{1-\alpha_y} A_i(y)} \left(\frac{w_{u,i}}{w_{s,i}}\right)^{\alpha_x - \alpha_y} \frac{\sum_n \Lambda_{in}(x) Q_n P_n}{\sum_n \Lambda_{in}(y) Q_n P_n}$$

so that

$$k_i = \left(\frac{w_{u,i}^T/w_{s,i}^T}{w_{u,i}^A/w_{s,i}^A}\right)^{\alpha_x - \alpha_y} \frac{\sum_n \Lambda_{in}^T(x) Q_n P_n}{\sum_n \Lambda_{in}^T(y) Q_n P_n} \quad (47)$$

Re-expressing Equation (47) as a function of the  $\Lambda$ s and  $\Delta$ s, using equation (16), yields

$$k_1(\vartheta_1) = \left(\frac{1}{\alpha_y} \frac{2}{\frac{\alpha_x}{\alpha_y} + 1} - 1\right)^{\alpha_y - \alpha_x} \left(\frac{1}{\alpha_y} \frac{\vartheta_1 + 1}{\frac{\alpha_x}{\alpha_y} \vartheta_1 + 1} - 1\right)^{\alpha_x - \alpha_y} \vartheta_1. \quad (48)$$

Hence,  $k_1(\vartheta'_1) > k_1(\vartheta_1)$  if and only if

$$\left(\frac{\alpha_y + \alpha_x \vartheta_1}{\alpha_y + \alpha_x \vartheta'_1} \frac{1 - \alpha_y^2 + \vartheta'_1(1 - \alpha_x \alpha_y)}{1 - \alpha_y^2 + \vartheta_1(1 - \alpha_x \alpha_y)}\right)^{\alpha_x - \alpha_y} \frac{\vartheta'_1}{\vartheta_1} > 1 \quad (49)$$

If  $\vartheta'_1 > \vartheta_1$ , then

$$\left(\frac{\vartheta_1}{\vartheta'_1}\right)^{\alpha_x - \alpha_y} < \left(\frac{\alpha_y + \alpha_x \vartheta_1}{\alpha_y + \alpha_x \vartheta'_1} \frac{1 - \alpha_y^2 + \vartheta'_1(1 - \alpha_x \alpha_y)}{1 - \alpha_y^2 + \vartheta_1(1 - \alpha_x \alpha_y)}\right)^{\alpha_x - \alpha_y}. \quad (50)$$

Condition (50) and  $0 \leq \alpha_x - \alpha_y \leq 1$  imply Condition (49), so that  $k_1$  is increasing in  $\vartheta_1$ .

**Step 2.** If  $\Delta'_1 = \Delta_1$ ,  $\Delta'_2 = \Delta_2$ , and  $\Theta'_1 \geq \Theta_1$ , then  $\vartheta'_1 = \vartheta_1(\Theta'_1) \geq \vartheta_1(\Theta_1) = \vartheta_1$ .

Choose  $Q_1 P_1$  as the numeraire, which implies that  $Q_2 P_2$  is fixed given fixed trade shares and that  $\Lambda_{1n}(x) Q_n P_n + \Lambda_{1n}(y) Q_n P_n$  is fixed. Hence, a sufficient condition under which  $\vartheta'_1 \geq \vartheta_1$  is

$\Lambda'_{1n}(x) \geq \Lambda_{1n}(x)$  for  $n = 1, 2$ . We have  $\Lambda'_{11}(x) = 1 - \Delta'_1 + \frac{1}{2}\Theta'_1 \geq 1 - \Delta_1 + \frac{1}{2}\Theta_1 = \Lambda_{11}(x)$ , so that it only remains to show that  $\Lambda'_{12}(x) \geq \Lambda_{12}(x)$ . We have

$$\Lambda_{12}(x) = \left[ 1 + \left( \frac{1}{\Delta_1 - \frac{1}{2}\Theta_1} - 1 \right) t \right]^{-1},$$

so that  $\Lambda'_{12}(x) \geq \Lambda_{12}(x)$  if and only if

$$\phi'_1 t' \leq \phi_1 t$$

where

$$\phi_1 = \left( \frac{1}{\Delta_1 - \frac{1}{2}\Theta_1} - 1 \right)^{-1}$$

Moreover,  $\Delta_2 = \Delta'_2$  implies

$$(1 + \phi_1 t)^{-1} + (1 + \phi_2 t)^{-1} = (1 + \phi'_1 t')^{-1} + (1 + \phi'_2 t')^{-1} \quad (51)$$

where

$$\phi_2 = \left( \frac{1}{\Delta_1 + \frac{1}{2}\Theta_1} - 1 \right)^{-1}.$$

To obtain a contradiction, suppose that  $\phi'_1 t' > \phi_1 t$ . Then Equation (51) implies  $\phi'_2 t' < \phi_2 t$ . Hence,  $\phi'_2/\phi_2 < \phi'_1/\phi_1$ , which is equivalent to

$$\frac{\frac{1}{\Delta_1 - \frac{1}{2}\Theta'_1} - 1}{\frac{1}{\Delta_1 + \frac{1}{2}\Theta'_1} - 1} < \frac{\frac{1}{\Delta_1 - \frac{1}{2}\Theta_1} - 1}{\frac{1}{\Delta_1 + \frac{1}{2}\Theta_1} - 1}$$

which is violated. Hence,  $\phi'_1 t' \leq \phi_1 t$ , which is equivalent to  $\Lambda'_{12}(x) \geq \Lambda_{12}(x)$ . Hence,  $\Delta'_1 = \Delta_1$ ,  $\Delta'_2 = \Delta_2$  and  $\Theta'_1 \geq \Theta_1$  imply  $\vartheta'_1 \geq \vartheta_1$ .

We now use Steps 1 and 2 to conclude the proof of Proposition 5. Here we compare across two specifications, one in which entry is exogenous and one in which entry is endogenous, where the endogenous entry case is denoted by  $'$ . We proceed by contradiction. Suppose that  $\Theta_1 \geq \Theta'_1 > 0$ ,  $\Delta_1 = \Delta'_1 > 0$ , and  $\Delta_2 = \Delta'_2 > 0$ . Using similar logic that lead to Condition (39),  $\Theta_1 \geq \Theta'_1$  implies

$$\frac{M'_1(y) M'_2(x)}{M'_1(x) M'_2(y)} \left[ \frac{v'_1(x)/v'_1(y)}{v'_2(x)/v'_2(y)} \right]^{\frac{\eta\gamma - \eta + 1}{\eta - 1}} \geq \frac{M_1(y) M_2(x)}{M_1(x) M_2(y)} \left[ \frac{v_1(x)/v_1(y)}{v_2(x)/v_2(y)} \right]^{\frac{\eta\gamma - \eta + 1}{\eta - 1}} \quad (52)$$

Moreover, Proposition 3 and  $\Theta_1 \geq \Theta'_1 > 0$  imply

$$\frac{v_1(x)/v_1(y)}{v_2(x)/v_2(y)} \geq \frac{v'_1(x)/v'_1(y)}{v'_2(x)/v'_2(y)}.$$

Hence, Condition (52) requires

$$\frac{M'_2(x)/M'_2(y)}{M_2(x)/M_2(y)} \geq \frac{M'_1(x)/M'_1(y)}{M_1(x)/M_1(y)}. \quad (53)$$

Imposing  $M_i(j) = M_i$  for  $j = x, y$  in the exogenous entry case, Condition (53) is equivalent to

$$\frac{M_2^{A'}(x)}{M_2^{A'}(y)} \frac{M'_2(x)/M'_2(y)}{M_2(x)/M_2(y)} \geq \frac{M_1^{A'}(x)}{M_1^{A'}(y)} \frac{M'_1(x)/M'_1(y)}{M_1(x)/M_1(y)}. \quad (54)$$

With  $\Theta'_1 > 0$ , Steps 1 and 2 imply  $\frac{M'_1(x)/M'_1(y)}{M_1^{A'}(x)/M_1^{A'}(y)} > 1$  and  $\frac{M'_2(x)/M'_2(y)}{M_2^{A'}(x)/M_2^{A'}(y)} < 1$ . Hence, Condition (54) requires

$$M_2^{A'}(x)/M_2^{A'}(y) > M_1^{A'}(x)/M_1^{A'}(y) \quad (55)$$

We know that  $M_i^A(x)/M_i^A(y) = v_i^A(y)/v_i^A(x)$ , so that Condition (55) is equivalent to

$$v_2^{A'}(y)/v_2^{A'}(x) > v_1^{A'}(y)/v_1^{A'}(x),$$

which implies that country 2 has a comparative advantage in  $x$  and contradicts  $\Theta'_1 > 0$ . Hence, we must have  $\Theta'_1 > \Theta_1 > 0$ . Proposition 3 then implies that the between effect is stronger with endogenous entry. **QED. ■**

**Proof of Proposition 7.** The result in which entry is endogenous and trade shares are positive follow from Part (a) of Proposition 4 in BRS, combined with the fact that domestic cutoffs are identical across countries in autarky. The proof with exogenous entry follows.

Equation (12) implies that relative domestic cutoffs are given by

$$\frac{z_{ii}^*(x)}{z_{ii}^*(y)} = \frac{P_i(y)}{P_i(x)} \left( \frac{v_i(x)}{v_i(y)} \right)^{\frac{\eta}{\eta-1}}. \quad (56)$$

The price level equation—Equation (29)—and Equation (56) imply  $z_{11}^*(x)/z_{11}^*(y) < z_{22}^*(x)/z_{22}^*(y)$  if and only if

$$\left( \frac{v_1(x)}{v_1(y)} \frac{v_2(y)}{v_2(x)} \right)^{\frac{\gamma\eta}{\eta-1}} < \frac{v_1(y)^{\frac{\eta\gamma+1-\eta}{1-\eta}} t + v_2(y)^{\frac{\eta\gamma+1-\eta}{1-\eta}}}{v_1(x)^{\frac{\eta\gamma+1-\eta}{1-\eta}} t + v_2(x)^{\frac{\eta\gamma+1-\eta}{1-\eta}}} \times \frac{v_1(x)^{\frac{\eta\gamma+1-\eta}{1-\eta}} + v_2(x)^{\frac{\eta\gamma+1-\eta}{1-\eta}} t}{v_1(y)^{\frac{\eta\gamma+1-\eta}{1-\eta}} + v_2(y)^{\frac{\eta\gamma+1-\eta}{1-\eta}} t} \quad (57)$$

which is equivalent to

$$\begin{aligned} \frac{v_2(x)}{v_1(x)} \frac{v_2(y)}{v_1(y)} \left[ \left( \frac{v_2(x)}{v_1(x)} \right)^{\frac{\eta\gamma}{1-\eta}} - \left( \frac{v_2(y)}{v_1(y)} \right)^{\frac{\eta\gamma}{1-\eta}} \right] t + \left[ \frac{v_2(y)}{v_1(y)} - \frac{v_2(x)}{v_1(x)} \right] t^2 + \\ \left[ \left( \frac{v_1(y)}{v_2(y)} \right)^{\frac{\gamma\eta}{1-\eta}} - \left( \frac{v_1(x)}{v_2(x)} \right)^{\frac{\gamma\eta}{1-\eta}} \right] t < \left( \frac{v_1(y)}{v_2(y)} \frac{v_2(x)}{v_1(x)} \right)^{\frac{\gamma\eta}{\eta-1}} \frac{v_2(y)}{v_1(y)} - \left( \frac{v_2(y)}{v_1(y)} \frac{v_1(x)}{v_2(x)} \right)^{\frac{\gamma\eta}{\eta-1}} \frac{v_2(x)}{v_1(x)} \end{aligned} \quad (58)$$

Country 1's comparative advantage in sector  $x$  implies  $v_2(x)/v_1(x) > v_2(y)/v_1(y)$  (without factor



price equalization). This implies that the left-hand side of Condition (58) is negative and that the right-hand side of Condition (58) is positive. Therefore, Condition (58) is satisfied, which is equivalent to  $z_{11}^*(x)/z_{11}^*(y) < z_{22}^*(x)/z_{22}^*(y)$ . **QED. ■**

## References

- Acemoglu, Daron.** 2002. "Directed Technical Change." *Review of Economic Studies*, 69: 781-809.
- Arkolakis, Costas.** Forthcoming. "Market Penetration Costs and the New Consumers Margin in International Trade." *Journal of Political Economy*.
- Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare.** Forthcoming. "New Trade Models, Same Old Gains?" *American Economic Review*.
- Atkeson, Andrew and Ariel Burstein.** 2010. "Innovation, Firm Dynamics, and International Trade." *Journal of Political Economy*, 118(3): 433-484.
- Bernard, Andrew B., Jonathan Eaton, J. Bradford Jensen, and Samuel Kortum.** 2003. "Plants and Productivity in International Trade." *American Economic Review*, 93(4):1268-1290.
- Bernard, Andrew B., Stephen J. Redding, and Peter K. Schott.** 2007. "Comparative Advantage and Heterogeneous Firms." *Review of Economic Studies*, 74(1): 31-66.
- Burstein, Ariel and Jonathan Vogel.** 2010. "Globalization, Technology, and the Skill Premium: A Quantitative Analysis." *Mimeo, UCLA*.
- Burstein, Ariel, Javier Cravino, and Jonathan Vogel.** 2010. "Importing Skill-Biased Technology." *Mimeo, UCLA*.
- Chaney, Thomas.** 2008. "Distorted Gravity: the Intensive and Extensive Margins of International Trade." *American Economic Review*, 98(4): 1707-1721.
- Costinot, Arnaud and Jonathan Vogel.** 2010. "Matching and Inequality in the World Economy." *Journal of Political Economy*, 118(4): 747-786.
- Davis, Donald and David Weinstein.** 2001. "An Account of Global Factor Trade." *American Economic Review*, 91(5): 1423-1453.
- Deardorff, Alan V.** 2000. "Factor Prices and the Factor Content of Trade Revisited: What's the Use?" *Journal of International Economics*, 50(1): 73-90.
- Deardorff, Alan V. and Robert W. Staiger.** 1988. "An Interpretation of the Factor Content of Trade." *Journal of International Economics*, 24(1-2): 93-107.
- Eaton, Jonathan and Samuel Kortum.** 2002. "Technology, Geography, and Trade." *Econometrica*, 70(5): 1741-1779.
- Eaton, Jonathan, Samuel Kortum, and Francis Kramarz.** Forthcoming. "An Anatomy of International Trade: Evidence from French Firms." *Econometrica*.
- Ethier, Wilfred J.** 1984. "Higher Dimensional Issues in Trade Theory." In *Handbook of International Economics*, edited by R. W. Jones, and P. B. Kenen, 131-84. Amsterdam: Elsevier.
- Epifani, Paolo and Gino Gancia.** 2006. "Increasing Returns, Imperfect Competition, and Factor Prices." *The Review of Economics and Statistics*, 88(4): 583-598.
- Epifani, Paolo and Gino Gancia.** 2008. "The Skill Bias of World Trade." *Economic Journal*, Vol. 118, No. 530, pp. 927-960.

**Feenstra, Robert C. and Gordon H. Hanson.** 2000. "Aggregation Bias in the Factor Content of Trade: Evidence from U.S. Manufacturing." *American Economic Review*, 90(2): 155–160.

**Goldberg, Pinelopi Koujianou and Nina Pavcnik.** 2007. "Distributional Effects of Globalization in Developing countries." *Journal of Economic Literature*, 45(1): 39–82.

**Helpman, Elhanan, Oleg Itskhoki, and Stephen J. Redding.** 2010. "Inequality and Unemployment in a Global Economy." *Econometrica*, 78(4): 1239–1283.

**Helpman, Elhanan and Paul R. Krugman.** 1985. "Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition and the International Economy." MIT Press.

**Ho, Giang T.** 2010. "Labor Market Policies and Misallocation in India." Mimeo, UCLA.

**Jones, Ronald W. and Jose A. Scheinkman.** 1977. "The Relevance of the Two-Sector Production Model in Trade Theory." *Journal of Political Economy*, 85(5): 909–35.

**Katz, Lawrence F. and Kevin M. Murphy.** 1992. "Changes in Relative Wages, 1963-1987: Supply and Demand Factors." *The Quarterly Journal of Economics*, 107(1): 35–78.

**Krugman, Paul R.** 1980. "Scale Economies, Product Differentiation, and the Pattern of Trade." *American Economic Review*, 70(5): 950-959.

**Krusell, Per, Lee E. Ohanian, José-Víctor Ríos-Rull, and Giovanni L. Violante.** 2000. "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis." *Econometrica*, 68(5): 1029–1053.

**Lu, Dan.** 2010. "Exceptional Exporter Performance? Evidence from Chinese Manufacturing Firms." Mimeo, University of Chicago.

**Matsuyama, Kiminori.** 2007. "Beyond Icebergs: Towards A Theory of Biased Globalization." *The Review of Economic Studies*, 74: 237–253.

**Melitz, Marc.** 2003. "The Impact of Trade on Aggregate Industry Productivity and Intra-Industry Reallocations." *Econometrica*, 71(6): 1695–1725.

**Parro, Fernando.** 2010. "Capital-Skill Complementarity and the Skill Premium in a Quantitative Model of Trade." Mimeo, University of Chicago.

**Romalis, John.** 2004. "Factor Proportions and the Structure of Commodity Trade." *American Economic Review*, 94(1): 67–97.

**Stolper, Wolfgang F. and Paul A. Samuelson.** 1941. "Protection and Real Wages." *Review of Economic Studies*, 9: 58–73.

**Trefler, Daniel.** 1993. "International Factor Price Differences: Leontief was Right!" *Journal of Political Economy*, 101(6): 961–987.

**Trefler, Daniel.** 1995. "The Case of the Missing Trade and Other Mysteries." *American Economic Review*, 85(5): 1029–1046.

**Trefler, Daniel and Susan Chun Zhu.** 2010. "The Structure of Factor Content Predictions." *Journal of International Economics* 82: 195–207.

**Yeaple, Stephen R.** 2005. "A Simple Model of Firm Heterogeneity, International Trade, and Wages." *Journal of International Economics*, 65(1): 1-20.