International Trade, Technology, and the Skill Premium

Ariel Burstein — UCLA and NBER

Jonathan Vogel — Columbia and NBER

May 2012

Introduction

- What are the consequences of reductions in international trade costs for the relative wage of skilled to unskilled workers, the *skill premium*?

Introduction

- What are the consequences of reductions in international trade costs for the relative wage of skilled to unskilled workers, the *skill premium*?

- Two mechanisms linking Δtrade to $\Delta s/w$

  - Heckscher-Ohlin (**H-O**)

- Skill intensity of country i, sector j producer

$$\frac{h}{l} = \left(\frac{w_i}{s_i}\right)^{\rho} \frac{\alpha_j}{1-\alpha_j}$$

Introduction

- What are the consequences of reductions in international trade costs for the relative wage of skilled to unskilled workers, the *skill premium*?
- Two mechanisms linking Δtrade to $\Delta s/w$
  - Heckscher-Ohlin (**H-O**)
  - Skill-biased technology

Introduction

- What are the consequences of reductions in international trade costs for the relative wage of skilled to unskilled workers, the *skill premium*?

- Two mechanisms linking Δtrade to $\Delta s/w$
  - Heckscher-Ohlin (**H-O**)
  - Skill-biased technology

- Skill intensity of country i, sector j producer with productivity z

$$\frac{h}{l} = \left(\frac{w_i}{s_i}\right)^{\rho} \frac{\alpha_j}{1-\alpha_j} z^{\varphi}$$

Introduction

- What are the consequences of reductions in international trade costs for the relative wage of skilled to unskilled workers, the *skill premium*?

- Two mechanisms linking Δtrade to $\Delta s/w$
  - Heckscher-Ohlin (**H-O**)
  - Skill-biased technology

- Skill intensity of country i, sector j producer with productivity z

$$\frac{h}{l} = \left(\frac{w_i}{s_i}\right)^{\rho} \frac{\alpha_j}{1-\alpha_j} z^{\varphi}$$

- Discipline skill-bias of techn, φ, firm-level data Mexico, Brazil, US

Introduction

- What are the consequences of reductions in international trade costs for the relative wage of skilled to unskilled workers, the *skill premium*?
- Two mechanisms linking Δtrade to $\Delta s/w$
  - Heckscher-Ohlin (**H-O**)
  - Skill-biased technology
- Skill intensity of country i, sector j producer with productivity z

$$\frac{h}{l} = \left(\frac{w_i}{s_i}\right)^{\rho} \frac{\alpha_j}{1-\alpha_j} z^{\varphi}$$

- Discipline skill-bias of techn, φ, firm-level data Mexico, Brazil, US
  - Need model with firms: BEJK

Introduction

- What are the consequences of reductions in international trade costs for the relative wage of skilled to unskilled workers, the *skill premium*?
- Two mechanisms linking Δtrade to $\Delta s/w$
  - Heckscher-Ohlin (**H-O**)
  - Skill-biased technology
- Skill intensity of country i, sector j producer with productivity z

$$\frac{h}{l} = \left(\frac{w_i}{s_i}\right)^{\rho} \frac{\alpha_j}{1-\alpha_j} z^{\varphi}$$

- Discipline skill-bias of techn, φ, firm-level data Mexico, Brazil, US
  - Need model with firms: BEJK
  - $\varphi > 0$: model does not yield analytic gravity — alternative approach to match bilateral trade flows 65-countries

Preview of quantitative results

- Counterfactuals: autarky and 10% reduction in trade costs
  - while real wages $\uparrow$ in most countries...

Preview of quantitative results

- Counterfactuals: autarky and 10% reduction in trade costs
  - while real wages $\uparrow$ in most countries...
  - real wage of skilled $\uparrow$ $2-3\times$ more than unskilled, median country
  - s/w $\uparrow$ in almost all countries, *Goldberg & Pavcnik 07*
  - $\Delta \log s/w$ greater in smaller countries and in more open countries, but not necessarily in skill-abundant countries like US

Preview of quantitative results

- Counterfactuals: autarky and 10% reduction in trade costs
  - while real wages ↑ in most countries...
  - real wage of skilled ↑ $2-3\times$ more than unskilled, median country
  - s/w ↑ in almost all countries, *Goldberg & Pavcnik 07*
  - $\Delta \log s/w$ greater in smaller countries and in more open countries, but not necessarily in skill-abundant countries like US
  - limited factor mobility (short run transition) ⇒ bigger effects
  - more within than btw sector factor reallocation, *Haltiwanger et. al. 04*
  - relative price of skill-intensive goods ↓, *Lawrence & Slaughter 93*

Preview of quantitative results

- Counterfactuals: autarky and 10% reduction in trade costs
  - while real wages $\uparrow$ in most countries...
  - real wage of skilled $\uparrow$ $2-3\times$ more than unskilled, median country
  - $s/w\uparrow$ in almost all countries, *Goldberg & Pavcnik 07*
  - $\Delta\log s/w$ greater in smaller countries and in more open countries, but not necessarily in skill-abundant countries like US
  - limited factor mobility (short run transition) $\Rightarrow$ bigger effects
  - more within than btw sector factor reallocation, *Haltiwanger et. al. 04*
  - relative price of skill-intensive goods $\downarrow$, *Lawrence & Slaughter 93*
- Revisit in model previous approaches to trade and s/w
  - underestimate $\uparrow s/w$ in skill-abundant, predict $\downarrow s/w$ in skill-scarce

Related literature

- Trade and skill-biased technology: A many-country model, combined with H-O, and quantitative evaluation
  - Theory: e.g., Acemoglu (03), Yeaple (05), Epifani and Gancia (06), Matsuyama (07)
  - Empirics: e.g., Bloom et. al. (11), Bustos (11), Verhoogen (08)
- Recent quantitative models:
  - Burstein et. al. (11), Parro (11): We have diff mechanism, firm-heterogeneity to discipline w/ cross-sectional firm-level data
  - Helpman et. al. (11): We have many countries & btw group inequality

Related literature

- Trade and skill-biased technology: A many-country model, combined with H-O, and quantitative evaluation
  - Theory: e.g., Acemoglu (03), Yeaple (05), Epifani and Gancia (06), Matsuyama (07)
  - Empirics: e.g., Bloom et. al. (11), Bustos (11), Verhoogen (08)
- Recent quantitative models:
  - Burstein et. al. (11), Parro (11): We have diff mechanism, firm-heterogeneity to discipline w/ cross-sectional firm-level data
  - Helpman et. al. (11): We have many countries & btw group inequality
- Generalized Heckscher-Ohlin: We focus on skill premium
  - e.g., Trefler (93) and (95), Davis and Weinstein (01), Romalis (04), Costinot (05), Bernard et. al. (07), Chor (08), Morrow (08)
- Factor content of trade (FCT) and other measures: We revisit previous approaches and show why they predict small effects
  - Theory: e.g., Deardorff and Staiger (88), Burstein and Vogel (11)
  - Empirics: e.g., Katz and Murphy (92), Berman et. al. (94), Krugman (95), Krugman (08)

Model

Utility

- N countries indexed by n
- Aggregate consumption from merchandise and service sectors

$$Q_n = \left(Q_n^M\right)^{\gamma_n} \left(Q_n^S\right)^{1-\gamma_n}$$

with

$$Q_n^M = \left(\sum_{j=1}^{J_M} Q_n(j)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

- Sector j consumption a CES aggregate of a continuum of varieties

$$Q_n(j) = \left(\int_0^1 q_n(\omega, j)^{\frac{\eta-1}{\eta}} \, d\omega\right)^{\frac{\eta}{\eta-1}}$$

- Within each (ω, j): 2 potential producers x country, Bertrand pricing

Firms

Production function

- A country n firm in (ω, j) with productivity z produces

$$y = A_n(j) \left[\alpha_j^{\frac{1}{\rho}} \left(z^{2\phi} h \right)^{\frac{\rho-1}{\rho}} + (1-\alpha_j)^{\frac{1}{\rho}} \left(z^{2(1-\phi)} l \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

  units of output, where

  - $A_n(j)$ is Hicks-neutral sectoral TFP
  - α_j determines the relative importance of skilled labor in sector j
  - ϕ determines skill bias of technology

Firms

Production function

- A country n firm in (ω, j) with productivity z produces

$$y = A_n(j) \left[\alpha_j^{\frac{1}{\rho}} \left(z^{2\phi} h \right)^{\frac{\rho-1}{\rho}} + (1-\alpha_j)^{\frac{1}{\rho}} \left(z^{2(1-\phi)} l \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

  units of output, where

  - $A_n(j)$ is Hicks-neutral sectoral TFP
  - α_j determines the relative importance of skilled labor in sector j
  - ϕ determines skill bias of technology
  - $z = u^{-\theta}$, where $u \sim \exp(1)$
    - $\theta > 0$ determines dispersion of productivities across firms

Firms

Production function

- A country n firm in (ω, j) with productivity z produces

$$y = A_n(j)\left[\alpha_j^{\frac{1}{\rho}}\left(z^{2\phi}h\right)^{\frac{\rho-1}{\rho}} + (1-\alpha_j)^{\frac{1}{\rho}}\left(z^{2(1-\phi)}l\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

  units of output, where

  - $A_n(j)$ is Hicks-neutral sectoral TFP
  - α_j determines the relative importance of skilled labor in sector j
  - ϕ determines skill bias of technology
  - $z = u^{-\theta}$, where $u \sim \exp(1)$
    - ★ $\theta > 0$ determines dispersion of productivities across firms

- With $\phi > 0.5$ and $\rho \neq 1$, no analytic gravity at any level of aggregation

Firms

Skill bias of technology

$$\frac{h}{l} = \left(\frac{w_i}{s_i}\right)^{\rho} \frac{\alpha_j}{1-\alpha_j} z^{\varphi}$$

- $\varphi \equiv 2\,(2\phi - 1)\,(\rho - 1)$ skill-bias of technology
  - if $\varphi = 0$ we say technology is Hicks neutral
  - if $\varphi > 0$ we say technology is skill biased
- Two ways reallocation affects demand for skill
  1. Across firms between sectors
  2. Across firms within sectors

Firms

Skill bias of technology

$$\frac{h}{l} = \left(\frac{w_i}{s_i}\right)^{\rho} \frac{\alpha_j}{1-\alpha_j} z^{\varphi}$$

- $\varphi \equiv 2\,(2\phi - 1)\,(\rho - 1)$ skill-bias of technology
  - if $\varphi = 0$ we say technology is Hicks neutral
  - if $\varphi > 0$ we say technology is skill biased
- Two ways reallocation affects demand for skill
  1. Across firms between sectors
  2. Across firms within sectors
- In an extension, we allow H-O to be active within sectors

Details and Analytics

International trade

- Merchandise sectors:
  - iceberg transport cost $\tau_{in} \geq 1$ of shipping from i to n
  - $\tau_{nn} = 1$ for all n
- Service sectors:
  - no international trade

▸ Costs and Prices

General equilibrium

- Goods-market clearing

$$y_i(\omega, j) = \sum_n \tau_{in} q_n(\omega, j)\, \mathbb{I}_{in}(\omega, j)$$

- Factor-market clearing with inelastic supplies H_i and L_i

$$L_i = \sum_j \int_0^1 l_i(\omega, j)\, d\omega \text{ and } H_i = \sum_j \int_0^1 h_i(\omega, j)\, d\omega$$

- Trade imbalances (where NX_i are net exports in i)

$$P_i Q_i = (s_i H_i + w_i L_i + \Pi_i)\left(1 - \frac{NX_i}{Output_i}\right)$$

  - We treat $NX_i / Output_i$ as a parameter

- Also consider no labor mobility between merchandise & service sectors

Parameterization

Parameters to choose

- Factor endowments: H_n / L_n and $H_n + L_n = 1$
- Merchandise share of absorption γ_n
- Elasticities
  - demand σ, η
  - between skilled and unskilled at firm level ρ
  - skill intensity to productivity φ
- Variability of firm-level productivity θ
- Sectoral skill intensities α_j
- Net-exports relative to output nx_n
- Trade costs τ_{in}

Parameters to choose

- Factor endowments: H_n / L_n and $H_n + L_n = 1$
- Merchandise share of absorption γ_n
- Elasticities
  - demand σ, η
  - between skilled and unskilled at firm level ρ
  - skill intensity to productivity φ
- Variability of firm-level productivity θ
- Sectoral skill intensities α_j
- Net-exports relative to output nx_n
- Trade costs τ_{in}
- Systematic productivities: set $A_n(j) = T_n \times T_n(j)$
  - $T_n(j) = 1$ for services
  - $T_n(j) = 1 + (\alpha_j - \bar{\alpha}) t_n$ for merchandise
  - normalize $\sum_i weight_i \times t_i = 0$ (Output weights)

Connecting model and data

- 64 countries + rest of the world ROW (aggregate of 89 countries)
  - 64 countries account for approx 93% of world GDP
- Data averaged over 2005-2007 (if possible)
- Skilled worker: completed tertiary degree (i.e. in US, college degree)

Connecting model and data

- 64 countries + rest of the world ROW (aggregate of 89 countries)
  - 64 countries account for approx 93% of world GDP
- Data averaged over 2005-2007 (if possible)
- Skilled worker: completed tertiary degree (i.e. in US, college degree)
- 98 merchandise sectors = goods producing industries
- 155 services industries include construction, exclude government
- Measures of merchandise trade and gross output in dollars
  - value added ⇒ gross output using OECD IO tables

Parameterization basics

- Parameters assigned directly from data
  - γ_n from IO tables
  - $H_n / (H_n + L_n) =$ % with tertiary degree from Barro Lee
  - $\alpha_j =$ % w/ tertiary degree in US, American Community Survey
  - nx_n ratio of merchandise net exports to total output
- $\sigma = \eta = 2.7$ median 5-digit SITC, Broda Weinstein
- Choose T_n, τ_{in} to match relative country size and bilateral trade
- $\rho, \theta, \varphi, t_n$ to target specific moments

Skill intensities

- Five most and least skill-intensive merchandise sectors

Most skill intensive	Intensity
Pharma. & medicine manuf.	.611
Aerospace product and parts manuf.	.561
Computer and peripheral equip. manuf.	.553
Commun., audio, & video equip. manuf.	.465
Forestry except logging	.455

Least skill intensive	Intensity
Logging	.040
Animal slaughtering, processing	.073
Fiber, yarn, and thread mills	.075
Carpets and rug mills	.085
Turned product, screw, nut, bolt manuf.	.086

Trade costs and productivities

- Parameters
  - $(N-1)$ Relative productivities, T_n / T_1
  - $N(N-1)$ Trade costs, τ_{in}
- Moments
  - N^2 $\text{Exports}_{in} / Output_i$
  - Can show:
    - Imply $Output_i / Output_n$
    - Only $N(N-1)$ are independent
- Issue: $N-1$ more parameters than moments
- Three aproaches:
  - Find one (among potentially many) $\{\tau_{in}, T_i / T_1\}$
  - Restrict $\tau_{iN} = \tau_{Ni}$
  - Restrict $\tau_{in} = \tau_{ni}$
    - Three approaches give almost identical results

Target moment 1

- Aggregate elasticity of substitution btw H_{US} and L_{US} in US, $\widehat{\rho} = 1.6$
  - Katz and Murphy 92 estimate elasticity $= 1.4$
  - Acemoglu and Autor 10 estimate elasticity $\in [1.6, 1.8]$
- In baseline parameterization, we $\uparrow H_{US}$ by 10% and calculate

$$\widehat{\rho} = \Delta \left[\log \left(\frac{H_{US}}{L_{US}} \right) \Big/ \log \left(\frac{w_{US}}{s_{US}} \right) \right]$$

- If $\varphi = 0$ and only one sector $\Rightarrow \widehat{\rho} = \rho$
- With $\varphi > 0$ and many sectors $\Rightarrow \rho = 1.4$

Target moment 2

- Elasticity of trade with respect to variable trade cost, $\widehat{\varepsilon} = 5$
  - Eaton and Kortum 2002 preferred estimate 8.28
  - Donaldson 2010 preferred estimate 4
  - Simonovska and Waugh 2011 estimate $[2.47, 5.51]$
  - Eaton, Kortum, and Kramarz 2011 preferred estimate 5
  - Costinot, Donaldson, and Komunjer 2012 preferred estimate 6.53
- Run a gravity equation on data generated by our model

$$\log\left(Exp_{in}\right) = Importer_n FE + Exporter_i FE - \widehat{\varepsilon} \ln \tau_{in}$$

- If $\varphi = 0 \Rightarrow \theta = 1/\widehat{\varepsilon}$
- With $\varphi > 0 \Rightarrow \theta = 0.25$

Target moment 3

$$\log\left[\frac{h_i}{h_i + l_i}\right] = \beta_0 + \beta_1 \log sales_i + IndustryFE_i + \varepsilon_i$$

- In Mexico, $\beta_1 = 0.136$; unreported result from Verhoogen (2008)
  - 1998 *Encuesta Industrial Anual* (EIA) w/ large manufacturing plants
- In the model: $\varphi = 0 \Rightarrow \beta_1 = 0$
  - β_1 is increasing in φ

φ	0	0.08	0.24	$\mathbf{0.4}$ ($\phi = 0.75$)	0.64	0.72
Elasticity	0	0.05	0.085	**0.139**	0.213	0.23

- Note: If $\varphi = 0$ and αs vary within sector, then elasticity in skill-scarce countries is negative

Target moment 4

Between sector trade patterns

- For each $n = 1, ..., 64$, regress

$$\frac{\text{Net exports}_n(j)}{\text{Exports}_n(j) + \text{Imports}_n(j)} = \beta_{0i} + \beta_n \frac{H_{US}(j)}{H_{US}(j) + L_{US}(j)} + \varepsilon_n(j)$$

Target moment 4

Between sector trade patterns

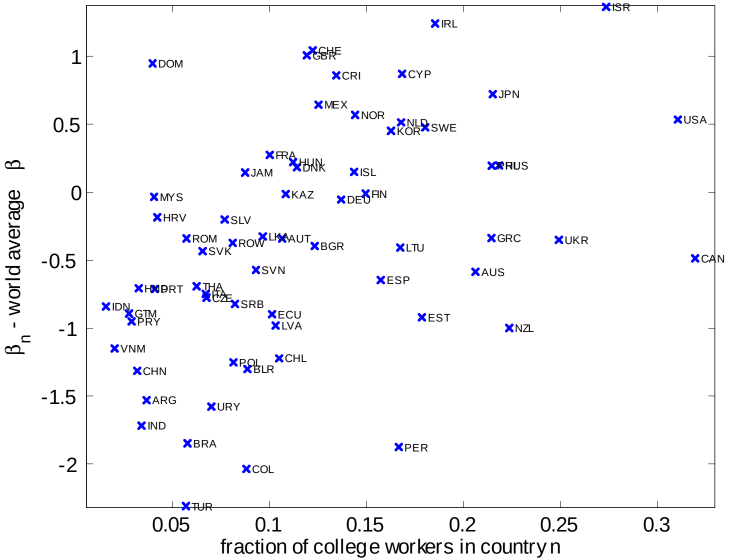
- For each $n = 1, ..., 64$, regress

$$\frac{\text{Net exports}_n(j)}{\text{Exports}_n(j) + \text{Imports}_n(j)} = \beta_{0i} + \beta_n \frac{H_{US}(j)}{H_{US}(j) + L_{US}(j)} + \varepsilon_n(j)$$

- Comparative advantage determined by H_n / L_n and *relative* t_n's
- Alternative 1: Choose t_n to match $\beta_n - \sum_i w_i^{out} \beta_i$
- Alternative 2: Choose $t_n = 0$ (Morrow 2010)

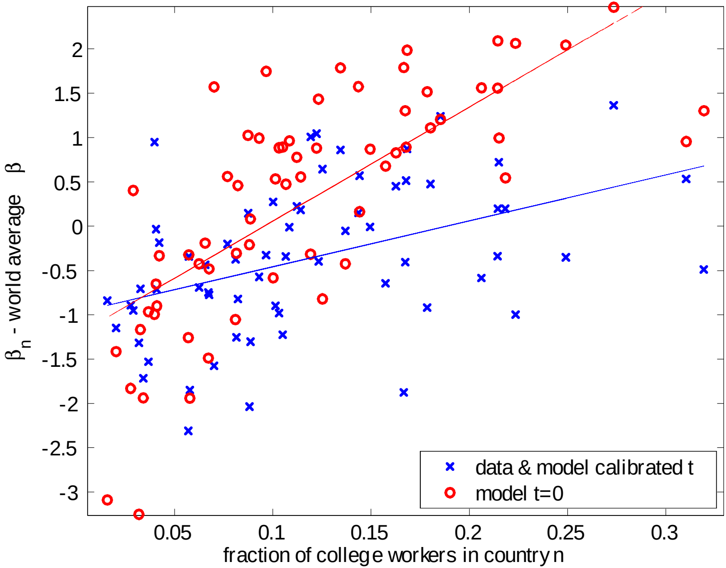
Target moment 4

Between sector trade patterns in the data and matched in the model



Target moment 4

Between sector trade pattern if we do and if we do not target moment 4



Solution Algorithm

Solution Algorithm

Overview of three loops

- Outer loop: iterate over φ, θ, ρ
- Middle loop: iterate over τ_{in}, T_n, t_n
  - Match $Exports_{in} / \left(Out_i + Out_n\right)$, $Out_n / World\ Out$, and target moment 4
  - Update τ_{in} using excess bilateral exports data - model
  - Update T_n using excess output$_n$ data - model
  - Update t_n using excess β_n data - model
- Inner loop: iterate over w_n, s_n, π_n
  - Extends Alvarez and Lucas
    - ★ no analytic gravity, 2 factors, $\Pi_n \neq 0$, & trade imbalances
  - no proof of uniqueness
  - numerical demonstration of existence

Inner loop: factor prices and profit shares

Inner loop k_I: given $\varphi, \theta, \rho, \tau, T_n, t_n$

- Initial guesses $\{w_n, s_n, \pi_n\}$ from inner loop $(k_I - 1)$

Inner loop: factor prices and profit shares

Inner loop k_I: given $\varphi, \theta, \rho, \tau, T_n, t_n$

- Initial guesses $\{w_n, s_n, \pi_n\}$ from inner loop $(k_I - 1)$
- Solve for
  - $P_n Q_n = \left(w_n L_n^d + s_n H_n^d\right)(1 + \pi_n)\left(1 - nx_n^d\right)$
  - prices, quantities demanded, ...

To inner loop details

Inner loop: factor prices and profit shares

- d = data and m = model
- Generate factor demand, output, aggregate profits, aggregate exports, and aggregate imports

$$
\begin{aligned}
L_n^m &= \sum_j \int_0^1 l_n(\omega, j)\, d\omega \quad \text{and } H_n^m = \sum_j \int_0^1 h_n(\omega, j)\, d\omega \\
Output_n^m &= \sum_i \sum_j \int_0^1 p_i(\omega, j)\, q_i(\omega, j)\, \mathbb{I}_{ni}(\omega, j)\, d\omega \\
\Pi_n^m &= Output_n^m - w_n L_n^m - s_n H_n^m \\
Exp_n^m &= \sum_{i \neq n} \sum_j \int_0^1 p_i(\omega, j)\, q_i(\omega, j)\, \mathbb{I}_{ni}(\omega, j)\, d\omega \\
Imp_n^m &= \sum_{i \neq n} \sum_j \int_0^1 p_n(\omega, j)\, q_n(\omega, j)\, \mathbb{I}_{in}(\omega, j)\, d\omega
\end{aligned}
$$

Inner loop: factor prices and profit shares

- Excess relative H/L and net exports equations

$$f_n^1 = \left(\frac{H_n^m}{L_n^m} - \frac{H_n^d}{L_n^d} \right) \Big/ \left(\frac{H_n^d}{L_n^d} \right)$$

$$f_n^2 = \left(NX_n^m - NX_n \right) \Big/ Exp_n^m$$

Inner loop: factor prices and profit shares

- Excess relative H/L and net exports equations

$$f_n^1 = \left(\frac{H_n^m}{L_n^m} - \frac{H_n^d}{L_n^d} \right) \Big/ \left(\frac{H_n^d}{L_n^d} \right)$$

$$f_n^2 = \left(NX_n^m - NX_n \right) / \, Exp_n^m$$

- New guesses satisfy

$$\begin{aligned} \left(\frac{s_n}{w_n} \right)^{k_I+1} &= \left(\frac{s_n}{w_n} \right)^{k_I} \times \left(1 + \Delta^{s/w} f_n^1 \right) \\ w_n^{k_I+1} &= w_n^{k_I} \times \left(1 + \Delta^{w} f_n^2 \right) \\ \left(\pi_n \right)^{k_I+1} &= \Pi_n^m \Big/ \left(s_n^{k_I} H_n^d + w_n^{k_I} L_n^d \right) \end{aligned}$$

re-scaling factor prices s.t. $\sum_n \left(s_n^{k_I+1} H_n + w_n^{k_I+1} L_n \right) = \overline{W}$

- Stop when $\left| f_n^1 \right|$, $\left| f_n^2 \right|$, $\left| \left(\pi_n \right)^{k_I+1} - \left(\pi_n \right)^{k_I} \right|$ are small

Middle loop: trade costs and productivities

Middle loop k_M: given φ, θ, ρ

- Initial guesses $\{\tau_{in}, T_i, t_i\}$ from middle loop $(k_M - 1)$
- Excess $Exp_{in}, Output_i, \beta_i$ equations are

$$f_{in}^{\tau} = \frac{\frac{Exp_{in}^m}{(Output_i^m)^{merch} + (Output_n^m)^{merch}}}{\frac{Exp_{in}^d}{\left(Output_i^d\right)^{merch} + (Output_n^d)^{merch}}} - 1$$

$$\begin{aligned} f_n^T &= \left(\frac{Output_n^m}{\sum_i Output_i^m}\right) \Big/ \left(\frac{Output_n^d}{\sum_i Output_i^d}\right) - 1 \\ f_n^{\beta} &= \beta_n^m - \overline{\beta}^m - \left(\beta_n^d - \overline{\beta}^d\right) \end{aligned}$$

Middle loop: trade costs and productivities

- New guesses satisfy

$$
\begin{aligned}
\tau_{ni}^{k_M+1} &= 1 + \left(\tau_{ni}^{k_M} - 1\right) \times \left[1 + \Delta^{\tau}\left(f_{ni}^{\tau} - 1\right)\right] \\
T_n^{k_M+1} &= T_n^{k_M}\left[1 + \Delta^{T}\left(f_n^{T} - 1\right)\right] \\
t_n^{k_M+1} &= t_n^{k_M} + \Delta^{\beta} f_n^{\beta}
\end{aligned}
$$

- Stop when $\left|f_i^T\right|$, $\left|f_i^{\beta}\right|$, $\left|f_{in}^{\tau}\right|$ small

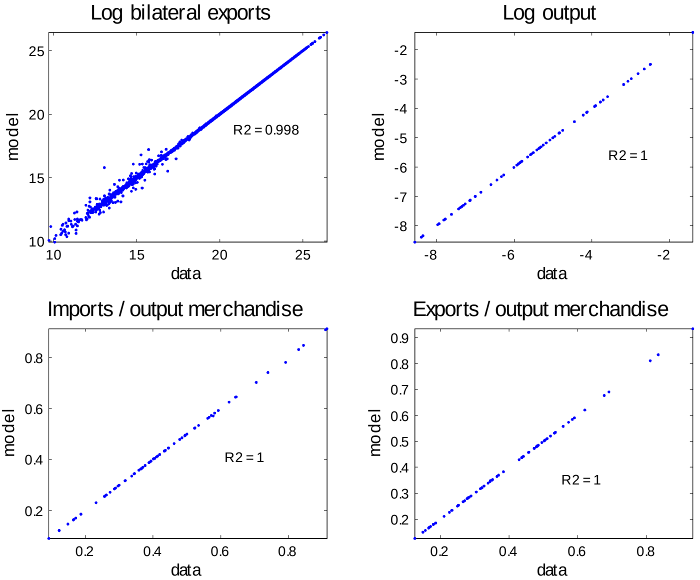
Outer loop: theta, rho, and phi

- Choose θ, ρ, φ to match target moments 1, 2, and 3

Moments targeted and not targeted

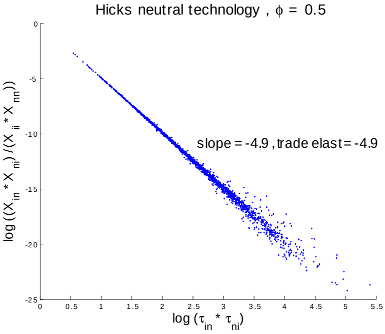
Trade flows and output: Data versus model

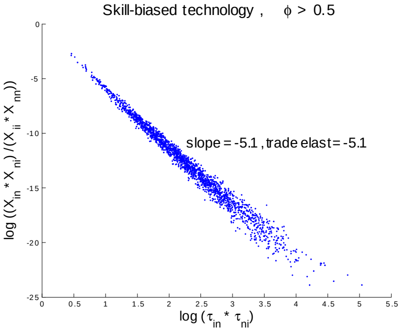
▸ To H/L and trade flows



Gravity

- Plot $\log\left[X_{in}X_{ni}/\left(X_{ii}X_{nn}\right)\right]$ and $\log\left(\tau_{in}\tau_{ni}\right)$
  With $\varphi = 0$, constant elasticity





Trade costs

- We project τ_{in} onto standard "gravity" variables
  - distance, distance squared, common language, common border, exporter and importer FEs
    - only using those τ_{in}s not set to $+\infty$

$\Rightarrow R^2 = 0.74$ with expected signs and statistical significance

Trade costs

- We project τ_{in} onto standard "gravity" variables
  - distance, distance squared, common language, common border, exporter and importer FEs
    - only using those τ_{in}s not set to $+\infty$

  $\Rightarrow R^2 = 0.74$ with expected signs and statistical significance

- Do poor countries face higher export and/or import costs conditioning on other observables?
  - Regressing importer FEs on importer GDP per capita $\Rightarrow$ negative coefficient highly significant
  - Regressing exporter FEs on exporter GDP per capita $\Rightarrow$ negative coefficient significant at 10% level

- Similar results if we directly include exporter & importer GDP per capita in gravity regression

Other moments not targeted: Mexico

- Exporter skill-intensity premium, controlling for industry

$$\ln\left[\frac{h_i}{h_i + l_i}\right] = \beta_0 + \beta_1 Exporter_i + IndustryFE_i + \varepsilon_i$$

  - in model $\beta_1 = 0.25$ in merchandise
  - in data $\beta_1 = 0.21$, 1998 EIA unreported from Verhoogen (2008)

Other moments not targeted: Brazil

- Elasticity of skill intensity to firm *i* size controlling for industry

$$\log\left[\frac{h_i}{h_i + l_i}\right] = \beta_0 + \beta_1 \log sales_i + IndustryFE_i + \varepsilon_i$$

  - in model $\beta_1 = 0.24$ in merchandise
  - in data $\beta_1 = 0.36$, 1995 *Pesquisa Industrial Anual* (PIA) sample (large manuf firms) unreported from Menezes-Filho et. al. (2008)

- Elasticity of skill intensity to domestic sales controlling for industry

$$\log\left[\frac{h_i}{h_i + l_i}\right] = \beta_0 + \beta_1 \log\left(\text{domestic sales}\right)_i + IndustryFE_i + \varepsilon_i$$

  - in model $\beta_1 = 0.34$ in merchandise
  - in data $\beta_1 = 0.34$, 1995 PIA sample unreported from Menezes-Filho et. al. (2008)

Other moments not targeted: US

- % of exporters = 0.51 too high, as in **BEJK**
  - need fixed cost
- However
  - share of aggregate revenues by exporters
    - in model = 65% in merchandise
    - in data = 60%, 1992 Census of Manuf, **BEJK**
  - VA per worker exporter premium in US

$$\ln\left(VA \text{ per worker}_i\right) = \beta_0 + \beta_1 Exporter_i + IndustryFE_i + \varepsilon_i$$

    - in model $\beta_1 = 0.135$ in merchandise
    - in data $\beta_1 = 0.11$, 2002 Census of Manuf, Bernard et. al. (2007)

Other moments not targeted: US

- Exporter skill-intensity premium, controlling for industry

$$\ln\left[\frac{h_i}{h_i + l_i}\right] = \beta_0 + \beta_1 Exporter_i + IndustryFE_i + \varepsilon_i$$

  - in model $\beta_1 = 0.14$ in merchandise
  - in data $\beta_1 = 0.11$, 2002 Census of Manuf, Bernard et. al. (2007)
  - Imperfect comparison: Bernard et. al. (2007) use non-production worker share

Other moments not targeted: US

- Regress $\frac{\text{Exp}_{US}(j)+\text{Imp}_{US}(j)}{\text{Absorption}_{US}(j)}$ on j skill intensity in US merchandise js
  - in data, coefficient on skill intensity $= 0.70$
    - significant at 1% level
    - use BEA's detailed IO tables for 2002 Benchmark
  - in model, coefficient on skill intensity $= 0.88$
  - re-parameterize model imposing $\phi = 1/2$, coefficient $= -0.06$

Other moments not targeted: US

- Regress $\frac{\text{Exp}_{US}(j)+\text{Imp}_{US}(j)}{\text{Absorption}_{US}(j)}$ on j skill intensity in US merchandise js
  - in data, coefficient on skill intensity $= 0.70$
    - significant at 1% level
    - use BEA's detailed IO tables for 2002 Benchmark
  - in model, coefficient on skill intensity $= 0.88$
  - re-parameterize model imposing $\phi = 1/2$, coefficient $= -0.06$
- **Intuition**: interaction between the two mechanisms

  $\phi > 1/2 \Rightarrow$ unit costs more sensitive to z in high α_j sectors

$$\frac{d}{d\alpha_j}\left|\frac{d\log\left[\text{unit cost}\,(\omega, j)\right]}{d\log z}\right| > 0 \Leftrightarrow \phi > 1/2$$

  $\Rightarrow$ more dispersed distribution of unit costs in high α_j sectors

  - even though same distribution of productivities across sectors

  $\Rightarrow$ more trade in high α_j sectors

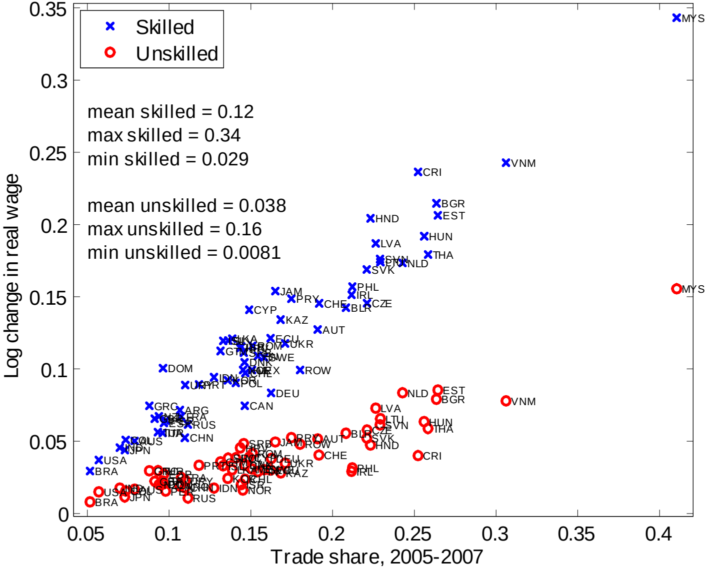
Counterfactuals

Counterfactuals

- We consider a range of counterfactuals:
  - autarky
  - 10% reduction in trade costs
    - Both with factor mobility and limited factor mobility, labor fixed in merchandise and services at baseline levels
    - In 10% experiment, keep $(\text{Net Exports})_i \, / \, Output_i$ fixed
- We revisit previous approaches using data generated by model and show why they would predict small effects of trade

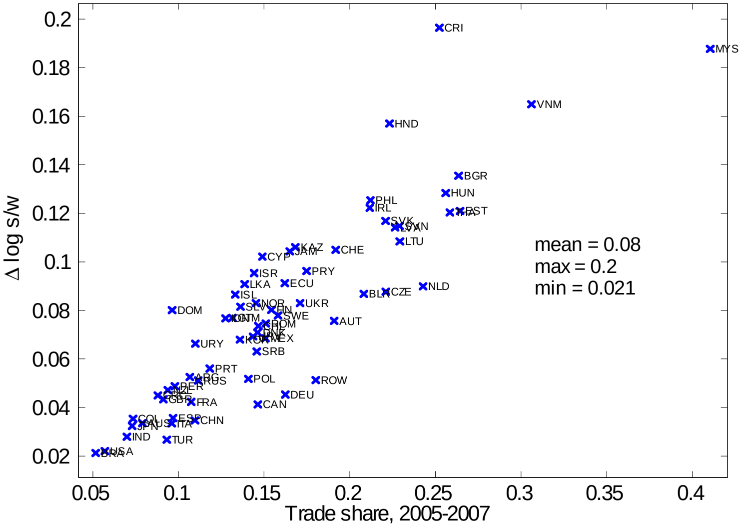
Welfare changes from autarky to baseline parameterization

Large differences in real wage changes across factors



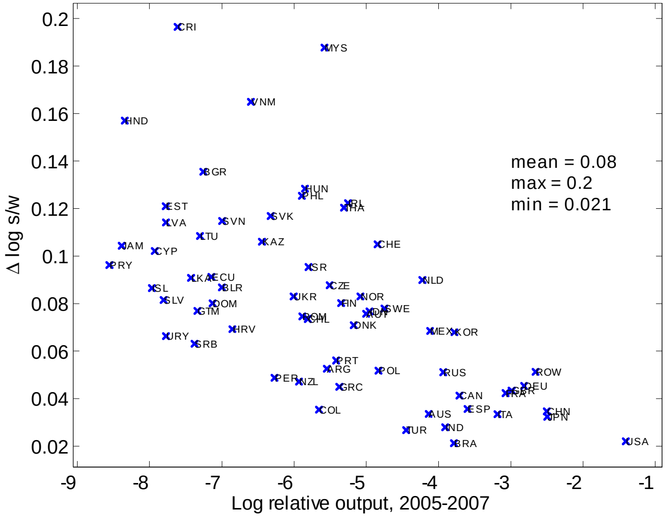
From autarky to baseline parameterization

Change in skill premium vs 2005-07 trade share, correlation = 0.70



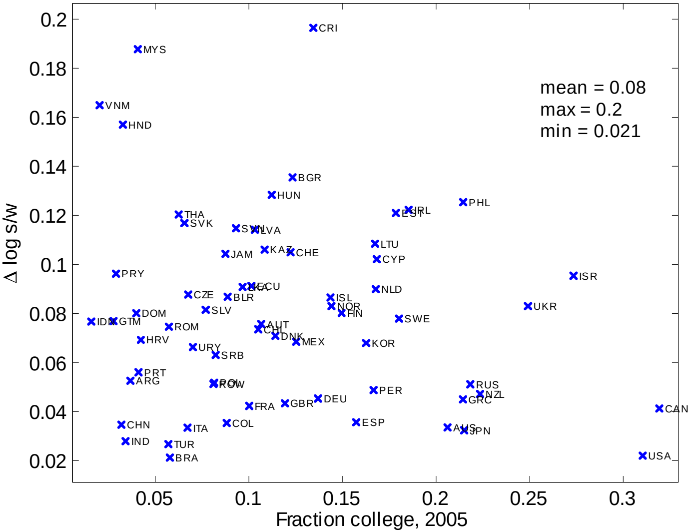
From autarky to baseline parameterization

Change in skill premium vs 2005-07 country size, correlation = -0.62



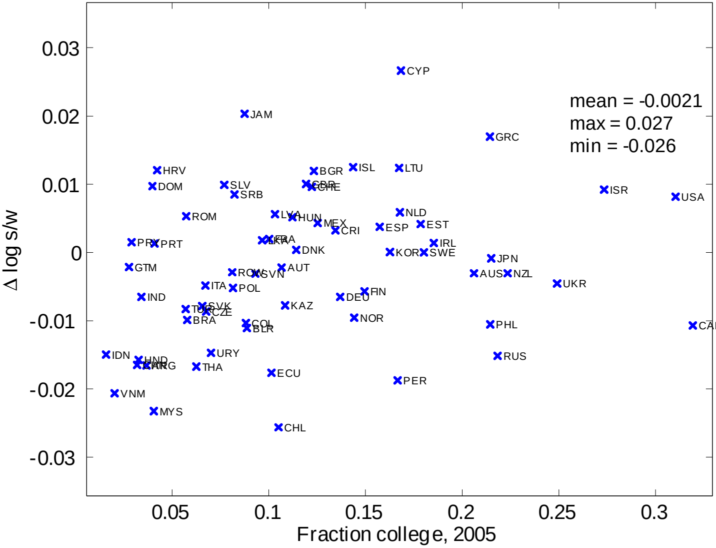
From autarky to baseline: strength of H-O

Correlation change skill premium & H/L = -0.16



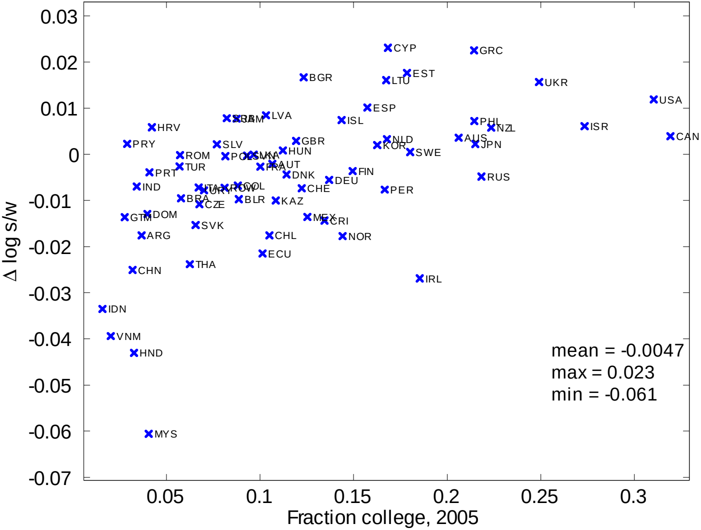
From autarky to baseline: strength of H-O

No skill bias, high prod dispersion, calib tn: correl change skill premium & H/L = 0.25



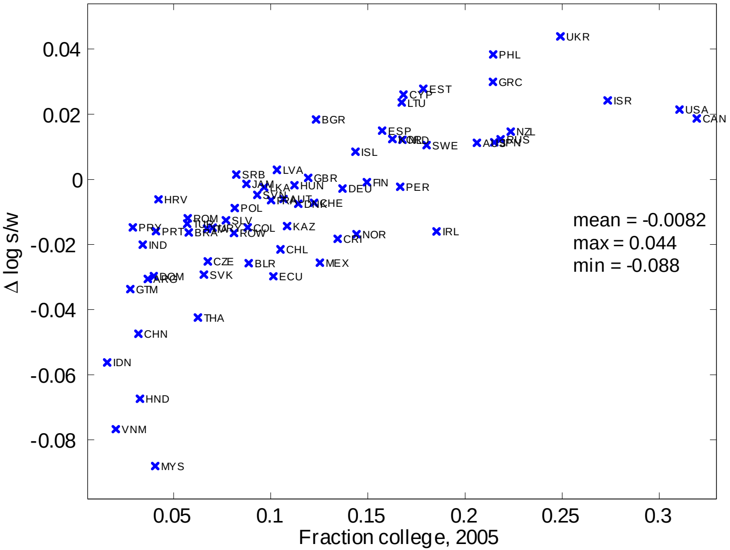
From autarky to baseline: strength of H-O

No skill bias, high prod dispersion, tn=0: correl change skill premium & H/L = 0.53



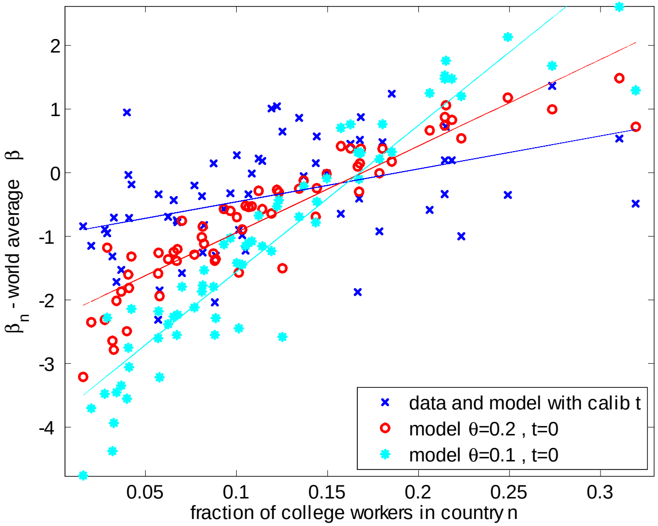
From autarky to baseline: strength of H-O

No skill bias, low prctivity dispersion, tn=0: correl change skill premium & H/L = 0.75



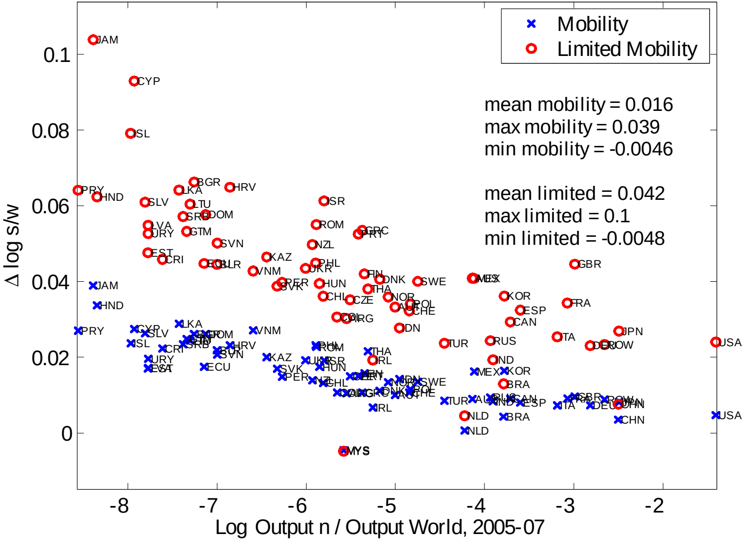
Target moment 4: Alternative parameterizations

No skill bias



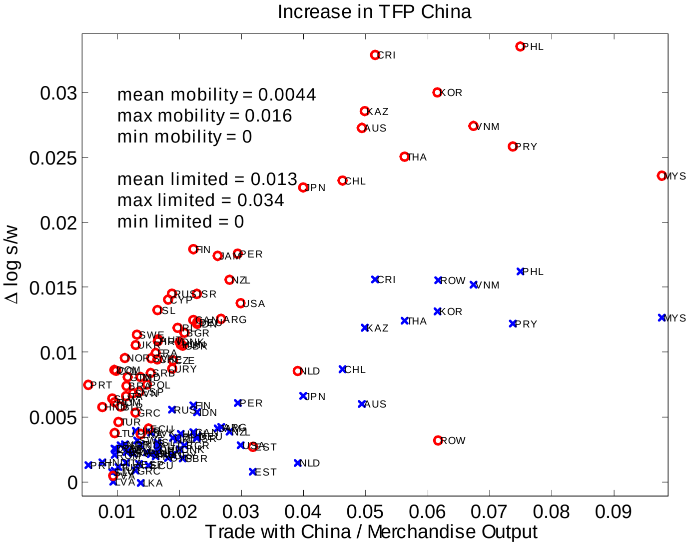
10% fall in trade costs from baseline parameterization

Skill premium with full and limited mobility



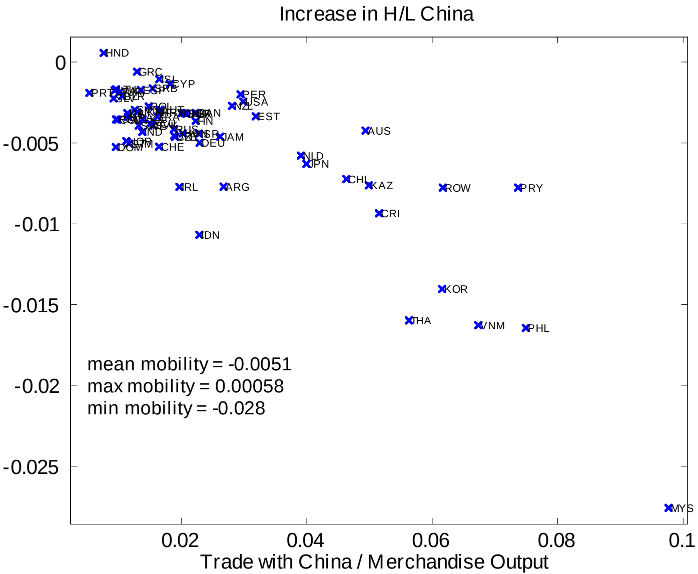
Three-fold increase in China's TFP

Skill premium change in China's trading partners, with full and limited mobility



Increase in China's skill abundance to equal US

Skill premium change in China's trading partners, with full mobility



Other approaches

Other approaches

- Factor content of trade (FCT)

- Between-sector price changes

- Between-sector factor reallocation

Factor content of trade (FCT)

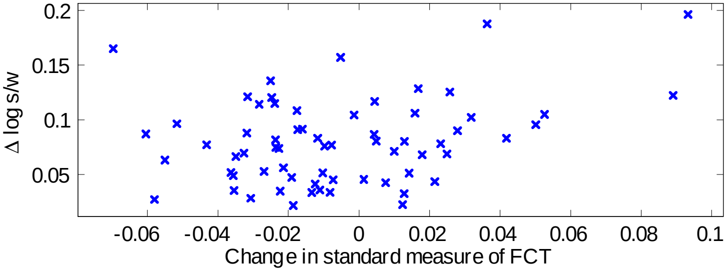
- Measure changes in the "factor content of trade" (FCT) to infer impact of trade on s/w details
  - e.g., Katz and Murphy (92), Berman et. al. (94), Krugman (95), Krugman (08)
- Under **strong** assumptions
  1. Cobb Douglas production and utility
  2. All producers w/in sector share common factor intensity

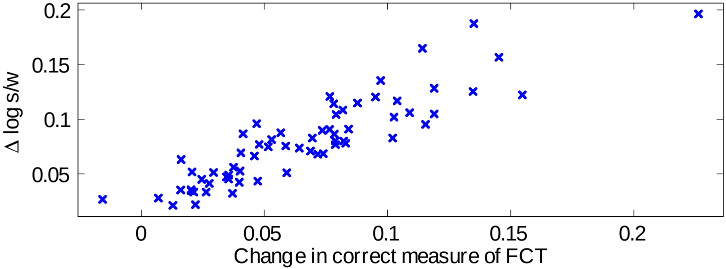
$$\frac{s_i}{w_i} = \frac{L_i - FCT_i(L)}{H_i - FCT_i(H)}$$

$$FCT_i(L) = \sum_j (\text{Employment of } L \text{ in sector } j) \frac{\text{Net Exp}_i(j)}{\text{Revenue}_i(j)}$$

- Correct measure of FCT adjusts for fact that exporters are more skill intensive than non-exporters

Factor content of trade (FCT)





Changes in prices

- Price = unit cost * markup
- Previous approach:

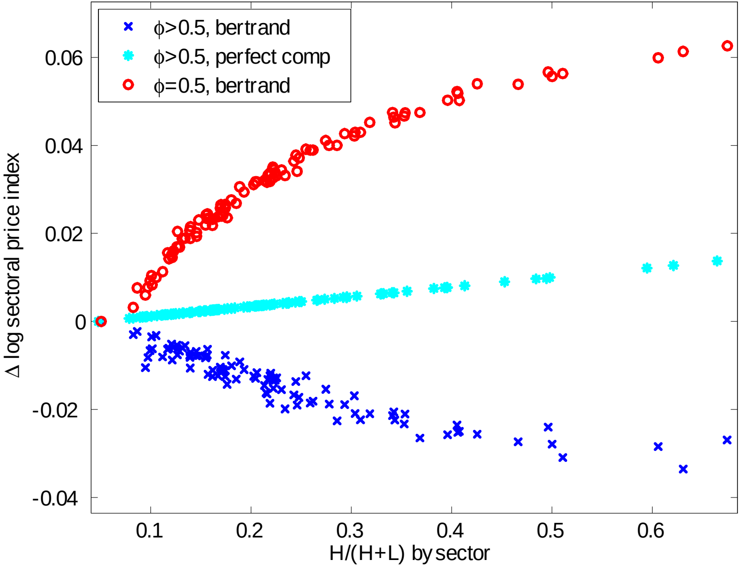
  1. project Δtrade on Δ producer prices
  2. infer Δwages from Δprices assuming constant markups
     If international trade $\uparrow$ relative price of skill intensive goods, it $\uparrow$ s/w

  - e.g. Lawrence and Slaughter 93, Sachs and Shatz 94, Feenstra and Hanson 99

Changes in prices

- Price = unit cost * markup
- Previous approach:
  1. project Δtrade on Δ producer prices
  2. infer Δwages from Δprices assuming constant markups
     If international trade $\uparrow$ relative price of skill intensive goods, it $\uparrow$ s/w
  - e.g. Lawrence and Slaughter 93, Sachs and Shatz 94, Feenstra and Hanson 99
- In our model with Bertrand competition and $\varphi > 0$: markups fall more in more skill-intensive sectors
  - unit costs more dispersed & imports $\uparrow$ most in these sectors
- Can get $\uparrow$ in s/w and $\downarrow$ in relative price of skill intensive goods
- Δprices underestimate $\uparrow$ in s/w

Changes in domestic prices by sector



Between sector factor reallocation

- H-O mechanism, factors reallocate towards comparative advantage sectors
- Empirically: little btw sector reallocation in countries where $s/w \uparrow$
  - e.g. Revenga (1997), Hanson and Harrison (1999), Attanasio et. al. (2004), Topalova (2004)...

Between sector factor reallocation

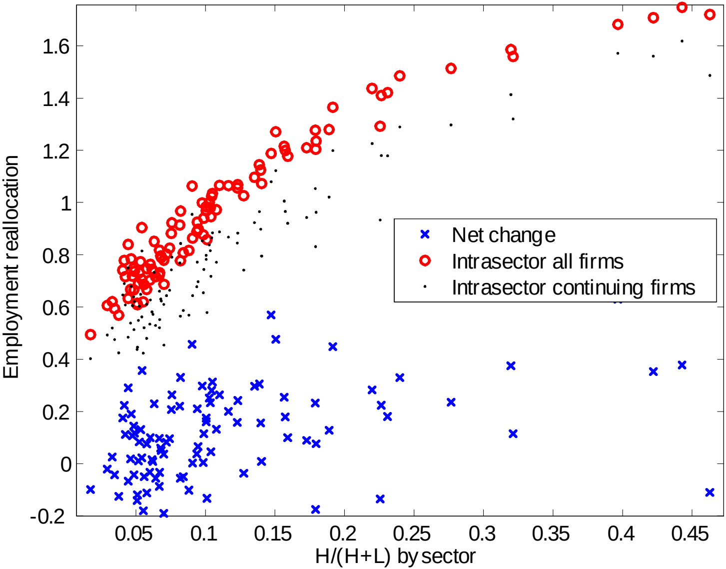
- H-O mechanism, factors reallocate towards comparative advantage sectors
- Empirically: little btw sector reallocation in countries where $s/w \uparrow$
  - e.g. Revenga (1997), Hanson and Harrison (1999), Attanasio et. al. (2004), Topalova (2004)...
- Firm heterogeneity within sector $\Rightarrow$ within sector reallocation
- $\theta \uparrow \Rightarrow$ more within sector reallocation & less between
- Large impact on s/w consistent, no substantial between-sector reallocation

Between sector factor reallocation

- H-O mechanism, factors reallocate towards comparative advantage sectors
- Empirically: little btw sector reallocation in countries where $s/w \uparrow$
  - e.g. Revenga (1997), Hanson and Harrison (1999), Attanasio et. al. (2004), Topalova (2004)...
- Firm heterogeneity within sector $\Rightarrow$ within sector reallocation
- $\theta \uparrow \Rightarrow$ more within sector reallocation & less between
- Large impact on s/w consistent, no substantial between-sector reallocation
- Skill-biased technology, $\varphi > 0$
  - Trade rises more in skill-intensive sectors, btw sector reallocation towards these sectors, contributing to $s/w \uparrow$
  - More within-sector reallocation in skilled-intensive sectors

Between sector factor reallocation

Model's implication for Chile: from autarky to baseline (s/w rises 7.5%)



Sensitivity

Perfect competition

Same $\{\rho, \varphi, \theta\}$, redo middle and inner loops

Move countries to autarky, full factor mobility, change in skill premium (%)

	Baseline	Perfect competition
mean	+8.00	+7.89
max	+19.65	+19.82
min	+2.12	+1.88

Alternative trade cost parameterization

Same $\{\rho, \varphi, \theta\}$, redo middle and inner loops
Move countries to autarky, full factor mobility, change in skill premium (%)

	Baseline	symm trade costs in ROW	symm trade costs in US	symm trade costs in all n
mean	$+8.00$	$+8.00$	$+8.00$	$+8.08$
max	$+19.65$	$+19.63$	$+19.63$	$+19.47$
min	$+2.12$	$+2.12$	$+2.12$	$+2.12$

Sectoral comparative advantage

Same $\{\rho,\varphi,\theta\}$, redo middle and inner loops
From 2006 parameterization, move countries to autarky, full factor mobility, change in skill premium (%)

	Baseline	Setting $t_i = 0$
mean	$+8.00$	$+9.27$
max	$+19.65$	$+23.23$
min	$+2.12$	$+0.81$

Measure of skill endowment

Same $\{\rho, \varphi, \theta\}$, redo middle and inner loops
From 2006 parameterization, move countries to autarky, full factor mobility, change in skill premium (%)

	Baseline	$\frac{H_i}{L_i}$ avg yrs of educ.	$\frac{H_i}{L_i}$ avg yrs of educ. and setting $t_i = 0$
mean	+8.00	+7.90	+9.80
max	+19.65	+19.40	+22.63
min	+2.12	+2.01	+1.84

Skill bias of technology

Same $\{\rho,\theta\}$, redo middle and inner loops
From 2006 parameterization, move countries to autarky, full factor mobility, change in skill premium (%)

	Baseline $\varphi = 0.4$	$\varphi = 0$	$\varphi = 0.08$	$\varphi = 0.24$	$\varphi = 0.64$	$\varphi = 0.72$
mean	$+8.00$	-0.2	$+1.14$	$+4.28$	$+13.83$	$+15.64$
max	$+19.65$	$+2.67$	$+4.05$	$+11.41$	$+33.19$	$+37.99$
min	$+2.12$	-2.56	-1.01	$+0.6$	$+3.04$	$+3.28$

Heterogeneity of productivity within sectors

Same $\{\rho,\varphi\}$, redo middle and inner loops
From 2006 parameterization, move countries to autarky, full factor mobility, change in skill premium (%)

	Baseline $\theta = 0.25$	$\theta = 0.125$	$\theta = 0.17$	$\theta = 0.3$
mean	$+8.00$	$+3.60$	$+5.15$	$+9.74$
max	$+19.65$	$+10.34$	$+13.56$	$+23.20$
min	$+2.12$	0	$+0.93$	$+2.45$

Heterogeneity of alpha within sectors

- Aggregation bias in skill intensities: Feenstra 2010

- $\alpha_j(\omega) = \min\{\bar{\alpha}_j \exp(\varepsilon), 1\}$
- $\varepsilon \sim \ln \mathcal{N}(0, \sigma_\alpha)$

- Stronger H-O mechanism (now also operates within sector)
- If impose $\varphi = 0$, exporters exhibit *low h/l* in high s/w countries
  - *Negative* elasticity of firm's skill intensity to firm's sales

Heterogeneity of alpha within sectors

- $\alpha_j(\omega) = \min\{\max\{0, \bar{\alpha}_j \exp(\varepsilon)\}, 1\}, \varepsilon \sim N(0, \sigma_\alpha)$
- Redo outer, middle and inner loops
  - Require lower ρ (more within reallocation)
- From 2006 parameterization, move countries to autarky, full factor mobility, change in skill premium (%)

	Baseline $\sigma_\alpha = 0$	$\sigma_\alpha = 0.05$	$\sigma_\alpha = 0.1$	$\sigma_\alpha = 0.2$
St. dev log h/l: (median sector within) / btw	0.21	0.66	2	4.2
mean	+8.00	+8.32	+9.64	+10.84
max	+19.65	+20.26	+24.07	+28.62
min	+2.12	+2.09	+1.73	−1.67

Elasticity of substitution across goods

Lower σ $\downarrow$ btw sector reallocation induced by SBT effect

Redo outer, middle and inner loops, keeping $\eta = 2.7$.
From 2006 parameterization, move countries to autarky, full factor mobility, change in skill premium (%)

	Baseline $\sigma = \eta = 2.7$	$\sigma = 2.2$ $\eta = 2.7$ BW 3 digits	$\sigma = 1$ $\eta = 2.7$	$\sigma = 1$ $\eta = 2.7$ base ρ ($\widehat{\rho} = 1.38$)
mean	$+8.00$	$+6.94$	$+3.96$	$+6.24$
max	$+19.65$	$+17.9$	$+12.10$	$+18.21$
min	$+2.12$	$+1.53$	-0.3	$+0.3$

Conclusion

- Embed into otherwise standard quantitative trade model, 2 central mechanisms in theoretical and empirical trade literature through which trade shapes skill premium

- Much of gains from trade accrue to skilled labor bc skill premium in most countries in response to changes in trade costs

- Use computational approach to accurately match bilateral exports, does not require analytic gravity at any level of aggregation

Conclusion

- Embed into otherwise standard quantitative trade model, 2 central mechanisms in theoretical and empirical trade literature through which trade shapes skill premium
- Much of gains from trade accrue to skilled labor bc skill premium in most countries in response to changes in trade costs
- Use computational approach to accurately match bilateral exports, does not require analytic gravity at any level of aggregation
- Multinational production is another major form of globalization
  - MP may strengthen H-O mechanism, high productivity firms can produce in countries with comparative advantage in their sector
  - MP may strengthen SBT mechanism, promotes international diffusion of best technologies

Skill premium decomposition

- Define:
  - $L_{k,i}$ = employment of factor k in country i
  - $L_{k,in}(j)$ = employment of k in country i sector j used in goods bound for country n
  - $w_{k,i}$ avg wage paid to factor k in country i
  - $FCT_i(k) = \sum_j \sum_n \left[L_{k,in}(j) - L_{k,ii}(j) \frac{\Lambda_{ni}(j)}{\Lambda_{ii}(j)} \frac{w_{k,ii}(j)}{w_{k,i}} \right]$
    - ★ $w_{k,ii}(j)$ = wage paid to factor k employed in sector j used to supply domestic mkt
    - ★ $\Lambda_{ni}(j)$ share of i's expenditure in sector j from country n
  - $\Phi_i(k) = \sum_j [w_{k,ii}(j) L_{k,ii}(j)] / \Lambda_{ii}(j)$
- Accounting identity $L_{k,i} = \sum_j \sum_n L_{k,in}(j)$ implies

$$w_{k,i} L_{k,i} = w_{k,i} FCT_i(k) + \Phi_i(k)$$

Skill premium decomposition

- Can express $\Phi_i(k)$ and $FCT_i(k)$ as

$$\Phi_i(k) = \sum_j \lambda_{ii}(j)\,\alpha_{k,ii}(j)\,E_i(j)$$
$$w_{k,i}FCT_i(k) = \sum_{j,n}\left[\begin{array}{r}\alpha_{k,in}(j)\,\lambda_{in}(j)\,\Lambda_{in}(j)\,E_n(j) \\ -\,\alpha_{k,ii}(j)\,\lambda_{ii}(j)\,\Lambda_{ni}(j)\,E_i(j)\end{array}\right]$$

  - $\alpha_{k,in}(j)$ = share of factor payments paid to k, in j prodn bound for n
  - $\lambda_{in}(j)$ = share of i sales in country n in sector j paid to all factors
  - $E_n(j)$ = n's expenditure in j

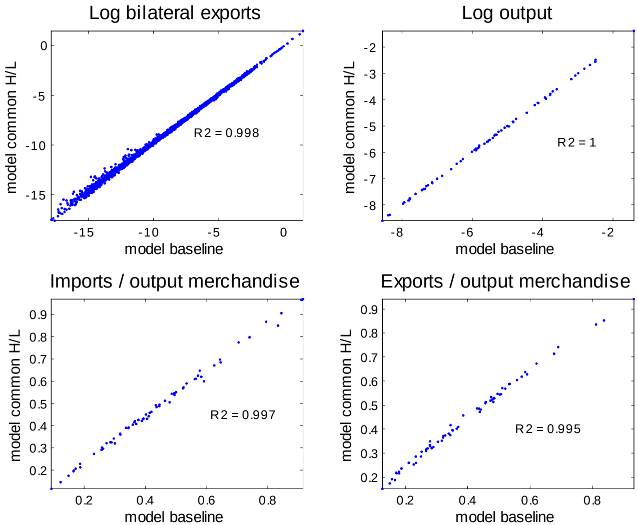
- If $\alpha_{k,in}(j)$ and $\lambda_{in}(j)$ fixed across destinations
  $\Rightarrow FCT_i(k) = \sum_j L_{k,i}(j)\,\omega_i(j)$
  - $\omega_i(j) = (\text{Net Exp}_i(j)) / (\text{Rev}_i(j))$
  - $\Rightarrow$ Component 1 easily measured using sector-level data

- If $\lambda_{ii}(j)$ and $\alpha_{k,ii}(j)$ fixed and $E_i(j)/E_i(j')$ fixed $\Rightarrow$
  $\Rightarrow$ Component 2 constant across equilibria

back to FCT

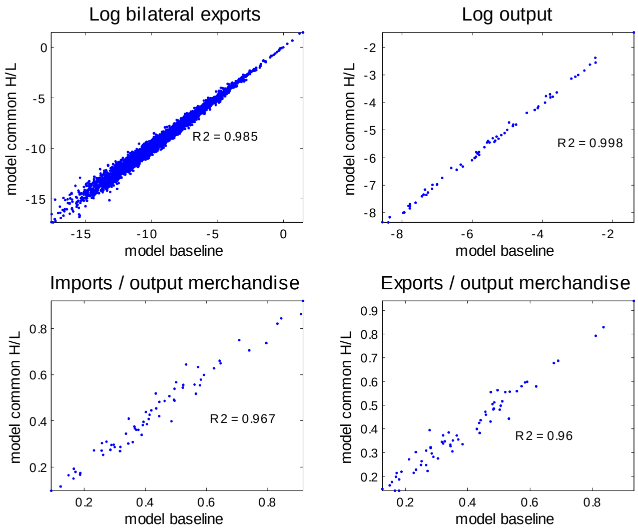
Do H/L's play large role in shaping bilateral exports?

- Set $H_n / L_n = H_{world} / L_{world}$ for all n, keep $H_n + L_n = 1$
- Other parameters (incl. calibrated t_n, T_n, τ_{in}) unchanged



Do H/L's play large role in shaping bilateral exports?

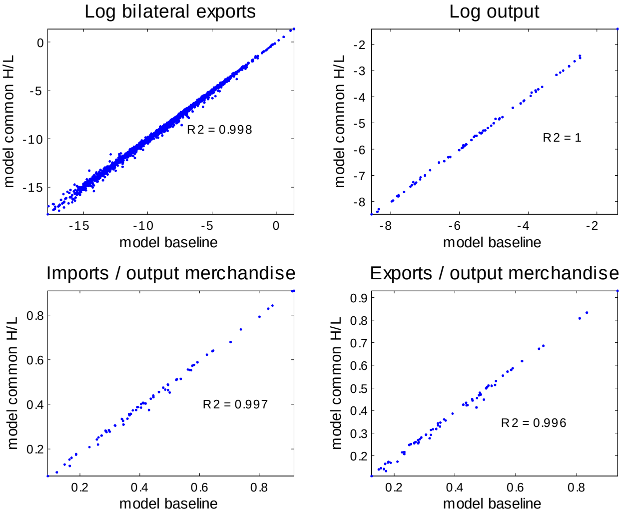
- Set $H_n / L_n = H_{world} / L_{world}$ for all n, keep $H_n + L_n = 1$
- Other parameters (incl. $t_n = 0$, T_n, τ_{in}) unchanged



Do H/L's play large role in shaping bilateral exports?

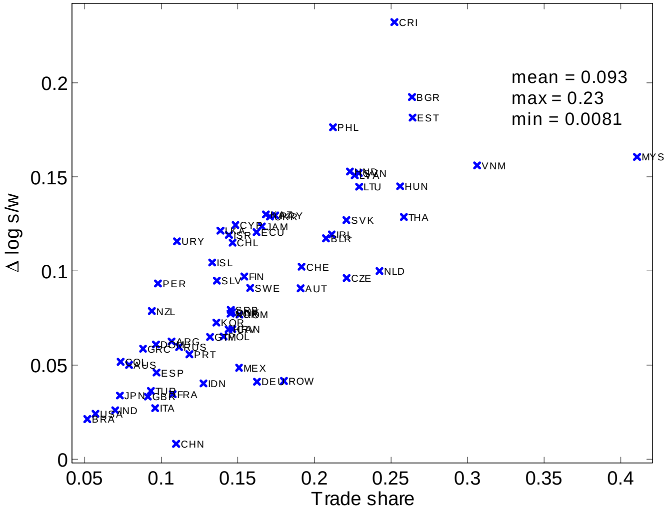
- Set $H_n / L_n = H_{world} / L_{world}$ for all n, keep $H_n + L_n = 1$
- Other parameters (incl. $t_n = 0$, $\theta = 0.1$, $\varphi = 0$, T_n, τ_{in}) unchanged

back to trade flows



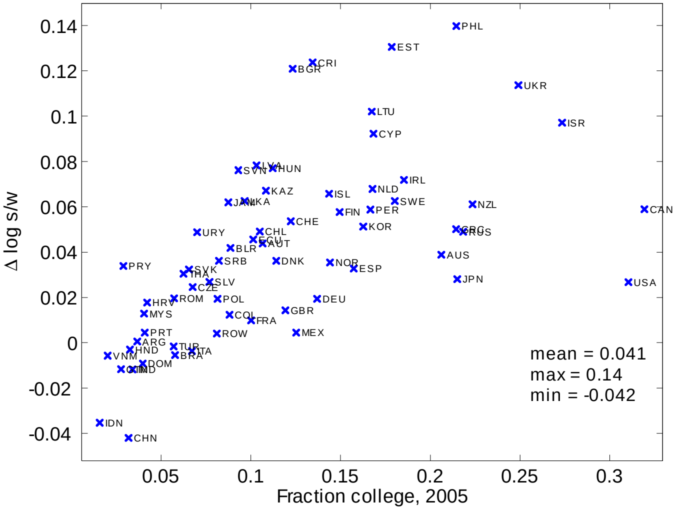
From autarky to baseline: strength of H-O

Skill bias, high prctivity dispersion, tn=0: correl change skill premium & H/L = 0.04



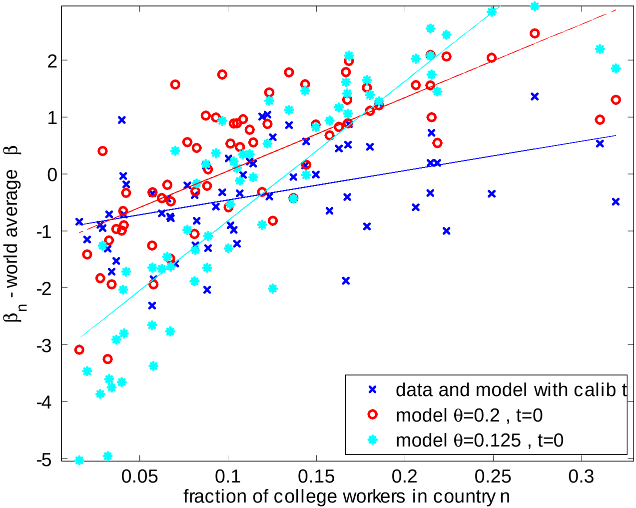
From autarky to baseline: strength of H-O

Skill bias, low prctivity dispersion, tn=0: correl change skill premium & H/L = 0.60



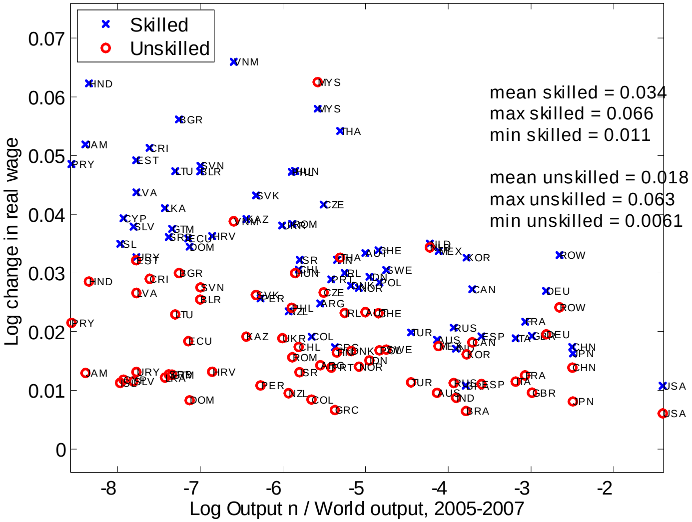
Target moment 4: Alternative parameterizations

Skill bias



10% fall in trade costs from baseline parameterization

Real wages: large difference between skilled & unskilled workers



Costs and prices

- Let $c_{ink}(\omega, j)$ denote $\tau_{in} \times$ the unit cost of production of the k'th most productive (ω, j) firm in country i

$$c_{ink}(\omega, j) = \frac{\tau_{in}}{A_i(j)} \left[\alpha_j z^{\frac{\varphi}{2}+\rho-1} s_i^{1-\rho} + (1-\alpha_j) z^{\rho-1-\frac{\varphi}{2}} w_i^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

where z is the productivity of this firm

- Denote 1^{st}- and 2^{nd}-lowest costs of supplying (ω, j) to n by

$$\begin{aligned} C_{1n}(\omega, j) &= \min_i \{c_{in1}(\omega, j)\} \\ C_{2n}(\omega, j) &= \min \left\{ c_{i^*n2}, \min_{i \neq i^*} \{c_{in1}(\omega, j)\} \right\} \end{aligned}$$

where i^* satisfies $C_{1n}(\omega, j) = c_{i^*n1}(\omega, j)$

- Price of (ω, j) in country n is

$$p_n(\omega, j) = \min \left\{ C_{2n}(\omega, j), \frac{\eta}{\eta - 1} C_{1n}(\omega, j) \right\}$$

back to Trade

The strength of the mechanisms

What determines strength of H-O mechanism?

- If $\varphi = 0$, then only H-O mechanism is active
- Assume marginal cost pricing; $i = 1, 2$; $j = x, y$; & $\sigma = \rho = 1$
  - Let $i = 1$ have comparative advantage in skill-intensive sector x

The strength of the mechanisms

What determines strength of H-O mechanism?

- If $\varphi = 0$, then only H-O mechanism is active
- Assume marginal cost pricing; $i = 1, 2$; $j = x, y$; & $\sigma = \rho = 1$
  - Let $i = 1$ have comparative advantage in skill-intensive sector x
- **Proposition**: Rise (fall) in s_1 / w_1 (s_2 / w_2) caused by moving from autarky to fixed trade share decreasing in θ & increasing in $A_1(x) A_2(y) / [A_1(y) A_2(x)]$
- **Intuition 1**: Higher $\theta \Rightarrow$ firm productivities more dispersed
  $\Rightarrow$ in relative firm costs, z more important vs. $A_i(j)$ and wages
  $\Rightarrow$ comparative advantage mitigated
  $\Rightarrow$ less btw sector reallocation $\Rightarrow$ smaller wage changes

The strength of the mechanisms

What determines strength of H-O mechanism?

- If $\varphi = 0$, then only H-O mechanism is active
- Assume marginal cost pricing; $i = 1, 2$; $j = x, y$; & $\sigma = \rho = 1$
  - Let $i = 1$ have comparative advantage in skill-intensive sector x
- **Proposition**: Rise (fall) in s_1 / w_1 (s_2 / w_2) caused by moving from autarky to fixed trade share decreasing in θ & increasing in $A_1(x) A_2(y) / [A_1(y) A_2(x)]$
- **Intuition 1**: Higher $\theta \Rightarrow$ firm productivities more dispersed
  $\Rightarrow$ in relative firm costs, z more important vs. $A_i(j)$ and wages
  $\Rightarrow$ comparative advantage mitigated
  $\Rightarrow$ less btw sector reallocation $\Rightarrow$ smaller wage changes
- **Intuition 2**: Higher $A_1(x) A_2(y) / [A_1(y) A_2(x)]$ strengthens 1's comparative advantage in x
  $\Rightarrow$ more btw sector reallocation $\Rightarrow$ bigger wage changes

The strength of the mechanisms

Skill-biased technology and trade

- If $\varphi > 0$ then skill-biased technology and trade interact

$$\frac{h}{l} = \left(\frac{w_i}{s_i}\right)^{\rho} \frac{\alpha_j}{1-\alpha_j} z^{\varphi}$$

- What shapes the strength of this mechanism?
  - $\frac{h(z')}{l(z')} \Big/ \frac{h(z)}{l(z)}$ is increasing in φ for all $z' > z$
  - avg difference btw expanding z' & contracting z increasing in θ

- Shown quantitatively: strength of mechanism $\uparrow$ in θ and φ

▸ Back to h/l

Inner loop: factor prices and profit shares

Inner loop k_I: given $\varphi, \theta, \rho, \tau, T_n, t_n$

- Initial guesses $\{w_n, s_n, \pi_n\}$ from inner loop $(k_I - 1)$
- Solve for
  - $P_n Q_n = \left(w_n L_n^d + s_n H_n^d\right)(1 + \pi_n)\left(1 - nx_n^d\right)$
  - $p_n(\omega, j), \mathbb{I}_{in}(\omega, j), P_n(j), P_n \Leftarrow$ price equations
  - $Q_n, q_n(\omega, j) \Leftarrow$ price and demand equations
  - $y_n(\omega, j), l_n(\omega, j), h_n(\omega, j) \Leftarrow$ production fcn, h/l, $q_n(\omega, j)$, $\mathbb{I}_{in}(\omega, j)$

Back to inner loop