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An Elementary Theory of Global Supply Chains

ARNAUD COSTINOT
MIT and NBER
JONATHAN VOGEL
Columbia University and NBER
and
SU WANG
MIT

This article develops an elementary theory of global supply chains. We consider a world economy with an arbitrary number of countries, one factor of production, a continuum of intermediate goods and one final good. Production of the final good is sequential and subject to mistakes. In the unique free trade equilibrium, countries with lower probabilities of making mistakes at all stages specialize in later stages of production. Using this simple theoretical framework, we offer a first look at how vertical specialization shapes the interdependence of nations.

Keywords: global supply chains, vertical specialization, interdependence of nations.

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> 'One man draws out the wire, another straights it, a third cuts it, a fourth points it, a fifth grinds it at the top for receiving the head; to make the head requires two or three distinct operations; to put it on, is a peculiar business, to whiten the pins is another; it is even a trade by itself to put them into the paper; and the important business of making a pin is, in this manner, divided into about eighteen distinct operations, which, in some manufactories, are all performed by distinct hands, though in others the same man will sometimes perform two or three of them.' Adam Smith (1776)

1. INTRODUCTION

Most production processes consist of a large number of sequential stages. In this regard the production of pins in late Eighteenth century England is no different from today's production of tee-shirts, cars, computers or semiconductors. Today, however, production processes increasingly involve global supply chains spanning multiple countries, with each country specializing in particular stages of a good's production sequence, a phenomenon which Hummels *et al.* (2001) refer to as vertical specialization.

This worldwide phenomenon has attracted a lot of attention among policy makers, business leaders and trade economists alike. On the academic side of this debate, a large literature has emerged to investigate how the possibility to fragment production processes across borders may affect the volume, pattern and consequences of international trade; see e.g. Feenstra and Hanson (1996), Yi (2003) and Grossman and Rossi-Hansberg (2008). In this article, we propose to take a first look at a distinct, but equally important question: Conditional on production processes being fragmented across borders, how does technological change, either global or local, affect different countries participating in the same supply chain? In other words, how does vertical specialization shape the interdependence of nations?

From a theoretical standpoint, this is not an easy question. General equilibrium models with an arbitrary number of goods and countries—with or without sequential production—rarely provide sharp and intuitive comparative static predictions.[1] To make progress, we therefore start by proposing a simple theory of trade with sequential production. In Section 2 we consider a world economy with multiple countries, one factor of production (labor) and one final good. Production is sequential and subject to mistakes, as in Sobel (1992) and Kremer (1993). Production of the final good requires a continuum of intermediate stages. At each of these stages, production of one unit of an intermediate good requires one unit of labor and one unit of the intermediate good produced in the previous stage. Mistakes occur along the supply chain at a constant Poisson rate, which is an exogenous technological characteristic of a country. When a mistake occurs at some stage, the intermediate good is entirely lost. By these stark assumptions, we aim to capture the more general idea that because of less skilled workers, worse infrastructure or inferior contractual enforcement, both costly defects and delays in production are more likely in some countries than in others.

Section 3 describes the properties of the free trade equilibrium in our basic environment. Although our model allows for any finite number of countries and a continuum of stages, the unique free trade equilibrium is fully characterized by a simple system of first-order non-linear difference equations. This system can be solved recursively by first determining the assignment of countries to different stages of production and then computing the wages and export prices sustaining that allocation as an equilibrium outcome. In our model, the free trade equilibrium always exhibits vertical specialization: countries with a lower probability of making mistakes, at *all* stages, specialize in later stages of production, where mistakes are more costly. Because of the sequential nature of production, absolute productivity differences are a source of comparative advantage among nations.

Using this simple model, the rest of our article offers a comprehensive exploration of how technological change, either global or local, affects different countries participating in the same global supply chain. Section 4 analyzes the consequences of global technological change. We investigate how an increase in the length of production processes, which we refer to as an increase in "complexity", and a uniform decrease in failure rates worldwide, which we refer to as "standardization", may affect the pattern of vertical specialization and the world income distribution. Building solely on the idea that labor markets must clear both before and after a given technological change, we demonstrate that although both an increase in complexity and standardization lead all countries to "move up" the supply chain, they have opposite effects on inequality between nations. While an increase in complexity increases inequality around the world, standardization benefits poor countries, i.e. countries with higher failure rates, disproportionately more. According to our model, standardization may even lead to a welfare loss in the most technologically advanced country, a form of immiserizing growth.

Section 5 focuses on how local technological change may spill over, through terms-of-trade effects, to other countries participating in the same supply chain. We consider two forms of local technological change: (i) labor-augmenting technical progress; and (ii) a decrease in a country's failure rate, which we refer to as "routinization." In a world with sequential production, we show

1. Ethier (1984) offers a review of theoretical results in high-dimensional trade models.

that local technological changes tend to spillover very differently at the bottom and the top of the chain. At the bottom, depending on the nature of technological changes, all countries either move up or down, but whatever they do, movements along the chain fully determine changes in inequality between nations. At the top of the chain, in contrast, local technological progress always leads all countries to move up, but even conditioning on the nature of technological change, inequality between nations may either fall or rise. Perhaps surprisingly, while richer countries at the bottom of the chain benefit disproportionately more from being pushed into later stages of production, this is not always true at the top.

Section 6 demonstrates how more realistic features of global supply chains may easily be incorporated into our simple theoretical framework. Our first extension introduces trading frictions, which we refer to as "coordination costs". Among other things, we demonstrate that a decrease in coordination costs may lead to "overshooting": more stages of production may be offshored to a small country at intermediate levels of coordination costs than under perfectly free trade. Our second extension allows for the existence of multiple parts, each produced sequentially and then assembled, with equal productivity in each country, into a unique final good using labor. In this environment, the poorest countries tend to specialize in assembly, whereas the richest countries tend to specialize in the later stages of the most complex parts. Our third extension allows for imperfect observability of mistakes. In this situation, we show how differences in failure rates and "quality control" across countries jointly determine the pattern of vertical specialization. We conclude by providing sufficient conditions such that our cross-sectional predictions remain unchanged for more general production functions.

Our article is related to several strands of the literature. First, we draw some ideas from the literature on hierarchies in closed-economy (and mostly partial-equilibrium) models. Important contributions include Lucas (1978), Rosen (1982), Sobel (1992), Kremer (1993), Garicano (2000) and Garicano and Rossi-Hansberg (2006). As in Sobel (1992) and Kremer (1993), we focus on an environment in which production is sequential and subject to mistakes, though we do so in a general equilibrium, open-economy setup. Models of hierarchies have been applied to the study of international trade issues before, but with very different goals in mind. For instance, Antràs *et al*. (2006) use the knowledge economy model developed by Garicano (2000) to study the matching of agents with heterogeneous abilities across borders and its consequences for within-country inequality. Instead, countries are populated by homogeneous workers in our model.[2]

In terms of techniques, our article is also related to a growing literature using assignment or matching models in an international context; see, for example, Grossman and Maggi (2000), Grossman (2004), Yeaple (2005), Ohnsorge and Trefler (2007), Nocke and Yeaple (2008), Costinot (2009), Blanchard and Willmann (2010) and Costinot and Vogel (2010). Here, like in some of our earlier work, we exploit the fact that the assignment of countries to stages of production exhibits positive assortative matching, i.e. more productive countries are assigned to later stages of production, to generate strong and intuitive comparative static predictions in an environment with a large number of goods and countries.

In terms of focus, our article is motivated by the recent literature documenting the importance of vertical specialization in world trade. On the empirical side, this literature builds on the influential work of Hummels *et al*. (1998), Hummels *et al*. (2001), and Hanson *et al*.(2005). Our focus on how vertical specialization shapes the interdependence of nations is also related to the work of Kose and Yi (2001, 2006), Burstein *et al*. (2008), and Bergin *et al*. (2009) who study how production sharing affects the transmission of shocks at business cycle frequency.

2. Other examples of trade articles using hierarchy models to study within-country inequality include Kremer and Maskin (2006), Sly (2010), Monte (2010) and Sampson (2010).

On the theoretical side, the literature on fragmentation is large and diverse; see Antràs and Rossi-Hansberg (2009) for a recent overview. Our theoretical framework is most closely related to Dixit and Grossman (1982), Sanyal (1983), Yi (2003, 2010), Harms *et al*. (2009), Baldwin and Venables (2010) and Antràs and Chor (2011) who also develop trade models with sequential production. None of these articles, however, investigate how technological change, either global or local, may differentially impact countries located at different stages of the same supply chain. This is the main focus of our analysis.

2. BASIC ENVIRONMENT

We consider a world economy with multiple countries, indexed by $c \in \mathcal{C} \equiv \{1,...,C\}$, one factor of production, labor, and one final good. Labor is inelastically supplied and immobile across countries. L_c and w_c denote the endowment of labor and wage in country c, respectively. Production of the final good is sequential and subject to mistakes. To produce the final good, a continuum of stages $s \in \mathcal{S} \equiv (0,S]$ must be performed. At each stage, producing one unit of intermediate good requires one unit of the intermediate good produced in the previous stage and one unit of labor.

Mistakes occur along the supply chain at a constant Poisson rate, $\lambda_c > 0$, which is an exogenous technological characteristic of a country. Countries are ordered so that λ_c is strictly decreasing in c. When a mistake occurs on a unit of intermediate good at some stage, that intermediate good is entirely lost. Formally, consider two consecutive stages, s and $s+ds$, with ds infinitesimal. If a firm from country c combines $q(s)$ units of intermediate good s with $q(s)ds$ units of labor, its output of intermediate good $s+ds$ is given by

$$q(s+ds) = (1-\lambda_c ds)\, q(s). \tag{1}$$

Note that letting $q'(s) \equiv [q(s+ds)-q(s)]/ds$, Equation (1) can be written as $q'(s)/q(s) = -\lambda_c$. In other words, moving along the supply chain in country c, potential units of the final good get destroyed at a constant rate, λ_c. In the rest of our analysis, we often refer to λ_c as a measure of total factor productivity.[3] Since λ_c is strictly decreasing in c, countries with a higher index c are more productive.

All markets are perfectly competitive and all goods are freely traded. $p(s)$ denotes the world price of intermediate good s. For expositional purposes, we assume that "intermediate good 0" is in infinite supply and has zero price, $p(0)=0$. "Intermediate good S" corresponds to the unique final good mentioned before, which we use as our numeraire, $p(S)=1$. For technical reasons, we further assume that if a firm produces intermediate good $s+ds$, then it necessarily produces a measure $\Delta > 0$ of intermediate goods around that stage. Formally, for any intermediate good $s+ds$, we assume the existence of $s_\Delta < s+ds \leq s_\Delta + \Delta$ such that if $q(s+ds) > 0$, then $q\left(s'\right) > 0$ for all $s' \in (s_\Delta, s_\Delta + \Delta]$. This implies that each unit of the final good is produced by a finite, though possibly arbitrarily large number of firms.

3. Although labor is the only primary factor of production, λ_c is not a measure of labor productivity. Instead it measures how much output at each stage can be produced by one unit of labor and one unit of intermediate good from the previous stage. In an environment with a discrete number of stages, the production function corresponding to Equation (1) would be simply given by a Leontief production function $q(s+1) = e^{-\lambda_c} \min\{q(s), l(s+1)\}$, where $q(s)$ and $l(s+1)$ are the inputs used in stage $s+1$. We come back to this issue in Section 6.4.

3. FREE TRADE EQUILIBRIUM

3.1. *Definition*

In a free trade equilibrium, all firms maximize their profits taking world prices as given and all markets clear. Profit maximization requires that for all $c \in \mathcal{C}$,

$$\begin{aligned} &p(s+ds) \leq (1+\lambda_c ds)p(s)+w_c ds, \\ &p(s+ds) = (1+\lambda_c ds)p(s)+w_c ds, \text{ if } Q_c\left(s'\right) > 0 \text{ for all } s' \in (s, s+ds], \end{aligned} \tag{2}$$

where $Q_c\left(s'\right)$ denotes total output at stage s' in country c. Condition (2) states that the price of intermediate good $s+ds$ must be weakly less than its unit cost of production, with equality if intermediate good $s+ds$ is actually produced by a firm from country c. To see this, note that the production of one unit of intermediate good $s+ds$ requires $1/(1-\lambda_c ds)$ units of intermediate good s as well as labor for all intermediate stages in $(s, s+ds]$. Thus the unit cost of production of intermediate good $s+ds$ is given by $[p(s)+w_c ds]/(1-\lambda_c ds)$. Since ds is infinitesimal, this is equal to $(1+\lambda_c ds)p(s)+w_c ds$.

Good and labor market clearing further require that

$$\sum_{c=1}^{C} Q_c(s_2) - \sum_{c=1}^{C} Q_c(s_1) = -\int_{s_1}^{s_2} \sum_{c=1}^{C} \lambda_c Q_c(s)\, ds, \text{ for all } s_1 \leq s_2, \tag{3}$$

$$\int_0^S Q_c(s)\, ds = L_c, \text{ for all } c \in \mathcal{C}. \tag{4}$$

Equation (3) states that the change in the world supply of intermediate goods between stages s_1 and s_2 must be equal to the amount of intermediate goods lost due to mistakes in all countries between these two stages. Equation (4) states that the total amount of labor used across all stages must be equal to the total supply of labor in country c. In the rest of this article, we formally define a free trade equilibrium as follows.

Definition 1. *A free trade equilibrium corresponds to output levels $Q_c(\cdot): \mathcal{S} \longrightarrow \mathbb{R}^+$ for all $c \in \mathcal{C}$, wages $w_c \in \mathbb{R}^+$ for all $c \in \mathcal{C}$, and intermediate good prices $p(\cdot): \mathcal{S} \longrightarrow \mathbb{R}^+$ such that Conditions* (2)–(4) *hold.*

3.2. *Existence and uniqueness*

We first characterize the pattern of international specialization in any free trade equilibrium.

Proposition 1. *In any free trade equilibrium, there exists a sequence of stages $S_0 \equiv 0 < S_1 < \ldots < S_C = S$ such that for all $s \in \mathcal{S}$ and $c \in \mathcal{C}$, $Q_c(s) > 0$ if and only if $s \in (S_{c-1}, S_c]$.*

According to Proposition 1, there is vertical specialization in any free trade equilibrium with more productive countries producing and exporting at later stages of production. The formal proof as well as all subsequent proofs can be found in the Appendices.[4] The intuition behind Proposition 1 can be understood in two ways. One possibility is to look at Proposition 1 through the lens of the hierarchy literature; see e.g. Lucas (1978), Rosen (1982), and Garicano (2000).

4. A result similar to Lemma 1 in an environment with a discrete number of stages can also be found in Sobel (1992) and Kremer (1993). In Section 6 we extend it to more general production functions.

Since countries that are producing at later stages can leverage their productivity on larger amounts of inputs, efficiency requires countries to be more productive at the top. Another possibility is to note that since new intermediate goods require both intermediate goods produced in previous stages and labor, prices must be increasing along the supply chain. Thus the labor cost share is relatively lower in the production of intermediate goods produced at later stages, which makes them relatively cheaper to produce in countries with higher wages. In our model these are the countries with higher productivity in all stages. Because of the sequential nature of production, absolute productivity differences—here in the form of uniformly lower failure rates at *all* stages of production—are a source of comparative advantage among nations.[5]

We refer to the vector $(S_1,...,S_C)$ as the *"pattern of vertical specialization"* and denote by $Q_c \equiv Q_c(S_c)$ the total amount of intermediate good S_c produced and exported by country c. Using the previous notation, the pattern of vertical specialization and export levels can be jointly characterized as follows.

Lemma 1. *In any free trade equilibrium, the pattern of vertical specialization and export levels satisfy the following system of first-order non-linear difference equations:*

$$S_c = S_{c-1} - \left(\frac{1}{\lambda_c}\right) \ln\left(1 - \frac{\lambda_c L_c}{Q_{c-1}}\right), \text{ for all } c \in \mathcal{C}, \tag{5}$$

$$Q_c = e^{-\lambda_c (S_c - S_{c-1})} Q_{c-1}, \text{ for all } c \in \mathcal{C}, \tag{6}$$

with boundary conditions $S_0 = 0$ *and* $S_C = S$.

Lemma 1 derives from the goods and labor market clearing Conditions (3) and (4). Equation (5) reflects the fact that the exogenous supply of labor in country c must equal the amount of labor demanded to perform all stages from S_{c-1} to S_c. This amount of labor depends both on the rate of mistakes λ_c as well as the total amount Q_{c-1} of intermediate good S_{c-1} imported from country $c-1$. Equation (6) reflects the fact that intermediate goods get lost at a constant rate at each stage when produced in country c.

In the rest of this article, we refer to the vector of wages $(w_1,...,w_C)$ as the "*world income distribution*" and to $p_c \equiv p(S_c)$ as the price of country c's exports (which is also the price of country $c+1$'s imports under free trade). Let $N_c \equiv S_c - S_{c-1}$ denote the measure of stages performed by country c within the supply chain. In the next lemma, we show how the measures of stages being performed in all countries $(N_1,...,N_C)$ shape the world income distribution.

5. In his early work on fragmentation, Jones (1980) pointed out that if some factors of production are internationally mobile, then absolute advantage may affect the pattern of international specialization. The basic idea is that if physical capital is perfectly mobile and one country has an absolute advantage in producing capital services, then it will specialize in capital intensive goods. The logic of our results is very different and intimately related to the sequential nature of production. Mathematically, a simple way to understand why sequential production processes make absolute productivity differences a source of comparative advantage is to consider the cumulative amount of labor necessary to produce all stages from 0 to $s \le S$ in country c for a potential unit of the final good. By Equation (1), this is equal to $e^{\lambda_c s}$ which is log-supermodular in (λ_c, s). This is the exact same form of complementarity that determines the pattern of international specialization in standard Ricardian models; see Costinot (2009).

Lemma 2. *In any free trade equilibrium, the world income distribution and export prices satisfy the following system of first-order linear difference equations:*

$$w_{c+1} = w_c + \left(\lambda_c - \lambda_{c+1}\right) p_c, \text{ for all } c < C, \tag{7}$$

$$p_c = e^{\lambda_c N_c} p_{c-1} + \left(e^{\lambda_c N_c} - 1\right)(w_c/\lambda_c), \text{ for all } c \in \mathcal{C}, \tag{8}$$

with boundary conditions $p_0 = 0$ *and* $p_C = 1$.

Lemma 2 derives from the zero-profit Condition (2). Equation (7) reflects the fact that for the "cutoff" good, S_c, the unit cost of production in country c, $(1+\lambda_c ds)p_c + w_c ds$, must be equal to the unit cost of production in country $c+1$, $\left(1+\lambda_{c+1} ds\right)p_c + w_{c+1} ds$. Equation (8) directly derives from the zero-profit Condition (2) and the definitions of N_c and p_c. It illustrates the fact that the price of the last intermediate good produced by country c depends on the price of the intermediate good imported from country $c-1$ as well as the total labor cost in country c.

Combining Proposition 1 with Lemmas 1 and 2, we can establish the existence of a unique free trade equilibrium and characterize its main properties.

Proposition 2. *There exists a unique free trade equilibrium. In this equilibrium, the pattern of vertical specialization and export levels are given by Equations* (5) *and* (6)*, and the world income distribution and export prices are given by Equations* (7) *and* (8).

The proof of Proposition 2 formally proceeds in two steps. First, we use Lemma 1 to construct the unique pattern of vertical specialization and vector of export levels. In Equations (5) and (6), we have one degree of freedom, Q_0, which corresponds to total input used at the initial stage of production. Since S_C is decreasing in Q_0, it can be set to satisfy the final boundary condition $S_C = S$. Once $(S_1, ..., S_C)$ and $\left(Q_0, ..., Q_{C-1}\right)$ have been determined, all other output levels can be computed using Equation (1) and Proposition 1. Second, we use Lemma 2 together with the equilibrium measure of stages computed before, $(N_1, ..., N_C)$, to characterize the unique world income distribution and vector of export prices. In Equations (7) and (8), we still have one degree of freedom, w_1. Given the monotonicity of p_C in w_1, it can be used to satisfy the other final boundary condition, $p_C = 1$. Finally, once $(w_1, ..., w_C)$ and $(p_1, ..., p_C)$ have been determined, all other prices can be computed using the zero-profit Condition (2) and Proposition 1.

3.3. *Discussion*

As a first step toward analyzing how vertical specialization shapes the interdependence of nations, we have provided a full characterization of the free trade equilibrium in a simple trade model with sequential production. Before turning to our comparative static exercises, we briefly discuss the cross-sectional implications that have emerged from this characterization.

First, since rich countries specialize in later stages of production whereas poor countries specialize in earlier stages, our model implies that rich countries tend to trade relatively more with other rich countries (from whom they import their intermediates and to whom they export their output) whereas poor countries tend to trade relatively more with other poor countries, as documented by Hallak (2010). Second, since intermediate goods produced in later stages have higher prices and countries producing in these stages have higher wages, our model implies that rich countries both tend to import goods with higher unit values, as documented by Hallak (2006), and to export goods with higher unit values, as documented by Schott (2004), Hummels and Klenow (2005) and Hallak and Schott (2010).

Following Linder (1961), the two previous stylized facts have traditionally been rationalized using non-homothetic preferences; see e.g. Markusen (1986), Flam and Helpman (1987), Bergstrand (1990), Stokey (1991), Murphy and Shleifer (1997), Matsuyama (2000), Fieler (2010) and Fajgelbaum *et al*. (2009). The common starting point of the previous articles is that rich countries' preferences are skewed toward high-quality goods, so they tend to import goods with higher unit values. Under the assumption that rich countries are also relatively better at producing high-quality goods, these models can further explain why rich countries tend to export goods with higher unit values and why countries with similar levels of GDP per capita tend to trade more with each other.[6]

The complementary explanation offered by our elementary theory of global supply chains is based purely on supply considerations. According to our model, countries with similar per-capita incomes are more likely to trade with one another because they specialize in nearby regions of the same supply chain. Similarly, countries with higher levels of GDP per capita tend to have higher unit values of imports and exports because they specialize in higher stages in the supply chain, for which inputs and outputs are more costly. Note that our supply-side explanation also suggests new testable implications. Since our model only applies to sectors characterized by sequential production and vertical specialization, if our theoretical explanation is empirically relevant, one would therefore expect "Linder effects" —i.e. the extent of trade between countries with similar levels of GDP per capita—to be higher, all else equal, in sectors in which production processes are vertically fragmented across borders in practice.

The previous cross-sectional predictions, of course, should be interpreted with caution. Our theory is admittedly stylized. In Section 6, we will discuss how the previous results may be affected (or not) by the introduction of more realistic features of global supply chains.

4. GLOBAL TECHNOLOGICAL CHANGE

Many technological innovations, from the discovery of electricity to the internet, have impacted production processes worldwide. Our first series of comparative static exercises focuses on the impact of global technological changes on different countries participating in the same supply chain. Our goal is to investigate how an increase in the length of production processes, perhaps associated with the development of higher quality goods, as well as a uniform decrease in failure rates worldwide, perhaps due to the standardization of production processes, may affect the pattern of vertical specialization and the world income distribution.

4.1. *Definitions*

It is useful to introduce first some formal definitions describing the changes in the pattern of vertical specialization and the world income distribution in which we will be interested.

Definition 2. *Let $\left(S'_1,...,S'_C\right)$ denote the pattern of vertical specialization in a counterfactual free trade equilibrium. A country $c \in \mathcal{C}$ is moving up (resp. down) the supply chain relative to the initial free trade equilibrium if $S'_c \geq S_c$ and $S'_{c-1} \geq S_{c-1}$ (resp. $S'_c \leq S_c$ and $S'_{c-1} \leq S_{c-1}$).*

According to Definition 2, a country is moving up or down the supply chain if we can rank the set of stages that it performs in the initial and counterfactual free trade equilibria in terms of the

6. In Fajgelbaum *et al*. (2009), such predictions are obtained in the absence of any exogenous relative productivity differences. In their model, a higher relative demand for high-quality goods translates into a higher relative supply of these goods through a "home-market" effect.

strong set order. Among other things, this simple mathematical notion will allow us to formalize a major concern of policy makers and business leaders in developed countries, namely the fact that China and other developing countries are "moving up the value chain"; see e.g. OECD (2007).

Definition 3. *Let $(w'_1,...,w'_C)$ denote the world income distribution in a counterfactual free trade equilibrium. Inequality is increasing (resp. decreasing) among a given group $\{c_1,...c_n\}$ of adjacent countries if $w'_{c+1}/w'_c \geq w_{c+1}/w_c$ (resp. $w'_{c+1}/w'_c \leq w_{c+1}/w_c$) for all $c_1 \leq c \leq c_{n-1}$.*

According to Definition 3, inequality is increasing (resp. decreasing) within a given group of adjacent countries, if for any pair of countries within that group, the relative wage of the richer country is increasing (resp. decreasing). This property offers a simple way to conceptualize changes in the world income distribution in our model.[7]

4.2. *Increase in complexity*

At the end of the Eighteenth century, Adam Smith famously noted that making a pin was divided into about 18 distinct operations. Today, as mentioned by Levine (2010), making a Boeing 747 requires more than 6,000,000 parts, each of them requiring many more operations. In this section we analyze the consequences of an increase in the measure of stages S necessary to produce a final good, which we simply refer to as an "increase in complexity".[8]

Our approach, like in subsequent sections, proceeds in two steps. We characterize first the changes in the pattern of vertical specialization and second the associated changes in the world income distribution. Our first comparative static results can be stated as follows.

Proposition 3. *An increase in complexity leads all countries to move up the supply chain and increases inequality between countries around the world.*

The changes in the pattern of vertical specialization and the world income distribution associated with an increase in complexity are illustrated in Figure 1. The broad intuition behind changes in the pattern of vertical specialization is simple. An increase in complexity tends to decrease total output at all stages of production. Since labor supply must remain equal to labor demand, this decrease in output levels must be accompanied by an increase in the measure N_c of stages performed in all countries. Proceeding by iteration from the bottom of the supply chain, we can then show that this change in N_c can only occur if all countries move up.

The logic behind the changes in the world income distribution is more subtle. From Equation (7) in Lemma 2, we know that relative wages of countries $c+1$ and c must equate the unit cost of production of the cutoff good S_c,

$$\frac{w_{c+1}}{w_c} = 1 + \frac{\lambda_c - \lambda_{c+1}}{(w_c/p_c)}, \text{ for all } c < C. \tag{9}$$

7. For expositional purposes, we have chosen to state all definitions and propositions in Sections 4 and 5 using weak inequalities. Most of our comparative static results also hold as strict inequalities; see Appendices.

8. For simplicity, we abstract from any utility gains that may be associated with the production of more complex goods in practice. Our analytical results on the pattern of vertical specialization and inequality between nations do not depend on this simplification. But it should be clear that changes in real wages depend on it. Here an increase in complexity necessarily lowers total output, and in turn, real wages. If the utility level associated with the consumption of one unit of the final good were allowed to increase with the complexity of the production process, then an increase in complexity may very well raise real wages.

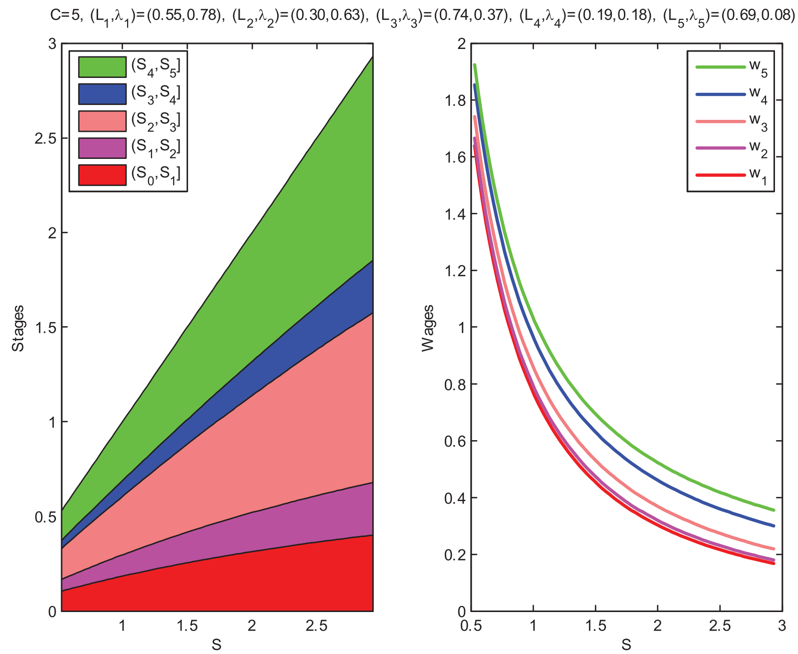

FIGURE 1

Consequences of an increase in complexity

Thus, w_{c+1}/w_c is decreasing in w_c/p_c, which we refer to as the labor cost share of country c's exports.[9] This is reminiscent of the mechanism underlying terms-of-trade effects in a Ricardian model; see e.g. Dornbusch *et al*. (1977) and Krugman (1986). From an economic standpoint, Equation (9) captures the basic idea that the wage of country $c+1$ should increase relative to the wage of country c if and only if $c+1$ moves into sectors in which it has a comparative advantage. In our model, since country $c+1$ has a higher wage, these are the sectors with lower labor cost shares. In a standard Ricardian model, these would be the sectors in which country $c+1$ is relatively more productive instead.

There is, however, one important difference between a standard Ricardian model and our model with sequential production. In a standard Ricardian model, the pattern of comparative advantage only depends on exogenous productivity differences. In our model, the same pattern depends on endogenous differences in labor cost shares across stages. According to Equation (8) in Lemma 2, we have

$$\frac{w_c}{p_c}=\left[\left(e^{\lambda_c N_c}\right)\left(\frac{p_{c-1}}{w_c}\right)+\left(\frac{e^{\lambda_c N_c}-1}{\lambda_c}\right)\right]^{-1}. \tag{10}$$

Hence the labor cost share of country c's exports depends on: (i) the price of country c's imports, p_{c-1}; (ii) the volume of imports necessary to produce one unit of export, $e^{\lambda_c N_c}$; and (iii) the associated amount of labor necessary to transform imports into exports, $\left(e^{\lambda_c N_c}-1\right)/\lambda_c$. Having characterized how an increase in complexity affects the pattern of vertical specialization, it is

9. We slightly abuse terminology. Strictly speaking, the share of wages in the unit cost of production incurred at stage S_c, which is the stage at which country c exports, is equal to $(w_c/p_s)ds$.

fairly easy to evaluate how it affects each of these three components. From the first part of Proposition 3, we know that countries are both moving up into higher stages and performing more stages. Moving up into higher stages tends to increase import prices and hence, p_{c-1}/w_c. Performing more stages also increases the amount of labor necessary to transform imports into exports, $\left(e^{\lambda_c N_c}-1\right)/\lambda_c$. In addition, it raises the volume of imports necessary to produce one unit of export, $e^{\lambda_c N_c}$. In this situation, all three effects tend to lower the labor cost share of intermediate goods that are being traded, which explains why inequality between nations increases.

4.3. *Standardization*

In most industries, production processes become more standardized as goods mature over time. To study the potential implications of this particular type of technological change within our theoretical framework, we now consider a uniform decrease in failure rates from λ_c to $\lambda'_c \equiv \beta\lambda_c$ for all $c \in \mathcal{C}$, with $\beta < 1$, which we simply refer to as "standardization". The consequences of standardization on the pattern of vertical specialization and the world income distribution can be described as follows.

Proposition 4. *Standardization leads all countries to move up the supply chain and decreases inequality between countries around the world.*

The consequences of standardization are illustrated in Figure 2. For a given pattern of vertical specialization, standardization tends to raise total output—and, therefore, the demand for labor—at all stages of production. Since labor supply must remain equal to labor demand, this increase in output levels must be partially offset by a reduction of output at earlier stages of production. Hence, poor countries must increase the measure of stages that they perform, pushing all countries up the supply chain.

Like in our first comparative static exercise, the logic behind the changes in the world income distribution is more subtle. The direct effect of standardization on relative wages is to decrease inequality. By construction, for any pair countries c_2 and c_1 such that $c_2 > c_1$, we have $\lambda'_{c_1} - \lambda'_{c_2} = \beta\left(\lambda_{c_1} - \lambda_{c_2}\right) > 0$. Thus the productivity gap between poor and rich countries is lower for any $\beta \in (0,1)$. In the extreme case in which $\beta = 0$, having a lower rate of mistakes λ_c does not provide any benefit. There is, however, an indirect, general equilibrium effect associated with changes in the pattern of vertical specialization. To establish that the direct effect necessarily dominates the indirect one, the basic idea behind our proof is to normalize the measures of stages performed and export prices by β. Under this normalization, standardization is equivalent to a reduction in complexity: because both tend to reduce output lost to mistakes, they both require countries to move down the (normalized) supply chain, which leads to a fall in inequality between countries.[10] It is interesting to note that while standardization and an increase in complexity both cause all countries to move up the supply chain, they have opposite effects on inequality between nations.

The previous comparative static results are reminiscent of Vernon's (1966) "product cycle hypothesis"; see also Grossman and Helpman (1991) and Antràs (2005). In our model, as a particular production process becomes more standardized, less productive countries start performing a broader set of stages. As this happens, our analysis demonstrates that inequality

10. Formally, whereas N_c rises for poor countries and falls for rich countries, βN_c falls for all countries. Hence, whereas all countries move up the chain, they move down the normalized chain. Under this normalization, countries: (i) are performing fewer stages, and (ii) are moving down into lower stages. Both effects tend to lower the normalized price βp_c of intermediate goods that are being traded, and in turn, to increase their labor cost share. This explains why inequality between nations decreases.

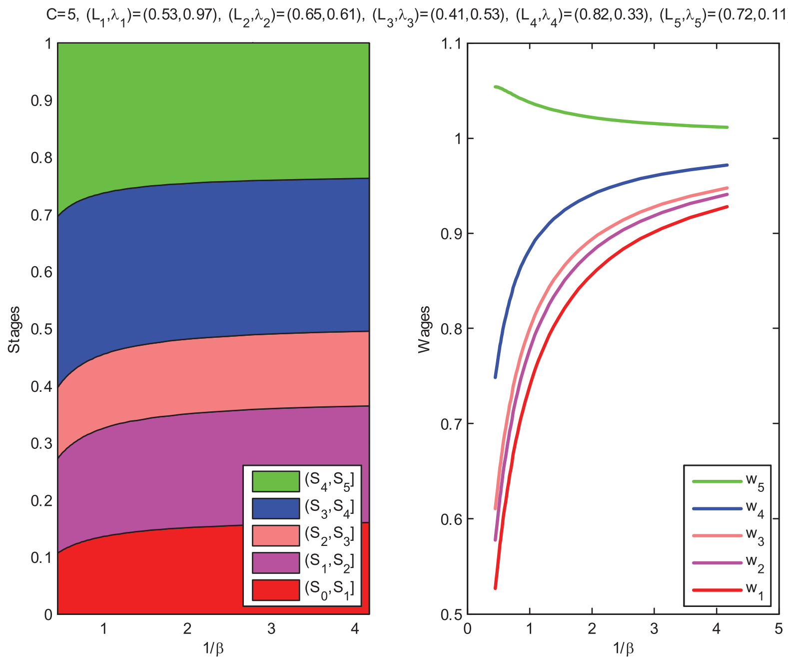

FIGURE 2

Consequences of standardization.

between nations decreases around the world. Figure 2 also illustrates that although the direct effect of standardization is to increase output in all countries, welfare may fall in the most technologically advanced countries through a terms-of-trade deterioration. This is reminiscent of Bhagwati's (1958) "immiserizing growth". Two key differences, however, need to be highlighted. First, standardization increases productivity in all countries in the global supply chain, whereas Bhagwati's (1958) immiserizing growth occurs in response to an outward shift in the production possibility frontier in a single country. Second, and more importantly, standardization proportionately increases productivity at all stages of production, whereas Bhagwati's (1958) immiserizing growth occurs in response to an outward shift in the export sector. In our model, it is the sequential nature of production that makes uniform productivity growth endogenously act as export-biased technological change in the more technologically advanced countries.[11]

5. LOCAL TECHNOLOGICAL CHANGE

Two of the major changes in today's world economy are: (i) the increased fragmentation of the production process, which Baldwin (2006) refers to as the "Great Unbundling"; and (ii) the rise of China and other developing countries, such as India and Brazil. Although both phenomena have been studied separately, we know very little about their interaction, either theoretically or empirically. The goal of this section is to use our elementary theory of global supply chains to

11. Like in Bhagwati's original article, however, it should be clear that immiserizing growth arises in this environment because of strong complementarities between goods. In our model, producing one unit of intermediate good always requires one unit of the intermediate good produced in the previous stage and one unit of labor. This explains why technological changes may have large (and adverse) terms-of-trade effects.

take a first stab at this issue. To do so, our second series of comparative static exercises focuses on the impact of labor-augmenting technical progress and routinization in one country and describes how they spill over to other countries in the same supply chain through terms-of-trade effects.

5.1. *Labor-augmenting technical progress*

We first study the impact of labor-augmenting technical progress, which increases the total efficiency units of labor in a given country c_0 from L_{c_0} to $L'_{c_0} > L_{c_0}$. Following the same two-step logic as in Section 4, the consequences of labor-augmenting technical progress can be described as follows.[12]

Proposition 5. *Labor-augmenting technical progress in country c_0 leads all countries $c < c_0$ to move down the supply chain and all countries $c > c_0$ to move up. This decreases inequality among countries $c \in \{1,...,c_0\}$, increases inequality among countries $c \in \{c_0,...,c_1\}$, and decreases inequality among countries $c \in \{c_1,...,C\}$, with $c_1 \in \{c_0+1,...,C\}$.*

The spillover effects associated with labor-augmenting technical progress are illustrated in Figure 3. The broad intuition behind the changing patterns of specialization is simple. An increase in the supply of labor (in efficiency units) in one country tends to raise total output at all stages of production. Since labor supply must remain equal to labor demand, this increase in output levels must be accompanied by a decrease in the measure of stages N_c performed in each country $c \neq c_0$. Proceeding by iteration from the bottom and the top of the supply chain, we can then show that this change in N_c can only occur if all countries below c_0 move down and all countries above c_0 move up. Finally, since the total measure of stages remains constant, the measure of stages N_{c_0} performed in country c_0 must increase.

Changes in the pattern of vertical specialization naturally translate into changes in the world income distribution. Countries at the bottom of the chain are moving down into lower stages and performing fewer stages. Both changes tend to increase the labor cost share of intermediate goods that are being traded and decrease inequality between nations at the bottom of the chain. The non-monotonic effects on inequality at the top of the chain reflect two conflicting forces. On the one hand, countries at the top of the chain are moving up. This tends to increase their import prices, reduce the labor cost share of their exports, and in turn, increase inequality. On the other hand, countries at the top of the chain are performing fewer stages. This decreases both the volume of imports and amount of labor necessary to produce one unit of their exports, which tends to raise the labor cost share of their exports, and in turn, decrease inequality.[13]

The previous non-monotonic effects stand in sharp contrast to the predictions of standard Ricardian models and illustrate nicely the importance of modeling the sequential nature of production for understanding the consequences of technological changes in developing and developed countries on their trading partners worldwide. To see this, consider a Ricardian model without sequential production in which there is a ladder of countries with poor countries at the bottom and rich countries at the top. Krugman (1986) is a well-known two-country example. In

12. In line with our previous comparative static exercises, when discussing changes in the world income distribution, we focus on changes in wages per efficiency units. For all countries $c \neq c_0$, this is equivalent to changes in wages per worker. For country c_0, however, one should keep in mind that the two types of changes are distinct. If M_{c_0} denotes population size in country c_0, then the wage per worker is $w_{c_0}L_{c_0}/M_{c_0}$. Thus a decrease in w_{c_0} does not necessarily imply a decrease in wages per worker in that country.

13. Note that since $c_1 \in \{c_0+1,...,C\}$, the third group of countries, $\{c_1,...,C-1\}$, is non-empty if $c_1 < C$, but empty if $c_1 = C$. We have encountered both cases in our simulations.

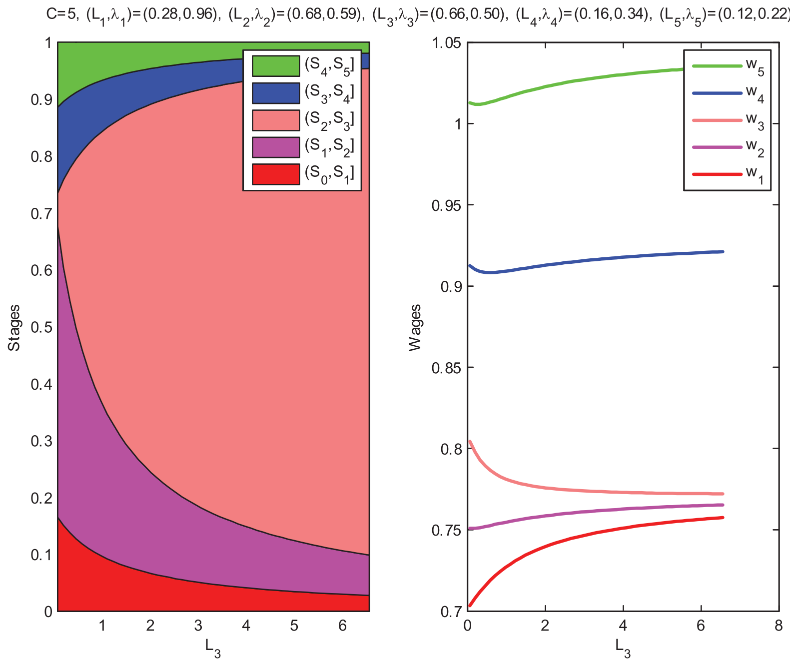

FIGURE 3

Consequences of labor-augmenting technical progress in country 3

such an environment, if foreign labor-augmenting technical progress leads the richest countries to move up, inequality among these countries necessarily increases. The reason is simple. On the one hand, the relative wage of two adjacent countries is equal to their relative productivity in the "cutoff" sector. On the other hand, richer countries are relatively more productive in sectors higher up the ladder (otherwise they would not be specializing in these sectors in equilibrium). In contrast, Proposition 5 predicts that as the richest countries move up, inequality may decrease at the very top of the chain. This counterintuitive result derives from the fact that the pattern of comparative advantage in a model with sequential production is not exogenously given, but depends instead on endogenous labor cost shares along the supply chain. This subtle distinction breaks the monotonic relationship between the pattern of international specialization and inequality between nations. Although later stages necessarily have lower labor cost shares in a given equilibrium, the labor cost shares of later stages in the new equilibrium may be higher than the labor cost shares of earlier stages in the initial equilibrium. At the top of the chain, poorer countries may therefore benefit disproportionately more from being pushed into later stages of production.

5.2. *Routinization*

We now turn our attention to the consequences of a decrease in the failure rate λ_{c_0} of a given country c_0, which we refer to as "routinization". For simplicity we restrict ourselves to a small change in λ_{c_0}, in the sense that it does not affect the ranking of countries in terms of failure rates. The consequences of routinization can be described as follows.

Proposition 6. *Routinization in country c_0 leads all countries to move up the supply chain, increases inequality among countries $c \in \{1,\ldots,c_0\}$, decreases inequality among countries*

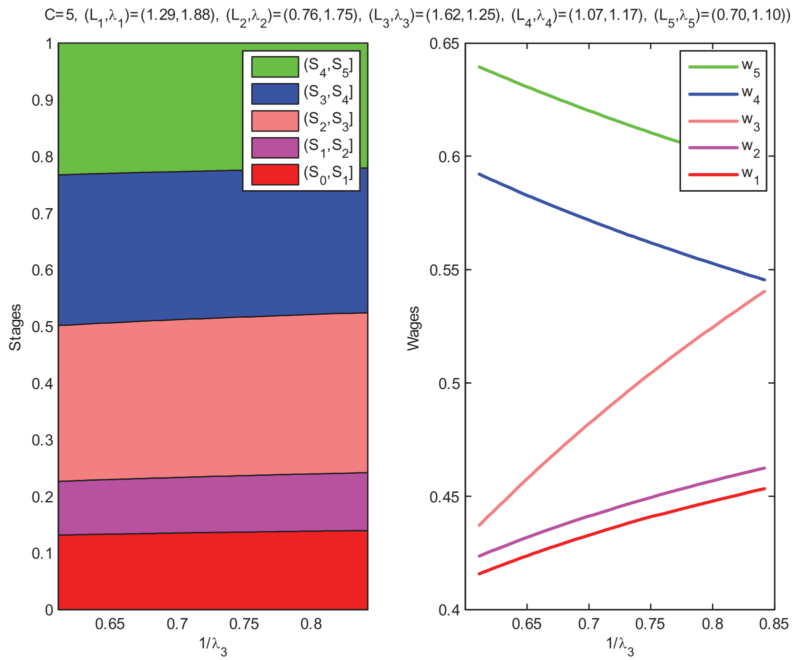

FIGURE 4

Consequences of routinization in country 3

$c \in \{c_0, c_0+1\}$, increases inequality among countries $c \in \{c_0+1, ..., c_1\}$, and decreases inequality among countries $c \in \{c_1, ..., C\}$, with $c_1 \in \{c_0+1, ..., C\}$.

The spillover effects associated with routinization are illustrated in Figure 4. According to Proposition 6, all countries move up the supply chain. In this respect, the consequences of routinization are the same as the consequences of labor-augmenting technical progress at the top of the chain, but the exact opposite at the bottom.

To understand this result, consider first countries located at the top of the chain. Since total output of the final good must rise in response to a lower failure rate in country c_0, countries at the top of the chain must perform fewer stages for labor markets to clear. By a simple iterative argument, these countries must therefore move further up the supply chain, just like in Proposition 5. At the top of the chain, the consequences of routinization for inequality are the same as the consequences of labor-augmenting technical progress. The non-monotonicity—with inequality rising among countries $c \in \{c_0+1, ..., c_1\}$ and decreasing among countries $c \in \{c_1, ..., C\}$—arises from the same conflicting forces: countries move up the chain but produce fewer stages.

At the bottom of the chain, the broad intuition behind the opposite effects of labor-augmenting technical progress and routinization for changes in the pattern of specialization can be understood as follows. Holding the pattern of vertical specialization fixed, labor-augmenting technical progress in country c_0 increases the total labor supply of countries $c \geq c_0$, but leaves their labor demand unchanged. Thus labor market clearing requires countries at the bottom of the chain to reduce the number of stages they perform, to move down the chain, and to increase their output, thereby offsetting the excess labor supply at the top. In contrast, routinization in country c_0 increases the total labor demand of countries $c \geq c_0$ (since country c_0 now produces more output at each stage), but leaves their labor supply unchanged. As a result, countries at the bottom of the chain now need to increase the number of stages they perform, to move up the chain, and to reduce their output to offset the excess labor demand at the top. The consequences for inequality follow from the same logic as in the previous section.[14]

Our goal in this section was to take a first stab at exploring theoretically the relationship between vertical specialization and the recent emergence of developing countries like China. Much remains to be done to assess whether the effects identified in this article are empirically important. Nevertheless, we view our theoretical analysis as a useful first step toward understanding how vertical specialization shapes the interdependence of nations. A key insight that has emerged is that because of sequential production, local technological changes tend to spillover very differently at the bottom and the top of the chain. At the bottom of the chain, depending on the nature of technological changes, countries may move up or down, but regardless of the nature of technological changes, movements along the chain fully determine changes in the world income distribution within that region. At the top of the chain, in contrast, local technological progress always leads countries to move up, but even conditioning on the nature of technological change, inequality between nations within that region may fall or rise. Perhaps surprisingly, while richer countries at the bottom of the chain benefit disproportionately more from being pushed into later stages of production, this is not always true at the top.

6. EXTENSIONS

Our elementary theory of global supply chains is special along several dimensions. First, all intermediate goods are freely traded. Second, production is purely sequential. Third, mistakes are perfectly observable. Fourth, labor and intermediate goods are assumed to be perfect complements and all stages of production are subject to the same failure rates. In this section we demonstrate how more realistic features of global supply chains may be incorporated into our theoretical framework. To save on space, we focus on sketching alternative environments and summarizing their main implications. A detailed analysis can be found in our online Addendum.

6.1. *Coordination costs*

An important insight of the recent trade literature is that changes in trade costs affect the pattern and consequences of international trade not only by affecting final goods trade, but also by affecting the extent of production fragmentation across borders; see e.g. Feenstra and Hanson (1996), Yi (2003) and Grossman and Rossi-Hansberg (2008). We now discuss how the introduction of trading frictions in our simple environment would affect the geographic structure of global supply chains, and in turn, the interdependence of nations.

A natural way to introduce trading frictions in our model is to assume that the likelihood of a defect in the final good is increasing in the number of times the intermediate goods used in its production have crossed a border. We refer to such costs, which are distinct from standard iceberg trade costs, as "coordination costs". Formally, if the production of a given unit u of the final good involves n international transactions—i.e. export and import at stages $0 < S_1^u \leq S_2^u \leq \cdots \leq S_n^u < S$—then the final good is defect free with probability $(1-\tau)^n$. The parameter $\tau \in (0,1)$ measures the extent of coordination costs. Section 2 corresponds to the limit case when coordination costs go to

14. The only difference is that in the middle of the chain, inequality decreases among countries $c \in \{c_0, c_0+1\}$ because of the direct effect of a reduction in λ_{c_0}, which tends to decrease inequality between c_0 and c_0+1, as seen in Equation (9). This force was absent from our previous comparative static exercise since labor endowments (in efficiency units) did not directly affect zero-profit conditions.

zero. Upon completion of each unit of the final good, we assume that consumers perfectly observe whether the unit is defect free or not. A unit with a defect has zero price. Like in Section 2, we assume that the (defect-free) final good is freely traded and we use it as our numeraire. Finally, we assume that all international transactions are perfectly observable by all firms so that two units of the same intermediate good s may, in principle, command two different prices if their production requires a different number of international transactions. Accordingly, competitive equilibria remain Pareto optimal in the presence of coordination costs.

The analysis of this generalized version of our model is considerably simplified by the fact that, in spite of coordination costs, a weaker version of vertical specialization must still hold in any competitive equilibrium. Let $c^u(s)$ denote the country in which stage s has been performed for the production of a given unit u. Using the previous notation, the pattern of international specialization can be characterized as follows.

Proposition 1 [Coordination costs] *In any competitive equilibrium, the allocation of stages to countries, $c^u : \mathcal{S} \to \mathcal{C}$, is increasing in s for all $u \in \left[0, \sum_{c \in \mathcal{C}} Q_c(S)\right]$.*

According to this variation of Proposition 1, for any unit of the final good, production must still involve vertical specialization, with less productive countries specializing in earlier stages of production. This result is weaker, however, than the one derived in Section 2 in that it does not require $c^u(\cdot)$ to be the same for all units. This should be intuitive. Consider the extreme case in which τ is arbitrarily close to one. In this situation all countries will remain under autarky in a competitive equilibrium. Thus the same stages of production will be performed in different countries. In the presence of coordination costs, one can therefore only expect vertical specialization to hold within each supply chain, whether or not all chains are identical, which is what our new proposition establishes. Armed with this proposition, we can characterize competitive equilibria using the same approach as in Section 2. The only difference is that we now need to guess first the structure of the equilibrium (e.g. some units are produced entirely in country 1, whereas all other units are produced jointly in all countries) and then verify ex post that our guess is correct.

Figure 5 illustrates how the structure of competitive equilibria varies with the magnitude of coordination costs in the two-country case.[15] There are three distinct regions. For sufficiently high-coordination costs, all stages are being performed in both countries and there is no trade. Conversely, for low enough coordination costs, the pattern of vertical specialization is the same as under free trade. In this region, reductions in coordination costs have no effect on the pattern of specialization, but raise wages in all countries. The most interesting case arises when coordination costs are in an intermediate range. In this region, the large country (country 2) is incompletely specialized, whereas the small country (country 1) is completely specialized in a subset of stages. As can easily be shown analytically, the set of stages that are being offshored to the small country is necessarily increasing in the level of coordination costs over that range. Hence starting from autarky and decreasing coordination costs, there will be "overshooting": a broader set of stages will be performed in the poor country at intermediate levels of coordination costs than under perfectly free trade. This pattern of overshooting does not arise from coordination failures, heterogeneity in trade costs or the imperfect tradability of the final good, as discussed in Baldwin and Venables (2010). It simply reflects the fact that in a perfectly competitive model with sequential production and trading frictions, a sufficiently large set of stages must be performed in

15. Details about the construction of these competitive equilibria are available upon request.

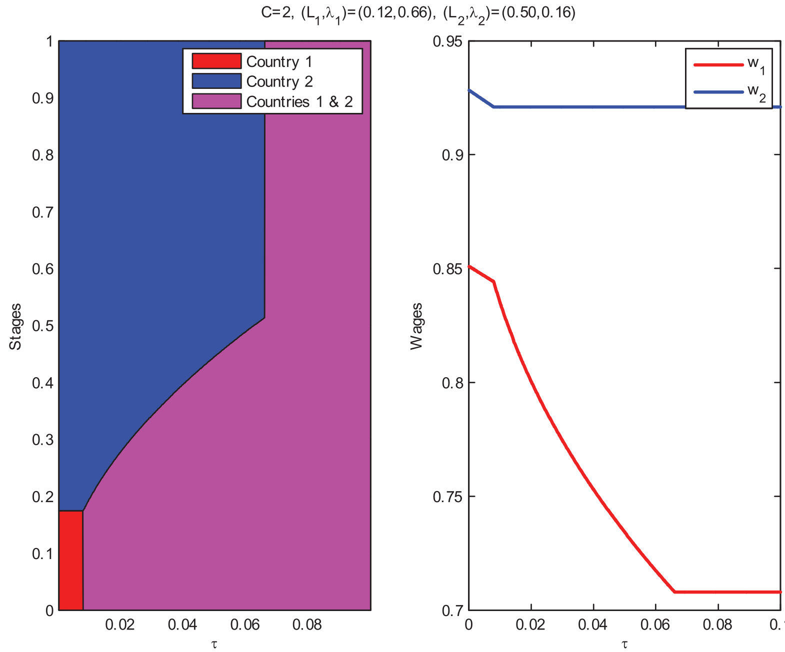

FIGURE 5

Consequences of a change in coordination costs

the small country for firms to find it profitable to fragment production across borders. Accordingly, the larger the coordination costs, the larger the set of stages being performed in the small country!

Figure 5 also illustrates that sequential production does not hinder the ability of smaller countries to benefit from international trade. On the contrary, smaller countries tend to benefit more from freer trade. In the above example, a decrease in coordination costs either only benefits the small country (for intermediate levels of coordination costs) or affects real wages in both countries in the same proportional manner (for low enough coordination costs). Finally, Figure 5 highlights that how many stages of the production process are being offshored to a poor country may be a very poor indicator of the interdependence of nations. Here, when the measure of stages being offshored is the largest, the rich country is completely insulated from (small) technological shocks in the poor country.

6.2. *Simultaneous versus sequential production*

Most production processes are neither purely sequential, as assumed in Section 2, nor purely simultaneous, as assumed in most of the existing literature. Producing an aircraft, for example, requires multiple parts, e.g. fuselage, stabilizer, landing gears, entry doors, seats and windows. Many of these parts are produced simultaneously before being assembled, but each of these parts requires a large number of sequential stages. For instance, the construction of a mid-fuselage section for the Boeing 787 Dreamliner in the USA involves the fabrication of wing-to-body fairing in Canada using panels made in China; see Gates (2005). More generally, extraction of raw materials comes before refining, which itself comes before manufacturing.

With this is mind, we turn to a generalization of our original model in which there are multiple supply chains, indexed by $n \in \mathcal{N} \equiv \{1,...,N\}$, each associated with the production of a part. We allow supply chains to differ in terms of their complexity, S^n, but for simplicity, we require failure rates to be constant across chains and given by λ_c, as in Section 2. Hence countries do not have a comparative advantage in particular parts. Parts are ordered such that S^n is weakly increasing in n, so that parts with a higher index n are more complex.

Parts are assembled into a unique final good using labor. Formally, the output Y_c of the final good in country c is given by

$$Y_c = F\left(X_c^1, \ldots, X_c^N, A_c\right),$$

where $F(\cdot)$ is a production function with constant returns to scale, X_c^n is the amount of part n used in the production of the final good in country c, and $A_c \leq L_c$ corresponds to the amount of labor used for assembly in country c. Note that the production function $F(\cdot)$ is assumed to be identical across countries, thereby capturing the idea that assembly is sufficiently standardized for mistakes in this activity to be equally unlikely in all countries. Note also that by relabelling each part n as a distinct final good and the production function $F(\cdot)$ as a utility function, with $F(\cdot)$ independent of A_c, this section can also be interpreted as a multi-sector extension of our baseline framework.[16]

In this generalized version of our model, the pattern of international specialization still takes a very simple form, as the next proposition demonstrates.

Proposition 1 [Simultaneous production] *In any free trade equilibrium, there exists a sequence of stages* $S_0 \equiv 0 \leq S_1 \leq \ldots \leq S_C = S^N$ *such that for all* $n \in \mathcal{N}$, $s \in (0, S^n]$, *and* $c \in \mathcal{C}$, $Q_c^n(s) > 0$ *if and only if* $s \in (S_{c-1}, S_c]$. *Furthermore, if country c is engaged in parts production,* $A_c < L_c$, *then all countries* $c' > c$ *are only involved in parts production,* $A_{c'} = 0$.

This strict generalization of Proposition 1 imposes three restrictions on the pattern of international specialization. First, the poorest countries tend to specialize in assembly, whereas the richest countries tend to specialize in parts production. This directly derives from the higher relative productivity of the poorest countries in assembly. Second, among the countries that produce parts, richer countries produce and export at later stages of production. This result also held in Section 3, and the intuition is unchanged. Third, whereas middle-income countries tend to produce all parts, the richest countries tend to specialize in only the most complex ones. Intuitively, even the final stage S^n of a simple part has a sufficiently high labor cost share that high-wage, high-productivity countries are less competitive at that stage. Viewed through the lens of the hierarchy literature, the final output of a simple chain does not embody a large enough amount of inputs to merit, from an efficiency standpoint, leveraging the productivity of the most productive countries.

Compared with the simple model analyzed in Section 3, the present model suggests additional cross-sectional predictions. Here, trade is more likely to be concentrated among countries with similar levels of GDP per capita if exports and imports tend to occur along the supply chain associated with particular parts rather than at the top between "part producers" and "assemblers". Accordingly, one should expect trade to be more concentrated among countries with similar levels of GDP per capita in industries in which the production process consists of very complex parts.

16. More generally, one could interpret the present model as a multi-sector economy with one "outside" good, that can be produced one-to-one from labor in all countries, and multiple "sequential" goods, whose production is as described in Section 2. Under this interpretation, A_c would be the amount of labor allocated to the outside good in country c.

6.3. *Imperfect observability of mistakes*

In our benchmark model, all mistakes are perfectly observable so that each country's exports are equally and fully reliable. In practice, quality concerns are a major determinant of the organization of global supply chains; see e.g. Manuj and Mentzer (2008). To capture such considerations within our framework, we now generalize our model to allow countries to differ not only in terms of the rate at which they make mistakes, but also the probability with which mistakes are observed, i.e. their "quality control".

As before, when a mistake occurs on a given unit u at stage s, any intermediate good produced after stage s using unit u is also defective and the associated final good is worthless. Our only point of departure from our benchmark model is that mistakes are imperfectly observed with a country-specific probability $\beta_c \in [0,1]$. The location in which different stages associated with a given unit have been performed is public information. All markets are perfectly competitive and all goods are freely traded. Thus different units of a given intermediate good s produced at different locations may command different prices depending on their "qualities", i.e. the commonly known probabilities that they are defect free.

In this environment, if a firm from country c combines $q[s,\theta(s)]$ units of intermediate good s with quality $\theta(s)$ with $q[s,\theta(s)]ds$ units of labor, its output at stage $s+ds$ is given by

$$q[s+ds,\theta(s+ds)] = (1-\beta_c\lambda_c ds)\,q[s,\theta(s)]. \tag{11}$$

Using Bayes' rule and a first-order Taylor expansion, the quality at stage $s+ds$ can be computed as

$$\theta(s+ds) = [1-(1-\beta_c)\lambda_c ds]\theta(s). \tag{12}$$

For simplicity, we restrict ourselves to "symmetric" free trade equilibria in which all units of the final good are produced in the same manner. The next proposition demonstrates how both β_c and λ_c shape the pattern of international specialization in this situation.

Proposition 1 [Imperfect observability of mistakes] *Suppose that $\beta_c\lambda_c$ is strictly decreasing in c and λ_c is weakly decreasing in c. Then in any symmetric free trade equilibrium, there exists a sequence of stages $S_0 \equiv 0 < S_1 < \ldots < S_C = S$ such that for all $s \in \mathcal{S}$ and $c \in \mathcal{C}$, $Q_c(s) > 0$ if and only if $s \in (S_{c-1}, S_c]$.*

This strict generalization of Proposition 1 states that countries with higher failures rates and better quality controls tend to specialize in the earlier stages of global supply chains. The tendency for less productive countries to produce and export at earlier stages of production is driven by the same forces as in our benchmark model. But there is now also a tendency for countries with better quality controls to specialize in the same stages. Intuitively, if a country makes an unobserved mistake in producing a unit of output u in stage s, which occurs at rate $\lambda_c(1-\beta_c)$, then firms will continue to add labor to this defective unit as it moves up the chain. If this unobserved mistake occurs lower down the chain, then more labor gets wasted. Hence, it is efficient to have countries with better quality control, all else equal, at the bottom of the chain. This again leads to more nuanced cross-sectional predictions than our benchmark model. Since richer countries are likely to have both lower failure rates and better quality control, the relative importance of these two considerations in different industries may determine whether or not they tend to operate at the top of global supply chains in practice.

6.4. *General production functions*

To focus attention in the simplest possible way on the novel aspects of an environment with sequential production, we have focused on a very stylized production process. Stages only differ in the order in which they are performed, and countries only differ in the rates at which mistakes occur along the supply chain. Formally, the production process underlying Equation (1) corresponds to the limit, when δs goes to zero, of the following Leontief production function:

$$q(s+\delta s)=e^{-\lambda_c \delta s}\min\{q(s),l(s)/\delta s\}.$$

In this final subsection, we relax the three main features of the above production function: perfect complementarity between labor and intermediate goods at all stages of production, symmetry between all stages of production, and production that is subject to mistakes ($\lambda_c>0$). Instead we assume the following CES production function:

$$q(s+\delta s)=e^{-\lambda_c(s)\delta s}\left\{(1-\delta s)q(s)^{\frac{\sigma-1}{\sigma}}+\delta s[l(s)/\delta s]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}},$$

where $\sigma \geq 0$ denotes the elasticity of substitution at all stages of production and $\lambda_c(s)\in\mathbb{R}$ is a measure of country c's total factor productivity at stage s. As demonstrated in our online Addendum, starting from the previous production function and substituting in the optimal labor demand, we obtain the following generalization of Equation (1):

$$q(s+ds)=\left\{1-\left[\lambda_c(s)-\frac{\sigma}{1-\sigma}\left(1-(w_c/p(s))^{1-\sigma}\right)\right]ds\right\}q(s).$$

Equation (1) corresponds to the special case: $\sigma=0$ and $\lambda_c(s)\equiv\lambda_c>0$. For technical reasons, we further assume that to produce one unit of intermediate good 0, firms need to hire a small number $\varepsilon>0$ of workers. Thus perfect competition guarantees that $p(0)>0$.[17] In the next proposition we provide sufficient conditions under which the pattern of international specialization can still be described as in Proposition 1 in this more general environment.

Proposition 1 [General production function] *Suppose that $\sigma<1$ and that $\lambda_c(s)$ is strictly decreasing in c, differentiable in s with either $\lambda_c'(s)>0$ or $\lambda_c'(s)=0$ for all s, and weakly submodular in (s,c). Then in any free trade equilibrium, there exists a sequence of stages $S_0\equiv 0<S_1<\cdots<S_C=S$ such that for all $s\in\mathcal{S}$ and $c\in\mathcal{C}$, $Q_c(s)>0$ if and only if $s\in(S_{c-1},S_c]$.*

According to this strict generalization of Proposition 1, our cross-sectional predictions are unchanged if: (i) the elasticity of substitution between labor and intermediate goods is not too high, $\sigma<1$; (ii) later stages of production tend to be more costly to produce, $\lambda_c'(s)>0$ or $=0$; and (iii) more productive countries (at all stages) are relatively more productive in later stages of production, $\lambda_c(s)$ is weakly submodular. To understand this result, it is useful to go back to the intuition behind Proposition 1. In Section 3 prices were increasing along the supply chain. Thus the cost share of labor was relatively lower in the production of intermediate goods produced at later stages, which made them relatively cheaper to produce in countries with higher wages. Conditions (i)–(iii) guarantee that the same logic applies. By Condition (ii), prices are still increasing along the supply chain. By Condition (i), this implies that the cost share of labor remains relatively lower in the production of intermediate goods produced at later stages. Thus absent any comparative advantage across stages, more productive countries should tend to specialize in later stages of production. By Condition (iii), the previous pattern of international specialization is reinforced by the comparative advantage of more productive countries in later stages.

17. If $p(0)=0$ and $\sigma>0$, firms producing intermediate good ds have zero labor demand, zero costs, and in turn, $p(ds)=0$. Iterating, this implies zero prices at all stages.

Although the basic forces emphasized in Sections 4 and 5 still shape the interdependence of nations, deriving comparative static predictions in this more general environment is more involved. Consider the smallest departure from our baseline model: assuming that production is no longer subject to mistakes, $\lambda_c < 0$ for all c. In this situation, all of the comparative static results on vertical specialization derived in Sections 4 and 5 continue to hold. Nevertheless, the implications for the world income distribution are more subtle as changes in the measures of stages performed in each country are no longer sufficient to predict how changes in the pattern of vertical specialization affect inequality between nations.[18] A similar issue arises in environments in which failure rates are no longer constant across stages of production. Finally, without perfect complementarity between labor and intermediate goods, our model is no longer block-recursive: prices affect labor demand, and thus, the assignment of countries to stages of production.

These issues notwithstanding, the pattern of international specialization in this extension, as well as the extensions presented in Sections 6.1–6.3, always exhibits vertical specialization. This implies that the free trade equilibrium remains characterized by a simple system of non-linear difference equations, akin to the ones presented in Lemmas 1 and 2. Accordingly, it is still easy to use simulations to investigate how global supply chains shape the interdependence of nations. We hope that this appealing feature of our theoretical framework will make it a useful guide for future work on global supply chains in richer, more realistic environments.

7. CONCLUDING REMARKS

In this article, we have developed an elementary theory of global supply chains. The key feature of our theory is that production is sequential and subject to mistakes. In the unique free trade equilibrium, countries with lower probabilities of making mistakes at all stages specialize in later stages of production. Because of the sequential nature of production, absolute productivity differences are a source of comparative advantage among nations.

Using this simple theoretical framework, we have taken a first step toward analyzing how vertical specialization shapes the interdependence of nations. Among other things, we have shown that local technological changes tend to spillover very differently at the bottom and the top of the chain. At the bottom of the chain, depending on the nature of technological changes, countries may move up or down, but whatever they do, movements along the chain fully determine changes in the world income distribution within that region. At the top of the chain, in contrast, local technological progress always leads countries to move up, but even conditioning on the nature of technological change, inequality between nations within that region may fall or rise. Perhaps surprisingly, while richer countries at the bottom of the chain benefit disproportionately more from being pushed into later stages of production, this is not always true at the top.

Our model is admittedly stylized, but we believe that its tractability lends itself to a variety of extensions and applications. The previous section has explored some of them. There are many others. For instance, we have focused on a perfectly competitive environment. It would be interesting to extend our framework to allow for monopolistic competition and endogenous innovation. We have also ignored any policy-related issues. It would be interesting to investigate how global supply chains may affect countries' commercial policies, and in turn, the optimal design of international trade agreements. One could also analyze competition between countries—perhaps through infrastructure spending or education policies—to capture higher positions in

18. Specifically, if $\lambda_c < 0$, then while increasing the measure of stages performed in a country c still increases the amount of labor necessary to transform imports into one unit of export, $\left(e^{\lambda_c N_c} - 1\right)/\lambda_c$, it now decreases the volume of imports necessary to produce one unit of export, $e^{\lambda_c N_c}$, thereby pushing labor cost shares, and in turn, relative wages in the opposite direction.

supply chains. Last but not least, it would be interesting to incorporate our simple model with sequential production into a standard quantitative trade model, such as Eaton and Kortum (2002), to explore the quantitative implications of global supply chains for the interdependence of nations. Such a model would be ideally suited to organize recent empirical evidence on trade in value added (e.g. Johnson and Noguera, 2011).

Although we have emphasized the consequences of vertical specialization for the interdependence of nations, we believe that our general results also have useful applications outside of international trade. Sequential production processes are pervasive in practice. They may involve workers of different skills, as emphasized in the labor and organizations literature. They may also involve firms of different productivities, as in the industrial organization literature. Whatever the particular context may be, our theoretical analysis may help shed a new light on how vertical specialization shapes the interdependence between different actors of a given supply chain.

APPENDICES

A. *Proofs (I): free trade equilibrium*

Proof of Proposition 1. As mentioned in the main text, if a firm in country c produces intermediate good s, then it necessarily produces a measure $\Delta > 0$ of intermediate goods around that stage. Specifically, there exists an $s_\Delta < s \leq s_\Delta + \Delta$ such that $Q_c(s') > 0$ for all $s' \in (s_\Delta, s_\Delta + \Delta]$. Throughout this proof we define $\Delta(s) \equiv (s_\Delta, s_\Delta + \Delta)$, for some s_Δ satisfying the previous conditions. The local properties that follow do not depend on which exact s_Δ we choose. We proceed in four steps.

Step 1: $p(\cdot)$ is continuous.

Consider a stage $s_0 \in (0,S]$. By the goods market clearing condition, we know that there must be at least one country, call it c_0, producing intermediate good s_0, which requires $Q_{c_0}(s) > 0$ for all $s \in \Delta(s_0)$. By Condition (2), we therefore have

$$p(s) = \left(1 + \lambda_{c_0} ds\right) p(s - ds) + w_{c_0} ds, \text{ for all } s \in \Delta(s_0),$$

which implies

$$\frac{dp(s)}{ds} = \lambda_{c_0} p(s) + w_{c_0}, \text{ for all } s \in \Delta(s_0). \tag{13}$$

Thus $p(\cdot)$ is piecewise differentiable over $(0,S]$, and in turn, continuous almost everywhere. To conclude let us show that p cannot have any jump. We proceed by contradiction. Suppose that there exists $s_0 \in (0,S)$ such that $p\left(s_0^+\right) \neq p\left(s_0^-\right)$. Then there must exist $c_0 \neq c_1$ such that firms in country c_0 produce intermediate good s_0 and sell it to firms in country c_1. If $p\left(s_0^+\right) > p\left(s_0^-\right)$, then $p\left(s_0^+\right) > \left(1 + \lambda_{c_0} ds\right) p(s_0 - ds) + w_{c_0} ds$, which violates Condition (2). If instead $p\left(s_0^+\right) < p\left(s_0^-\right)$, then $p(s_0 + ds) > \left(1 + \lambda_{c_1} ds\right) p\left(s_0^-\right) + w_{c_1} ds$, which also violates Condition (2).

Step 2: If $s_2 > s_1$, then $p(s_2) > p(s_1)$.

By Step 1 we know that $p(\cdot)$ is continuous. By Step 1 we also know that for any stage $s \in (0,S]$, there is a small neighborhood $\Delta(s)$ of s such that $p(\cdot)$ is a solution of Equation (13). Thus $p'(s) > 0$ for almost all $s \in (0,S]$. The fact that $p(\cdot)$ is strictly increasing directly derives from these two observations.

Step 3: If $c_2 > c_1$, then $w_{c_2} > w_{c_1}$.

Consider two countries c_2 and c_1 with $c_2 > c_1$. Factor market clearing requires country c_1 to produce at least one intermediate good in $(0,S]$, call it s_1. By assumption, this requires $Q_{c_1}(s) > 0$ for all $s \in \Delta(s_1)$. Thus Condition (2) implies

$$p(s_1) = \left(1 + \lambda_{c_1} ds\right) p(s_1 - ds) + w_{c_1} ds, \tag{14}$$

$$p(s_1) \leq \left(1 + \lambda_{c_2} ds\right) p(s_1 - ds) + w_{c_2} ds. \tag{15}$$

Since $\lambda_{c_2} < \lambda_{c_1}$, Equation (14) and Inequality (15) imply $w_{c_2} > w_{c_1}$.

Step 4: If $c_2 > c_1$ and $Q_{c_1}(s_1) > 0$, then $Q_{c_2}(s) = 0$ for all $s < s_1$.

We proceed by contradiction. Suppose that there exist two countries, $c_2 > c_1$, and two intermediate goods, $s_1 > s_2 > 0$, such that c_1 produces s_1 and c_2 produces s_2. By assumption, this requires $Q_{c_1}(s) > 0$ for all $s \in \Delta(s_1)$ and $Q_{c_2}(s) > 0$ for all $s \in \Delta(s_2)$. Thus Condition (2) implies

$$\begin{aligned} p(s_1) &= \left(1+\lambda_{c_1} ds\right) p(s_1 - ds) + w_{c_1} ds, \\ p(s_2) &= \left(1+\lambda_{c_2} ds\right) p(s_2 - ds) + w_{c_2} ds, \\ p(s_1) &\leq \left(1+\lambda_{c_2} ds\right) p(s_1 - ds) + w_{c_2} ds, \\ p(s_2) &\leq \left(1+\lambda_{c_1} ds\right) p(s_2 - ds) + w_{c_1} ds. \end{aligned}$$

Combining the four previous expressions, we get

$$\begin{aligned} &\left[\left(1+\lambda_{c_2} ds\right) p(s_1 - ds) + w_{c_2} ds\right]\left[\left(1+\lambda_{c_1} ds\right) p(s_2 - ds) + w_{c_1} ds\right] \\ &\geq \left[\left(1+\lambda_{c_1} ds\right) p(s_1 - ds) + w_{c_1} ds\right]\left[\left(1+\lambda_{c_2} ds\right) p(s_2 - ds) + w_{c_2} ds\right], \end{aligned}$$

which can be rearranged as

$$\left(1+\lambda_{c_2} ds\right)\left[p(s_1 - ds) - p(s_2 - ds)\right] w_{c_1} \geq \left(1+\lambda_{c_1} ds\right)\left[p(s_1 - ds) - p(s_2 - ds)\right] w_{c_2}.$$

By Step 2, we know that $p(s_1 - ds) - p(s_2 - ds) > 0$. Thus the previous inequality implies

$$\left(1+\lambda_{c_2} ds\right) w_{c_1} \geq \left(1+\lambda_{c_1} ds\right) w_{c_2}. \tag{16}$$

Since $\lambda_{c_2} < \lambda_{c_1}$, Inequality (16) implies $w_{c_1} > w_{c_2}$, which contradicts Step 3.

To conclude the proof of Proposition 1, let us define $S_c \equiv \sup\{s \in \mathcal{S} | Q_c(s) > 0\}$ for all $c \in \mathcal{C}$. By Step 4, we must have $S_0 \equiv 0 < S_1 < \cdots < S_C = S$, and for all $s \in \mathcal{S}$ and $c \in \mathcal{C}$, $Q_c(s) > 0$ if $S_{c-1} < s < S_c$ and $Q_c(s) = 0$ if $s < S_{c-1}$ or $s > S_c$. Since $Q_c(s) > 0$ requires $Q_c\left(s'\right) > 0$ for all $s' \in (s - ds, s]$, we must also have $Q_c(S_c) > 0$ and $Q_c(S_{c-1}) = 0$ for all $c \in \mathcal{C}$. Thus $Q_c(s) > 0$ if and only if $s \in (S_{c-1}, S_c]$. Finally, by the goods market clearing condition, country C must produce stage S, so that $S_C = S$. ‖

Proof of Lemma 1. We first consider Equation (6). Proposition 1 and Equation (3) imply

$$Q_c(s_2) - Q_c(s_1) = -\lambda_c \int_{s_1}^{s_2} Q_c(s) ds, \text{ for all } s_1, s_2 \in (S_{c-1}, S_c]. \tag{17}$$

Taking the derivative of the previous expression with respect to s_2, we get

$$\frac{dQ_c(s)}{ds} = -\lambda_c Q_c(s), \text{ for all } s \in (S_{c-1}, S_c).$$

The solution of the previous differential equation must satisfy

$$Q_c(S_c) = e^{-\lambda_c (S_c - S_{c-1})} \lim_{s \to S_{c-1}^+} Q_c(s). \tag{18}$$

Proposition 1 and Equation (3) also imply

$$Q_c(S_{c-1} + ds) - Q_{c-1}(S_{c-1} - ds) = -\left[\lambda_c \lim_{s \to S_{c-1}^+} Q_c(s) + \lambda_{c-1} Q_{c-1}(S_{c-1} - ds)\right] ds.$$

Since ds is infinitesimal, this further implies

$$\lim_{s \to S_{c-1}^+} Q_c(s) = \lim_{s \to S_{c-1}^-} Q_{c-1}(s) = Q_{c-1}(S_{c-1}). \tag{19}$$

Equation (6) derives from Equations (18) and (19) and the definition of $Q_c \equiv Q_c(S_c)$.

Let us now turn to Equation (5). By Proposition 1 and Equation (4), we know that

$$\int_{S_{c-1}}^{S_c} Q_c(s) ds = L_c, \text{ for all } c \in \mathcal{C}. \tag{20}$$

By Equations (17) and (19), we also know that

$$\int_{S_{c-1}}^{S_c} Q_c(s) ds = \frac{1}{\lambda_c}\left[Q_{c-1}(S_{c-1}) - Q_c(S_c)\right]. \tag{21}$$

Equations (20) and (21) imply

$$L_c = \frac{1}{\lambda_c}\left[Q_{c-1}(S_{c-1}) - Q_c(S_c)\right], \text{ for all } c \in \mathcal{C}. \tag{22}$$

Equation (5) derives from Equations (6) and (22) and the definition of $Q_c \equiv Q_c(S_c)$. The boundary conditions $S_0 = 0$ and $S_C = S$ have already been established in the proof of Proposition 1. ‖

Proof of Lemma 2. We first consider Equation (7). Proposition 1 and Condition (2) imply

$$p(S_c + ds) - (1 + \lambda_{c+1} ds) p(S_c) - w_{c+1} ds \geq p(S_c + ds) - (1 + \lambda_c ds) p(S_c) - w_c ds,$$
$$p(S_c) - (1 + \lambda_c ds) p(S_c - ds) - w_c ds \geq p(S_c) - (1 + \lambda_{c+1} ds) p(S_c - ds) - w_{c+1} ds,$$

for any $c < C$. After simplifications, the two previous inequalities can be rearranged as

$$(\lambda_c - \lambda_{c+1}) p(S_c) \geq w_{c+1} - w_c \geq (\lambda_c - \lambda_{c+1}) p(S_c - ds).$$

Since p is continuous and ds is infinitesimal, we get

$$w_{c+1} - w_c = (\lambda_c - \lambda_{c+1}) p(S_c), \text{ for all } c < C.$$

which is equivalent to Equation (7) by the definition of $p_c \equiv p(S_c)$.

Let us now turn to Equation (8). Proposition 1 and Condition (2) imply

$$p(s + ds) = (1 + \lambda_c ds) p(s) + w_c ds, \text{ for all } s \in (S_{c-1}, S_c],$$

which further implies

$$\frac{dp(s)}{ds} = \lambda_c p(s) + w_c, \text{ for all } s \in (S_{c-1}, S_c).$$

The solution of the previous differential Equation must satisfy

$$p(S_c) = e^{\lambda_c (S_c - S_{c-1})} \lim_{s \to S_{c-1}^+} p(S_{c-1}) + \left[e^{\lambda_c (S_c - S_{c-1})} - 1 \right] (w_c / \lambda_c),$$

which is equivalent to Equation (8) by the continuity of $p(\cdot)$ and the definitions of $N_c \equiv S_c - S_{c-1}$ and $p_c \equiv p(S_c)$. The boundary conditions derive from the fact that $p_0 = p(S_0) = p(0) = 0$ and $p_C = p(S_C) = p(S) = 1$. ‖

Proof of Proposition 2. We proceed in four steps.

Step 1: $(S_0, \ldots, S_C)$ and $(Q_0, \ldots, Q_C)$ satisfy Equations (5) and (6) if and only if

$$S_c = S_0 + \sum_{c'=1}^{c} \left(\frac{1}{\lambda_{c'}} \right) \ln \left[\frac{Q_0 - \sum_{c''=1}^{c'-1} \lambda_{c''} L_{c''}}{Q_0 - \sum_{c''=1}^{c'} \lambda_{c''} L_{c''}} \right], \text{ for all } c \in \mathcal{C}, \tag{23}$$

$$Q_c = Q_0 - \sum_{c'=1}^{c} \lambda_{c'} L_{c'}, \text{ for all } c \in \mathcal{C}. \tag{24}$$

Let us first show that if $(S_0, \ldots, S_C)$ and $(Q_0, \ldots, Q_C)$ satisfy Equations (5) and (6), then they satisfy Equations (23) and (24). Consider Equation (24). Equations (5) and (6) imply

$$Q_c = Q_{c-1} - \lambda_c L_c, \text{ for all } c \in \mathcal{C},$$

By iteration we therefore have

$$Q_c = Q_0 - \sum_{c'=1}^{c} \lambda_{c'} L_{c'}, \text{ for all } c \in \mathcal{C}.$$

Now consider Equation (23). Starting from Equation (5) and iterating we get

$$S_c = S_0 - \sum_{c'=1}^{c} \left(\frac{1}{\lambda_{c'}} \right) \ln \left(1 - \frac{\lambda_{c'} L_{c'}}{Q_{c'-1}} \right), \text{ for all } c \in \mathcal{C}.$$

Equation (23) directly derives from the previous expression and Equation (24). It is a matter of simple algebra to check that if $(S_0, \ldots, S_C)$ and $(Q_0, \ldots, Q_C)$ satisfy Equations (23) and (24), then they satisfy Equations (5) and (6).

Step 2: There exists a unique pair of vectors $(S_0, \ldots, S_C)$ and $(Q_0, \ldots, Q_C)$ satisfying Equations (5) and (6) and the boundary conditions: $S_0 = 0$ and $S_C = S$.

Let $\underline{Q}_0 \equiv \sum_{c=1}^{C} \lambda_c L_c$. By Step 1, if $Q_0 \leq \underline{Q}_0$, then there does not exist a pair of vectors $(S_0, \ldots, S_C)$ and $(Q_0, \ldots, Q_C)$ that satisfy Equations (5) and (6). Otherwise $(Q_0, \ldots, Q_C)$ and $(S_0, \ldots, S_C)$ would also satisfy Equations (23) and (24), which cannot be the case if $Q_0 \leq \underline{Q}_0$. Now consider $Q_0 > \underline{Q}_0$. From Equation (23), it is easy to check that $\partial S_C / \partial Q_0 < 0$ for all $Q_0 > \underline{Q}_0$; $\lim_{Q_0 \to \underline{Q}_0^+} S_C = +\infty$; and $\lim_{Q_0 \to +\infty} S_C = S_0$. Thus conditional on having set $S_0 = 0$, there exists a unique $Q_0 > \underline{Q}_0$ such that $(S_0, \ldots, S_C)$ and $(Q_0, \ldots, Q_C)$ satisfy Equations (23) and (24) and $S_C = S$. Step 2 derives from Step 1 and the previous observation.

Step 3: For any $(N_1,...,N_C)$ there exists a unique pair of vectors $(w_1,...,w_C)$ and $(p_0,...,p_C)$ satisfying Equations (7) and (8) and the boundary conditions: $p_0=0$ and $p_C=1$.

For any $(N_1,...,N_C)$ w_1, and p_0, there trivially exists a unique pair of vectors $(w_2,...,w_C)$ and $(p_1,...,p_C)$ that satisfy Equations (7) and (8). Thus taking $(N_1,...,N_C)$ as given and having set $p_0=0$, we only need to check that there exists a unique w_1 such that $p_C=1$. To do so, we first establish that p_C is strictly increasing in w_1. We proceed by iteration. By Equation (8), we know that p_1 is strictly increasing in w_1. Thus by Equation (7), w_2 must be strictly increasing in w_1 as well. Now suppose that p_{c-1} and w_c are strictly increasing in w_1 for $c<C$. Then p_c must be strictly increasing in w_1, by Equation (8), and w_{c+1} must be strictly increasing in w_1, by Equation (7). At this point we have established, by iteration, that p_{C-1} and w_C are strictly increasing in w_1. Combining this observation with Equation (8), we obtain that p_C is strictly increasing in w_1. To conclude, let us note that, by Equations (7) and (8), we also have $\lim_{w_1\to 0}p_C=0$ and $\lim_{w_1\to+\infty}p_C=+\infty$. Since p_C is strictly increasing in w_1, there therefore exists a unique w_1 such that $p_C=1$.

Steps 1–3 imply the existence and uniqueness of $(S_0,...,S_C)$, $(Q_0,...,Q_C)$, $(w_1,...,w_C)$ and $(p_0,...,p_C)$ that satisfy Equations (5)–(8) with boundary conditions $S_0=0$, $S_C=S$, $p_0=0$ and $p_C=1$. Now consider the following output levels and intermediate good prices

$$\begin{aligned} Q_c(s) &= e^{-\lambda_c(s-S_{c-1})}Q_{c-1}, \text{ for all } s\in(S_{c-1},S_c], \\ p(s) &= e^{\lambda_c(s-S_{c-1})}p_{c-1}+\left[e^{\lambda_c(s-S_{c-1})}-1\right](w_c/\lambda_c), \text{ for all } s\in(S_{c-1},S_c]. \end{aligned}$$

By construction, $[Q_1(\cdot),...,Q_C(\cdot)]$, $(w_1,...,w_C)$, and $p(\cdot)$ satisfy Conditions (2)–(4). Thus a free trade equilibrium exists. Since $(S_0,...,S_C)$, $(Q_0,...,Q_C)$, $(w_1,...,w_C)$ and $(p_0,...,p_C)$ are unique, the free trade equilibrium is unique as well by Proposition 1 and Lemmas 1 and 2. ||

B. *Proofs (II): global technological change*

Proof of Proposition 3. We decompose the proof of Proposition 3 into three parts. First, we show that an increase in S increases the measure of stages N_c performed in all countries. Second, we show that an increase in S leads all countries to move up the supply chain. Third, we show that an increase in S increases inequality between countries around the world.

Part I: If $S'>S$, then $N'_c>N_c$ for all $c\in\mathcal{C}$.

We first show that $N'_1>N_1$ by contradiction. Suppose that $N'_1\le N_1$. By Equation (5), Equation (6) and the definition of $N_c\equiv S_c-S_{c-1}$, we know that

$$N_c=-\left(\frac{1}{\lambda_c}\right)\ln\left[1-\left(\frac{\lambda_cL_c}{\lambda_{c-1}L_{c-1}}\right)\left(e^{\lambda_{c-1}N_{c-1}}-1\right)\right], \text{ for all } c>1. \tag{25}$$

According to Equation (25), we have $\partial N_c/\partial N_{c-1}>0$. Thus by iteration, $N'_1\le N_1$ implies $N'_c\le N_c$ for all countries $c\in\mathcal{C}$. This further implies $\sum_{c=1}^{C}N'_c=S'_C-S'_0\le S_C-S_0=\sum_{c=1}^{C}N_c$, which contradicts $S'_C-S'_0>S_C-S_0$ by Lemma 1. Starting from $N'_1>N_1$, we can then use Equation (25) again to show by iteration that $N'_c>N_c$ for all $c\in\mathcal{C}$.

Part II: If $S'>S$, then $S'_c>S_c$ for all $c\in\mathcal{C}$.

We proceed by iteration. By Lemma 1 and Part I, we know that $S'_1=N'_1>S_1=N_1$. Thus $S'_c>S_c$ is satisfied for $c=1$. Let us now show that if $S'_c>S_c$ for $1\le c<C$, then $S'_{c+1}>S_{c+1}$. By definition, we know that $S'_{c+1}=S'_c+N'_{c+1}$ and $S_{c+1}=S_c+N_{c+1}$. By Part I, we also know that $N'_{c+1}>N_{c+1}$. Thus $S'_c>S_c$ implies $S'_{c+1}>S_{c+1}$.

Part III: If $S'>S$, then $(w_{c+1}/w_c)'>(w_{c+1}/w_c)$ for all $c<C$.

The proof of Part III proceeds in two steps.

Step 1: If $N'_1>N_1$, then $(w_2/w_1)'>(w_2/w_1)$.

Since $p_0=0$, we know from Equations (7) and (8) that

$$\frac{w_2}{w_1}=1+\frac{1}{\lambda_1}(\lambda_1-\lambda_2)\left(e^{\lambda_1N_1}-1\right). \tag{26}$$

Combining Equation (26) and $N'_1>N_1$, we obtain $(w_2/w_1)'>(w_2/w_1)$. This completes the proof of Step 1.

Step 2: For any country $1<c<C$, if $N'_c>N_c$ and $(w_c/w_{c-1})'\ge(w_c/w_{c-1})$, then $(w_{c+1}/w_c)'>(w_{c+1}/w_c)$.

Consider a country $1<c<C$. Equations (7) and (8) imply

$$\frac{w_{c+1}}{w_c}=1+(\lambda_c-\lambda_{c+1})\left[\left(\frac{e^{\lambda_c N_c}-1}{\lambda_c}\right)+e^{\lambda_c N_c}\left(\frac{w_{c-1}}{w_c}\right)\left(\frac{p_{c-1}}{w_{c-1}}\right)\right]. \tag{27}$$

By Equation (7), we also know that

$$\frac{w_c}{w_{c-1}}=1+(\lambda_{c-1}-\lambda_c)\left(\frac{p_{c-1}}{w_{c-1}}\right), \tag{28}$$

which further implies

$$\left(\frac{w_{c-1}}{w_c}\right)\left(\frac{p_{c-1}}{w_{c-1}}\right)=\frac{(p_{c-1}/w_{c-1})}{1+(\lambda_{c-1}-\lambda_c)(p_{c-1}/w_{c-1})}. \tag{29}$$

Since $(w_c/w_{c-1})'\geq(w_c/w_{c-1})$ and $\lambda_{c-1}>\lambda_c$, Equation (28) immediately implies

$$\left(\frac{p_{c-1}}{w_{c-1}}\right)'\geq\left(\frac{p_{c-1}}{w_{c-1}}\right).$$

Combining this observation with Equation (29)—the right-hand side of which is increasing in (p_{c-1}/w_{c-1})—we obtain

$$\left(\frac{w_{c-1}}{w_c}\right)'\left(\frac{p_{c-1}}{w_{c-1}}\right)'\geq\left(\frac{w_{c-1}}{w_c}\right)\left(\frac{p_{c-1}}{w_{c-1}}\right). \tag{30}$$

To conclude, note that $N'_c>N_c$ implies $e^{\lambda_c N'_c}>e^{\lambda_c N_c}$. Thus Equation (27) and Inequality (30) imply $(w_{c+1}/w_c)'>(w_{c+1}/w_c)$. This completes the proof of Step 2. Combining Part I with Steps 1 and 2, it is then easy to establish Part III by iteration. ‖

Proof of Proposition 4. We decompose the proof of Proposition 4 into three parts. First, we show that a decrease in β increases the measure of stages N_c performed by all countries $c<c_1$ and decreases the measure of stages N_c performed by all countries $c\geq c_1$, with $1<c_1\leq C$. Second, we show that a decrease in β leads all countries to move up. Third, we show that a decrease in β decreases inequality between countries around the world.

Part I: If $\beta'<\beta$, then there exists $1<c_1\leq C$ such that $N'_c>N_c$ if $c<c_1$, $N'_{c_1}\leq N_{c_1}$ and $N'_c<N_c$ if $c>c_1$.

Equation (5), Equation (6) and the definition of N_c imply

$$N_c=-\left(\frac{1}{\beta\lambda_c}\right)\ln\left[1-\left(\frac{\lambda_c L_c}{\lambda_{c-1}L_{c-1}}\right)\left(e^{\beta\lambda_{c-1}N_{c-1}}-1\right)\right],\text{ for all } c>1. \tag{31}$$

After some algebra, one can check that $\partial N_c/\partial N_{c-1}>0$ and $\partial N_c/\partial\beta>0$. Since $\beta'<\beta$, Equation (31) implies that if $N'_{c-1}\leq N_{c-1}$ for $c>1$, then $N'_c<N_c$. This further implies the existence of $1\leq c_1\leq C+1$ such that $N'_c>N_c$ if $c<c_1$, $N'_{c_1}\leq N_{c_1}$, and $N'_c<N_c$ if $c>c_1$. To conclude the proof of Part I, note that if $c_1=1$, then $\sum_{c=1}^C N'_c=S'_C-S'_0<S_C-S_0=\sum_{c=1}^C N_c$, which contradicts $S'_C-S'_0=S_C-S_0$ by Lemma 1. Similarly, if $c_1=C+1$, then $\sum_{c=1}^C N'_c=S'_C-S'_0>S_C-S_0=\sum_{c=1}^C N_c$, which also contradicts $S'_C-S'_0=S_C-S_0$ by Lemma 1.

Part II: If $\beta'<\beta$, then $S'_c>S_c$ for all $c\in\{1,\ldots,C-1\}$.

We first show by iteration that $S'_c>S_c$ if $c<c_1$. By Lemma 1 and Part I, we know that $S'_1=N'_1>S_1=N_1$. Thus $S'_c>S_c$ is satisfied for $c=1$. Let us now show that if $S'_c>S_c$ for $1\leq c<c_1-1$, then $S'_{c+1}>S_{c+1}$. By definition, we know that $S'_{c+1}=S'_c+N'_{c+1}$ and $S_{c+1}=S_c+N_{c+1}$. By Part I, we also know that $N'_{c+1}>N_{c+1}$ if $c<c_1-1$. Thus $S'_c>S_c$ for $1\leq c<c_1-1$ implies $S'_{c+1}>S_{c+1}$. This establishes that $S'_c>S_c$ if $c<c_1$. If $c_1=C$, our proof is complete. If instead $c_1<C$, we still need to show that $S'_c>S_c$ if $C-1\geq c\geq c_1$. We again proceed by iteration. For country $C-1$, we know from Part I that $S'_{C-1}=S-N'_C>S-N_C=S_{C-1}$. Thus $S'_c>S_c$ is satisfied for $c=C-1$. Now suppose that $S'_c>S_c$ for $c>c_1+1$. Then $S'_{c-1}=S'_c-N'_c>S_c-N_c=S_{c-1}$, since $S'_c>S_c$ by assumption and $N'_c<N_c$ by Part I. This establishes that $S'_c>S_c$ for all $C-1\geq c\geq c_1$, which completes the proof of Part II.

Part III: If $\beta'<\beta$, then $(w_{c+1}/w_c)'<(w_{c+1}/w_c)$ for all $c<C$.

Throughout this part of the proof, we let $\widetilde{N}_c\equiv\beta N_c$ and $\widetilde{p}_c\equiv\beta p_c$. The proof of Part III proceeds in two steps.

Step 1: If $\beta'<\beta$, then $\widetilde{N}'_c<\widetilde{N}_c$ for all countries $c\in\mathcal{C}$.

The proof is similar to Part I of the proof of Proposition 3. We first show that $\widetilde{N}'_1<\widetilde{N}_1$ by contradiction. Suppose that $\widetilde{N}'_1\geq\widetilde{N}_1$. By Equation (5), Equation (6) and the definition of $\widetilde{N}_c$, we know that

$$\widetilde{N}_c=-\left(\frac{1}{\lambda_c}\right)\ln\left[1-\left(\frac{\lambda_c L_c}{\lambda_{c-1}L_{c-1}}\right)\left(e^{\lambda_{c-1}\widetilde{N}_{c-1}}-1\right)\right],\text{ for all } c>1, \tag{32}$$

where $\partial\widetilde{N}_c/\partial\widetilde{N}_{c-1}>0$. Thus by iteration, $\widetilde{N}'_1\geq\widetilde{N}_1$ implies $N'_c\geq N_c$ for all countries $c\in\mathcal{C}$. This implies $\beta'\left(\sum_{c=1}^C N'_c\right)=\sum_{c=1}^C\widetilde{N}'_c\geq\sum_{c=1}^C\widetilde{N}_c=\beta\left(\sum_{c=1}^C N_c\right)$. Since $\beta'<\beta$, this further implies $\sum_{c=1}^C N'_c=S'_C-S'_0>S_C-S_0=\sum_{c=1}^C N_c$, which contradicts $S'_C - S'_0 = S_C - S_0$ by Lemma 1. Starting from $\widetilde{N}'_1 < \widetilde{N}_1$, we can now use Equation (32) to show by iteration that $\widetilde{N}'_c < \widetilde{N}_c$ for all $c \in \mathcal{C}$.

Step 2: If $\widetilde{N}'_c < \widetilde{N}_c$ for all countries $c \in \mathcal{C}$, then $(w_{c+1}/w_c)' < (w_{c+1}/w_c)$ for all $c < C$.

Equations (7) and (8) can be rearranged as

$$w_{c+1} = w_c + (\lambda_c - \lambda_{c+1})\widetilde{p}_c, \text{ for all } c < C, \tag{33}$$

$$\widetilde{p}_c = e^{\lambda_c \widetilde{N}_c} \widetilde{p}_{c-1} + \left(e^{\lambda_c \widetilde{N}_c} - 1\right)(w_c/\lambda_c), \text{ for all } c \in \mathcal{C}. \tag{34}$$

Following the exact same strategy as in Part III of the proof of Proposition 3, it is then easy to show by iteration that $(w_{c+1}/w_c)' < (w_{c+1}/w_c)$ for all $c < C$. Part II directly follows from Steps 1 and 2. ‖

C. *Proofs (III): local technological change*

Proof of Proposition 5. We decompose the proof of Proposition 5 into three parts. First, we show that an increase in L_{c_0} increases the measure of stages performed in country c_0 and decreases the measure of stages performed in any other country. Second, we show that an increase in L_{c_0} leads all countries all countries $c < c_0$ to move down and all countries $c > c_0$ move up. Third, we show that an increase in L_{c_0} decreases inequality among countries $c \in \{1, ..., c_0\}$, increases inequality among countries $c \in \{c_0, ..., c_1\}$, and decreases inequality among countries $c \in \{c_1, ..., C\}$, with $c_1 \in \{c_0 + 1, ..., C\}$.

Part I: If $L'_{c_0} > L_{c_0}$, then $N'_{c_0} > N_{c_0}$ and $N'_c < N_c$ for all $c \neq c_0$.

Like in our previous proofs, we will repeatedly use the following relationship

$$N_c = -\left(\frac{1}{\lambda_c}\right) \ln\left[1 - \left(\frac{\lambda_c L_c}{\lambda_{c-1} L_{c-1}}\right)\left(e^{\lambda_{c-1} N_{c-1}} - 1\right)\right], \text{ for all } c > 1, \tag{23}$$

where $\partial N_c / \partial N_{c-1} > 0$ and $\partial N_c / \partial L_c > 0$. The proof of Part I proceeds in two steps.

Step 1: If $L'_{c_0} > L_{c_0}$, then $N'_c < N_c$ for all $c > c_0$.

Let us first establish that $Q'_C > Q_C$. By Proposition 1, we know that $Q'_C \equiv Q'_C(S) = \sum_{c=1}^{C} Q'_c(S)$. By the First Welfare Theorem, we also know that the allocation in a free trade equilibrium is Pareto optimal. Thus Q'_C must be the maximum output level of the final good attainable given the new resource and technological constraints, i.e.,

$$Q'_C = \underset{\widetilde{Q}_1(\cdot), ..., \widetilde{Q}_C(\cdot)}{\arg\max} \quad \sum_{c=1}^{C} \widetilde{Q}_c(S),$$

subject to

$$\sum_{c=1}^{C} \widetilde{Q}_c(s_2) - \sum_{c=1}^{C} \widetilde{Q}_c(s_1) \leq -\int_{s_1}^{s_2} \sum_{c=1}^{C} \lambda_c \widetilde{Q}_c(s) ds, \text{ for all } s_1 \leq s_2, \tag{35}$$

$$\int_0^S \widetilde{Q}_c(s) ds \leq L'_c, \text{ for all } c \in \mathcal{C}, \tag{36}$$

where $L'_{c_0} > L_{c_0}$ and $L'_c = L_c$ for all $c \neq c_0$. Now consider $\widetilde{Q}_1(\cdot), ..., \widetilde{Q}_C(\cdot)$ such that

$$\widetilde{Q}_{c_0}(s) \equiv Q_{c_0}(s) + \left(\frac{\lambda_{c_0} e^{-\lambda_{c_0} s}}{1 - e^{-\lambda_{c_0} S}}\right)\left(L'_{c_0} - L_{c_0}\right), \text{ for all } s \in \mathcal{S},$$

and

$$\widetilde{Q}_c(s) \equiv Q_c(s), \text{ for all } s \in \mathcal{S} \text{ and } c \neq c_0.$$

Since $Q_1(\cdot), ..., Q_C(\cdot)$ satisfies the initial resource and technological constraints, as described by Conditions (3) and (4), $\widetilde{Q}_1(\cdot), ..., \widetilde{Q}_C(\cdot)$ must satisfy, by construction, the new resource and technological constraints, as described by Conditions (35) and (36). Since $L'_{c_0} > L_{c_0}$, we must also have

$$\widetilde{Q}_{c_0}(S) + \widetilde{Q}_C(S) = \left(\frac{\lambda_{c_0} e^{-\lambda_{c_0} S}}{1 - e^{-\lambda_{c_0} S}}\right)\left(L'_{c_0} - L_{c_0}\right) + Q_C > Q_C.$$

Since $Q'_C \geq \widetilde{Q}_{c_0}(S) + \widetilde{Q}_C(S)$, the previous inequality implies $Q'_C > Q_C$. By Equation (5), Equation (6) and the definition of N_c, we also know that

$$N_C = \left(\frac{1}{\lambda_c}\right) \ln\left(1 + \frac{\lambda_C L_C}{Q_C}\right).$$

Thus if $C > c_0$, $Q'_C > Q_C$ and $L'_C = L_C$ imply $N'_C < N_C$. To conclude the proof of Step 1, note that if $N'_c < N_c$ for $c > c_0 + 1$, then $L'_{c-1} = L_{c-1}$ and Equation (25) imply $N'_{c-1} < N_{c-1}$. Thus by iteration, $N'_c < N_c$ for all $c > c_0$.

Step 2: If $L'_{c_0} > L_{c_0}$, then $N'_c < N_c$ for all $c < c_0$.

We first show by contradiction that if $L'_{c_0} > L_{c_0}$ and $c_0 > 1$, then $N'_1 < N_1$. Suppose that $N'_1 \geq N_1$. Since $L'_c = L_c$ for all $c < c_0$, we can use Equation (25)—the fact that $\partial N_c / \partial N_{c-1} > 0$—to establish by iteration that $N'_c \geq N_c$ for all $c < c_0$. Since $L'_{c_0} > L_{c_0}$ and $L'_{c_0-1} = L_{c_0-1}$, we can further use Equation (25)—the facts that $\partial N_c / \partial N_{c-1} > 0$ and that $\partial N_c / \partial L_c > 0$—to establish that $N'_{c_0} > N_{c_0}$. To show that $N'_{c_0+1} > N_{c_0+1}$, we use the two following relationships:

$$Q_{c-1} = \frac{\lambda_c L_c}{1 - e^{-\lambda_c N_c}}, \text{ for all } c \in \mathcal{C}, \tag{37}$$

$$Q_c = \frac{\lambda_c L_c e^{-\lambda_c N_c}}{1 - e^{-\lambda_c N_c}}, \text{ for all } c \in \mathcal{C}. \tag{38}$$

Equation (37) derives from Equation (5) and the definition of $N_c \equiv S_c - S_{c-1}$. Equation (38) further uses Equation (6). Since $N'_{c_0-1} > N_{c_0-1}$ and $L'_{c_0-1} = L_{c_0-1}$, Equation (38)—in particular, the fact that $\partial Q_c / \partial N_c < 0$—implies $Q'_{c_0-1} < Q_{c_0-1}$. Since $Q'_{c_0-1} < Q_{c_0-1}$ and $N'_{c_0} > N_{c_0}$, Equation (6) implies $Q'_{c_0} < Q_{c_0}$. Finally, since $Q'_{c_0} < Q_{c_0}$ and $L'_{c_0+1} = L_{c_0+1}$, Equation (37)—in particular, the fact that $\partial Q_{c-1} / \partial N_c < 0$—implies $N'_{c_0+1} > N_{c_0+1}$, which contradicts Step 1. At this point, we have established that if $L'_{c_0} > L_{c_0}$ and $c_0 > 1$, then $N'_1 < N_1$. To conclude the proof of Step 2, note that if $N'_c < N_c$ for $c < c_0 - 1$, then $L'_{c+1} = L_{c+1}$ and Equation (25) imply $N'_{c+1} < N_{c+1}$. Thus by iteration, $N'_c < N_c$ for all $c < c_0$. Part I directly derives from Step 1, Step 2 and the fact that $\sum_{c=1}^{C} N'_c = \sum_{c=1}^{C} N_c = S$, by Lemma 1.

Part II: If $L'_{c_0} > L_{c_0}$, then $S'_c < S_c$ for all $c < c_0$ and $S'_c > S_c$ for all $C - 1 \geq c \geq c_0$.

The proof of Part II proceeds in two steps.

Step 1: If $L'_{c_0} > L_{c_0}$, then $S'_c > S_c$ for all $C - 1 \geq c \geq c_0$.

We proceed by iteration. Suppose that $c_0 \leq C - 1$. For country $C - 1$, we know from Part I that $S'_{C-1} = S - N'_C > S - N_C = S_{C-1}$. Thus $S'_c > S_c$ is satisfied for $c = C - 1$. Now suppose that $S'_c > S_c$ for $c > c_0 + 1$. Then $S'_{c-1} = S'_c - N'_c > S_c - N_c = S_{c-1}$, since $S'_c > S_c$ by assumption and $N'_c < N_c$ by Part I. This establishes that $S'_c > S_c$ for all $C - 1 \geq c \geq c_0$, which completes the proof of Step 1.

Step 2: If $L'_{c_0} > L_{c_0}$, then $S'_c < S_c$ for all $c < c_0$.

We proceed by iteration. Suppose that $c_0 > 1$. For country $c = 1$, we know from Part I that $S'_1 = N'_1 < N_1 = S_1$. Thus, $S'_c < S_c$ is satisfied for $c = 1$. Now suppose that $S'_{c-1} < S_{c-1}$ for $c < c_0$. Then $S'_c = N'_c + S'_{c-1} < N_c + S_{c-1} = S_c$, since $S'_{c-1} < S_{c-1}$ by assumption and $N'_c < N_c$ by Part I. This establishes that $S'_c < S_c$ for all $c < c_0$, which completes the proof of Step 2. Part II directly follows from Steps 1 and 2.

Part III: If $L'_{c_0} > L_{c_0}$, then there exists $c_0 + 1 \leq c_1 \leq C$ such that $(w_{c+1}/w_c)' < w_{c+1}/w_c$ for all $1 \leq c < c_0$; $(w_{c+1}/w_c)' > w_{c+1}/w_c$ for all $c_0 \leq c < c_1 - 1$; $\left(w_{c_1}/w_{c_1-1}\right)' \geq w_{c_1}/w_{c_1-1}$; and $(w_{c+1}/w_c)' < w_{c+1}/w_c$ for all $c_1 \leq c < C$.

The proof of Part III proceeds in three steps.

Step 1: If $L'_{c_0} > L_{c_0}$, then $(w_{c+1}/w_c)' < (w_{c+1}/w_c)$ for all $c < c_0$.

By Part I, we know that $N'_c < N_c$ for all $c < c_0$. Thus we can use the same argument as in Part III of the proof of Proposition 3 to show that $(w_{c+1}/w_c)' < (w_{c+1}/w_c)$ for all $c < c_0$.

Step 2: If $L'_{c_0} > L_{c_0}$, then there exists $c_0 \leq c_1 \leq C$ such that $(w_{c+1}/w_c)' > w_{c+1}/w_c$ for all $c_0 \leq c < c_1 - 1$; $\left(w_{c_1}/w_{c_1-1}\right)' \geq w_{c_1}/w_{c_1-1}$; and $(w_{c+1}/w_c)' < w_{c+1}/w_c$ for all $c_1 \leq c < C$.

By Part I, we know that $N'_c < N_c$ for all $c > c_0$. Thus we can again use the same argument as in Part III of the proof of Proposition 3 to show that if there exists $\widetilde{c} \geq c_0$ such that $(w_{\widetilde{c}+1}/w_{\widetilde{c}})' \leq (w_{\widetilde{c}+1}/w_{\widetilde{c}})$, then $(w_{c+1}/w_c)' < (w_{c+1}/w_c)$ for all $\widetilde{c} \leq c < C$. To conclude the proof of Step 2, let us just define $c_1 \equiv \inf\left\{c \geq c_0 | (w_{c+1}/w_c)' < (w_{c+1}/w_c)\right\}$. By construction, w_{c+1}/w_c rises for all $c_0 \leq c < c_1$ and falls for all $c_1 \leq c < C$. To complete the proof of Part III, the only thing left to show is that $c_1 > c_0$, which is what we establish in our final step.

Step 3: If $L'_{c_0} > L_{c_0}$ and $c_0 \neq C$, then $\left(w_{c_0+1}/w_{c_0}\right)' > \left(w_{c_0+1}/w_{c_0}\right)$.

By Part II, we already know that $S'_c < S_c$ for all $c \in \{1, ..., c_0 - 1\}$ and $S'_c > S_c$ $\{c_0, ..., C-1\}$. Since the optimal allocation is the solution of a well-behaved planning problem, the maximum theorem implies the continuity of the pattern of vertical specialization in L_{c_0}. Hence if the change from L_{c_0} to $L'_{c_0} > L_{c_0}$ is small enough, the following chain of inequalities must hold:

$$S'_1 < S_1 < S'_2 < \cdots < S'_{c_0-1} < S_{c_0-1} < S_{c_0} < S'_{c_0} < S_{c_0+1} < \cdots < S'_{C-2} < S_{C-1} < S'_{C-1}.$$

We first focus on this situation. For any $c \in \{1,...,c_0-2\}$, since $S'_c < S_c < S'_{c+1} < S_{c+1}$, condition (2) implies

$$\frac{p'\left(S'_{c+1}\right)}{w'_{c+1}} = \frac{e^{\lambda_{c+1}\left(S'_{c+1}-S_c\right)}p'\left(S_c\right)}{w'_{c+1}} + \frac{e^{\lambda_{c+1}\left(S'_{c+1}-S_c\right)}-1}{\lambda_{c+1}},$$

$$\frac{p\left(S'_{c+1}\right)}{w_{c+1}} = \frac{e^{\lambda_{c+1}\left(S'_{c+1}-S_c\right)}p\left(S_c\right)}{w_{c+1}} + \frac{e^{\lambda_{c+1}\left(S'_{c+1}-S_c\right)}-1}{\lambda_{c+1}}.$$

Since $S'_{c+1} > S_c$, the two previous equations further imply that for any $c \in \{1,...,c_0-2\}$,

$$p'\left(S_c\right)/w'_{c+1} \geq p\left(S_c\right)/w_{c+1} \Rightarrow p'\left(S'_{c+1}\right)/w'_{c+1} \geq p\left(S'_{c+1}\right)/w_{c+1}. \tag{39}$$

In addition, for any $c \in \{1,...,c_0-1\}$, since $S_{c-1} < S'_c < S_c$, condition (2) also implies that

$$\frac{p'\left(S_c\right)}{w'_{c+1}} = \frac{e^{\lambda_{c+1}\left(S_c-S'_c\right)}p'\left(S'_c\right)}{w'_{c+1}} + \frac{e^{\lambda_{c+1}\left(S_c-S'_c\right)}-1}{\lambda_{c+1}}, \tag{40}$$

$$\frac{p\left(S_c\right)}{w_{c+1}} = \frac{e^{\lambda_c\left(S_c-S'_c\right)}p\left(S'_c\right)}{w_{c+1}} + \frac{e^{\lambda_c\left(S_c-S'_c\right)}-1}{\lambda_c}\left(\frac{w_c}{w_{c+1}}\right). \tag{41}$$

Let us now show that if $p'\left(S'_c\right)/w'_c \geq p\left(S'_c\right)/w_c$, then

$$\frac{e^{\lambda_{c+1}\left(S_c-S'_c\right)}p'\left(S'_c\right)}{w'_{c+1}} \geq \frac{e^{\lambda_c\left(S_c-S'_c\right)}p\left(S'_c\right)}{w_{c+1}}, \tag{42}$$

$$\frac{e^{\lambda_{c+1}\left(S_c-S'_c\right)}-1}{\lambda_{c+1}} \geq \frac{e^{\lambda_c\left(S_c-S'_c\right)}-1}{\lambda_c}\left(\frac{w_c}{w_{c+1}}\right). \tag{43}$$

We start with Inequality (42), which can be rearranged as

$$e^{\lambda_{c+1}\left(S_c-S'_c\right)}\frac{w'_c}{w'_{c+1}}\frac{p'\left(S'_c\right)}{w'_c} \geq e^{\lambda_c\left(S_c-S'_c\right)}\frac{w_c}{w_{c+1}}\frac{p\left(S'_c\right)}{w_c}.$$

By Equation (7), we know that

$$e^{\lambda_c\left(S_c-S'_c\right)}\frac{w_c}{w_{c+1}}\frac{p\left(S'_c\right)}{w_c} = \frac{\frac{p\left(S'_c\right)}{w_c}}{1+\left(\lambda_c-\lambda_{c+1}\right)\frac{p\left(S_c\right)}{w_c}}, \tag{44}$$

$$e^{\lambda_{c+1}\left(S_c-S'_c\right)}\frac{w'_c}{w'_{c+1}}\frac{p'\left(S'_c\right)}{w'_c} = \frac{\frac{p'\left(S'_c\right)}{w'_c}}{1+\left(\lambda_c-\lambda_{c+1}\right)\frac{p'\left(S'_c\right)}{w'_c}}. \tag{45}$$

Under the assumption that $p'\left(S'_c\right)/w'_c \geq p\left(S'_c\right)/w_c$, Equations (44) and (45) imply

$$e^{\lambda_{c+1}\left(S_c-S'_c\right)}\frac{w'_c}{w'_{c+1}}\frac{p'\left(S'_c\right)}{w'_c} \geq \frac{\frac{p\left(S'_c\right)}{w_c}}{1+\left(\lambda_c-\lambda_{c+1}\right)\frac{p\left(S'_c\right)}{w_c}} > e^{\lambda_c\left(S_c-S'_c\right)}\frac{w_c}{w_{c+1}}\frac{p\left(S'_c\right)}{w_c},$$

where the second inequality also uses the fact that $S'_c < S_c$. Thus Inequality (42) holds. Let us now consider inequality (43), which can be rearranged as

$$\frac{\lambda_c\left(e^{\lambda_{c+1}\left(S_c-S'_c\right)}-1\right)}{\lambda_{c+1}\left(e^{\lambda_c\left(S_c-S'_c\right)}-1\right)} \geq \frac{w_c}{w_{c+1}}. \tag{46}$$

Since $S_{c-1} < S'_c$ for any $c \in \{1,...,c_0-1\}$, Condition (2) implies $p\left(S_c\right)/w_c \geq \left[e^{\lambda_c\left(S_c-S_{c-1}\right)}-1\right]/\lambda_c > \left[e^{\lambda_c\left(S_c-S'_c\right)}-1\right]/\lambda_c$. Combining the previous inequality with Equation (7), we obtain

$$\frac{w_c}{w_{c+1}} = \frac{1}{1+\left(\lambda_c-\lambda_{c+1}\right)\frac{p\left(S_c\right)}{w_c}} < \frac{\lambda_c}{\lambda_c+\left(\lambda_c-\lambda_{c+1}\right)\left[e^{\lambda_c\left(S_c-S'_c\right)}-1\right]}. \tag{47}$$

By Inequalities (46) and (47), a sufficient condition for Inequality (43) to hold is

$$\frac{\lambda_c\left[e^{\lambda_{c+1}\left(S_c-S'_c\right)}-1\right]}{\lambda_{c+1}\left[e^{\lambda_c\left(S_c-S'_c\right)}-1\right]} \geq \frac{\lambda_c}{\lambda_c+\left(\lambda_c-\lambda_{c+1}\right)\left[e^{\lambda_c\left(S_c-S'_c\right)}-1\right]},$$

which can be rearranged as $\lambda_c/\left[1-e^{-\lambda_c\left(S_c-S'_c\right)}\right] \geq \lambda_{c+1}/\left[1-e^{-\lambda_{c+1}\left(S_c-S'_c\right)}\right]$. The previous inequality necessarily holds since $f(x) \equiv \frac{x}{1-e^{-tx}}$ is increasing in x for $t>0$. At this point, we have established that Inequalities (42) and (43) hold if $p'\left(S'_c\right)/w'_c \geq p\left(S'_c\right)/w_c$. Combining this observation with Equations (40) and (41), we further have that for any $c \in \{1,...,c_0-1\}$,

$$p'\left(S'_c\right)/w'_c \geq p\left(S'_c\right)/w_c \Rightarrow p'\left(S_c\right)/w'_{c+1} \geq p\left(S_c\right)/w_{c+1}. \tag{48}$$

Since $p'(0)=p(0)=0$, we know that $p'(S_0)/w'_1 \geq p(S_0)/w_1$. Thus we can use implications (39) and (48) to establish, by iteration, that

$$\frac{p'\left(S_{c_0-1}\right)}{w'_{c_0}} \geq \frac{p\left(S_{c_0-1}\right)}{w_{c_0}}. \tag{49}$$

Since $S_{c_0} < S'_{c_0}$, we know from Condition (2) that

$$\frac{p'\left(S_{c_0}\right)}{w'_{c_0}} = \frac{e^{\lambda_{c_0}\left(S_{c_0}-S_{c_0-1}\right)}p'\left(S_{c_0-1}\right)}{w'_{c_0}} + \frac{e^{\lambda_{c_0}\left(S_{c_0}-S_{c_0-1}\right)}-1}{\lambda_{c_0}}, \tag{50}$$

$$\frac{p\left(S_{c_0}\right)}{w_{c_0}} = \frac{e^{\lambda_{c_0}\left(S_{c_0}-S_{c_0-1}\right)}p\left(S_{c_0-1}\right)}{w_{c_0}} + \frac{e^{\lambda_{c_0}\left(S_{c_0}-S_{c_0-1}\right)}-1}{\lambda_{c_0}}. \tag{51}$$

Inequality (49) and Equations (50) and (51) imply $p'\left(S_{c_0}\right)/w'_{c_0} \geq p\left(S_{c_0}\right)/w_{c_0}$. Finally, since $S_{c_0} < S'_{c_0}$, we also know that $p'\left(S'_{c_0}\right)/w'_{c_0} > p'\left(S_{c_0}\right)/w'_{c_0}$. Combining these two observations, we get $p'\left(S'_{c_0}\right)/w'_{c_0} > p\left(S_{c_0}\right)/w_{c_0}$. Together with Equation (7), the previous inequality implies $\left(w_{c_0+1}/w_{c_0}\right)' > \left(w_{c_0+1}/w_{c_0}\right)$, which implies $c_1 \equiv \inf\left\{c \geq c_0 | (w_{c+1}/w_c)' < (w_{c+1}/w_c)\right\} > c_0$. This completes the proof of Step 3 for a small enough change from L_{c_0} to $L'_{c_0} > L_{c_0}$. Since the previous result holds for any initial value of L_{c_0}, it must hold for large changes as well. As mentioned above, Part III directly follows from Steps 1–3. ‖

Proof of Proposition 6. We decompose the proof of Proposition 6 into three parts. First, we show that a decrease in λ_{c_0} increases the measure of stages N_c performed in all countries $c < c_0$ and decreases the measure of stages N_c performed in all countries $c > c_0$. Second, we show that a decrease in λ_{c_0} leads all countries to move up. Third, we show that a decrease in λ_{c_0} increases inequality among countries $c \in \{1,...,c_0\}$, decreases inequality among countries $c \in \{c_0, c_0+1\}$, increases inequality among countries $c \in \{c_0+1,...,c_1\}$, and decreases inequality among countries $c \in \{c_1,...,C\}$, with $c_1 \in \{c_0+1,...,C\}$.

Part I: If $\lambda'_{c_0} < \lambda_{c_0}$, then $N'_c > N_c$ for all $c < c_0$ and $N'_c < N_c$ for all $c > c_0$.

Like in our previous proofs, we will repeatedly use the following relationship

$$N_c = -\left(\frac{1}{\lambda_c}\right)\ln\left[1-\left(\frac{\lambda_c L_c}{\lambda_{c-1}L_{c-1}}\right)\left(e^{\lambda_{c-1}N_{c-1}}-1\right)\right], \text{ for all } c>1, \tag{23}$$

where $\partial N_c/\partial N_{c-1} > 0$. The proof of Part I proceeds in two steps.

Step 1: If $\lambda'_{c_0} < \lambda_{c_0}$, then $N'_c < N_c$ for all $c > c_0$.

Let us first establish that $Q'_C > Q_C$. By the same argument as in Step 1 of Proposition 5, Q'_C must be such that

$$Q'_C = \underset{\widetilde{Q}_1(\cdot),...,\widetilde{Q}_C(\cdot)}{\arg\max} \quad \textstyle\sum_{c=1}^{C}\widetilde{Q}_c(S),$$

subject to

$$\textstyle\sum_{c=1}^{C}\widetilde{Q}_c(s_2) - \sum_{c=1}^{C}\widetilde{Q}_c(s_1) \;\leq\; -\int_{s_1}^{s_2}\sum_{c=1}^{C}\lambda'_c\widetilde{Q}_c(s)ds, \text{ for all } s_1 \leq s_2, \tag{52}$$

$$\int_0^S \widetilde{Q}_c(s)ds \;\leq\; L_c, \text{ for all } c \in \mathcal{C}, \tag{53}$$

where $\lambda'_{c_0} < \lambda_{c_0}$ and $\lambda'_c = \lambda_c$ for all $c \neq c_0$. Now consider $\widetilde{Q}_1(\cdot),...,\widetilde{Q}_C(\cdot)$ such that

$$\begin{aligned}\widetilde{Q}_{c_0}(s) \equiv\; & e^{-\left(\lambda_{c_0}-\lambda'_{c_0}\right)\left(S_{c_0}-s\right)}Q_{c_0}(s)\\ &+\left(\frac{\lambda'_{c_0}e^{-\lambda'_{c_0}s}}{1-e^{-\lambda'_{c_0}S}}\right)\int_{S_{c_0-1}}^{S_{c_0}}\left[1-e^{-\left(\lambda_{c_0}-\lambda'_{c_0}\right)\left(S_{c_0}-t\right)}\right]Q_{c_0}(t)dt, \text{ for all } s \in \mathcal{S},\end{aligned}$$

and

$$\widetilde{Q}_c(s) \equiv Q_c(s), \text{ for all } s \in \mathcal{S} \text{ and } c \neq c_0.$$

Since $Q_1(\cdot),\ldots,Q_C(\cdot)$ satisfy the initial resource and technological constraints, as described by Equations (3) and (4), $\widetilde{Q}_1(\cdot),\ldots,\widetilde{Q}_C(\cdot)$ must satisfy, by construction, the new resource and technological constraints, as described by Equations (52) and (53). Since $\lambda'_{c_0} < \lambda_{c_0}$, we must also have

$$\widetilde{Q}_{c_0}(S)+\widetilde{Q}_C(S)=\left(\frac{\lambda_{c_0}e^{-\lambda_{c_0}S}}{1-e^{-\lambda_{c_0}S}}\right)\int_{S_{c_0-1}}^{S_{c_0}}\left[1-e^{-\left(\lambda_{c_0}-\lambda'_{c_0}\right)(S_{c_0}-t)}\right]Q_{c_0}(t)\,dt+Q_C>Q_C.$$

Since $Q'_C \geq \widetilde{Q}_{c_0}(S)+\widetilde{Q}_C(S)$, the previous inequality implies $Q'_C > Q_C$. Combining this observation with Equation (38) and the fact that $c_0 \neq C$, which implies $\lambda'_C=\lambda_C$, we get $N'_C < N_C$. To conclude the proof of Step 1, note that if $N'_c < N_c$ for $c > c_0+1$, then $\lambda'_{c-1}=\lambda_{c-1}$ and Equation (25)—the fact that $\partial N_c/\partial N_{c-1}>0$—imply $N'_{c-1} < N_{c-1}$. Thus by iteration, $N'_c < N_c$ for all $c > c_0$.

Step 2: If $\lambda'_{c_0} < \lambda_{c_0}$, then $N'_c > N_c$ for all $c < c_0$.

We first show by contradiction that if $\lambda'_{c_0} < \lambda_{c_0}$ and $c_0>1$, then $N'_1 > N_1$. Suppose that $N'_1 \leq N_1$. Since $\lambda'_c=\lambda_c$ for all $c<c_0$, we can use Equation (25)—the fact that $\partial N_c/\partial N_{c-1}>0$—to establish by iteration that $N'_c \leq N_c$ for all $c<c_0$. Since λ_c is strictly decreasing in c, Equation (3) therefore implies

$$\sum_{c=1}^{C} Q'_c(s) \geq \sum_{c=1}^{C} Q_c(s), \text{ for all } s \leq S'_{c_0-1}.$$

By Step 1, we also know that $N'_c < N_c$ for all $c > c_0$. Thus $N'_{c_0}=S-\sum_{c\neq c_0}N'_c > S-\sum_{c\neq c_0}N_c=N_{c_0}$. Since $\lambda'_{c_0}<\lambda_{c_0}$, Equation (3) therefore also implies

$$\sum_{c=1}^{C} Q'_c(s) > \sum_{c=1}^{C} Q_c(s), \text{ for all } S'_{c_0-1} \leq s \leq S_{c_0}.$$

Using the fact that $N'_c \leq N_c$ for all $c<c_0$ and $N'_c<N_c$ for all $c>c_0$, one can easily establish by iteration, as we do in Part II, that $S'_{c_0-1} \leq S_{c_0-1}$ and $S'_{c_0} > S_{c_0}$. Combining these two observations with the previous inequality, we obtain

$$\int_{S'_{c_0-1}}^{S'_{c_0}} \sum_{c=1}^{C} Q'_c(s)ds > \int_{S_{c_0-1}}^{S_{c_0}} \sum_{c=1}^{C} Q'_c(s)ds > \int_{S_{c_0-1}}^{S_{c_0}} \sum_{c=1}^{C} Q_c(s)ds,$$

which contradicts the fact that $\int_{S'_{c_0-1}}^{S'_{c_0}}\sum_{c=1}^{C}Q'_c(s)ds=\int_{S_{c_0-1}}^{S_{c_0}}\sum_{c=1}^{C}Q_c(s)ds=L_{c_0}$, by Equation (4). At this point, we have established that if $\lambda'_{c_0}<\lambda_{c_0}$ and $c_0>1$, then $N'_1>N_1$. To conclude, note that if $N'_c>N_c$ for $c<c_0-1$, then $\lambda'_c=\lambda_c$, $\lambda'_{c+1}=\lambda_{c+1}$, and Equation (25) imply $N'_{c+1}>N_{c+1}$. Thus by iteration, $N'_c>N_c$ for all $c<c_0$. Part I directly derives from Steps 1 and 2.

Part II: If $\lambda'_{c_0}<\lambda_{c_0}$, then $S'_c>S_c$ for all $c\in\{1,\ldots,C-1\}$.

The proof of Part II proceeds in two steps.

Step 1: If $\lambda'_{c_0}<\lambda_{c_0}$, then $S'_c>S_c$ for all $c\in\{c_0,\ldots,C-1\}$.

The proof is identical to the proof of Step 1 Part II of Proposition 5 and omitted.

Step 2: If $\lambda'_{c_0}<\lambda_{c_0}$, then $S'_c>S_c$ for all $c\in\{1,\ldots,c_0-1\}$.

We proceed by iteration. Suppose that $c_0>1$. For country $c=1$, we know from Part I that $S'_1=N'_1>N_1=S_1$. Thus, $S'_c>S_c$ is satisfied for $c=1$. Now suppose that $S'_{c-1}>S_{c-1}$ for $c<c_0$. Then $S'_c=N'_c+S'_{c-1}>N_c+S_{c-1}=S_c$, since $S'_{c-1}>S_{c-1}$ by assumption and $N'_c>N_c$ by Part I. This establishes that $S'_c>S_c$ for all $c<c_0$, which completes the proof of Step 2. Part II directly follows from Steps 1 and 2.

Part III: If $\lambda'_{c_0}<\lambda_{c_0}$, then there exist $c_0<c_1\leq C$ such that $(w_{c+1}/w_c)'>w_{c+1}/w_c$ for all $c<c_0$; $\left(w_{c_0+1}/w_{c_0}\right)'<w_{c_0+1}/w_{c_0}$; $(w_{c+1}/w_c)'>w_{c+1}/w_c$ for all $c_0<c<c_1-1$; $\left(w_{c_1+1}/w_{c_1}\right)'\geq w_{c_1+1}/w_{c_1}$; and $(w_{c+1}/w_c)'<w_{c+1}/w_c$ for all $c_1\leq c<C$.

The proof of Part III proceeds in four steps.

Step 1: If $\lambda'_{c_0}<\lambda_{c_0}$, then $(w_{c+1}/w_c)'>(w_{c+1}/w_c)$ for all $c<c_0$.

By Part I, we know that $N'_c>N_c$ for all $c<c_0$. Thus we can use the same argument as in Part III of the proof of Proposition 3 to show that $(w_{c+1}/w_c)'>(w_{c+1}/w_c)$ for all $c<c_0$.

Step 2: If $\lambda'_{c_0} < \lambda_{c_0}$ and $c_0 > 1$, then $\lambda'_{c_0}\left(S'_{c_0} - S'_{c_0-1}\right) < \lambda_{c_0}(S_{c_0} - S'_{c_0-1})$.

By Part II, we already know that $S'_c > S_c$ for all $c \in \{1,...,C-1\}$. Since the optimal allocation is the solution of a well-behaved planning problem, the maximum theorem implies the continuity of the pattern of vertical specialization in λ_{c_0}. Hence if the change from λ_{c_0} to $\lambda'_{c_0} < \lambda_{c_0}$ is small enough, the following chain of inequalities must hold:

$$S_1 < S'_1 < S_2 < \cdots < S'_{C-2} < S_{C-1} < S'_{C-1}.$$

We first focus on this situation. In the proof of Step 2, we let $Q'(s)$ and $p'(s)$, denote the output at stage s and the price of stage s if the failure rate in country c_0 is equal to λ'_{c_0}. From Equation (38), we have

$$Q_1 = \frac{\lambda_1 L_1 e^{-\lambda_1 N_1}}{1 - e^{-\lambda_1 N_1}}.$$

Similarly, we have

$$Q'(S_1) = \frac{\lambda_1 L'_1 e^{-\lambda_1 N'_1}}{1 - e^{-\lambda_1 N'_1}},$$

where L'_1 is the amount of labor from country 1 used to perform stages $(0, S_1)$ when the failure rate of country c_0 is equal to λ'_{c_0}. The two previous equations, together with $N'_1 > N_1$ and $L'_1 < L_1$, therefore, imply $Q'(S_1) < Q_1$. Assume that $Q'(S_c) < Q_c$ holds for some $1 \le c \le c_0 - 2$. Since $c+1 < c_0$, we have

$$\begin{aligned} Q_{c+1} &= e^{-\lambda_{c+1} N_{c+1}} Q_c > e^{-\lambda_{c+1}(S_{c+1} - S'_c + S'_c - S_c)} Q'(S_c) \\ &\ge e^{-\lambda_c (S'_c - S_c) - \lambda_{c+1}(S_{c+1} - S'_c)} Q'(S_c) = e^{-\lambda_{c+1}(S_{c+1} - S'_c)} Q'(S'_c) = Q'(S_{c+1}). \end{aligned}$$

Therefore, by iteration, we obtain

$$Q'(S_c) < Q_c \text{ for all } 1 \le c \le c_0 - 1. \tag{54}$$

By Equation (54), which implies $Q'(S_{c_0-1}) < Q_{c_0-1}$, and Equation (1), we have

$$\begin{aligned} Q'_{c_0} &= e^{-\lambda'_{c_0}(S'_{c_0} - S'_{c_0-1}) - \lambda_{c_0-1}(S'_{c_0-1} - S_{c_0-1})} Q'(S_{c_0-1}) \\ &< e^{-\lambda'_{c_0}(S'_{c_0} - S'_{c_0-1}) - \lambda_{c_0-1}(S'_{c_0-1} - S_{c_0-1})} Q_{c_0-1}. \end{aligned} \tag{55}$$

Equation (6) implies

$$Q_{c_0} = e^{-\lambda_{c_0}(S_{c_0} - S_{c_0-1})} Q_{c_0-1}. \tag{56}$$

By the proof of Part I of Proposition 6, we have $N'_{c_0+1} < N_{c_0+1}$. Equation (37) and $N'_{c_0+1} < N_{c_0+1}$ imply $Q'_{c_0} > Q_{c_0}$. Equation (55), Equation (56), and $Q'_{c_0} > Q_{c_0}$ imply

$$e^{-\lambda'_{c_0}\left(S'_{c_0} - S'_{c_0-1}\right) - \lambda_{c_0-1}\left(S'_{c_0-1} - S_{c_0-1}\right)} > e^{-\lambda_{c_0}\left(S_{c_0} - S_{c_0-1}\right)},$$

which implies

$$S'_{c_0} - S'_{c_0-1} < \frac{\lambda_{c_0}}{\lambda'_{c_0}}(S_{c_0} - S'_{c_0-1}) - \frac{\lambda_{c_0-1} - \lambda_{c_0}}{\lambda'_{c_0}}(S'_{c_0-1} - S_{c_0-1}) \le \frac{\lambda_{c_0}}{\lambda'_{c_0}}(S_{c_0} - S'_{c_0-1}),$$

concluding the proof of Step 2 for a small enough change from λ_{c_0} to $\lambda'_{c_0} < \lambda_{c_0}$. Since the previous result holds for any initial value of λ_{c_0}, it must hold for large changes as well.

Step 3: If $\lambda'_{c_0} < \lambda_{c_0}$ and $c_0 < C$, then $\left(w_{c_0+1}/w_{c_0}\right)' < w_{c_0+1}/w_{c_0}$.

Like in the previous step, we first consider a change from λ_{c_0} to $\lambda'_{c_0} < \lambda_{c_0}$ small enough for the following chain of inequalities to hold:

$$S_1 < S'_1 < S_2 < ... < S'_{C-2} < S_{C-1} < S'_{C-1}.$$

In the same way as we have proceeded in Part III Step 3 of Proposition 5, one can show by iteration that

$$\frac{p'(S'_{c_0-1})}{w'_{c_0}} \le \frac{p(S'_{c_0-1})}{w_{c_0}}.$$

By Condition (2) and Equation (7), we know that

$$\frac{w'_{c_0+1}}{w'_{c_0}} - 1 = (\lambda'_{c_0} - \lambda_{c_0+1}) \left[\frac{p'\left(S'_{c_0-1}\right)}{w'_{c_0}} e^{\lambda'_{c_0}\left(S'_{c_0} - S'_{c_0-1}\right)} + \frac{e^{\lambda'_{c_0}\left(S'_{c_0} - S'_{c_0-1}\right)} - 1}{\lambda'_{c_0}} \right].$$

Combining the two previous expressions with Step 2, we therefore get

$$\frac{w'_{c_0+1}}{w'_{c_0}}-1<(\lambda'_{c_0}-\lambda_{c_0+1})\left[\frac{p\left(S'_{c_0-1}\right)}{w_{c_0}}e^{\lambda_{c_0}\left(S_{c_0}-S'_{c_0-1}\right)}+\frac{e^{\lambda_{c_0}\left(S_{c_0}-S'_{c_0-1}\right)}-1}{\lambda'_{c_0}}\right]$$

Since $\lambda'_{c_0}<\lambda_{c_0}$, this implies

$$\frac{w'_{c_0+1}}{w'_{c_0}}-1<(\lambda_{c_0}-\lambda_{c_0+1})\left[\frac{p(S'_{c_0-1})}{w_{c_0}}e^{\lambda_{c_0}(S_{c_0}-S'_{c_0-1})}+\frac{e^{\lambda_{c_0}(S_{c_0}-S'_{c_0-1})}-1}{\lambda_{c_0}}\right]. \tag{57}$$

But by Condition (2) and Equation (7), we also know that

$$\frac{w_{c_0+1}}{w_{c_0}}-1=(\lambda_{c_0}-\lambda_{c_0+1})\left[\frac{p\left(S'_{c_0-1}\right)}{w_{c_0}}e^{\lambda_{c_0}\left(S_{c_0}-S'_{c_0-1}\right)}+\frac{e^{\lambda_{c_0}\left(S_{c_0}-S'_{c_0-1}\right)}-1}{\lambda_{c_0}}\right]. \tag{58}$$

Equations (57) and (58) imply $w'_{c_0+1}/w'_{c_0}<w_{c_0+1}/w_{c_0}$. This completes the proof of Step 3 for a small enough change from λ_{c_0} to $\lambda'_{c_0}<\lambda_{c_0}$. Since the previous result holds for any initial value of λ_{c_0}, it must hold for large changes as well.

Step 4: If $\lambda'_{c_0}<\lambda_{c_0}$, then there exists $c_0<c_1\le C$ such that $(w_{c+1}/w_c)'>w_{c+1}/w_c$ for all $c_0<c<c_1-1$; $\left(w_{c_1}/w_{c_1-1}\right)'\ge w_{c_1}/w_{c_1-1}$; and $(w_{c+1}/w_c)'<w_{c+1}/w_c$ for all $c_1\le c<C$.

By Part I, we also know that $N'_c<N_c$ for all $c>c_0$. Thus we can again use the same argument as in Part III of the proof of Proposition 3 to show that if there exists $\widetilde{c}>c_0$ such that $(w_{\widetilde{c}+1}/w_{\widetilde{c}})'\le(w_{\widetilde{c}+1}/w_{\widetilde{c}})$, then $(w_{c+1}/w_c)'<(w_{c+1}/w_c)$ for all $\widetilde{c}\le c<C$. To conclude the proof of Step 4, let us just define $c_1\equiv\inf\left\{c>c_0|(w_{c+1}/w_c)'<(w_{c+1}/w_c)\right\}$. By construction, w_{c+1}/w_c rises for all $c_0<c<c_1$ and falls for all $c_1\le c<C$. This concludes the proof of Part III. ‖

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