BILINEAR SPACE-TIME ESTIMATES FOR LINEARIZED KP-TYPE EQUATIONS WITH SEMIPERIODIC AND PERIODIC DATA

AXEL GRÜNROCK, UNIVERSITÄT BONN; ON JOINT WORK WITH M. PANTHEE AND J. SILVA

We consider the initial value problem for the linearized Kadomtsev-Petviashvili type equation

$$\begin{cases} \partial_t u - i\phi(D_x, D_y)u := \partial_t u - i\phi_0(D_x)u + \partial_x^{-1}\Delta_y u = 0 \\ u(0,x,y) = u_0(x,y), \end{cases} \tag{1}$$

where $(x, y) \in \mathbb{T} \times \mathbb{R}$, $\mathbb{T} \times \mathbb{R}^2$, or $\mathbb{T} \times \mathbb{T}^2$. The data are assumed to belong to an anisotropic Sobolev space $H_x^s H_y^\varepsilon$ and to satisfy a mean zero condition in the x-variable. For the phase function ϕ_0 we usually have $\phi_0(k) = k|k|^\alpha$ with $\alpha \geq 2$, but several arguments work for an arbitrary odd function ϕ_0.

The product uv of two such solutions is estimated in L^2_{xyt} - partially local in time - by $\|u_0\|_{H_x^{s_1} H_y^{\varepsilon_1}} \|v_0\|_{H_x^{s_2} H_y^{\varepsilon_2}}$, with essentially optimal Sobolev exponents. The argument relies heavily on the Strichartz estimates for the free Schrödinger equation.

Several applications to local well-posedness of dispersion generalized KP-II equations are discussed. In two dimensions and for higher dispersion in three dimensions the local results can be combined with the conservation of the L^2-norm to obtain global well-posedness.