A Centre-Stable Manifold in $H^{\frac{1}{2}}$ for the $H^{\frac{1}{2}}$ Critical NLS

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Consider the equation
$$i\partial_t\psi + \Delta\psi + |\psi|^2\psi = 0.$$
It admits an eight-dimensional manifold of periodic solutions called solitons
$$e^{i(\Gamma+vx-t|v|^2+\alpha^2 t)}\phi(x-2tv-D,\alpha),$$
where $\phi(x,\alpha)$ is a positive ground state solution of the semilinear elliptic equation
$$-\Delta\phi + \alpha^2\phi = \phi^3.$$
We prove that in the neighborhood of the soliton manifold there exists a $H^{\frac{1}{2}}$ Lipschitz manifold N of asymptotically stable solutions, meaning they are the sum of a moving soliton and a dispersive term.
Furthermore, a solution starting on N remains on N for all positive time and for some finite negative time and N can be identified as the centre-stable manifold for this equation.
The result depends on a spectral assumption concerning the absence of embedded eigenvalues.