Logical Idealism & Einstein's Theory of Relativity

Schlick's Critique of Cassirer's Monograph *Zur Einsteinschen Relativitätstheorie* (1921)

Jacob Portes
PHIL G9062: Philosophy of Space and Time
June 30, 2015

When Einstein's general theory of relativity was formulated during the second decade of the 20th century, philosophers of both the neo-Kantian and logical empiricist variety scrambled to fit the revolutionary theory into their respective philosophical frameworks. A few even went so far as to claim that the new theory was an unambiguous confirmation of their particular philosophy of science. One of the first serious arguments of this kind was published in a 1921 monograph by the neo-Kantian Ernst Cassirer entitled *Zur Einsteinschen Relativitätstheorie* ("Einstein's Theory of Relativity").[1] This book was also the subject of a decisive critique, published soon afterwards by the champion of logical empiricism Moritz Schlick, which dismissed it as a worthy but ultimately unsuccessful attempt to incorporate the new relativity into Kantian epistemology.[2] Interestingly, some write that this article was in many ways responsible for the rise of logical empiricism and the precipitous decline of neo-Kantian thought among philosophers of science (logical idealism); that the debate between neo-Kantian and empiricist philosophy over relativity theory "effectively ended with Schlick's essay"[3] and that the article "may well be regarded as the point of departure of a new direction for scientific philosophy [i.e. logical empiricism]."[4]

A few scholars have noted, however, that Schlick's argument does not address Cassirer's neo-Kantian epistemology and instead attacks the straw man of traditional Kantian epistemology – an easy target indeed, considering that Kant relied heavily on "outdated" ideas from Euclidean geometry and Newtonian mechanics. More specifically, Schlick argues that the new theory of relativity abolishes all old notions of the synthetic a priori and that Cassirer fails to specify *new* notions of the synthetic a priori in the new theory. However, he defines synthetic a priori in a traditional Kantian manner, and essentially dismisses Cassirer's widely accepted neo-Kantian (Marburg school) understanding of the synthetic a priori outlined in his book and in previous publications as "transcending the region of critical [i.e. Kantian] philosophy proper."[5] This paper attempts to systematically argue that (1) Cassirer does indeed fail to incorporate Einstein's theory of relativity into the traditional Kantian notion of constitutive synthetic a priori (as argued by Schlick), but that (2) Cassirer successfully incorporates Einstein's theory of relativity into his neo-Kantian notion of regulative synthetic a priori.

This paper tentatively concludes that the oft-stated claim that Einstein's theory of general relativity decisively refutes neo-Kantian philosophy of science in favor logical empiricism is, to a large extent, unwarranted.[6] The question of whether general relativity is an unambiguous confirmation of logical idealism is left for a future discussion. In order to lay the groundwork for arguments (1) and (2), I briefly review the broad historical and philosophical contexts that led to the development of the theory of general relativity, including the substantivalist/relationalist divide, the developments of non-Euclidean geometry, and the theory of general relativity itself. I then touch on the traditional Kantian notions of the synthetic a priori, the role of space and time

---

[1] Ernst Cassirer, *Substance and Function and Einstein's Theory of Relativity*, trans. William and Marie Swabey (Chicago: Dover Publications, Inc., 1923). The translated edition contains both monographs in a single volume.

[2] Moritz Schlick, *Kritizistische oder empiristische Deutung der neuen Physik?*, Kant-Studien 26 (1921) 96-11. Translated by Peter Heath, "Critical or Empiricist Interpretation of Modern Physics?: Remarks on Ernst Cassirer's *Einstein's Theory of Relativity*" reprinted in *Moritz Schlick: Philosophical Papers Volume I (1909-1922)* Henk L. Mulder, 1979

[3] Thomas Ryckman, *The Reign of Relativity: Philosophy in Physics, 1915-1925* (Oxford University Press, 2005), 50

[4] Ibid.

[5] Schlick, *Philosophical Papers*, 332. Cassirer did slightly innovate in this regard, although Schlick also ignores this.

[6] I would like to note here that the motivation to write this paper came from a short discussion of the Cassirer-Schlick debate in Ryckman, *The Reign of Relativity,* pp. 47-59

in the Transcendental Aesthetic,[7] the innovations of the Marburg school and of Cassirer, and Cassirer's 1921 monograph. Finally, I critique Schlick's 1921 article.

Historical and Philosophical Context of General Relativity: Substantivalism vs. Relationalism

Einstein's theories of special and general relativity were developed with explicit philosophical motivations – in particular, Einstein was motivated by Leibnizean relationalism and Machian empiricism. Informally put, relationalism is the general view that the only quantities of motion are relative quantities, relative velocity, acceleration and so on. In particular, according to this view, *space is nothing more than a set of relations between particular objects*. Relationalism is often associated with Leibniz, as he espoused a particular form of relationalism in his famous debate with Newton supporter Samuel Clarke.[8] Substantivalism, on the other hand, is the view espoused by Newton (among others) that space is an *absolute object in and of itself*, and is a sort of *privileged frame of reference to which all motion refers*.[9] Kant, while at times alternating between substantivalism and relationalism, ultimately argues for an understanding of space very similar to Leibniz's relationalism (more on that later).

A watered down version of Leibniz's well-known argument for relationalism is this: according to Newton's absolutist theory, there is no way to distinguish between a particular arrangement of objects in the universe, and the same exact arrangement of objects in the universe *all shifted by an arbitrary distance*. In fact there are an infinite number of distinct ways of spatially locating the same exact arrangement of objects in the universe. However, even according to Newton, there is no way to detect these differences. Therefore, it is unnecessary to treat the universe as located in absolute space.[10] This argument is simple enough to understand: absolute space seems to be an unobservable, unnecessary assumption. So why posit a "real," absolute space?

In order to understand one of the main textbook arguments *for* substantivalism, it is important to understand what relative motion means to the relationalist and the substantivalist. According to a relationalist, there is no such thing as an object being at "absolute rest" or in absolute inertial motion (i.e. absolute constant velocity); objects can only be at rest relative to another object or objects, and in inertial motion relative to another object or objects. This can be put informally in the following scenario: Alice and Bob are in outer space, and can see nothing but each other. A relationalist would claim that the scenario where Alice is "absolutely at rest" in outer space and Bob is moving at "absolute constant velocity" is indistinguishable from the scenario where Alice is moving at constant velocity and Bob is at "absolute rest." The reason they are indistinguishable is that the scenarios give rise to the same, observable effects; in other words, the laws of physics are the same for both Alice and Bob. A relationalist would therefore say that the best way to describe this scenario is to say that inertial motion is relative to the

---

[7] Transcendental Aesthetic is the first section of the *Transcendental Doctrine of Elements* in the *Critique of Pure Reason* (1781). It is loosely defined in Kantian epistemology as the study of space and time as the a priori forms of perception. Transcendental = a priori (presupposed and necessary to experience in the traditional sense) and Aesthesis = sensation, sensitive to perception.

[8] See the Leibniz-Clarke correspondence, 1715-1716

[9] Huggett, Nick and Hoefer, Carl, "Absolute and Relational Theories of Space and Motion", *The Stanford Encyclopedia of Philosophy* (Spring 2015 Edition), Edward N. Zalta (ed.), <http://plato.stanford.edu/archives/spr2015/entries/spacetime-theories/>.

[10] Friedman, Michael. *Foundations of Space-Time Theories: Relativistic Physics and Philosophy of Science*. (Princeton, NJ: Princeton University Press, 1983). 219

objects in question – and that the relation between objects is all that is needed to describe the world. That is, all we need to say is that Bob is moving relative to Alice and Alice is still (in Alice's frame of reference), and Alice is moving relative to Bob and Bob is still (in Bob's frame of reference). Now, this particular formulation also happens to be consistent with Newton's mathematics/physics (the difference is that substantivalism allows for relative inertial motions as well as absolute motions). Why then, does he posit "substantivalist" absolute motion, that is, motion relative to a privileged "absolute" frame of reference?

The problem lies with *non-inertial*, i.e. arbitrarily *accelerating* frames of reference. Whereas with inertial motion, Alice is moving at constant velocity with respect to Bob, and Bob is moving at constant velocity with respect to Alice, *the analogous statement for acceleration does not hold.* There are certain scenarios where we *cannot* simply state that Alice is accelerating with respect to Bob, and Bob is accelerating with respect to Alice. In such scenarios we are forced to say that Bob is accelerating, and that Alice is not accelerating, period. How does this occur?

The following scenario is a bastardized version of Einstein's rotating globes scenario, which in turn is a modernization of the famous "Newton bucket experiment." According to classical (Newton's) laws of physics, we can have two forms of uniform acceleration: linear acceleration and uniform rotation. While linear acceleration always requires the application of an external force, uniform rotation can exist independently of an external force *due to the conservation of angular momentum*. Thus we can have the following scenario: let Alice be on a large, elastic globe S_1 and Bob be on a "materially" identical elastic globe S_2. Let's say that Alice sees that Bob's ball is spinning about the (imaginary) axis running through the poles of both S_1 and S_2. She measures the radius of S_2 from her perspective and notices that it has a larger radius than her radius of S_1, and that it has the shape of a bulging ellipse. She explains this by claiming that with respect to S_1, S_2 is rotating – and therefore accelerating, and that S_2 itself is experiencing centrifugal forces which cause it to morph from a sphere to an ellipsoid.

According to Bob on S_2, however, things are slightly different. Bob looks at S_1 and sees Alice rotating. Indeed, she *is* spinning with respect to Bob. However, when he measures the radius of S_1, it is in fact smaller than the radius of S_2. We can say that Bob is experiencing centrifugal forces, which is why his radius S_2 is larger – but how can we explain the difference between Alice and Bob's observations? We *cannot* just say that Alice is accelerating with respect to Bob, and Bob is accelerating with respect to Alice, and nothing more – the laws of physics expose a difference between them. Newton addresses this quagmire by positing that with regards to accelerating frames of reference, we must evoke acceleration with respect to a particular, privileged frame of reference, i.e. *absolute space.* Therefore, we must say that Alice is *not* rotating with respect to absolute space, and Bob *is* rotating with respect to absolute space, in order to fully understand the scenario. This, of course, proved problematic for relationalism (unfortunately Leibniz died in 1716, and the issue was not satisfactorily resolved).

Interestingly, Kant was well versed in Newton's physics and well aware of the substantivalist/relationalist debates of the time. However, he does not deal with the above debates in the Transcendental Aesthetic, as he argues that the debates deal with motion as an empirical concept, and not a pure a priori conception of space. Instead, he throws his hat into the ring in the Metaphysical Foundations, where he tries to resolve Newton's bucket argument (about absolute acceleration) by appealing to the center of mass of the solar system as a way to distinguish "relative" motion from "true" motion, and thereby avoid all forms of "absolute" unobservable space (more on Kant's Transcendental Aesthetic later).[11] This essentially claims that Bob on S_2 is rotating with respect to the distant center of mass of universe, and that in any empirical scenario we will be able to appeal to this argument (a consequence of this is that the total angular momentum of the universe must be zero – which is certainly respectable and self-consistent from a relationalist perspective). Mach famously took a similar approach, and it was Mach who influenced Einstein most directly.

Einstein was acutely aware of the philosophical debates, and, not satisfied with Newton's appeal to "absolute space," began his famous 1916 paper on general relativity with a version of the above thought experiment in order to overcome this "epistemological defect."[12] Simply put, Einstein argues that we can resolve this issue by claiming that S_1 and S_2 are partly conditioned by distant masses that we have not included in the system under consideration. "These different masses and their motions relative to S_1 and S_2" he writes "must then be regarded as the seat of the causes (which must be susceptible to observation) of the different behavior of our two bodies S_1 and S_2."[13] According to this line of reasoning, we can therefore avoid the need for absolute space in our ontology by appealing to distance masses that are a third, and more important frame of reference. Interestingly, Einstein combines this form of Machianization of space (i.e. avoiding the appeal to absolute space by claiming that rotation must be relative to distant bodies) with a desire to *require all the laws of physics to apply equally and indistinguishably to both inertial and accelerating frames of reference*. He continues: "The general laws of nature are to be expressed by equations which hold good for all systems of co-ordinates, that is, are co-variant with respect to any substitutions whatever (generally co-variant)."[14] Whether general relativity *succeeds* in this Machianization and this strong form of "general covariance" is another question – both Cassirer and Schlick believed that it did (see Friedman for a contemporary analysis of this question – he answers strongly in the negative).

Before continuing, it is important to fit the theory of special relativity (1905) in the context of the substantivalist/relationalist debate. Classical physics, up until Einstein, was based on Newtonian mechanics as well as more contemporary electrodynamics, and essentially made implicit use of absolute motion. A new challenge appeared in the 19th century when paradoxical experimental results seemed to imply that a privileged inertial frame of reference both *did* and *did not* exist. On the one hand, the speed of electromagnetic waves (including light) did not seem to obey the classical rules of Galilean relativity, implying that the motion of these waves was "absolute" with respect to the ether. This ether was a contemporary version of the substantivalist's real, privileged, absolute frame of reference. However, on the other hand, the ether seemed to fail all predictions of empirical observation.[15] This is where Einstein's theory of special relativity came in; Einstein was able to resolve this paradox and do away with the ether by relativizing length, time and momentum (I avoid the details here for the sake of brevity). However, Einstein recognized that special relativity fell short of his Machian aspirations, as it still made the distinction between inertial frames and accelerated frames. While "absolute" velocity is eliminated (by postulating that the speed of light is constant in *all* frames of reference and not one privileged frame of reference), absolute acceleration is unfortunately maintained.

---

[11] Janiak, Andrew, "Kant's Views on Space and Time", *The Stanford Encyclopedia of Philosophy* (Winter 2012 Edition), Edward N. Zalta (ed.), <http://plato.stanford.edu/archives/win2012/entries/kant-spacetime/>.
[12] Friedman, *Foundations of Space-Time Theories*, 205
[13] Ibid., 207
[14] Ibid.
[15] See the Michelson-Morley 1887 experiment for more details

We can therefore summarize Einstein's agenda with *general relativity* as twofold:

1. Einstein wanted to eliminate a privileged class of inertial frames, and to formulate the laws of motion such that they were valid in arbitrary reference frames (both inertial and accelerated). He wanted the rules of physics to be *generally covariant*, in such a way that they would be preserved by all admissible transformations. This would (hopefully) implement a rigorous "equivalence" or "indistinguishability" of all states of motion.
2. In addition, as per Kant and Mach, Einstein wanted to account for distorting centrifugal effects experienced by "absolutely" rotating objects *in terms of their relative rotation with respect to the total distribution of matter in the universe.* Doing so would avoid (just as in 1.) any need to appeal to some privileged, unobservable inertial frame.[16]

Historical and Philosophical Context of General Relativity: Euclidean vs. non-Euclidean Geometry

We now move on to the geometrical foundations of general relativity, namely, the non-Euclidean geometry of Gauss and Riemann. Before Gauss, it was generally understood that the world could be described by the rules of Euclid.[17] And while curved objects such as spheres and hyperboloids can be described using various coordinates (e.g. Cartesian or polar coordinates), the fundamental *geometry* is Euclidean (in particular, an arbitrary line element in Euclidean geometry can always be put in terms of the Cartesian line element).

By discarding the parallel postulate, however, Gauss was able to describe arbitrary curved surfaces using line elements that *could not* be reduced to Cartesian line elements. In non-Euclidean geometries, space itself is curved. While a particular geometry can be described using various different coordinates, different geometries have different properties completely independent of any particular coordinatization. So the distinction between Euclidean geometries and non-Euclidean geometries is simple but fundamental – for non-Euclidean geometries, Cartesian coordinates simply do not exist. Characterized differently, a given intrinsic feature of a surface (i.e. whether it is Euclidean or non-Euclidean) corresponds to the existence of coordinate systems with certain extrinsic features.

Riemann generalized this new way of formulating geometries to arbitrary spaces/manifolds of n-dimensions. Points on a manifold can be represented by n-tuples of real numbers x_1, x_2, x_3, … x_n, and the n-dimensional line element (or metric tensor) becomes $ds^2 = \sum_{i,j=1}^{n} g_{ij} dx_i dx_j$. In particular, for a Riemannian metric tensor, the following two conditions are met: (1) $g_{ij} = g_{ji}$ (symmetry) and (2) $ds^2 > 0$ (positive definiteness). For a semi-Riemannian metric, only the symmetry condition is met (this is the metric for special relativity, as we shall see). Thus for flat/Euclidean (and Riemannian) manifold, $g_{ij} = g_{ji} = 1$, whereas for a semi-Riemannian manifold $g_{ij} = \pm 1$. For a non-semi-Riemannian metric (e.g. general relativity), neither condition is met. Finally, to reiterate the difference between Euclidean and non-Euclidean geometries, when we change coordinates *within* a geometry, the *line element* is still preserved:

[16] Friedman, pp. 205-206

[17] Euclid's postulates: (1) To draw a straight line from any point to any point (2) to produce [extend] a finite straight line continuously in a straight line (3) to describe a circle with any centre and distance [radius] (4) that all right angles are equal to one another, and (4) [the succinct parallel postulate] *in a plane, given a line and a point not on it, at most one line parallel to the given line can be drawn through the point*.

$ds^2 = ds'^2 = \sum g'_{ij} dx'_i dx'_j = \sum g_{ij} dx_i dx_j$. But if we change *geometries* from a flat, Euclidean geometry to a curved, non-Euclidean geometry, we change to a new metric tensor where the line element is *not* preserved: $ds^{*2} = \sum g^{*}_{ij} dx_i dx_j$, where no coordinates exist such that $g^{*}_{ij} = 1$. All this fits into the general framework of n-dimensional manifolds.

Newton's geometry is a flat, three-dimensional Euclidean geometry, where relative motions are composed for primary, absolute motions in the privileged frame of reference. The geometry of special relativity (due to Minkowski) is a four dimensional semi-Euclidean manifold, with time as the first (or fourth) dimension. The geometry of general relativity is a bit more complicated, and can be briefly described in the following manner:

1. Start with a Minkowski manifold, with line element $ds^2 = dx_0{}^2 - dx_1{}^2 - dx_2{}^2 - dx_3{}^2$
2. Then add variable curvature to the manifold, where the degree of curvature depends on the distribution of mass and energy (inspired by Mach's approach)
3. In order to do this, one needs a more comprehensive line element $ds^2 = \sum_{i,j=0}^{3} g_{ij} dx_i dx_j$, where g_{ij} is not constant
4. Interpret g_{ij} as potentials of the gravitational field (*equivalence principle of gravitational and inertial mass*).[18] The gravitational field determines the trajectories of freely falling particles via a geodesic law of motion (shortest possible line between two points on a sphere or curved surface)

The theory of general relativity therefore describes a four-dimensional *differentiable* manifold, where its ("gravitational") field equations and laws of motion are written in *generally covariant* tensor form. The field equations and laws of motion together define the geometrical structure of space-time.[19]

The Kantian and Neo-Kantian Synthetic A Priori

Both Cassirer and Schlick shared very similar attitudes towards general relativity (it just so happens that they were both quite friendly with Einstein himself). Where they disagree, however, is with regards to the definition of a synthetic a priori statement and the process of synthetic a priori thought. Schlick rightly states that the synthetic a priori is essentially the defining feature of Kantian epistemology; he is also quick to claim that it is the central point of conflict between Kantian transcendental philosophy and his form of logical empiricism (which would claim that all definitions of the synthetic a priori are meaningless, and that what a Kantian might call a priori, an positivist would call a hypothesis or convention). It is therefore appropriate to briefly explain the traditional Kantian notion of the synthetic a priori (Schlick's interpretation of Kant), and Cassirer's neo-Kantian reformulation of the synthetic a priori.[20]

---

[18] The above paragraphs were a quick review of the first two chapters of Michael Friedman's *Foundations of Space Time Theories*. I highly recommend referring to those chapters for more detail.

[19] Friedman, *Foundations of Space-Time Theories,* 177. Note that a *differentiable manifold* is a manifold that is locally similar enough to a linear space such that integration and differentiation is valid.

[20] The Kantian concepts mentioned here have been the subjects of much scholarship. My hope is only to give a liberal description of each of these concepts sufficient for an intelligible discussion.

Kantian epistemology is fundamentally built on the synthetic/analytic and a priori/a posteriori divisions of knowledge. Liberally put, a proposition in which the predicate is contained in the subject is *analytic* (such as "all bachelors are male"), while a proposition is which the predicate is not contained in the subject is *synthetic* ("all bachelors are unhappy"). In addition, an *a priori* proposition is a proposition that can be logically deduced without any recourse to experiment or measurement, while an *a posteriori* proposition relies on experiential verification for validity. Thus a *synthetic a priori* proposition has a predicate that is not logically or analytically contained in the subject, and is verifiable independent of experience. Kant's classic example of a synthetic a priori statement is "7 + 5 = 12" – it is synthetic because the notion of "12" is not contained in the definition of "7", "+" or "5," and a priori since the truth of the statement is evident without experiential verification. It follows that synthetic a priori propositions are underdetermined by experience (non-empirical) and necessarily true (non-contingent). It also follows that their denial can be logically true and that their truths are intuition based.

The obvious question is, then, how is synthetic a priori reasoning possible? In Kant's classical formulation, the process of synthetic a priori reasoning involves the combination of two distinct types of a priori principles: *constitutive principles* (e.g. Euclidean geometry and Newtonian mechanics) which are apodictic – clearly established – non-empirical facts about the world, and *regulative principles* (e.g. unity of nature, maximal simplicity) which can never be fully realized in experience but are nevertheless guiding ideals.[21] The combination of these two cognitive principles forms the basis of synthetic a priori judgment. As we shall soon see, Schlick focuses on the constitutive definition of a synthetic a priori proposition, and argues that such apodictic propositions cannot be found in Einstein's new theory, while Cassirer focuses on the regulative process of synthetic a priori reasoning and argues that general relativity is a clear manifestation of this. Cassirer in fact mostly rejects the constitutive principles (admitting that we must move *beyond* Kant in this regard) and reduces the synthetic a priori to the regulative realm.

While Kant changed his views on space and time somewhat over the course of his scholarship, the general consensus is that Kant treats space and time as *mental schemes* for coordinating the external world. For example, in his *Inaugural Dissertation* (1770) he succinctly expresses his views:

> *Space is not something objective and real, nor a substance, nor an accident, nor a relation; instead, it is subjective and ideal, and originates from the mind's nature in accord with a stable law as a scheme, as it were, for coordinating everything sensed externally.* (Ak 2: 403)[22]

According to Kant, space and time are not objects per se, but "conditions of the possibility of experience" that originate in the "mind's nature."[23] He explicitly dismisses the main claims of substantivalism and relationalism (although we might argue that his conception is very similar to the standard Leibnizean relationalism). As Cassirer writes in his general explanation of the traditional Kantian framework, concepts like space and time that allow for the

[21] Michael Friedman, "Ernst Cassirer and the Philosophy of Science" in *Continental Philosophy of Science* ed. Gary Gutting (Malden, MA: Blackwell Publ., 2005)
[22] His Transcendental Aesthetic is dealt more extensively in *The Critique of Pure Reason (1781, 1787)*. I avoid a more rigorous discussion here for brevity.
[23] Cassirer, *Einstein's Theory of Relativity*, 411

positing of objects cannot be treated as objects themselves, as "the *forms* of possible experience, the forms of intuition as well as the pure concepts of the understanding, are not met again as *contents* of real experience."[24] That is, space and time are a priori (i.e. necessary) conditions on all empirical intuitions, and are themselves intuitions that precede all experience.[25] Kant continues to argue that space is the source of the synthetic a priori propositions of Euclidean geometry, and that time is the source of synthetic a priori propositions of arithmetic. It follows that scientific theories based on Euclidean geometry and arithmetic, such as Newtonian mechanics, are combinations of synthetic a priori propositions and empirical observations (the synthetic a posteriori).[26] It is important to note that while Kant essentially sides with Leibniz in the substantivalist/relationalist debate, space and time play a unique role in Kant's epistemology.

Cassirer was a proponent of the well-known Marburg school of neo-Kantian epistemology, which differed from traditional Kantian epistemology in certain areas of logic and science. While Kant wrote with Euclidean geometry and Newtonian mechanics in mind, the philosophers of the Marburg school took it upon themselves to generalize Kant's approach for the modern sciences including non-Euclidean geometry, Einstein's theory of special relativity, and in Cassirer's case, general relativity.[27] These new mathematical and scientific theories undermined the supposed *apodictic properties of the constitutive a priori principles* (e.g. non-Euclidean geometry challenged the apodictic nature of Euclidean geometry). The Marburg school responded to these challenges by shifting the main focus of the synthetic a priori from constitutive to regulative. The Marburg "genetic conception" of synthetic a priori reasoning is a process of active generation whereby the mind continuously re-determines the objects of physics that constitute reality. These objects of reality are not separate from this synthetic process; as Friedman explains, they are to be conceived as *"the necessary endpoint or limit towards which the continuous serial process exemplified in modern mathematical natural scientific knowledge is converging."*[28]

Cassirer took this one step further in his book *Substanzbegriff und Funktionsbegriff* ("Substance and Function," 1910, trans. 1923), where he criticizes the theory of concept formation (i.e. that general concepts are arrived at by ascending inductively from sensory particulars) as too committed to the idea of "substance" as the ultimate substratum of reality. He instead argues that the matching of metaphysics with substances (i.e. the "copy" theory of knowledge) should be replaced by the matching of metaphysics *with mathematical relational structures, or functions*.[29]

Cassirer's Zur Einsteinschen Relativitätstheorie (1921)

All these elements – the traditional Kantian framework combined with the Marburg genetic conception theory as well as Cassirer's emphasis on mathematical relational structures – form the foundation of Cassirer's 1921 monograph on general relativity *Zur Einsteinschen*

---

[24] Ibid., 411-412. He continues to claim that "What they can accomplish as presuppositions in this connection can be exactly determined by transcendental criticism; what they are as things in themselves is a vain and fundamentally unintelligible question"

[25] A more rigorous discussion would clarify Kant's distinction between intuitions and concepts – I avoid this here for purposes of brevity. See http://plato.stanford.edu/entries/kant-spacetime/

[26] I won't go into more details here, as Cassirer's interpretation of Kant is the main focus of this paper

[27] Cassirer, *Einstein's Theory of Relativity*, 353

[28] Friedman, "Ernst Cassirer and the Philosophy of Science," 72

[29] Ibid., 77

Relativitätstheorie ("Einstein's Theory of Relativity," trans. 1923). Cassirer begins with a simple question: does the Transcendental Aesthetic offer a foundation that is strong enough to bear the weight of modern physics, i.e. general relativity?[30] He answers in the affirmative, emphasizing that the most fundamental (physical) theories have a *very close connection to epistemology*, and that general relativity, more than any other theory, extends into the field of epistemological problems from the very beginning.[31]

Cassirer starts by examining the empirical and conceptual foundations of the theory of relativity in order to decide whether the theory in its origin and development "is to be taken as an example and witness of the critical [i.e. neo-Kantian] or of the sensualistic concept of experience."[32] He divides the issue into two classically opposing views: is knowledge of the natural world a simple registration of facts (broadly espoused by extreme empiricists), or are there independent criteria of judgment involved as well? He rejects the extreme empiricist view and argues for the importance of the synthetic a priori criteria of judgment as a general form of natural law, which is *invariant* and is the real logical framework of nature in general.[33] He then clarifies his regulative ideal of the synthetic a priori:

> *In this sense, the critical theory of experience actually aims to construct a universal invariant theory of experience and thereby to fulfill a demand towards which the character of the inductive procedure itself ever more clearly presses…This goal may never be completely attained at any given stage of knowledge; nevertheless, it remains as a demand and determines a fixed direction in the continual unfolding and development of the system of experience itself…A cognition is called a priori, not because it lies in any sense before experience, but rather because, and in so far as, it is contained in every valid judgment about facts as a necessary premise.*[34]

This scientific progression is seen as a series or sequence of abstract formal structures, which must also have the property of approximate backwards-directed inclusion (e.g., non-Euclidean geometries must contain the older geometry of Euclid as an approximated limiting case).[35] Similar to the genetic conception, this is a regulative ideal that is never quite reached. Reality, according to this viewpoint, is therefore the endpoint of the converging series of formal structures.

Cassirer uses Einstein's early innovation, the special theory of relativity, as his first example of this regulative ideal. The special theory of relativity is essentially based on two basic assumptions: (a) that propagating light in a vacuum has a maximum speed of propagation, and (b) that the same laws of physics apply to all systems in inertial (non-accelerating) reference frames. While these two assumptions were motivated by experiment and observation, they belonged to a slightly different stratum – they were fundamental postulates of a mathematical, relational nature.[36] As we saw earlier, special relativity did away with the unnecessary

---

[30] Cassirer, *Einstein's Theory of Relativity*, 352
[31] Ibid., 356
[32] Ibid., 367
[33] Ibid., 374
[34] Ibid., 355-356,
[35] Friedman, "Ernst Cassirer and the Philosophy of Science," 75
[36] Cassirer, *Einstein's Theory of Relativity*, 379. I would recommend reading this chapter carefully for a more rigorous analysis of special and general relativity.

metaphysical notion of the ether while simultaneously relativizing length, time, and momentum. Cassirer sees this as a triumph, writing that "the idea of the ether as an inexperienceable substance is excluded by the theory of relativity in order to give conceptual expression merely to the pure properties of empirical knowledge."[37] The removal of the ether was a blow to the "metaphysics of substances," and the relativization of length, time, and momentum a win for Cassirer's "metaphysics of structural relations." That is, a fundamental substance was deemed unnecessary, and new and more general mathematical/structural relations were deemed fundamental. Thus special relativity fits into Cassirer's regulative ideal of the synthetic a priori.

He then argues that even greater unity is achieved with the move from special relativity to general relativity. In general relativity, the law of constancy of the velocity of light in a vacuum (a) no longer holds; rather, it depends on mass. What is fundamental, now, is a new more insightful mathematical relation. While special relativity treats the speed of light in an inertial frame of reference as separate from the speed of light in a non-inertial frame of reference, general relativity explains both scenarios with a single assumption (one might think that general relativity *contradicts* special relativity in this respect; Cassirer argues that it *subsumes* special relativity within a more general mathematical framework). And while classical mechanics and special relativity specified which sorts of bodies and reference frames the laws of physics could be applied to (b), with a fundamental distinction between inertial and non-inertial frames, general relativity does away with that; the laws of nature require some definite system of reference, but there are *no limits* to the system of reference.[38]

Finally he explains how general relativity fits into his critical theory of knowledge based on mathematical structures: "[In Einstein's theory] no sort of things are truly invariant, but always only certain fundamental relations and functional dependencies retained in the symbolic language of our mathematics and physics, in certain equations."[39] All of general relativity is therefore the radical resolution of "things" into mere relations (with no privileged system of coordinates) that Cassirer championed in *Substanzbegriff und Funktionsbegriff* in 1910! He thus concludes that Einstein's new theory exemplifies the *regulative ideal of synthetic unity* by unifying the totality of observations into a single whole. In particular, general covariance is seen a manifestation of this ideal, since it is the collection of all particular systematic principles into the unity of a supreme postulate, "not of the constancy of things, but of the invariance of certain magnitudes and laws with regard to all transformations of the system of reference."[40]

Cassirer then addresses the philosophical concept of truth and the theory of relativity, arguing that the truly objective element in modern knowledge of nature is not so much *things* as *laws*.[41] But what guarantees the objectivity of these laws? This question lies at the heart of the idealist/empiricist debate, as empiricists like Schlick would claim that these laws are not objective and are simply hypotheses or conventions. Cassirer answers by evoking the Kantian critical concept; *that objectivity exists in the preservation of relations*.[42] As we have seen, the theory of relativity is not a picture, but a schema, a form of equations and relations, which are covariant with respect to arbitrary substitutions. The theory is therefore a pure doctrine of form and relation.

---

[37] Ibid., 407
[38] Ibid., 379
[39] Ibid.
[40] Ibid., 404
[41] Ibid., 388
[42] Cassirer, *Einstein's Theory of Relativity*, 392. "One fact expresses another when there exists between what can be said of the one, and of the other, a constant and regular relation."

Finally, Cassirer addresses how general relativity fits into Kant's overall critical philosophy. It would seem like general relativity destroys the unity of space and time demanded by Kant, as there are infinitely many space times dependent on state of motion of place from which measurement is made, and there is no one unified time. He responds by arguing that this view is erroneous, and that relativity actually grounds and confirms the Transcendental Aesthetic – for according to Kant, time is a schema that can fit with each and every *empirical* space-time.[43] The unity of space and time demanded by Kant lies in the form of "system of valid relations." And the a priori nature of space, which [physics] asserts as the condition of every physical theory, does not include, as has been shown, any assertion about a determinate particular structure of space, but is concerned only with the function of "spatiality in general."[44] Cassirer finally concludes that general relativity, in this respect displays fewer contradictions to the Kantian Transcendental Aesthetic than any earlier physics.[45] And with regard to the substantivalist/relationalist debate, it is clear that Cassirer is on the right side. He indeed champions mathematical relations over absolute space, and the unity of functional relations (e.g. general covariance) over arbitrary postulated distinctions (i.e. between inertial and non-inertial frames).[46]

Before delving into Schlick's critique, I would like to address Cassirer's philosophy with respect to Poincare's well-known conventionalism. Poincare essentially argued that certain incompatible geometries (e.g. Euclidean and non-Euclidean geometries) are actually equivalent descriptions of the same facts – that is, we can use both to describe the same empirical observations. The spatial/temporal congruence relations that seem to exist in one geometry and seem not to exist in another are not objectively real; rather, they "hold" or "fail" relative to one coordinative definition or another.[47] This is sort of a combination between ideological relationalism and theoretical underdetermination. And while there might be *pragmatic* reasons to choose one geometry over another, there is no *truth* or *reality* involved. While Cassirer does not explicitly address any forms of conventionalism, it can be understood from his text that his regulative ideal *demands* the most general and unifying of descriptions to be privileged over less unifying descriptions. This is really at the heart of his critical idealism, and is another difference between him and Schlick. Whereas for Schlick would treat the choice of one geometry of another as "convention," Cassirer would treat the choice of one geometry over another as *necessary*, without appeal to empirical measurement. Thus Schlick would suffer from the problem of underdetermination, Cassirer would not.

Schlick's Critique: Kritizistche oder empiristische Deutung der neuen Physik?(1921)

In response to Cassirer's monograph, Schlick sets himself the task of deciding, by epistemological analysis "whether the theory in its origin and development is to be taken as an example and witness of the critical or of the sensualistic concept of experience." He takes issue

---

[43] "For in Kant's manner of expression time is, like space, a pure form of intuition; a schema in which we must arrange events, so that in opposition to subjective and highly contingent perceptions they may gain objective meaning." Laue, 414
[44] Cassirer, *Einstein's Theory of Relativity*, 420
[45] Ibid., 396
[46] Cassirer, 407. *"For this purpose, however, according to GR, we don't need the fixed reference body used by Newton…no longer measures with rigid bodies of Euclidean geometry and classical mechanics, but builds more inclusive standpoint in determination of the universal linear element ds."*
[47] Friedman, *Foundations of Space-Time Theories*, 264.

with Cassirer's somewhat blunt reduction of all forms of empiricism to the rather extreme "sensualism," pointing out that the incompatibility between relativity and sensualism doesn't necessarily rule out other forms of empiricism. He then clarifies his understanding of the two sides by defining the synthetic a priori (in his formulation, the essence of the critical viewpoint) as constitutive principles that are apodictic – beyond dispute and clearly established. The empiricist can acknowledge the necessity of constitutive principles, but will deny that they are synthetic a priori; rather, according to the empiricist viewpoint, they are either *hypotheses* or *conventions*, neither of which is apodictic.

Already in the first few paragraphs of Schlick's critique we see that he has shifted the entire debate; so while Cassirer argued from a liberal neo-Kantian viewpoint against a strict sensualist viewpoint, Schlick is ready to argue from a moderate empiricist viewpoint against a more traditional Kantian one (and, as we shall see, the traditional Kantian straw man is easily defeated by the postulates of general relativity).

Schlick continues his line of reasoning by stating that anyone who accepts Einstein's new theory must reject Kant's theory in its original form, which relies heavily on Euclidean geometry and Newtonian mechanics. He demands that, in line with his strict definition of the synthetic a priori, the adherent to the critical philosophy must vindicate himself only by producing such a system of judgments; that is, in order to reconcile Einstein with Kant, one must unambiguously point to synthetic a priori principles embedded within the new theory itself. He notes that Cassirer has taken "one step beyond Kant," and proceeds to scour his writing for constitutive synthetic a priori principles dictated by general relativity.

Schlick finds two possible examples of such new synthetic a priori principles: (i) the concept of the coincidence of 'world points,' to which general relativity reduces all laws of nature, and (ii) the function of spatiality in general, which is expressed even in the general concept of the linear element.[48] He dismisses the concept of the coincidence of world points (i) as an empirical intuition by arguing that it is simply representative of a psychological experience of coming together, "much as the word 'yellow' designates a simple color experience that cannot be further defined."[49] And while it plays the intermediary role between reality and the scientific conceptual construction that the theory attributes to it, it is simply an empirical intuition, and is therefore not apodictic.[50]

Interestingly, Schlick addresses this issue of intuition and sensation in a footnote (he even writes that Schlick misunderstands Kant!). According to Cassirer, there is a difference between the immediate sensation of duration, and the measuring number used to categorize duration. Another example would be the sensation of warmth vs. temperature. Kant's space and time of pure intuition are never "sensed" or "perceived" space or time, but the *mathematical* space and time of Newton. They themselves are constructively generated, and form the foundation for further mathematical and physical construction. Pure intuition does not coincide with the "subjective" psychological, experienced space and time. Rather, subjective for Kant means subjectivity as the condition of the possibility of objectifying empirical knowledge.[51] Thus, according to Cassirer, pure intuition conditions the coincidence of world points as a starting point for the physical construction of a full theory. So while world points are not synthetic a

[48] Ibid.
[49] Schlick, "Critical or Empiricist Interpretation of Modern Physics?" 326
[50] Ibid.
[51] Ibid.

priori in the traditional Kantian sense, they *do* fit into Cassirer's theory in a self consistent manner.

Schlick further argues that any other choice of a synthetic a priori principle (for example, 'the speed of light depends on mass') is not guaranteed to be apodictic – for all we know, a new theory might come along that neglects this particular principle. Therefore, with respect to the function of spatiality (ii), Schlick argues that Cassirer is too vague by not specifying particular axioms, and even if he did, there is no seeing why just these should constitute the one necessary structure of space, "since others that are no less 'self-evident' have fallen victim to the progress of physics."[52]

With these two examples, we see that Schlick doesn't address Cassirer on his own terms. Cassirer was not attempting to find new constitutive synthetic a priori principles – such an approach was abandoned by neo-Kantians in the early days of the Marburg school. As we saw earlier, non-Euclidean geometry undermined the traditional Kantian claims that the postulates of Euclidean geometry were synthetic a priori principles, to which the neo-Kantians responded by shifting the emphasis away from the constitutive a priori towards the regulative a priori. Schlick recognizes this when he writes "Cassirer's observations appear to me to provide no convincing evidence of how we may heal the wound dealt to the original Kantian viewpoint by the overthrow of Euclidean physics." Special relativity dealt a similar blow to the constitutive a priori with regards to space and time. So it is no wonder that Schlick did not find any synthetic a priori principles in Cassirer's book that satisfied him.

However, what is even more revealing about the above quote is that Schlick does not seem to accept as valid any of the early neo-Kantian attempts to accommodate non-Euclidean geometry (he thinks the "wound" was never healed). He adds: "It would also be extraordinary if the Kantian theory of knowledge were held to stand in such clear contradiction to the Newtonian view of nature, whose philosophical vindication was one of its principal goals"[53] But while he doesn't agree with the premises that Cassirer employs, he doesn't even address this issue head on in his critique. Granted, it would be difficult to address the half a century worth of neo-Kantian scholarship dealing with non-Euclidean geometry in a single article. But it is somewhat surprising – and disappointing – that Schlick doesn't discuss Cassirer's book on his own terms and dismisses Cassirer's Marburg framework and premises without any serious philosophical argumentation.

Interestingly, Cassirer actually addresses the issues of Euclidean vs. non-Euclidean geometry, although Schlick does not reference this. Even though general relativity is a theory of non-Euclidean manifolds, he points to the fact that in the theory, Euclidean geometry is the geometry of infinitely small areas. We can therefore still salvage Euclidean geometry as the expression of certain elementary relations, (which can still be taken as the Kantian basis in thought), and then advance from Euclidean geometry to more complex non-Euclidean geometry! The implication, of course, is that the "wound dealt to the original Kantian viewpoint by the overthrow of Euclidean physics" is not as severe as Schlick implies. In addition, all physical theory and physical measurement cannot necessarily prove anything about the Euclidean or non-Euclidean character of *space*, since it is only concerned with the properties of *physical reality in space.* The step beyond Kant is to allow (on the basis of GR) the non-Euclidean axioms and laws to enter into this determination of the empirical and physical world.[54]

---

[52] Ibid.
[53] Schlick, "Critical or Empiricist Interpretation of Modern Physics?" 326
[54] Cassirer, 436

Schlick lays his final criticism on Cassirer in a slightly different manner by labeling his logical idealism as too general to be meaningful.[55] Schlick went so far as to write a letter to Cassirer pressing him to point out the synthetic a priori in Einstein's theory, to which Cassirer responded (consistent with his monograph) that the synthetic a priori principles of all science 'really consist only of the idea of the unity of nature, that is, the law abiding character of experience in general, or, more briefly perhaps, of the univocal nature of coordination.'[56] Schlick responds to this with the accusation that Cassirer has extended his definition of the synthetic a priori to something that is too broad. Unitary obedience to natural law, according to Schlick, is assuredly "the condition sine qua non of science;" even the strictest empiricist believes in the unity of nature and the law-abiding character of experience. The difference between the empiricist and the idealist, he continues to explain, is that the empiricist doesn't believe that the validity and objective necessity of the unity of nature can necessarily be proved in any other way, including via transcendental deduction. Schlick dramatically concludes that "the doctrine of synthetic a priori judgments as the constructive principles of exact science obtains no unambiguous confirmation from the new theory."[57]

If a priori is understood as (like Cassirer does) a regulative principle, then general relativity as a usurping theory of space and time seems to be a fulfillment of neo-Kantian critical idealism. For although empiricists assume the "objective necessity of the unity of nature," it doesn't *have* to be the case from an epistemological point of view – when unity occurs, it is simply a convenience. There are some imaginable cases where a shift in the theory might be so great that there is hardly any continuity left (e.g. quantum mechanics; Thomas Kuhn might argue similarly) – which would be problematic for a neo-Kantian but not necessarily for an empiricist. Ultimately, an empiricist might be content with a non-unified theory, while a logical idealist would demand unity on principle, which could also play out in the practice of science.

This was all a very long-winded way of arguing that general relativity did not undermine neo-Kantian epistemology in any significant way. Einstein's main philosophical motivations had to do with relativizing all forms of motion as well as the Machianization of space, both of which were perfectly in line with Cassirer's logical idealism. The real issue between Schlick and Cassirer was the definition of the synthetic a priori, and a good opportunity was missed in Schlick's critique to address Cassirer's updated, neo-Kantian version of the synthetic a priori (something which was pointed out by many, such as Friedman and Ryckman). Attacking neo-Kantian arguments via traditional Kantian epistemology is analogous to attacking logical empiricism by exposing discrepancies in traditional Humean empiricism. While successful in a certain sense, it is not particularly insightful with respect to Cassirer's frame of reference. All this seems to somewhat undermine the strength of Schlick's case against logical idealism, and to cast aside general relativity from the debate between logical empiricism and logical idealism.

---

[55] Schlick, 326
[56] Ibid.
[57] Ibid., 327

Bibliography

1. Cassirer, Ernst. *Substance and Function and Einstein's Theory of Relativity*, trans. William and Marie Swabey (Chicago: Dover Publications, Inc., 1923)
2. Friedman, Michael. *Foundations of Space-Time Theories: Relativistic Physics and Philosophy of Science*. (Princeton, NJ: Princeton University Press, 1983).
3. Friedman, Michael "Ernst Cassirer and the Philosophy of Science" in *Continental Philosophy of Science* ed. Gary Gutting (Malden, MA: Blackwell Publ., 2005).
4. Howard, Don "Einstein, Kant, and the Origins of Logical Empiricism." in *Logic, Language, and the Structure of Scientific Theories: Proceedings of the Carnap-Reichenbach Centennial, University of Konstanz, 21-24 May 1991* ed. Salmon, Wesley C., and Gereon Wolters. (Pittsburgh, Pa.: University of Pittsburgh Press, 1994) 51-55
5. Huggett, Nick and Hoefer, Carl, "Absolute and Relational Theories of Space and Motion", *The Stanford Encyclopedia of Philosophy* (Spring 2015 Edition), Edward N. Zalta (ed.), <http://plato.stanford.edu/archives/spr2015/entries/spacetime-theories/>.
6. Janiak, Andrew, "Kant's Views on Space and Time", *The Stanford Encyclopedia of Philosophy* (Winter 2012 Edition), Edward N. Zalta (ed.), <http://plato.stanford.edu/archives/win2012/entries/kant-spacetime/>.
7. Ryckman, Thomas. *The Reign of Relativity: Philosophy in Physics, 1915-1925.* (Oxford University Press, 2005).
8. Schlick, Moritz, "Critical or Empiricist Interpretation of Modern Physics?: Remarks on Ernst Cassirer's *Einstein's Theory of Relativity*" in *Philosophical Papers, Volume I (1909-1922)* trans. Peter Heath, ed. Henk L Mulder, and Barbara F. B. van de Velde-Schlick (Dordrecht: D. Reidel Pub. Co., 1978), 322-334.