Appendix to:

Lost in Transit:
Product Replacement Bias and Pricing to Market

Emi Nakamura and Jón Steinsson
Columbia University

September 9, 2009

A Log-Likelihood in the Presence of Unobserved Heterogeneity in the Frequency of Price Change

We assume that product i has a constant hazard of adjusting, f_i, in each month, where $f_i \sim \text{Beta}(a, b)$. Let us denote the product's lifetime by n_i. These assumptions imply that the total number of price changes in a product's lifetime is distributed according to the binomial distribution, $x_i \sim \text{Bin}(n_i, f_i)$. We assume, furthermore, that f_i is distributed according to the beta distribution, $f_i \sim \text{Beta}(a, b)$.

Given this model, we can write the likelihood of observing a product with length n_i and the total number of price changes x_i as,

$$L = \prod_{i=1}^{I} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} f_i^{a-1}(1-f_i)^{b-1} \binom{n_i}{x_i} f_i^{x_i}(1-f_i)^{n_i-x_i} \quad (1)$$

$$= \prod_{i=1}^{I} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} f_i^{x_i+a-1}(1-f_i)^{n_i-x_i+b-1} \binom{n_i}{x_i} \quad (2)$$

We can integrate out the $f_i's$ to get,

$$L = \prod_{i=1}^{I} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \binom{n_i}{x_i} \frac{\Gamma(a+x_i)\Gamma(b+n_i-x_i)}{\Gamma(a+b+n_i)}. \quad (3)$$

The log-likelihood function is, therefore,

$$\log L = n \log\Gamma(a+b) - n\log\Gamma(a) - n\log\Gamma(b)^n + \sum_{i=1}[\log n_i! - \log x_i! \quad (4)$$

$$-\log(n_i - x_i)! + \log\Gamma(a+x_i) + \log\Gamma(b+n_i-x_i) - \log\Gamma(a+b+n_i)]. \quad (5)$$