2.28 Let $x_1, x_2, x_3, x_4$ be the four input signals displayed in Figure P2.28-1, and let $y_1, y_2, y_3, y_4$ the corresponding outputs.

(a) Can the system be time-invariant? Yes. A system is just a mapping $S$ from inputs $x$ to outputs $y$. This system could be time invariant if there exists some mapping $S$ from inputs to outputs which is consistent with the observed data (1)-(4) and satisfies the time-invariance property $S\{D_k x\} = D_k S\{x\}$.

(b) Can the system be linear? No. Notice that $x_1 + x_2 = 2(x_3 + x_4)$, but $y_1 + y_2 \neq 2(y_3 + y_4)$.

(c) Notice that $\delta[n] = (1/2) \times (x_2 + D_{-1} x_3 - x_3)$, and so $h[n] = (1/2) \times (y_2 + D_{-1} y_3 - y_3)$.

(d) Let $x'$ denote the input in Figure P2.28-2. Notice that $x' = x_1 + D_2 x_1$. Hence, the output $y' = y_1 + D_2 y_1 = (\ldots, 0, 1, 0, 1, 1, 1, 1, 0, \ldots)$.

2.46 (a) We can use the time reversal property of the DTFT:

$$w[-n] \xrightarrow{\text{DTFT}} W(e^{-j\omega}). \quad (1)$$

Since

$$a^n u[n] \xrightarrow{\text{DTFT}} \frac{1}{1 - ae^{-j\omega}} \quad |a| < 1,$$  \quad (2)

$$a^{-n} u[-n] \xrightarrow{\text{DTFT}} \frac{1}{1 - ae^{j\omega}} \quad |a| < 1. \quad (3)$$

Let $v[n] = a^{-n} u[-n]$. From the time delay property of the DTFT,

$$-av[n + 1] \xrightarrow{\text{DTFT}} \frac{-ae^{j\omega}}{1 - ae^{j\omega}} \quad |a| < 1. \quad (4)$$

Since $-av[n + 1] = -a^{-n} u[-n - 1]$, writing $b = a^{-1}$ and noting the restriction $|b| > 1$, we have

$$-b^n u[-n - 1] \xrightarrow{\text{DTFT}} \frac{-b^{-1} e^{j\omega}}{1 - b^{-1} e^{j\omega}} \quad |b| > 1. \quad (5)$$

We can simplify this a little bit to

$$X(e^{j\omega}) = \frac{e^{j\omega}}{e^{j\omega} - b} \quad |b| > 1. \quad (6)$$

Note that this can also be obtained directly from the definition of the DTFT – this may actually be easier!
(b) There are several ways to do this – anything which produces a correct answer is acceptable. Here is one way. Notice that

\[
Y(e^{j\omega}) = \frac{2e^{-j\omega}}{1 + 2e^{-j\omega}} = \frac{1}{1 - (-\frac{1}{2})e^{j\omega}} = \text{DTFT}\{(-1/2)^{-n}u[-n]\}. 
\]

Above, we have used (3). Hence,

\[
y[n] = (-2)^nu[-n]. 
\]

Again, you can check this by using the definition of the DTFT to re-calculate \(Y(e^{j\omega})\).

2.62 Note that^1

\[
x[n] = \cos\left(\frac{15\pi n}{4} - \frac{\pi}{3}\right) \quad (11) 
\]

\[
x[n] = \cos\left(\frac{-\pi n}{4} - \frac{\pi}{3}\right) \quad (12) 
\]

\[
x[n] = \cos\left(\frac{\pi n}{4} + \frac{\pi}{3}\right). \quad (13)
\]

The DTFT of \(x\) is

\[
X(e^{j\omega}) = \pi e^{j\pi/3}\delta(\omega - \pi/4) + \pi e^{-j\pi/3}\delta(\omega + \pi/4) \quad -\pi < \omega \leq \pi. 
\]

The DTFT of \(y\) is

\[
Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}) 
\]

\[
= e^{-j\omega/2 + \pi/4} \left[\pi e^{j\pi/3}\delta(\omega - \pi/4) + \pi e^{-j\pi/3}\delta(\omega + \pi/4)\right] \quad -\pi < \omega \leq \pi. 
\]

Using the sifting property of the Dirac delta^2 to plug in \(\omega = \pm\frac{\pi}{4}\) in the term \(e^{-j[\omega/2 + \pi/4]}\), this becomes

\[
e^{-j\pi/4} \left[\pi e^{j\pi/3}e^{-j\pi/8}\delta(\omega - \pi/4) + \pi e^{-j\pi/3}e^{j\pi/8}\delta(\omega + \pi/4)\right] \quad (18) 
\]

\[
e^{-j\pi/4}\text{DTFT}\left[\cos\left(\frac{\pi}{8}n - \frac{\pi}{3}\right)\right]. \quad (19)
\]

So,

\[
y[n] = e^{-j\pi/4} \cos\left(\frac{\pi}{8}n + \frac{\pi}{3} - \frac{\pi}{8}\right). \quad (20)
\]

This should seem like a reasonable answer – \(H(e^{j\omega}) = e^{-j\omega/2 - j\pi/4}\) consists of a half-sample delay and a multiplication by a complex exponential! Any expression for \(y\) which is equivalent to this one will receive full credit.

---

^1(11) is true because \(15\pi n/4 = 16\pi n/4 - \pi n/4\), and for any integer \(n\), \(16\pi n/4\) is an integer multiple of \(2\pi\). This manipulation is helpful, because it makes the frequency fall between \(-\pi\) and \(\pi\), which allows us to immediately write down the DTFT of \(x\).

^2For continuous \(f\), \(f(t)\delta(t-\tau) = f(\tau)\delta(t-\tau)\).