ELEN 4810 Homework 6

Analytical Questions

5.34 The correct output is \( y_2[n] \). The high frequency component of the input is attenuated. The medium frequency component \( \omega \sim 0.3 \) is delayed by 80 samples. The low frequency component is delayed by 40 samples.

5.45 (a) B, C, D, E. Pole zero diagrams of IIR systems possess poles in \( \mathbb{C} \setminus \{0\} \); these systems satisfy this property.

(b) A, F. Pole zero diagrams of FIR systems possess poles only (perhaps) at 0 and \( \infty \). A and F are the only diagrams with this property.

(c) A, B, C, E, F. Stable, causal systems have all poles inside the unit circle.

(d) E. Minimum phase systems have all poles and zeros strictly inside the unit circle.

(e) A, F. Generalized linear phase systems have zeros on the unit circle, and in conjugate reciprocal pairs.

(f) C. All pass systems have pole-zero pairs at conjugate reciprocal positions.

(g) E. A system has a stable causal inverse if all of its zeros are strictly inside the unit circle.

(h) F. Only A and F are FIR. Because the system is causal, the length of the filter simply \( \text{deg}(H(z)) + 1 \); for A this is 12, for F it is 7.

(i) A, F. The systems B, D, and E attenuate frequencies near zero. C is all-pass. This leaves A and F, both of which have a larger magnitude response near zero.

(j) E. System E is minimum phase; this implies the minimum group delay property.

7.23 (a) Using the transformation \( s = \frac{z^{-1}}{z+1} \), we obtain

\[
H(z) = \frac{z + 1}{z - 1} = 1 - \frac{2}{z - 1} = 1 + \frac{2z^{-1}}{1 - z^{-1}}
\]

(1)

(2)

(3)

Using the \( Z \)-transform relationships \( \delta[n] \to 1 \) and \( u[n] \to \frac{1}{1-z^{-1}} \), we can identify the impulse response as

\[
h[n] = \delta[n] + 2u[n - 1].
\]

(4)
(b) Re-expressing $H(z)$ as $H(z) = \frac{1+z^{-1}}{1-z^{-1}}$, we obtain the difference equation

$$y[n] = y[n-1] + x[n] + x[n-1]. \quad (5)$$

The major problem with this system is that it is not stable.

(c) The discrete-time version has

$$H(e^{j\omega}) = e^{j\omega} + 1 \over e^{j\omega} - 1 = \cos(\omega/2) \over j \sin(\omega/2) \quad (6)$$

$$= -j \cot(\omega/2). \quad (7)$$

The continuous time version has $H_c(j\Omega) = {1 \over j\Omega}$. These should be plotted and compared; the comparison reveals that they behave similarly for low-frequency inputs.

(d) We simply have

$$G(z) = {z - 1 \over z + 1} \quad (9)$$

$$= 1 - 2z^{-1} \over 1 + z^{-1}, \quad (10)$$

and so

$$g[n] = \delta[n] - 2(-1)^{-1}u[n-1]. \quad (11)$$

(e) We have

$$G(e^{j\omega}) = e^{j\omega} - 1 \over e^{j\omega} + 1 \quad (12)$$

$$= j \sin(\omega/2) \over \cos(\omega/2) \quad (13)$$

$$= j \tan(\omega/2). \quad (14)$$

The continuous time version has $G_c(j\Omega) = j\Omega$. These should be plotted and compared; the comparison reveals that they behave similarly for low-frequency inputs.

(f) Yes, in the sense that $H(z)G(z) = 1$, and so for any input $X(z)$ in the intersection of the region of convergence of $H$ and that of $G$, $g \ast h \ast x = x$. The behavior outside the intersection of the ROC’s is not guaranteed – e.g., $H(z)$ has a zero at $z = -1$, which corresponds to frequency $\pi$, i.e., input $x[n] = e^{j\pi n} = (-1)^n$. For this input, $h \ast x = 0$, and so it is not true that $g \ast (h \ast x) = x$. 

[Note: if you used the more general form of the bilinear transformation given in the text, you would end up with a prefecture of $\frac{Td}{2}$. This is also correct.]
7.36  (a) As follows:

- $i = 1$: $A_1(e^{j\omega})$ could correspond to a Type I. It does not possess zeros at 0 or $\pi$, and so it cannot be a Type II, III, or IV.

- $i = 2$: $A_2(e^{j\omega})$ could correspond to a Type I or II. It does not possess a zero at 0, and so it cannot be a Type III or IV.

- $i = 3$: $A_3(e^{j\omega})$ could correspond to a Type I. It does not possess zeros at 0 or $\pi$, and so it cannot be Type II-IV.

- $i = 4$: $A_4(e^{j\omega})$ could correspond to a Type I. It does not possess zeros at 0 or $\pi$, and so it cannot be Type II-IV.

(b) As follows:

- $i = 1$: 7 alternations
- $i = 2$: 7 alternations
- $i = 3$: 4 alternations
- $i = 4$: 6 alternations

(c) As follows:

- $i = 1$: Yes.
- $i = 2$: Yes.
- $i = 3$: No. No alternation at $\omega_s$.
- $i = 4$: No. There must be an alternation at 0, $\pi$, or both.

(d) As follows:

- $i = 1$: There are 7 alternations. We know that either $L + 2 = 7$, or $L + 3 = 7$, where $L$ is the degree of the polynomial in the Chebyshev approximation problem. Because there is no alternation at $\pi$, we know that the only possibility is $L + 2 = 7$, whence $L = 5$. The length of the impulse response is $2L + 1 = 11$ (or, put equivalently, $L = M/2$, where $M$ is the index of the last nonzero in the impulse response; the length is $M + 1$).

- $i = 2$: There are again 7 alternations. Via the same reasoning as $i = 1$, the length is 11.