ELEN 4810 Homework 6

**Analytical Questions**

5.22  (a) For \( h[n] \) real, any poles which are not pure real occur in complex conjugate pairs. Any zeros which are not pure real also occur in complex conjugate pairs.

(b) There is a pole at \( \rho = 0 \), whose multiplicity is the order of the system. The region of convergence is \( \{ z \mid 0 < |z| \leq +\infty \} \).

(c) The impulse response is symmetric about \( \alpha \). The system is FIR. It has a pole at \( \rho = 0 \) whose multiplicity is the is the order 2\( \alpha \) of the system. Its zeros occur in reciprocal pairs: if \( \zeta \) is a zero, so is \( 1/\zeta \). If \( \alpha \) is a half integer (i.e., 2\( \alpha \) is odd), there is necessarily a zero at \( \zeta = -1 \).

(d) A minimum phase system has all of its poles and zeros strictly inside the unit circle.

(e) An all-pass system has poles and zeros which occur in conjugate reciprocal pairs, i.e., a pole at \( \rho \) is paired with a zero at \( 1/\rho^* \).

5.34 \( y_2[n] \) is the correct output of the system. The input signal consists of three frequency packets. The highest frequency packet is almost completely attenuated. The lowest frequency packet is amplified by about a factor of 1.75, and delayed by about 40 samples. The medium frequency packet is amplified by about a factor of 1.5, and delayed by 80 samples.

5.36 Factor \( H(z) \) in terms of its poles and zeros, to give

\[
H(z) = \frac{(1 - 2jz^{-1})(1 + 2jz^{-1})}{(1 - 3/4z^{-1})(1 + 3/2z^{-1})}.
\]  

Thus, the system has poles at 3/4 and \(-1/2\), and zeros at \( 2j \) and \(-2j \). Because the system is stable, the ROC must contain the unit circle, and so

\[
\text{ROC}(h) = \{ z \mid |z| > 3/4 \}.
\]

The system is not minimum phase, because the two zeros occur outside the unit circle. We “reflect” them back into the unit circle, by introducing pole-zero cancellations at \( 1/2j \) and \(-1/2j \),
giving

\[ H_{\text{min}}(z) = \frac{(1 - \frac{1}{2} j z^{-1})(1 + \frac{1}{2} j z^{-1})}{(1 - \frac{3}{4} z^{-1})(1 + \frac{3}{2} z^{-1})} \]  
\[ = \frac{1 - \frac{1}{2} j z^{-1}}{1 - \frac{3}{4} z^{-1}}. \]  
\[ H_{\text{ap}}(z) = \frac{(1 - 2 j z^{-1})(1 + 2 j z^{-1})}{(1 - \frac{1}{2} j z^{-1})(1 + \frac{1}{2} j z^{-1})}. \]  

We should associate regions of convergence with \( H_{\text{min}} \) and \( H_{\text{ap}} \) such that the region of convergence of the product \( H_{\text{min}} H_{\text{ap}} \) is the region of convergence of \( H(z) \). Thus, we associate with \( H_{\text{min}} \) the region of convergence

\[ \text{ROC}(h_{\text{min}}) = \{ z \mid |z| > 3/4 \}, \]  
and with \( H_{\text{ap}} \) the region of convergence

\[ \text{ROC}(h_{\text{ap}}) = \{ z \mid |z| > 1/2 \}. \]  

I leave it to you to sketch the picture based on the expressions given for \( H_{\text{min}}, H_{\text{ap}} \) and the regions of convergence.