Option Value of Cash*

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Abstract

This paper presents a dynamic model of heterogeneous beliefs (where investors agree to disagree) to study the positive price-volume correlation during a housing downturn. It shows: (i) beliefs may diverge, which prevents some pessimists from buying; (ii) in the case that beliefs cross (i.e., buyers become more optimistic than the sellers), home sales occur but are delayed due to the buyers' option to sell cash higher (using house as numeraire) if the downturn worsens. Such option to wait also has implications for the velocity of money during deflation, troubled assets since 2007, takeover bids, IPO waves, and fire sales.

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1 Introduction

In recent years, the U.S. home sales dropped significantly alongside home prices. According to the National Association of Realtors, existing home sales in the US is 24% lower in 2008 compared to 2006. Meanwhile, the S&P/Case-Shiller home price index, which tracks changes in the value of the residential real estate market in 20 metropolitan regions across the United States, show a home price decline of 26% from January 2006 to December 2008. Such positive price-volume correlation exists in other samples. Stein (1995), using data going back as far as 1968, also finds that home prices are correlated with trading volumes.

The literature on the price-volume relation in the housing market has largely focused on the supply side, i.e., the sellers. Stein (1995) and Genesove and Mayer (1997) show that the downpayment requirement makes a homeowner more reluctant to sell at a loss, which can imply less home sales. Genesove and Mayer (2001) document the loss aversion of homeowners as an explanation to the positive price-volume correlation in the housing market. It is natural to wonder whether the demand side (i.e., the buyers) plays any role in the price-volume relation.

This paper shows that the demand side can play an important role in reducing trading volume, which can inflict a potentially large cost to those homeowners who need to sell quickly.

To study trading, this paper uses a dynamic model of heterogeneous agents. In this model, home buyers and sellers have different prior beliefs regarding the house fundamental value. At the start of the housing slump, the existing homeowners are parameterized to be the initial optimists and the potential buyers the initial pessimists. Agents Bayesian-update their beliefs over time using common new information regarding the state of the housing market. In another word, the investors agree to disagree (Aumann (1976)). Potential trading opportunity arrives when beliefs cross, i.e., when a buyer becomes more optimistic than a seller regarding the fundamental value of a house.

Heterogeneity is modeled here by heterogeneity in prior beliefs. Since a nationwide house price slump like the one since 2007 is unprecedented in the past 120 years, investors have few experiences with similar events.[1] This prevents learning from eliminating differences in priors. Many of the paper's implications can also apply to other heterogeneities, for example, in preferences, wealth fluctuations, user costs, or other constraints, as long as such heterogeneities generate belief crossing.

[1]Home prices since 1890 are available from Robert Shiller's website.

This paper points out two reasons that reduce trading volume. First, for some buyers and sellers, beliefs may not cross and may instead diverge. I.e., the buyers become increasingly more pessimistic relative to the sellers. Such divergence occurs in an important parameter region — when the buyers and sellers' priors have the same precision. This is seemingly contradictory to the prediction from Bayesian learning. Bayesian learning implies that the posterior estimates will increasingly be determined by the data, which implies convergent mean estimates in large sample (Savage (1972)). The explanation lies in the nonlinear relation between price and belief parameter. In this model, agents disagree over the severity of the housing downturn. To be exact, they disagree over the recovery probability, which is the probability that the downturn ends within given time horizon. However, the fundamental value relates to the expected length of the downturn, which is the inverse of the recovery probability. In a prolonged downturn, Bayesian investors' posterior estimates of recovery probability converge, but towards zero. This, under certain conditions, leads to divergence in perceived fundamental values. Such divergence in fundamental values precludes some pessimistic investors from buying in the absence of a fire sale. It also prevents these pessimists from funding other agents' mortgages at a low rate.

For other parameter regions, belief crossing can happen. This includes the interesting case where the buyer is more experienced (i.e., the buyer has a more precise prior than the seller). When the downturn persists, both the buyer and the seller revise down their perceived fundamental values. However, the revision is slower for the buyer if she has a sharper prior. In an extreme case, if the buyer's prior is infinitely precise (i.e., the buyer knows the true recovery probability), the buyer's belief does not change and belief crossing occurs when the seller becomes sufficiently pessimistic. This can capture the bottom fishing behavior by some sophisticated investors who hold cash and wait to buy distressed assets.

After the beliefs cross, will the buyers (who are now more optimistic than the sellers) buy immediately? Interestingly, the answer is no. This is the other reason that reduces trading volume. By delaying and holding onto the cash, a buyer retains the opportunity (option) to buy at a bargain price in the future if the downturn worsens. Such option value of cash is consistent with the anecdote that "cash is king" and the occurrence of cash hoarding during crises. Numerical examples show that the option can result in substantial delays in a buyer's purchase.

The source of this option value is the temporary monopoly created by belief dispersion. This paper has a competitive setting with a continuum of buyers and sellers. However, for a buyer who is relatively more optimistic than other buyers, she knows she can wait for a bargain price because other buyers will not compete away the housing supply due to their pessimism. This implies that the most optimistic buyer is a monopoly (temporarily) and can deviate from the behavior in a competitive setting with homogeneous agents, which is to buy immediately when the price reaches perceived fundamental value. In another word, the monopoly due to belief dispersion allows a buyer to avoid suboptimal option exercise due to competition.[2] In practice, when a potential homebuyer finds house price attractive yet sees mounting "for sale" signs, it is feasible to infer that she is among the more optimistic buyers and is in a monopolistic position similar to the one described in this paper. Therefore, the homebuyer can afford to be strategic and wait for a better bargain, even if there are many other potential homebuyers. Such belief dispersion amplifies the "cash-in-the-market" pricing in Allen and Gale (1994). Even if there is ample cash on the sideline, buyers can delay if belief dispersion is large.

Such delay can be costly to a home seller who demands immediacy, such as a homeowner who is relocating for a new job. To attract buyers, the seller needs to cut the house price to compensate the buyer's option value of cash in addition to belief difference. Alternatively, the seller may decide to give up the new job and stay with an inferior job, which can also be costly. The total cost to a homeowner is the minimum of the option value of cash and the cost of giving up a better job. Numerical examples show that the option value of cash can be large, therefore potentially costly to a homeowner who needs to move.

The price-volume relation induced by the option value of cash can extend beyond the housing market. Potential applications to the velocity of money during deflation, the troubled assets in the crisis since 2007, takeover bids, and IPO waves are discussed. The option value of cash also extends the fire sale in Shleifer and Vishny (1992) to non-specialized assets.

Interestingly, the option value of cash in downturns relates to the speculative bubble in booms. Harrison and Kreps (1978), Morris (1996), and Scheinkman and Xiong (2003) show that an asset can be bid higher by speculators in anticipation of selling it to a "greater fool" in the future. Here, cash is valued higher by homebuyers in anticipation of selling cash higher in the future (i.e., when

[2] See Grenadier (2002), Lambrecht and Perraudin (2003), and Kondor (2007) for the effect of competition on option exercise when beliefs are homogeneous.

the house market deteriorates further, cash is worth more using house as numeraire). The key ingredients shared by this paper and the speculative bubble literature are belief crossing and the short-sales constraint of the higher-valued asset. In the current paper, the higher-valued asset is cash. Short-sales constraint in cash implies that investors cannot borrow money unlimitedly to buy houses. This is not unrealistic—the house price drop since 2007 coincides with an economy-wide deleveraging process. The belief dispersion also implies high mortgage cost to borrow from pessimists. This paper differs from the speculative bubble literature in that, when houses cannot be shorted, the option manifests in the trading volume rather than in the price. Such prediction of delay, therefore, also adds to the speculative bubble literature.

Heterogeneous beliefs can also come from asymmetric information, which is an important component in the market microstructure literature (e.g., Glosten and Milgrom (1985) and Kyle (1985)). To focus on its main contribution, this paper assumes away asymmetric information along with other important issues such as search for a new house, mortgage financing, taxes, etc. On the other hand, it shows that the option value of cash does not rely on these other mechanisms.

Section 2 presents the model. Section 3 discusses the model implications to housing and other markets. Section 4 concludes. The appendix contains the proofs.

2 The model

2.1 Assumptions

The model is in continuous time with infinite horizon. There is a risk-free asset (cash) and a risky asset (house). The risk-free term structure is flat at rate r. There are K homogeneous units of houses outstanding in the economy. A larger or better house may be viewed as having multiple such units. Each unit of house generates rental income of $d_t dt$ between time t and $t + dt$, where dt denotes an infinitesimal period. The rent switches between two regimes,

$$d_t = \begin{cases} D & \text{in normal market} \\ D - \delta & \text{in downturn} \end{cases}.$$

where $0 < \delta \leq D$. Time $t = 0$ is the beginning of the downturn. The regimes are time-varying according to the following transition probability matrix

	Normal at $t + dt$	Downturn at $t + dt$
Normal at t	1	0
Downturn at t	λdt	$1 - \lambda dt$

(1)

where λ is the recovery intensity which governs the transition probability from downturn to normal state.[3] For ease of notation, this paper will sometimes simply refer to λ as the recovery probability. Given λ, the downturn is expected to last $1/\lambda$. For simplicity, the normal state is absorbing—once the downturn ends, the regime stays at the normal state. This does not qualitatively affect the results, which are predictions during the downturn.

Investors do not observe λ, though they observe the current and past realized regimes (rental income is observable). Investors learn λ via Bayesian learning.[4] Each investor's prior of λ is assumed to come from the class of Gamma distributions, denoted by $Gamma(a, b)$ where the parameters $a > 1$ and b may differ across investors to reflect differences of opinion. A Gamma distribution $Gamma(a, b)$ has two parameters.[5] a and b are, respectively, the shape and scale parameters. To simplify notation during Bayesian updating, Gamma distribution in this paper is expressed in terms of the inverse scale. I.e., $Gamma(a, b)$ in this paper is equivalent to the typical definition of Gamma distribution with parameters a and $1/b$. Gamma distributions are commonly used to model wait time and allow a rich set of possibilities. For example, they include the exponential and χ^2 distributions as special cases.[6] Gamma priors allow closed-form solutions to the model. Section 2.6.8 discusses extensions to other distributions.

The beliefs are common knowledge, i.e., investors agree to disagree as in Aumann (1976). Investors are risk neutral and maximize expected discounted life-time income.

Assumption 1 (Optimists)**.** *There is a continuum of investors who collectively hold the K units*

[3]Such transition probability relates to the concept of hazard rate and has also been used to study crashes, currency de-pegs, or defaults. See Duffie and Singleton (2003) and Yu (2007) for additional details.

[4]Different from Barberis, Shleifer, and Vishny (1998), investors in the current paper use the correct model to update their beliefs.

[5]The probability density function of $Gamma(a, b)$ at λ is $\lambda^{a-1} e^{-b\lambda} b^a / \Gamma(a)$. $\Gamma(a) \equiv \int_0^\infty t^{a-1} e^{-t} dt$ is the Gamma function (see Lebedev and Silverman (1972) for additional details).

[6]$Gamma(1, \lambda)$ gives the exponential distribution with parameter λ. $Gamma(v/2, 1/2)$ is the χ^2 distribution with degree of freedom v.

of houses at the beginning of downturn. They are referred to as sellers in the paper. Each seller is identical and has prior $Gamma\,(a, b)$ *at* $t = 0$*.*

The assumption of homogeneous sellers simplifies the illustration and allows the focus on the option value of cash held by buyers. Extension to heterogeneous sellers is discussed in Section 2.6.7.

Assumption 2 (Pessimists)**.** *There is a continuum of investors indexed by* $i \in [0, 1]$ *who hold cash at the beginning of the downturn* $t = 0$*. They are referred to as buyers in the paper. Each buyer has sufficient capital to buy* M *units of houses.* $M > K$*, i.e., buyers collectively can absorb all the house supply. The buyers are assumed to have heterogeneous beliefs, all of which are from the Gamma class.*

2.2 Bayesian updating and the fundamental value

This section collects several useful results regarding Bayesian updating under Gamma priors and the resulting formula for the buy-and-hold fundamental value of houses.

Lemma 1 (Bayesian updating)**.** *For an investor whose prior of* λ *is* $Gamma\,(a, b)$*, the posterior after* Δ *periods of downturn is* $Gamma\,(a, b + \Delta)$*.*

Lemma 2 (Properties of Gamma distribution)**.** *If* λ *is distributed* $Gamma\,(a, b)$*,* $E\,(\lambda) = a\,/b$*,* $Var\,(\lambda) = a\,/b^2$*. When* $a > 1$*, the expected length of downturn is finite and given by* $E\,[1\,/\lambda] = b\,/(a - 1)$*.*

The precision of an agent's belief relates to $Var\,(\lambda)$. Larger $Var\,(\lambda)$ is associated with less precision. Lemmas 1 and 2 imply that, as the downturn lengthens, the posterior becomes more precise and expects lower λ. Figure 1 illustrates several examples of the Gamma probability density function. For an investor with prior $Gamma\,(20, 19)$, the belief peaks at $\lambda = 1$ and expects the downturn to last one year. The prior of $Gamma\,(20, 19)$ is more precise than the $Gamma\,(2, 1)$ prior, which also expects a one-year downturn. If the prior is $Gamma\,(2, 1)$, the posterior becomes $Gamma\,(2, 3)$ after observing two more periods of downturn. The Bayesian learning has two effects. First, lower recovery probability λ is considered more likely after observing longer downturn. Second, the posterior becomes more precise after more observations.

During the downturn, the fundamental value of a unit of house given λ is the net present value (NPV)

$$NPV(\lambda) = \int_0^{\infty} e^{-rt} D dt - \int_0^{\infty} \left(\int_0^t e^{-rs} \delta ds \right) \lambda e^{-\lambda t} dt \quad (2)$$
$$= \frac{D}{r} - \frac{\delta}{r+\lambda}$$

which is the fundamental value during the normal state minus the expected total losses during the downturn.

For an investor with belief $Gamma(a,b)$ regarding λ, the fundamental value of a unit of house is obtained by integrating (2) over the belief,

$$V(a,b) = E_{Gamma(a,b)}[NPV(\lambda)] \quad (3)$$
$$= D\left[\frac{1}{r} - \frac{\delta}{D} \cdot b^a e^{rb} r^{a-1} \Gamma(1-a, rb)\right]$$
$$\approx \frac{D}{r} - \delta \cdot \frac{b}{a-1} \qquad \text{when } r \text{ is small.}$$

(3) gives an exact formula and an approximation for the buy-and-hold fundamental value $V(a,b)$. The exact formula involves the incomplete gamma function $\Gamma(\cdot,\cdot)$, which is well defined if the second argument is positive and can be evaluated numerically to arbitrary precision.[7] Therefore, the exact valuation formula is essentially in closed form. The last step in (3) follows because, when r is small, the expected loss during the downturn is largely determined by the loss δ per period and the expected length of downturn (the expected length is $b/(a-1)$ by Lemma 2).

The propositions in this paper are proved using the exact as opposed to the approximate formula for $V(a,b)$ unless "when r is small" is explicitly stated. All the numerical examples and figures in this paper use the exact formula.

[7]Specifically, $\Gamma(x,y) \equiv \int_y^{\infty} t^{x-1} e^{-t} dt$. The software Mathematica can compute the incomplete gamma function to arbitrary precision using its command Gamma[·,·]. See Lebedev and Silverman (1972) for additional details on the incomplete gamma function.

2.3 Cases of initial conditions

Consider any two investors i and j and their Bayesian priors regarding λ at time $t = 0$. Assuming without loss of generality that investor i is more optimistic in the sense that $E_i(\lambda) > E_j(\lambda)$. The subscript specifies the investor whose belief is used to compute the expectation. There are three possibilities: (1) $Var_i(\lambda) = Var_j(\lambda)$ in which case both investors have the same prior precision; (2) $Var_i(\lambda) > Var_j(\lambda)$; (3) $Var_i(\lambda) < Var_j(\lambda)$. In case (2), investor i begins with less precise prior. It will be shown later that, as a result, her belief turns more pessimistic faster than investor j when bad news come in. After sufficiently many bad news, investor i, who has less precise prior, becomes more pessimistic than investor j. This corresponds to case (3) with the labels of i and j reversed. Therefore, case (3) is subsumed by case (2). The rest of this paper will focus on cases (1) and (2) in sections 2.4 and 2.5, respectively. It will turn out that case (1) corresponds to the case of belief divergence while case (2) corresponds to the case of belief crossing where the option value of cash emerges.

2.4 Case 1: Divergence of beliefs regarding fundamental

This section studies those buyers whose prior at time $t = 0$ is $Gamma(a_P, b_P)$ where a_P and b_P satisfy

$$E_P(\lambda) = \frac{a_P}{b_P} < \frac{a}{b} = E(\lambda) \tag{4}$$
$$Var_P(\lambda) = \frac{a_P}{b_P^2} = \frac{a}{b^2} = Var(\lambda).$$

Recall from Assumption 1 that the sellers' prior is $Gamma(a, b)$. In this section, the subscript P of a variable indicates that the variable applies to the buyers, who are initially pessimistic. Those variables without the subscript P apply to the sellers. Recall $V(\cdot, \cdot)$ is the posterior belief about fundamental value from equation (3).

Proposition 1. *When condition (4) holds,*

- *(Initial disagreement) If r is sufficiently small, $V(a, b) > V(a_P, b_P)$ at time $t = 0$.*
- *(Belief updating) After Δ periods of downturn, the sellers and the buyers believe that the fundamental value is $V(a, b + \Delta)$ and $V(a_P, b_P + \Delta)$, respectively.*

- *(Initial divergence in valuation) For given* Δ*, if* r *is sufficiently small,*

$$V(a_P, b_P + \Delta) - V(a_P, b_P) < V(a, b + \Delta) - V(a, b) < 0. \quad (5)$$

- *(Eventual convergence in valuation)* $\lim_{\Delta \to \infty} V(a, b + \Delta) = \lim_{\Delta \to \infty} V(a_P, b_P + \Delta) = (D - \delta)/r$.

(5) is interesting because the fundamental value believed by the buyers may diverge from that of the sellers, even controlling for their prior precisions. This is seemingly opposite of the Bayesian learning prediction that posterior mean estimates converge after common observations (Savage (1972)).[8] What explains the difference? To see the intuition, (2) shows that, given recovery intensity, the difference in beliefs of fundamental value between the seller and the buyer is

$$\left(\frac{1}{r + \lambda_P} - \frac{1}{r + \lambda}\right)\delta \approx \left(\frac{1}{\lambda_P} - \frac{1}{\lambda}\right)\delta \quad (6)$$

when r is small. λ and λ_P denote, respectively, recovery intensity in the eyes of the seller and the buyer. When the downturn persists, both investors' posterior expectations of recovery intensity converge as predicted by Bayesian learning. However, the posterior expectations converge towards zero, which can lead to divergence in (6) under certain conditions because the recovery intensity sits in the denominator.[9]

Proposition 1 proves for the case when r is small. Does it apply to r of realistic magnitude? Further, Proposition 1 shows that eventually the beliefs regarding fundamental converge, consistent with Bayesian learning. How long can the initial divergence last? Both questions are useful for assessing the empirical relevance of Proposition 1. Figure 2 shows that Proposition 1 can apply under reasonable value of r (0.5% per month). In Figure 2, the priors are such that the sellers (buyers) initially expect the downturn to last 1 month (6 months). This translates into a small difference in believed fundamental values at $t = 0$: \$199.12 for the sellers and \$197.52 for the buyers. The proportional difference is less than 1%. The difference increases to over 10% two years into the downturn (\$182.67 for the seller and \$164.10 for the buyer). Eventually, the difference converges back to zero. However, the convergence does not begin until after a long time (about 50 years and

[8] Though see Acemoglu, Chernozhukov, and Yildiz (2009) regarding the fragility of asymptotic agreement under small perturbations to Bayesian learning.

[9] Proposition 1 is conditioning on being in the downturn. Therefore, the beliefs do not exhibit the "eternal switching" in Morris (1996).

the difference peaks at over 25%). That the divergence can be so large and last for so long implies that these buyers characterized by Proposition 1 can be ruled out as meaningful buyers without a fire sale. Therefore, such buyers are eliminated from the potential buyers in Section 2.5 below.[10]

2.5 Case 2: Option value of cash and delayed purchase

This section studies the case where the sellers have less precise priors at time $t = 0$. It will be shown that beliefs cross in this case. I.e., the sellers' belief of fundamental value becomes lower than that of buyers after a string of bad news. When beliefs cross, the buyers will contemplate buying houses from the sellers and this section studies the timing of such purchase.

Assumption 3 (Pessimists' beliefs)**.** *The prior of a buyer $i \in [0,1]$ at the beginning of the downturn $t = 0$ is Gamma $(a_L, b_L + ig)$ where $g > 0$ captures belief dispersion among pessimists. Further,*

$$a_L > a, \; \frac{b_L}{a_L - 1} > \frac{b}{a-1}. \tag{7}$$

The buyers are assumed to have heterogeneous beliefs. Recall from Lemma 2 that, for an agent with belief *Gamma* (x, y), increasing y makes the agent more pessimistic holding everything else constant. Therefore, $i = 0$ is the most optimistic buyer and $i = 1$ corresponds to the most pessimistic buyer. (7) in Assumption 3 ensures that, at time $t = 0$, the buyers have more pessimistic and more precise priors than the sellers, as shown in the lemma below.

Lemma 3. *Under Assumptions 1–3, at the beginning of the downturn $t = 0$,*

$$Var_i(\lambda) < Var(\lambda) \quad \textit{for any } i \in [0,1]$$

where Var_i and Var denote the prior variances of buyer i and the sellers, respectively. Further, when r is sufficiently small,

$$V(a_L, b_L + ig) < V(a, b) \quad \textit{for any } i \in [0,1]$$

where $V(\cdot,\cdot)$ is the belief of fundamental value from equation (3). Recall that the sellers' prior is

[10] Proposition 1 shows that these buyers will not purchase for buy-and-only purpose. It can also be shown that these buyers will not buy for short-term speculation (see Proposition 2 and the discussion following it).

Gamma (a, b) *at* $t = 0$.

The buyers in this section can be interpreted as experienced investors who hold cash waiting for the opportunity of bottom fishing of distressed assets. Such investors may obtain more precise priors from, e.g., proprietary research. A sharper prior implies slower belief updating, which leads to the possibility of belief crossing. In an extreme case, if a buyer's belief is infinitely precise (i.e., the buyer knows the recovery intensity), the belief does not change with new information. Therefore, when the sellers become sufficiently pessimistic, belief crossing occurs.

Assumption 3 also implies that a more pessimistic buyer stays more pessimistic relative to a more optimistic buyer during the downturn, as shown in the following lemma.

Lemma 4. *For any two buyers* $i < j$, *the expected length of the downturn is smaller for* i *than* j *at any point in time during the downturn.*

Therefore, belief crossing occurs between the sellers and the buyers, but not among the buyers in this setting. Such layering of the buyers not only simplifies the illustration, but also can capture the various real estate investors — e.g., individuals looking for primary residence, individuals looking for second home/investment properties, institutional investors, or other entities, etc. Their differences in incentives, required rates of return, etc., can make these different clienteles step in at different times.

When the downturn progresses, the sellers gradually revise downward their belief of the fundamental value. Eventually, belief crossing occurs when the sellers' belief of fundamental drops below that of buyer $i = 0$. If the downturn persists, the sellers' belief will further cross below more pessimistic buyers. After beliefs cross, the buyers value the house higher from a buy-and-hold perspective than the sellers. Will the buyers buy? If so, when will they buy? This paper shows next that the answer to the first question is, perhaps naturally, yes. However, the answer to the second question is not "buy as soon as beliefs cross."

Before the sellers sell out of their house holdings, the sellers remain the marginal investor and the house price tracks the sellers' belief of fundamental value. For a buyer, this induces a dichotomy between buy-and-hold and short-term returns, which is documented in the following proposition.

Proposition 2. *Let* *Gamma* (α, β) *and* *Gamma* (A, B) *denote, respectively, the belief of sellers* *(marginal investor) and a buyer at some point in time during the downturn. The buyer's instantaneous expected return from investing in a house is above r if*

$$\frac{A}{B} \geq \frac{\alpha}{\beta}. \tag{8}$$

Further, when r is sufficiently small, the house price is below the buyer's buy-and-hold valuation if

$$\frac{A}{B} \geq \frac{\alpha}{\beta} - \left(\frac{1}{\beta} - \frac{1}{B}\right).$$

Note that $B > \beta$ because of (7). Therefore, for the buyer, there is a parameter region where,

$$\underbrace{\frac{\alpha}{\beta} - \left(\frac{1}{\beta} - \frac{1}{B}\right) <}_{\textit{buy-and-hold ret>r}} \frac{A}{B} \underbrace{< \quad \frac{\alpha}{\beta}}_{\textit{instantaneous ret<r}} . \tag{9}$$

In the situation described by (9), the house price is already below the buyer's buy-and-hold valuation. However, the buyer prefers to delay buying because the short-term expected return is too low. This is because, if the downturn persists, the sellers' belief of fundamental drops faster than the buyer's belief. Therefore, to the buyer, delaying purchase creates an opportunity to buy at a price below fundamental.

The intuition of the dichotomy between long-term buy-and-hold return and short-term return is that a buyer's perceived buy-and-hold return is determined by her own belief of the fundamental only. However, the short-term return depends also on the price fluctuation, which is affected by the other agents' beliefs.

In essence, beliefs cross twice here. First, the beliefs regarding buy-and-hold value cross. Then, the beliefs regarding instantaneous return cross. A buyer prefers to wait until the beliefs regarding instantaneous return cross, at which point the house price is sufficiently distressed that further waiting risks losing the surplus should the recovery takes place.

Competition from other buyers may prevent a buyer from waiting until the ideal entry condition (8) is met. Such effect from competition on option exercise is consistent with Grenadier (2002), Lambrecht and Perraudin (2003), and Kondor (2007). Interestingly, different from these studies, the differences-of-opinion that generate the option value of cash also hinder the competition from more pessimistic buyers, which is formalized in the following proposition.

Proposition 3. *For a buyer i, let $j(i)$ refer to the buyer whose buy-and-hold valuation equals the house price at the same time when i's instantaneous expected return from buying a house equals r. Those buyers in the range $(i, j(i))$ find house price attractive from a buy-and-hold perspective before buyer i's ideal entry time. Therefore, they can potentially compete with i. When r is sufficiently small,*

$$j(i) = i + \frac{b_L - b}{a-1}\frac{1}{g} + \frac{i}{a-1}.$$

Buyers $(i, j(i))$ collectively control capital $(j(i) - i) M$. For the most optimistic buyer $i = 0$,

$$j(0) M = \frac{b_L - b}{a-1}\frac{M}{g}. \tag{10}$$

Even if the total capital M is large, the buyer $i = 0$ can wait until her ideal entry time if the belief dispersion $g \to \infty$. On the contrary, when belief dispersion disappears ($g \to 0$), a house is purchased immediately after price equals buy-and-hold valuation.

Belief dispersion affects competition. Even if capital is plenty, larger belief dispersion implies more buyers will sit on the sideline due to their pessimism. This effectively reduces competition facing the relatively more optimistic buyers and can exacerbate their delay in purchase. Therefore, belief dispersion amplifies the "cash-in-the-market" pricing in Allen and Gale (1994).

Combining Proposition 2 and 3 characterizes the timing of house purchases. Note that Proposition 3 places an upper bound on the competition by assuming that the competitors step in at their buy-and-hold valuations even though these competitors themselves may prefer to wait. The equilibrium timing of purchase is shown below in Proposition 4.

Proposition 4 (Equilibrium: delayed house purchase)**.** *When r is sufficiently small, the time $t(i)$ when buyers $i \in [0, K/M]$ buys a house is*

$$t(i) = n(i) + \min\left(t_1^*(i), t_2^*(i)\right)$$

where

$$n(i) = \frac{(a-1)\,b_L - b\,(a_L - 1)}{a_L - a} + \frac{a-1}{a_L - a} ig$$
$$t_1^*(i) = \frac{b_L - b}{a_L - a} + \frac{i}{a_L - a} g$$
$$t_2^*(i) = g \frac{a-1}{a_L - a} \left(\frac{K}{M} - i \right).$$

$n(i)$ is the time when the house price reaches i's buy-and-hold valuation. t_1^* is the wait time until the instantaneous expected return becomes attractive. t_2^* is the wait time before competition absorbs all the supply of houses. n, t_1^*, t_2^*, and hence t are increasing in g (longer delay when beliefs are more dispersed).

Figure 3 visualizes the equilibrium. At the start of the downturn, the most optimistic buyer ($i = 0$) values a house for buy-and-hold purpose at \$196.11, just below the seller' belief of \$196.13 regarding the house fundamental. The house price equals the sellers' belief of fundamental until all sellers sell their houses. Belief crossing occurs between the sellers and buyer $i = 0$ almost immediately after the downturn starts. However, due to the option value of cash, buyer 0 does not buy until 3.24 months into the downturn at which point the house price is \$188.05 and buyer 0's buy-and-hold valuation is \$194.78. Buyer $i = 0$ delays for about 3 months and, upon purchase, locks in a surplus of \$6.73 which is over 3% of the house price. Similarly, the house price drops to the buy-and-hold valuation of buyer $i = 0.1$ at 10.94 months into the downturn (at this time, the house price is \$175.19). However, the buyer $i = 0.1$ does not step in until month 25.45 when all remaining buyers swarm in together. This is a delay of over a year. At the entry time (month 25.45), the buy-and-hold valuation of buyer $i = 0.1$ is \$171.16, which is substantially above the house price of \$159.06. Intuitively, more pessimistic buyers can afford to delay longer because they believe a longer downturn hence perceive a bigger chance of buying at bargain prices. Without the threat of competition, buyer $i = 0.1$ would have liked to wait even longer. Due to competition, those buyers $i \geq 0.084$ buy at the same time and, by doing so, absorb all the houses from the sellers. Other than the last buyer $i = 0.2$, none of the other buyers step in immediately after the house price reaches their buy-and-hold valuations.

The delay can reduce house transactions substantially. The second plot in Figure 3 shows the cumulative fraction of houses sold over time. The plot shows both the equilibrium in Proposition 4 and a hypothetical equilibrium where the option value of cash is assumed away and buyers are assumed to step in as soon as the house price reaches the buy-and-hold valuations. The market is essentially frozen until 3.24 months into the downturn. Without waiting, 18% of the houses would have changed hands by this time. One year into the downturn, only 17% of the houses are bought by the buyers compared to 54% without waiting. At the two-year point, more than 60% of the houses is still left with the sellers while, without waiting, the buyers would have absorbed over 95% of the houses.

Knowing the optimal entry time $t(i)$ of each buyer from Proposition 4, the option value of cash can be explicitly calculated. Let $\pi(i,t)$ denote the equilibrium expected profit per unit of house for buyer i at time t. At the actual entry time $t(i)$, the expected profit is the difference between the buy-and-hold valuation and the house price,

$$\pi(i,t(i)) = V(a_L, b_L + ig + t(i)) - V(a, b + t(i)). \tag{11}$$

Prior to entry, the expected profit is discounted for both the time value of money and the probability of recovery. I.e., for $t < t(i)$,

$$\pi(i,t) = \pi(i,t(i))\, e^{-r(t(i)-t)} P(\text{downturn lasts till } t(i))$$

where the probability is taken under buyer i's posterior at time t. The following proposition provides a closed-form expression of the expected profit.

Proposition 5. *For a buyer $i \in [0, K/M]$, when r is sufficiently small, the equilibrium expected profit per unit of house is*

$$\pi(i,t) = \pi(i,t(i))\, e^{-r(t(i)-t)} \left(\frac{b_L + ig + t}{b_L + ig + t(i)}\right)^{a_L} \tag{12}$$

per unit of house at time $t \leq t(i)$. $t(i)$ is the optimal entry time by buyer i in Proposition 4. $\pi(i,t(i))$ is the expected profit upon entry, which is characterized in closed form by (11) and (3).

Figure 4 plots the equilibrium expected profit (12). For a given buyer, the expected profit increases over time due to less discounting. At a given time in the downturn, the expected profit is hump shaped across potential buyers. Because more pessimistic buyers perceive a smaller probability of recovery, they tend to wait longer to buy at more distressed prices. This tends to increase their expected profit. However, for very pessimistic buyers, the threat of competition from other buyers dominates. This tends to decrease the expected profit. The last buyer $i = 0.2$ cannot afford to delay at all and has an expected profit of zero. In the example in Figure 4, the fundamental value of the house is always lower than \$200. The option value can be as high as about \$15, which is over 7% of the house value and quite substantial.

Each buyer buys when the house price drops to the buy-and-hold value minus the expected profit (12) from the option to wait, not when the house price drops to the buy-and-hold value. This is illustrated in first plot of Figure 3 for buyer $i = 0$.

2.5.1 Cost to the sellers of the option value of cash

The option value of cash leads to a delay in house sale. However, no cost to sellers from such delay has been explicitly modeled so far. It is conceivable that sometimes a seller may prefer to sell a house quickly because, for example, she receives a better job offer in a different town. In this case, delay in house sales can be costly. If the homeowner is unwilling to cut the price to compensate buyers' option value of cash, the homeowner may have her home equity stuck with the house for a while and have to incur additional costs to hire a third party to manage the house after the homeowner moves. If this is undesirable, the homeowner may choose to turn down the new job and not move at all, which can also be costly.

To sell the house quickly, a homeowner needs to cut the price by

$$\Delta V + \pi \tag{13}$$

where ΔV is the difference in belief about fundamental value between the seller and the most optimistic buyer and π is the option value of cash for the buyer. $\Delta V < 0$ if belief crossing has occurred and the buyer is already more optimistic than the seller. $\Delta V > 0$ prior to belief crossing. The belief divergence result in Section 2.4 is also captured by setting $\pi = 0$ and ΔV to the belief difference. If the fire-sale in (13) is too costly, the homeowner may choose to turn down the new job offer and does not move. Assume it costs the homeowner C to stay with an inferior job. The total cost is

$$\min\left(\Delta V + \pi, C\right). \tag{14}$$

Figure 4 and Figure 2 show that both the option value π and the belief difference ΔV can be large and can inflict potentially significant cost to the homeowner.

The discussion assumes those homeowners who need to move have a zero mass. One can allow lumpiness in the arrival of job offers. However, a town with concentrated auto industry may have very different dynamics in job change from another town. Therefore, these variations are not directly modeled. Lumpiness lowers ex-ante house valuations, but does not alter the intuition regarding the option value to wait. When lumpiness is anticipated, homeowners on average are compensated through ex-ante lower house price. However, an individual homeowner who needs to move ex-post is not insured against the cost related to the option value of cash. This lack of insurance is similar in spirit to Shiller (2003).

2.6 Discussions and extensions

2.6.1 Speculation

Section 2.5 shows that the buyers do not step in immediately after the house price drops to the buy-and-hold valuations. Rather, the buyers buy when the house price drops to the buy-and-hold valuation minus the option value of cash. Such option value of cash, interestingly, relates to the speculative bubble typically associated with boom times. For example, Harrison and Kreps (1978), Morris (1996), and Scheinkman and Xiong (2003) show a speculative bubble when a speculator bids up the price of an asset in anticipation of selling it to a "greater fool" in the future. The option value of cash in the current paper arises when the buyers hope to sell cash higher (using house as numeraire) in the future when the housing slump worsens. Due to heterogeneous beliefs, competition does not eliminate the option value, which is similar to Abreu and Brunnermeier (2003) though the current paper does not depart from common knowledge.

The key ingredient shared by the current paper and this speculative bubble literature is (1) belief crossing: the sellers (who are initially more optimistic) become more pessimistic than the buyers; (2) short-sales constraint.[11] Note that, in the current paper, it is the short-sales constraint in cash (instead of house) that matters. Since private investors cannot print money, this short-sales constraint is equivalent to a constraint on leverage, which is not unreasonable if the housing slump coexist with an economy-wide deleveraging process. Further, the belief dispersion that generates the option value also constrains borrowing, see Section 2.6.2.

Different from this speculative bubble literature, the option value is not directly manifested in the price. Recall that, in Section 2.5, the house price tracks the buy-and-hold valuation of the sellers (who are the marginal investors) before the buyers absorb the entire housing supply. The option value perceived by the buyers does not appear in the price, because individual houses are difficult to short. Instead, the option value manifests through lower trading volume. Such delay due to the option value of cash is a new addition to the speculative bubble literature.

2.6.2 Can the buyers borrow?

If the potential buyers can borrow to buy houses, the option value of cash can be eroded due to increased competition (higher M reduces t_2^* in Proposition 4). However, the differences of opinion that generate the option value of cash also limit borrowing. From whom can a buyer borrow? Pessimistic lenders are reluctant to lend at a low mortgage rate because the buyer appears to be buying an overvalued house. Optimistic lenders likely prefer to buy themselves to capture the expected profits.

2.6.3 House supply

The house supply is fixed in the model. Since housing construction tends to drop during housing market downturn albeit with a lag, the supply is unlikely to increase substantially. On the contrary, homeowners may have less incentive to maintain their houses. Therefore, the quality-adjusted house supply may decrease. The reduced supply can reduce the option value of cash due to increased competition among buyers (lower K reduces t_2^* in Proposition 4). However, short of a drastic decline in house supply, the option value of cash will not be eroded completely, particularly to the more optimistic buyers (those with i close to zero in Proposition 4).

[11] Though see Cao and Ou-Yang (2005) who show that short-sales constraint need not be essential for a speculative bubble when investors are risk averse.

2.6.4 User costs

Section 2.5 does not explicitly model user costs. To a homeowner, the user costs can include repair costs, property taxes, etc. See Poterba (1984) for more details on user costs. When different investors have different user costs (e.g., different tax statuses), they have different buy-and-hold valuations holding everything else the same. Time-series variations in the user costs in Poterba (1984) can lead to house valuation changes that differ across investors. The differences-of-opinion model in this paper serves as a reduced-form way to reflect the valuation changes associated with user cost variations. The option value of cash can similarly arise if the belief crossing is induced by variations in user costs.

2.6.5 Costly search

The housing market in this paper is assumed centralized and information regarding all houses is instantaneously available to all potential buyers. Costly search (e.g., Wheaton (1990)) is assumed away to illustrate that the delay due to the option value of cash can exist even with centralized trading. When costly search is present, the total delay will be the sum of the delay due to search and the delay due to option value of cash. The two elements can amplify each other. For example, a potential buyer will like wait longer if waiting not only allows her to gain from further price drop but also allows her more time to search for a better matching house. Therefore, the paper also adds to the costly search literature because the motive for speculation affects the search decision.

2.6.6 Short sales

As discussed in Section 2.6.1, it is the short-sales constraint of cash that is essential for the option value of cash. Short-sales constraint in house implies the option value manifests through trading volume. In markets other than housing, shorting is sometimes feasible. When short sales are allowed, the option value manifests directly as lower asset price (i.e., higher price of cash when the asset is numeraire). It can be shown that allowing shorting of the asset results in an equilibrium resembling predatory trading in Brunnermeier and Pedersen (2005). Specifically, some potential buyers may even short at a price below their belief of the fundamental. This equilibrium is observationally similar to that in Brunnermeier and Pedersen (2005) except that the predators here are not strategic. These results are suppressed and are available from the author.

2.6.7 Seller heterogeneity

Section 2.5 assumes homogeneity among the sellers to focus on the option value of cash for buyers. When the sellers are heterogeneous, there is also an option value to delay sales for the sellers, which is similar in intuition to the option value for buyers. To see this, consider a seller and a buyer at the point of belief crossing when both have the same buy-and-hold valuation. If the seller holds on to the house, the outcome next period is "heads I win" (when the downturn ends) or "tails you lose" (when the downturn persists, the buyer who has more precise belief becomes more optimistic than the seller and creates an opportunity for the seller to sell at a price above seller's belief of fundamental). This option is worthless when the sellers are homogeneous due to competition (see Proposition 3). The option value raises the sellers' reservation value to sell, consistent with the speculative bubble literature discussed in Section 2.6.1. The equilibrium can be solved similarly to the one in Section 2.5. A seller's reservation value to sell equals the buy-and-hold value plus the option value to hold onto the house. A buyer's reservation value to buy equals the buy-and-hold value minus the option value of cash. Trade occurs when the reservation values cross, not when the beliefs regarding buy-and-hold value cross. A closed-form equilibrium is, however, difficult to characterize. Therefore, seller heterogeneity is not explicitly modeled.

2.6.8 Other belief distributions

Sections 2.4 and 2.5 use Gamma priors to obtain closed-form solutions. The tractability is due to Gamma distribution being the conjugate prior of exponential distributions (the length of the downturn is exponentially distributed here), which allows Bayesian updating in closed form. However, the intuition applies more generally. Specifically, similar results are obtained from a discrete-time transition matrix (instead of the continuous-time transition matrix (1)) and priors from the class of Beta distributions (the conjugate prior of Bernoulli transition probability in discrete time). These results are omitted for brevity.

Information theory also suggests that the result can extend to other distributions.[12] Let $f_1(\lambda)$ and $f_2(\lambda)$ be the probability density function (pdf) of the priors of a buyer and a seller, respectively.

[12] See Pierce (1980) for an introduction to information theory.

In Sections 2.4 and 2.5, f_1 and f_2 are from the Gamma distributions. In general, the difference between the two priors can be measured by the relative entropy (also known as the information divergence, or the Kullback-Leibler distance)

$$\text{entropy}\left(f_1, f_2\right) = \int f_1\left(\lambda\right) \log \frac{f_1\left(\lambda\right)}{f_2\left(\lambda\right)} d\lambda. \tag{15}$$

The entropy is always non-negative and equals zero if and only if the two densities are the same. Let f_1 be the pdf of $Gamma\left(a_L, b_L + \Delta\right)$ and f_2 be the pdf of $Gamma\left(a, b + \Delta\right)$. These are the posteriors of the buyer and the seller at time $t = \Delta$ in Sections 2.4 and 2.5 (buyer $i = 0$ in Section 2.5). Given the Gamma posteriors, the entropy varies over time according to

$$\frac{\partial}{\partial \Delta} \text{entropy}\left(f_1, f_2\right) = \frac{\left(b_L - b\right)}{\left(b + \Delta\right)\left(b_L + \Delta\right)^2} \left(a_L b - a b_L + \left(a_L - a\right) \Delta\right).$$

It can be calculated that the parameter region (4) for the no belief-crossing case in Section 2.4 corresponds to $\frac{\partial}{\partial \Delta}\text{entropy}(f_1, f_2) > 0$, i.e., a monotonically divergent entropy. On the other hand, for the belief crossing case in Section 2.5, (7) implies

$$\frac{\partial}{\partial \Delta} \text{entropy}\left(f_1, f_2\right) = \begin{cases} < 0 & \text{if } \Delta < \Delta^* \equiv \frac{a_L b - a b_L}{a_L - a} \\ \geq 0 & \text{if } \Delta \geq \Delta^* \end{cases}.$$

Therefore, the entropy first converges towards zero and then diverges. The time Δ^* when the entropy reaches its minimum corresponds exactly to the time when buyer $i = 0$ buys (i.e., $\Delta^* = t\left(0\right)$ in Proposition 4). At this time, buyer 0's posterior expectation of λ matches that of the seller. This turns out to imply buyer 0 finds the instantaneous return from investing in houses is attractive (see the proof of Proposition 2 for more details) hence it is her optimal purchase time. Afterwards, the entropy diverges again because buyer 0 becomes increasingly more optimistic than the seller. The mapping to entropy and information theory in general suggests that the option value of cash can extend to other belief distributions.[13]

[13] Entropy (15) is not symmetric between f_1 and f_2. However, the result is the same using entropy(f_2, f_1) or a symmetric version entropy(f_1, f_2) +entropy(f_2, f_1).

2.6.9 Multiple inferences

Sections 2.4 and 2.5 involve inference regarding the recovery intensity. Investors may need to infer additional unobservable variables. For example, there may be more than two regimes. Increasing the number of regimes allows finer approximation to the potentially continuous state of the housing market. Alternatively, the recovery intensity may be time varying and investors need to infer not only the level but also the rate of change of the recovery intensity, etc. If investors who are optimistic in recovery intensity are also optimistic in the other unobservable variables, the additional inferences exacerbate belief dispersion and its effect. On the contrary, if investors who are optimistic in recovery intensity are pessimistic in the other unobservable variables, the optimism and pessimism cancel out and belief dispersion is smaller. In general, more layers of inference allow more room for potential disagreement hence more room for the mechanism studied in this paper to appear.

2.6.10 Other preferences

Sections 2.4 and 2.5 simplify the analysis by assuming risk neutrality. Risk aversion or Knightian uncertainty (e.g., Epstein and Wang (1994)) can affect the buyers' valuations. However, the option value of cash in Section 2.5 can remain as long as beliefs (adjusted for risk/uncertainty aversion) cross.

3 Model implications

This section discusses the implication of the option value of cash for the housing market and potential applications to a number of interesting observations in other markets. The discussion for the other markets is subject to the caveat that the simple model in Section 2 cannot capture the full richness of each market and may omit other important mechanisms in play. Nonetheless, it is encouraging that the model can generate implications consistent with observations from these various markets. Enriching the model to address each individual market in detail is left for future research.

3.1 Price-volume correlation in the housing market

Section 2.5 implies that home sales are slower during housing market downturn. This is consistent with the recent housing market dynamics. According to the National Association of Realtors, the median (mean) sales price of existing homes in the US dropped 11% (10%) between 2006 and 2008. The S&P/Case-Shiller home price index, which tracks changes in the value of the residential real estate market in 20 metropolitan regions across the United States, show a home price decline of 26% from January 2006 to December 2008. The trading volume drops alongside the price decline. Existing home sales in the US is 24% lower in 2008 compared to 2006. Stein (1995), using data going back as far as 1968, also finds that home prices are correlated with trading volumes. The paper focuses on buyers in its explanation of the house price-volume correlation.[14] Therefore, it complements the existing literature, which focuses on sellers. It is likely that the sellers and the buyers are two sides of the same puzzle and together help explain the positive price-volume correlation in the real estate market.

3.2 Deflation and the velocity of money

The positive price-volume correlation from the option value of cash may extend beyond house purchases. For example, a consumer who expects deflation may prefer to wait and buy consumption goods in the future at a bargain. Holding constant the money stock, such delay implies lower velocity of money. The velocity of money is an important variable relating to both inflation and aggregate transactions. During the Great Depression, the consumer price index for all urban consumers (CPI-U series compiled by the Bureau of Labor Statistics) drops 23% from the beginning of 1929 to the end of 1932. Meanwhile, the velocity of money falls about 35% from 1929 to 1932 (see Chart 58 on page 641 of Friedman and Schwartz (1971)). Using a longer sample, Friedman and Schwartz (1971) (page 597) find that "Throughout almost the whole period from the Civil War through World War II, velocity ... tended to decline relative to its trend during the contraction phase of a cycle". Such drop in the velocity of money is consistent with the prediction from the option value of holding onto cash.

[14] Though the current paper can also speak to the sellers, see Section 2.6.7.

3.3 Price-volume correlation in the financial market

3.3.1 Troubled assets in the crisis since 2007

During the financial crisis since 2007, the banks are stuck with the alphabet soup of troubled assets for a long time. These troubled assets include for example the mortgage-backed securities (MBS), collateralized debt obligations (CDO), etc. These assets are difficult to short in general. They are also difficult to value, which likely results in differences of opinion regarding their payoffs. The economy is also under a deleveraging process during the crisis. An option value of cash can arise under these conditions. The predicted delay in sales is consistent with the slow pace at which these assets are unloaded. Similar to Section 2.5.1, such delay from the option value of cash is costly to the banks, which are left vulnerable to potential runs (Diamond and Dybvig (1983)).

The option value of cash can be reduced if the government purchases some of the troubled asset. Less troubled asset increases the competition among potential buyers (i.e., lower K implies less delay for some buyers in Proposition 4). This can have implications on interventions such as the Troubled Asset Relief Program (TARP).[15] Other intervention policies may also affect the option value of cash, which can be evaluated using the model. However, such intervention should also be weighed against taxes used to finance the intervention, incentive issues, etc., which are beyond the scope of this paper.

The availability of analyst forecasts in the stock market allows the construction of belief proxy hence an examination of the belief divergence prediction in Section 2.4 during the crisis. Specifically, analyst target price forecast history is obtained from Bloomberg on Oct 28, 2008 for the five largest commercial banks and five largest investment banks according to equity market capitalization at the end of 2006. The ten companies include Bank of America, Citigroup, JPMorgan Chase, Wachovia, Wells Fargo, Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch, and Morgan Stanley. Target price forecasts are assumed to reveal analysts' valuations (target prices are shown to be informative by Brav and Lehavy (2003) and Asquith, Mikhail, and Au (2005)). There are 51 unique analysts and 213 unique analyst-company pairs.[16] Each analyst on average covers 4 companies and

[15] In October 2008, the U.S. signed into law a bill authorizing the Treasury department to purchase as much as $700 billion in troubled assets.

[16] Bloomberg provides, for each stock, a list of analysts currently following the stock. However, this list does not include analysts who provided forecasts in the past but subsequently dropped coverage, even though these past forecasts remain available and can be downloaded given the analyst ID. This affects especially Bear Stearns (taken over by JPMorgan Chase in March 2008) and Lehman Brothers (filed for Chapter 11 bankruptcy protection in

each financial stock on average has forecasts from 21 analysts. The data contain 4,824 target price forecasts during the sample period of 2003-2008 (1,298 forecasts in 2007 and 1,374 forecasts in 2008). Bloomberg provides a variable "Period" indicating the horizon of the target price forecasts. Among the 1,982 non-missing horizon indicators, 94% (1,864 observations) are one-year target prices. Those target prices for other horizons are excluded. If the horizon indicator is missing (2,842 observations), the target price forecast is included since it is likely a one-year forecast, judging from observations with non-missing horizon indicator. Due to the stock price fluctuation, the same target price issued at different time can have different implications (e.g., a target price of $40 issued when the stock price is $35 differs from another forecast of the same target price issued when the stock price is $45). Correspondingly, Bloomberg provides the date of the analyst report and the closing stock price on the same day. We construct the scaled target price forecast for stock s on day τ by analyst i as

$$F_{s,\tau,i} = \frac{\text{Target price}_{s,\tau,i}}{\text{Close price}_{s,\tau}}.$$

Such scaling by market price is consistent with Dokko and Edelstein (1989) and Brav and Lehavy (2003). It represents the rate of return implied by the target price. The resulting data contain monthly scaled target prices for the ten companies. If an analyst issues multiple forecasts for the same company in a month, only the last forecast before month end is used. Analysts do not issue forecasts in all months. When an analyst does not issue forecast in a month, her most recent forecast in the past is assumed to be in effect for that month.[17] For each stock at the end of 2006, we sort analysts into two groups based on their scaled target price. Those analysts whose forecasts are below (equal to or above) the median are classified as pessimists (optimists). There are a total of 20 groups: one optimist group and one pessimist group for each stock. If an analyst covers multiple stocks, it is possible that she is in the optimist group for one stock yet in the pessimist group for another stock. The classification remains fixed during the rest of the sample period to see the belief evolution of the analysts who are initially optimistic (or pessimistic).[18] Let $\overline{F}_{s,\tau,g}$

September 2008). To reduce the effect from dropped coverage, we search in Bloomberg for an analyst's past coverage of all ten stocks as long as the analyst is currently covering at least one of the ten stocks. This mitigates the effect of dropped coverage because an analyst covering one financial company likely covers some other financial companies, too.

[17]The 25%, 50%, and 75% quantiles of days between successive forecasts are 9, 21, and 49 calendar days respectively. The results are similar if forecasts older than 1 or 3 months are excluded. These results are omitted for brevity and are available from the author.

[18]Forecasts of Bear Stearns are included up to February 2008 and forecasts of Lehman Brothers are included up to

denote the average of the scaled price targets across analysts in group g (g is either optimist group or pessimist group) for stock s in month τ. The following regression examines the belief dynamics during 2007-08.

$$\overline{F}_{s,\tau,g} = \sum_{t=Dec2006}^{Sep2008} \beta_t \times PESSIMIST_{s,g} \times MONTHDUMMY_t \tag{16}$$
$$+ \sum_{t=Dec2006}^{Sep2008} \alpha_t \times MONTHDUMMY_t + \varepsilon_{s,\tau,g}$$

where the dummy variable $PESSIMIST_{s,g}$ equals 1 (0) for the pessimist (optimist) group of stock s. The month dummy $MONTHDUMMY_t$ equals 1 if the forecast month equals t and 0 otherwise. t ranges from December 2006 to September 2008. The coefficients β_t are the objects of interest, which measure the valuation difference between pessimists and optimists. Figure 5 shows a time-series plot of the β estimates in 2007-08. In early 2007, before the financial crisis becomes headline, the valuation difference between optimists and pessimists shrinks over time. This convergence is consistent with the prediction from Bayesian learning based on common information. However, the beliefs diverge after the crisis starts, consistent with the prediction in Section 2.4.[19] The valuation discount β is statistically significant during the crisis until June 2008. After June 2008, the estimates become noisier though the point estimates indicate even wider divergence (the statistical significance is suppressed and available from the author). Note that the belief divergence during 2007-08 holds on average. Among individual analysts, belief crossing discussed in Section 2.5 occurs in that some initial optimists subsequently become more pessimistic than some initial pessimists.

3.3.2 Takeover bids

The option value of cash can have implications for takeovers, particularly for the bidding dynamics involving multiple bidders.[20] Takeover offers are often at a substantial premium relative to the

August 2008, one month before their restructuring events. The result is similar if Bear Stearns and Lehman Brothers are excluded.

[19] As a robustness check, the analysis is repeated on data one year earlier. I.e., optimists/pessimists are classified at the end of 2005 and held fixed during 2006. In this case, valuation convergence is observed throughout 2006. Unlike the belief dynamics during the crisis in 2007-08, there is no valuation divergence in 2006. Therefore, the belief dynamics in 2007-08 are unlikely driven by seasonality (e.g., Hong and Yu (2008)).

[20] I thank Sugato Bhattacharyya for suggesting this application.

target firm's pre-announcement price (Jarrell, Brickley, and Netter (1988)). When multiple bidders compete, the competing bidder is eager enough to outbid the initial bidder yet somehow did not make any bid earlier. Such observation can be consistent with the implications from option value of cash under certain conditions. When an acquirer is optimistic in the target firm due to perceived synergy, managerial hubris (Roll (1986)), or something else, such optimism shields the acquirer from competition of less optimistic investors. Therefore, the acquirer may find it feasible to deviate from the prescribed behavior in a competitive setting and wait until the target price is sufficiently below the acquirer's perceived value before making an offer, at which point the acquirer can afford a premium. If, before the acquirer's ideal time to make an offer, another bidder emerges, the increased competition forces the acquirer to exercise the option early and therefore make competing bids.

3.3.3 IPO waves

It is known that the number of initial public offerings (IPOs) change over time. For example, Pástor and Veronesi (2005) document that 845 firms went public in the US in 1996, yet only 87 IPOs in 2002. The option value of cash can have implications on why IPOs dry up in a down market: buyers may prefer to wait hoping the prices drop even further and hence are less eager to buy IPO stocks immediately. The conditions for the option value of cash are likely met in the case of IPOs. There can be substantial disagreement regarding newly listed firms, as argued by Morris (1996). Short-sales constraint naturally holds for firms yet to be listed. The size of an IPO is likely small relative to the amount of available capital, so competition among buyers may reduce the option value of cash (see Proposition 4). However, firms listed at adjacent times may be similar (Jovanovic and Rousseau (2001)). For example, many IPOs around year 2000 are related to information technology. Such similarity reduces competition among buyers because a buyer who misses out on an IPO may get another chance for an IPO of a similar company. Such similarity can also imply less than perfect diversification. This can deter buyers from taking excessive leverage, which increases the option value of cash.

3.4 Fire sales

Shleifer and Vishny (1992) show that specialized assets are susceptible to fire sales. This is because when a distressed firm needs to sell assets (say a farmer who tries to sell the land), its industry peers (neighboring farmers) are likely experiencing problems, too. The option value of cash can extend such fire sales to assets that are non-specialized. As discussed in Section 2.5.1, a seller needs to cut the price to compensate a buyer's option value of cash in addition to belief difference, which can manifest as fire sales.

4 Conclusion

This paper provides a dynamic model of heterogeneous beliefs to illustrate two reasons for less home sales in a housing downturn. First, the beliefs of homeowners and some potential buyers may diverge, which keeps the pessimistic buyers on the sideline. Further, in the case when the buyers become more optimistic than the homeowners, this paper shows an option value of cash, which can result in significant delays in home sales. Such option value of cash can potentially inflict large cost to a homeowner that demands immediacy in home sales. The option value of cash also has implications for the velocity of money during deflation, troubled assets in the crisis since 2007, takeover bids, IPO waves, and fire sales.

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Appendix Proofs

Proof of Lemma 1 and 2: This follows from the Bayes rule by noticing the probability of staying in the downturn is $e^{-\lambda t}$ over a period of length t. The expected values are from integration over the Gamma distribution function. $\square$

Proof of Proposition 1: When $r \downarrow 0$, (3) implies

$$V(a,b) - V(a_P, b_P) = \delta \cdot \left(\frac{b_P}{a_P - 1} - \frac{b}{a-1} \right). \tag{17}$$

The assumption in equation (4) implies $b > b_P$ which, together with $\frac{a}{b} > \frac{a_P}{b_P}$ assumed in (4), further implies $a > a_P$ and $\frac{a-1}{b} > \frac{a_P - 1}{b_P}$. Therefore, $V(a,b) > V(a_P, b_P)$.

The belief updating follows from Lemma 1. After Δ periods of continuous downturn, the investor with prior $Gamma\,(a,b)$ updates her belief to $Gamma\,(a, b+\Delta)$, and the valuation change is

$$V(a, b+\Delta) - V(a,b) = \delta \cdot \left(\frac{b}{a-1} - \frac{b+\Delta}{a-1} \right) = -\delta \cdot \frac{\Delta}{a-1}$$

when r is sufficiently small. Both investors revise down their valuations after observing a prolonged downturn, but the seller revises less since $a > a_P > 1$.

The eventual convergence in valuation occurs because, as the downturn persists, asymptotically both buyers and sellers believe they will never get out of the downturn, hence both value the asset at $(D-\delta)/r$. Mathematically, it is because the limit of $b^a e^{rb} r^{a-1} \Gamma(1-a, rb)$ in (3) is $1/r$ when $b \uparrow \infty$. $\square$

Proof of Lemma 3: (7) implies $\frac{a_L}{b_L} < \frac{a}{b}$ and $\frac{a_L}{b_L^2} < \frac{a}{b^2}$. Therefore,

$$Var_i(\lambda) = \frac{a_L}{(b_L + ig)^2} < \frac{a}{b^2} = Var(\lambda).$$

When r is sufficiently small, (3) implies that

$$\begin{aligned} V(a_L, b_L + ig) - V(a,b) &= \delta \left(\frac{b}{a-1} - \frac{b_L + ig}{a_L - 1} \right) \\ &< 0 \end{aligned}$$

where the last step follows from (7). □

Proof of Lemma 4: A buyer i's posterior is $Gamma\left(a_L, b_L + ig + \Delta\right)$ after Δ periods of downturn. The expected length of downturn is $\left(b_L + ig + \Delta\right) / \left(a_L - 1\right)$, which is increasing in i. □

Proof of Proposition 2: The buyer's expected instantaneous payoff

$$
\begin{aligned}
&(D-\delta)\, dt + \int \left[(\lambda dt)\, \frac{D}{r} + (1-\lambda dt)\, V\left(\alpha, \beta + dt\right)\right] f_{Gamma(A,B)}\left(\lambda\right) d\lambda \\
&= (D-\delta)\, dt + \frac{D}{r}\left(\frac{A}{B}\right) dt + \left(1 - \frac{A}{B} dt\right) V\left(\alpha, \beta + dt\right) \qquad (18)
\end{aligned}
$$

where f is the posterior probability density of recovery intensity of the buyer. When the recovery occurs, price moves to D/r. Otherwise, the price reflects the sellers' valuation before they sell all the houses. The sellers (marginal investors) belief the expected instantaneous payoff is

$$
(D-\delta)\, dt + \frac{D}{r}\left(\frac{\alpha}{\beta}\right) dt + \left(1 - \frac{\alpha}{\beta} dt\right) V\left(\alpha, \beta + dt\right) \qquad (19)
$$

where, due to time consistency of Bayesian learning, the marginal investor's expected return is r. Intuitively, this is because a seller's perceived buy-and-hold and short-term returns both involve only the seller's belief hence both expected returns are consistent with each other (on the contrary, the short-term return perceived by a buyer depends on price fluctuation, which involves the sellers' belief). (18) and (19) imply that the buyer's expected instantaneous return is higher than r if and only if $\frac{A}{B} \geq \frac{\alpha}{\beta}$.

When r is sufficiently small, (3) implies that the sellers' valuation is below that of the buyer if $\frac{\beta}{\alpha-1} \geq \frac{B}{A-1}$. □

Proof of Proposition 3: After n-periods into the downturn, the buyer i's posterior is $Gamma\left(a_L, b_L + ig + n\right)$ and the sellers' posterior is $Gamma\left(a, b + n\right)$. By proposition 2, i buys for the long term if

$$
\frac{b+n}{a-1} = \frac{b_L + ig + n}{a_L - 1}. \qquad (20)
$$

If he waits t more periods until the instantaneous return is attractive, the entry time satisfies

$$\frac{a_L}{b_L + ig + n + t} = \frac{a}{b + n + t}. \tag{21}$$

Let j be the buyer whose buy-and-hold valuation at time $n+t$ equals market price, similar to (20),

$$\frac{b + n + t}{a - 1} = \frac{b_L + jg + n + t}{a_L - 1}. \tag{22}$$

Eliminating n and t from the above three equations yields

$$j - i = \frac{b_L - b}{a - 1}\frac{1}{g} + \frac{i}{a - 1}.$$

□

Proof of Proposition 4: The buyer $i = K/M$ who buys the last unit of house will buy as soon as his buy-and-hold valuation is reached. For investor $i < K/M$, his buy-and-hold entry time (n) and desired wait time (t_1^*) until the instantaneous expected return equals r can be solved from (20) and (21). However, due to competition, investor i must step in before the last buyer K/M does. The time until K/M steps in (which is t_2^*) can be calculated from (20) and (22) (specifically, solve for t by setting $j = K/M$ in the two equations). □

Proof of Proposition 5: Assuming the posterior regarding recovery intensity λ is $Gamma\,(A, B)$ at time t, the probability of no recovery before $t\,(i)$ is

$$\int e^{-\lambda((t(i)-t))} f_{Gamma(A,B)}\,(\lambda)\, d\lambda = \left(\frac{B}{B + t\,(i) - t}\right)^A$$

where, given intensity λ, $e^{-\lambda((t(i)-t))}$ is the probability of no recovery between t and $t\,(i)$. $f_{Gamma(A,B)}\,(\cdot)$ denotes the probability density function of $Gamma\,(A, B)$ distribution. The proposition follows because the posterior of buyer i at time t is $Gamma\,(a_L, b_L + ig + t)$. □

Figure 1: Examples of Gamma distributions

This figure shows the probability density functions of three Gamma distributions: $Gamma\,(20,19)$, $Gamma\,(2,1)$, and $Gamma\,(2,3)$. $Gamma\,(2,3)$ is the Bayesian posterior after observing two additional periods of downturn for an investor with prior $Gamma\,(2,1)$.

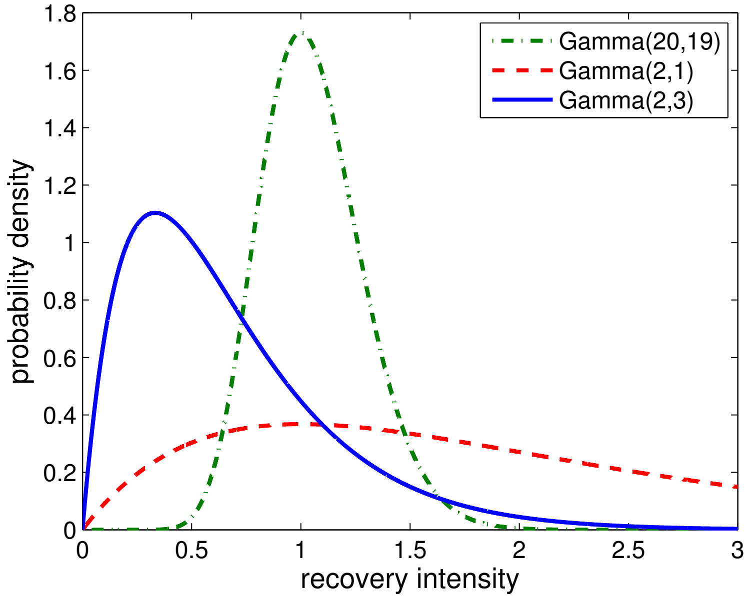


Figure 2: Divergence of buy-and-hold valuations

This figure shows the dynamics of the buy-and-hold valuations of the pessimist and the optimist when the downturn persists. The optimist's prior of recovery intensity is $Gamma\,(2,1)$ and the pessimist's prior is $Gamma\,(9/8,3/4)$. $r=0.5\%$ monthly. The dividend is $D=1$ and $D-\delta=1/10$ in the normal and down market, respectively. The first plot shows the valuations of the pessimist and the optimist during the first two years of the downturn. The second plot shows the valuation discount of the pessimist relative to the optimist up to 200 years into the downturn.

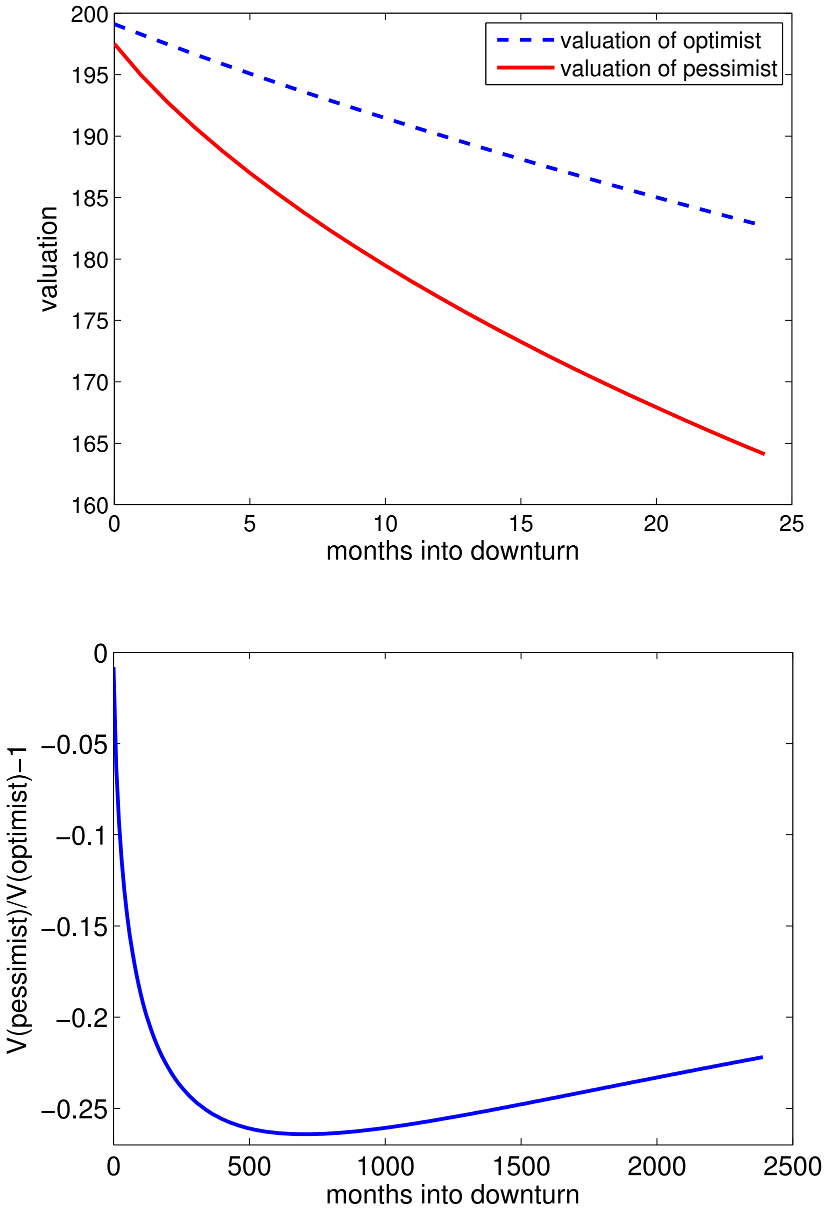
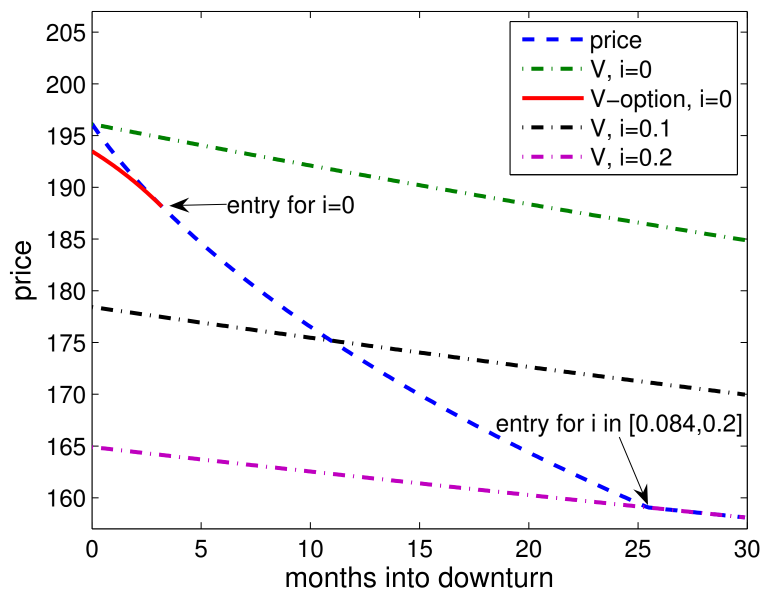


Figure 3: Delayed purchase

This figure illustrates the delay in buyers' purchases. The sellers' prior is $Gamma\,(26/25, 1)$. The buyers' priors are $Gamma\,(3, 9 + gi)$ for $i \in [0, 1]$. $g = 500$. Each buyer can buy $M = 5$ units of houses. The total supply of houses is normalized to $K = 1$. $r = 0.5\%$ monthly. The dividend is $D = 1$ and $D - \delta = 1/10$ in normal and down markets, respectively. The first plot shows the house price, along with the buy-and-hold valuations of buyers $i = 0$, 0.1, and 0.2. Also shown is the reservation value of buyer $i = 0$, which is the buy-and-hold value minus the option value of waiting in (12). The buyers absorb all the house after buyer $i = 0.2$ buys. At time 0, the most optimistic buyer $i = 0$ values the house at \$196.11, just below the sellers' valuation of \$196.13. Also shown are the equilibrium entry times for $i = 0$, and for $i \in [0.084, 0.2]$ who buy at the same time. The second plot compares the equilibrium cumulative fraction of houses sold to the hypothetical cumulative fraction when buyers do not wait and buy as soon as the house price drops to the buy-and-hold values.



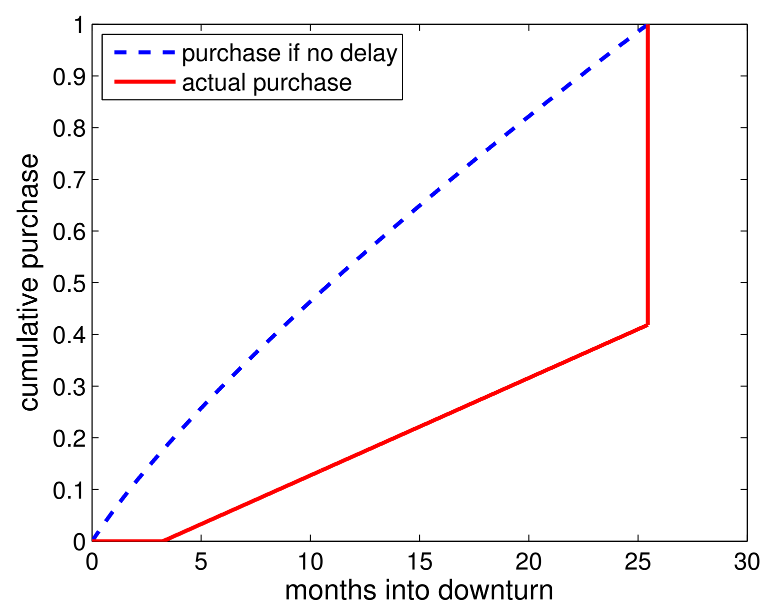


Figure 4: Option value

This figure plots the equilibrium expected profit (12) per unit of house from the option to delay purchase. The profit is plotted for buyers $i \in [0, 0.2]$ at different points in time before option exercise. The parameters are the same as those in Figure 3.

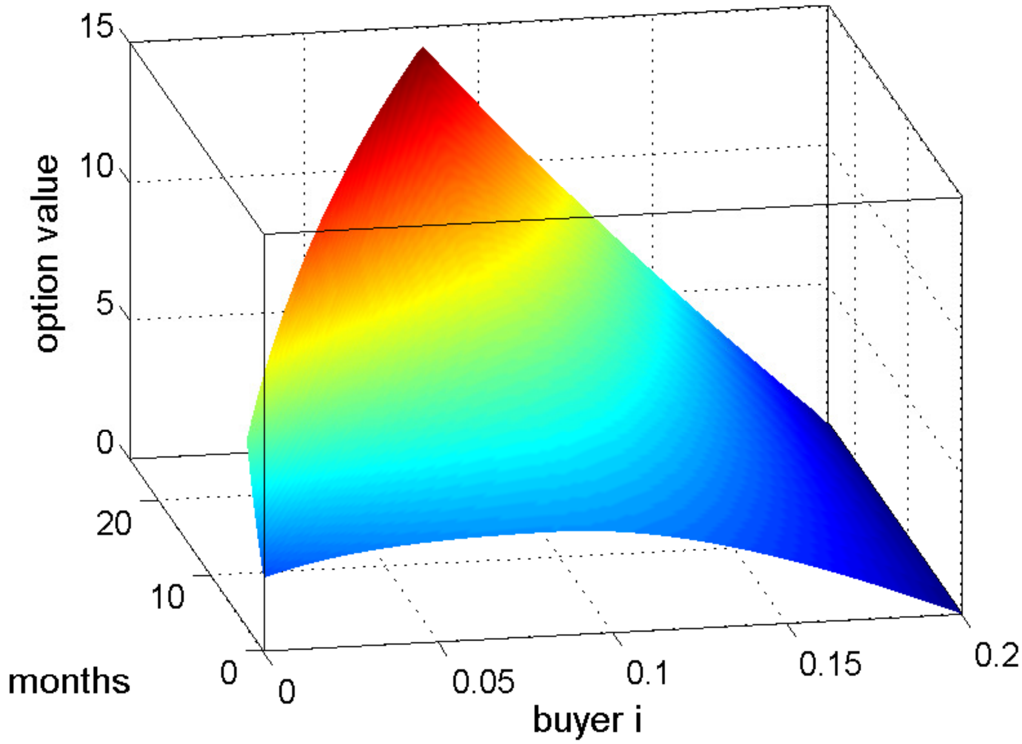


Figure 5: Belief dynamics for ten major financial stocks during 2007-08

The first plot shows the performance of S&P 500 index and an equal-weighted index of ten financial stocks in 2007-2008 (both indices normalized to 1 at the end of 2006). The ten stocks include Bank of America, Citigroup, JPMorgan Chase, Wachovia, Wells Fargo, Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch, and Morgan Stanley. The second plot shows β_t in regression (16) which is the time series of the valuation discount of pessimistic analysts relative to optimistic analysts. Optimists and pessimists are classified at the end of 2006 and held fixed during 2007-2008. The analyst valuation is defined as the analyst target price forecast divided by the stock close price on the day of analyst report.