IEOR 4630: Pricing models for financial engineering

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Lecture I: (09/06/07)

Banks borrow each other ⇒ LIBOR rate.

Math models model natural phenomena.

Special investment vehicle / special purpose vehicle

① | → borrow short term
  | → pay LIBOR + 25bps = 5.75 + 0.25 = 6 %.

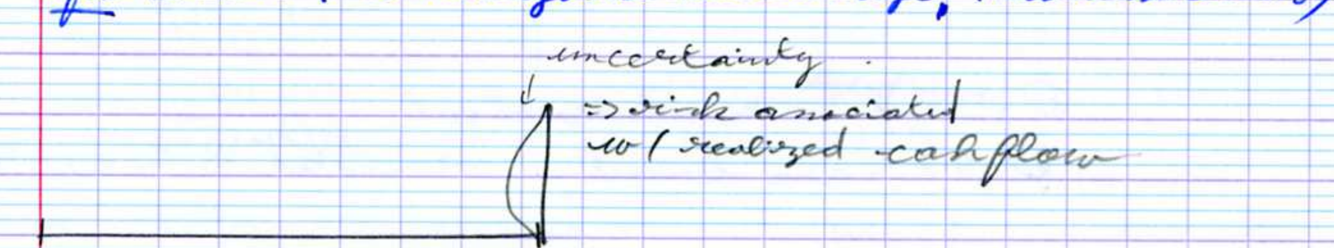
② → borrow CP = commercial paper ≤ 365.

back-stop facility: borrow from sb else (citi, JPM,) to repay a loan

- Relative value trades:

sell ⇒ convergence trade
buy ⚠ the 2 look different but they will converge because of their cash flows

cf: LTCM (convergence arbitrage, Russian crisis)



What is the PV of the cash flow?

At what rate should we discount the cash flow?

- arbitrage pricing.
- Martingales (change of measure)
- relationship between risk-neutral measure & obj measure

(1) consider an economy w/ and S and interest rate i such that in the next period the value of S is either S(H) or S(L)

$$S_0 \begin{cases} \xrightarrow{p} S(H) \\ \xrightarrow{1-p} S(L) \end{cases}$$

$\underline{t=0}$ $\quad$ $\underline{t=T}$

$1 \longrightarrow 1+i$

$B = PV(1\$) = \frac{1}{1+i}$

Suppose we know the price of S.

Consider all other assets in the economy such as G.

We know $G(H)$ & $G(L)$, then the price of G can be uniquely determined. Any deviation from this price results in arbitrage.

$$G \begin{cases} \xrightarrow{p} G(H) \\ \xrightarrow{1-p} G(L) \end{cases}$$

$$S \begin{cases} \xrightarrow{p} S(H) \\ \xrightarrow{1-p} S(L) \end{cases}$$

Consider the following "replicating portfolio"

$\Delta S + \lambda B$ such that at time T it has a cash flow exactly like G.

$$\Delta S(H) + \lambda = G(H)$$
$$\Delta S(L) + \lambda = G(L)$$

$$\Rightarrow \boxed{\Delta = \frac{G(H) - G(L)}{S(H) - S(L)}}$$

$$\boxed{\lambda = \frac{G(L)S(H) - G(H)S(L)}{S(H) - S(L)}}$$

(1) price of G does not depend on P.

(2) S & G are perfectly correlated $\Rightarrow S = a + bG$.

$$S \begin{cases} \xrightarrow{\frac{1}{2}} S(H) \\ \xrightarrow{\frac{1}{2}} S(L) \end{cases} \Rightarrow \frac{1}{2}\left(S(H) + S(L)\right) = E(S(T))$$

$$Var(S(T)) = \frac{1}{2}\left(S(H) - S(L)\right)^2$$
$$SD(S) = \frac{S(H) - S(L)}{\sqrt{2}}$$

Therefore:

$G = \Delta S + \lambda B$.

$$G = \frac{G(H) - G(L)}{S(H) - S(L)} S + \frac{S(H)G(L) - S(L)G(H)}{S(H) - S(L)} B.$$

$$\boxed{G = B\left[\frac{\frac{S}{B} - S(L)}{S(H) - S(L)} G(H) + \frac{S(H) - S/B}{S(H) - S(L)} G(L)\right]}$$

$\frac{S}{B} - S(L) \geq 0$ otherwise there is an arbitrage.

$\Leftrightarrow \frac{S}{B} \geq S(L)$.

and $S(H) - \frac{S}{B} \geq 0$ otherwise there is an arbitrage.

$\Leftrightarrow \underbrace{\frac{\frac{S}{B} - S(L)}{S(H)-S(L)}}_{Q} \geq 0 \ \lambda \ \underbrace{\frac{S(H) - \frac{S}{B}}{S(H)-S(L)}}_{1-Q} \geq 0$.

$\Rightarrow G = B\left[Q\, G(H) + (1-Q)\, G(L) \right]$.

↳ the value of G is the PV (E (cash flows)).

$\Rightarrow$ price of G = PV of expected payoff of asset G under prob. Q

Call options: (put with the call - put parity)

$$\boxed{p + S_0 = c + K e^{-rT}}$$

Most liquid options are close to the money, fairly shortdated.

The formula can be used to approx. the price of close to the money options.

$G = \Delta S + \lambda B$.

$\Delta = \frac{G(H) - G(L)}{S(H) - S(L)} \qquad \lambda = \frac{G(L) S(H) - G(H) S(L)}{S(H) - S(L)}$

ex: C dec 07 47.5 call

$C = 46.00 \qquad$ Annual vol. $= 31\% = \sigma$

today $\rightarrow$ dec = 3.5 months.

$v = \sigma \sqrt{T} = \sqrt{\frac{107}{365}} \cdot 31\% = 0.54 \cdot 31\% = 17\%$.

S (citi stock) 46 \$

$\quad\nearrow S(1.17) = S(1+\sigma\sqrt{T}) = 53.82$

$\quad\searrow S(0.83) = S(1-\sigma\sqrt{T}) = 38.18$.

G

$\quad\nearrow 53.82 - 47.5 = 6.32 = G(H)$

$\quad\searrow 0 = G(L)$

Strike $= X = 47.5 \Rightarrow \left| \begin{array}{l} G(H) = 6.32. \\ G(L) = 0 \end{array} \right.$

$X = (1+\alpha) S \Rightarrow \boxed{\alpha = 3\%}$

Io! $\left| \begin{array}{l} G(H) = S(1+\sigma\sqrt{t}) - (1+\alpha) S = (\sigma\sqrt{t} - \alpha) S. \\ G(L) = 0. \end{array} \right.$

$\rightarrow \Delta = \frac{(\sigma\sqrt{t} - \alpha) S}{S(1+\sigma\sqrt{t}) - S(1-\sigma\sqrt{t})} = \frac{\sigma\sqrt{t} - \alpha}{2\sigma\sqrt{t}}.$

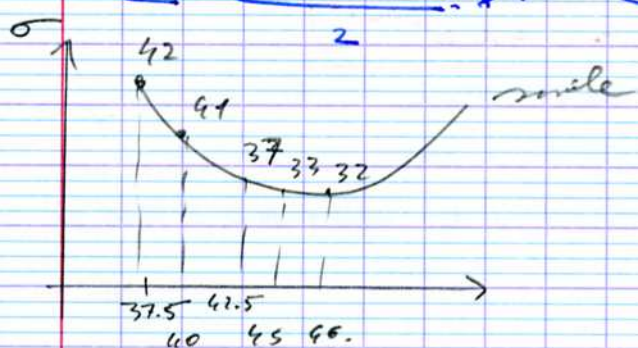
$\rightarrow \lambda = \frac{G(L) S(H) - G(H) S(L)}{S(H) - S(L)}.$ $\quad \boxed{v = \sigma\sqrt{t}.}$

since the maturity is short we can assume $B \simeq 1$

$\Rightarrow \lambda = -\frac{(v-\alpha) S (1-v) S}{2vS} = -\frac{(v-\alpha)(1-v) S}{2v}.$

$\Rightarrow \boxed{G = \frac{v-\alpha}{2} S.}$ ⊛ with $v = \sigma\sqrt{t}$, $X = (1+\alpha) S =$ strike.

And $\frac{0.17 - 0.03}{2} \times 46 = 3.22.$



⊛ $G = \Delta S + \lambda B$

$G = \frac{v-\alpha}{2v} S - \frac{(v-\alpha) S}{2v} + \frac{v(v-\alpha) S}{2v} = \frac{(v-\alpha) S}{2}$