Lecture XI: Credit derivatives

I – Cash Vs CDS

Negative basis
Z-spread
total-return swap
perfect asset swap
credit risk and rating agencies.
CDO's (credit debt obligation)
SIV's (special investment vehicules)

1°) CDS:

1 CDS = Insurance

$$\text{ABC} \xrightarrow{\text{premium}} \text{H}$$
$$\text{ABC} \xleftarrow{\text{par if credit event happens}} \text{H}$$

Reference entity:

If KLM $\xrightarrow{L+\Delta}$ that trades at par, then CDS premium $= \Delta$.

To imply the CDS, can use:

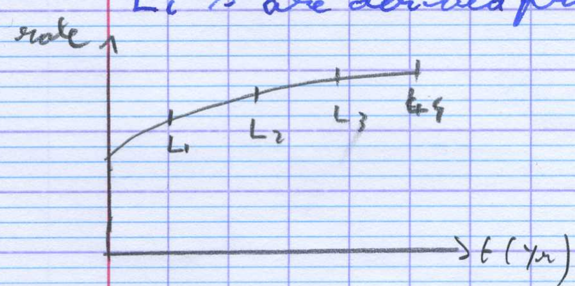
① Asset swap
② Z-spread (zero-volatility spread)

rem: $PV = \text{present value}(1\$) = \dfrac{1\$}{1+L+\Delta}$.

Suppose KLM has a fixed coupon bond traded at P (not necessarily par)

GM : 93.5
coupon = 7.2 // L_i that represent "riskless rates"

L_i's are derived from the swap-curve.



- We assume $\sigma = 0$ (zero - volatility) : forwards will be realized

discounted coupons:

$$\boxed{\frac{C}{1+L_1+\Delta} + \frac{C}{(1+L_1+\Delta)(1+L_2+\Delta)} + \dots + \frac{F}{(\ )(\ )(\ )} = PV}$$

ex: $\frac{7.2}{1+L_1+\Delta} + \frac{7.2}{(1+L_1+\Delta)(1+L_2+\Delta)} + \dots + \frac{100}{(1+L_1+\Delta)(\ \ )(\ \ )\dots}$

$= PV$ (price of the bond)

$\Rightarrow$ solve for Δ

Δ is called the z - spread

$$\boxed{\Delta_{tnet} \simeq \Delta_z \simeq \Delta_{OAS}.}$$

ex: GM $\longrightarrow$ C = coupon.

the yield of this bond is $y = L + \Delta$. with $\Delta = \Delta_{OAS}$.

here coupon = 7.2

If yield = 7.2% = C $\Rightarrow$ the bond would be traded at par $\Rightarrow$ $PV = F$ (present price = face value)

proof:

Let's take bond: | maturity T.
| # coupons: m / year.
| coupon = C
| yield = y.
| face value = F

note $n = Tm$.

then: $P = \sum_{k=1}^{Tm} \frac{FC/m}{(1+\frac{y}{m})^k} + \frac{F}{(1+\frac{y}{m})^n}$

$P = \frac{FC/m}{1+\frac{y}{m}} \sum_{k=0}^{n-1} \frac{1}{(1+\frac{y}{m})^k} + \frac{F}{(1+\frac{y}{m})^n}.$

$P = \frac{FC/m}{1+\frac{y}{m}} \frac{1-\frac{1}{(1+\frac{y}{m})^n}}{1-\frac{1}{1+\frac{y}{m}}} + \frac{F}{(1+\frac{y}{m})^n}.$

$P = F\frac{C}{m} \times \frac{1-\frac{1}{(1+\frac{y}{m})^n}}{\frac{y}{m}} + \frac{F}{(1+\frac{y}{m})^n}.$

$\boxed{Io: \text{ if } y = C \Rightarrow P = F}$

ex: If $y \simeq 7\%$, for 5 years

$\frac{700}{5} \simeq 140$ take discount

yield $7.2\% + 1.8\% = 9\%$.

Io yield = 9%

the yield of bond = T + Δ CDS

yield of treasury bond (need to know the spread over)

GM	↗	↘
ΔGM	↘	↗

view about the spread (ex: ΔGM) is going to tighten.
How do you express this view?

in the old days [- buy the bond in the cash market (OTC)
- have to hedge treasury risk ⇒ short the treasury

⇒ difficult: to find the bonds, the borrowing might not be there for the whole time of the trade.

⇒ simpler: In the CDS market we simply sell the CDS.

$ABC \xrightarrow{\text{CDS protection}} II$

2°) Basis: CDS premium - yield of the bond

If basis < 0

bond ⟶ 9%] buy the bond - receive the yield

CDS ⟶ 8.5%] buy CDS - you pay the CDS premium

Z-spread: more or less equivalent to CDS.

II - Total return swap

$GM \xrightarrow{7.2\%} 10{,}000{,}000$ { price = 93.5, effective yield = 9%.

Suppose we are bullish on GM, we think a spread is going to tighten, hence price ↑.
Because the return will be small, clients borrow some financial institutions balance sheet.

$$\text{Citi} \xrightarrow{C + \text{appreciation of the bond}} \text{ABC}$$
$$\text{Citi} \xleftarrow{L + \Delta + \text{any depreciation}} \text{ABC}$$

↳ total return swap spread.

buy the bond to issuing debt

$$\text{GM} \xrightarrow{C} \text{Citi} \qquad \text{Citi} \xrightarrow{L} \text{GM}$$

$P = 93.5$ GM

If bond price ↑ ⇒ ABC benefits.
ABC has bought a bond with the benefit of sb else's money.

bond price	92	93.5	95
pay for ABC	-1.5	0	Citi's going to pay ABC 1.5

III _ Credit linked note

_ cash security Vs "synthetic" contract.

$$\text{ABC} \xleftarrow{L + \Delta} \text{Citi} \qquad \text{ABC} \xrightarrow{100} \text{Citi}$$

payoff is "linked to the credit of GM"
ie: if GM defaults, the investor will lose par.

Citi issues this bond.
But pension funds can't invest in trades where they can lose their principal ⇒ invest in <u>principal - protected notes</u>

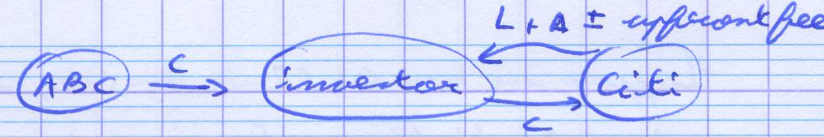
$$100 \xrightarrow{10\,Yr} 100 \text{ (maturity)}$$

Zero-coupon bond such that at maturity it is 100

IV _ Perfect asset swap _ or Extinguishing swap

$$\text{ABC} \xrightarrow{L + \Delta} \text{par} \quad \Rightarrow \quad \underline{\Delta = \Delta_{CDS}}$$

Now suppose

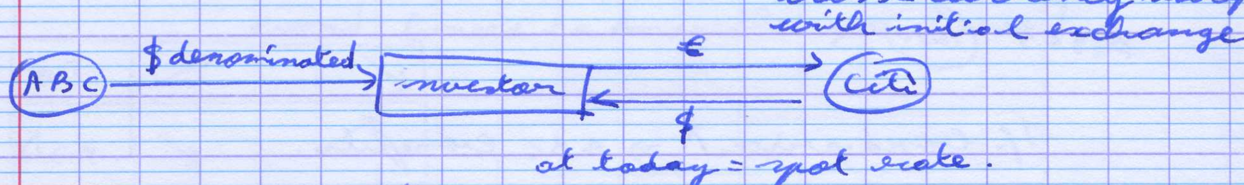


costs the investor 100.

→ citi adjust Δ to get 100 for investor.

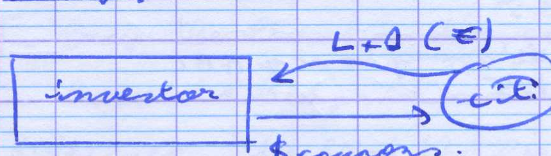
<u>Only risk</u>: if ABC defaults & the swap is out of the money

Investor still have to pay c even if ABC defaults.



ex: 100 € for 146 $.

<u>going forward</u>:

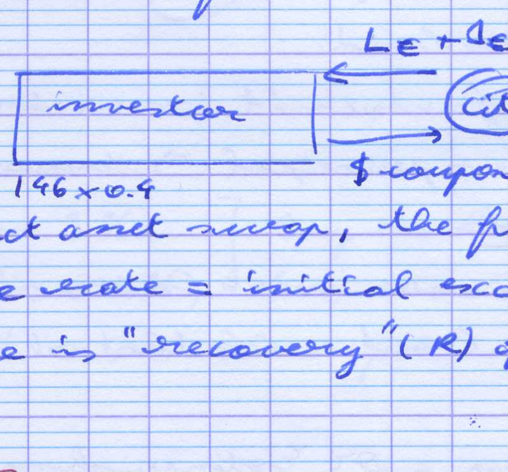


there is no default at maturity.

<u>What if there is a default</u>?

Suppose there is default:

$R = 0.4$



In a perfect asset swap, the final exchange accelerates

- exchange rate = initial exchange rate
- the size is "recovery" (R) of the bond.

V - Rating agencies

Moody's, S&P, Fitch

1°) Rating transition probabilities

ex: AA $\xrightarrow{5\,Yr}$ BBB. what probability?

- We can use the rating transition matrix to calculate probability of transition in 2, 3, 5 Yr ⟹ T×T×T...

rem: Indeed if:

T is the transition matrix.

$$T = \begin{pmatrix} T_{00} & T_{01} & T_{02} \\ T_{10} & \ddots & \\ T_{20} & & \ddots \end{pmatrix}$$ where T_{ij} = probability of going from rate i to j in 1 Yr.

⟹ these probabilities follow the Markov property.

⟹ T^2 = transition matrix in 2 steps (2 Yr)

T^n = ———————— n steps (n Years)

- Suppose we have a new and innovative instrument! how does the agency rate it?

Derive the distribution of loss in 5 Yr

⟹ take the matrix and raise to power 5: T^5

- S&P: looks at probability of 1$ loss

0.001 = proba to lose 1,000,000 or 100$

⟹ so same proba. for different sizes.

- Moody's calculates expected loss

⟹ { S&P rates for the 1,000,000 & 100 are the same.
Moody's doesn't.

⟹ That's why bonds usually get rated by both agencies.

VI - First to default basket

IBM
Merck
GE
JP Morgan

basket

$\xrightarrow{L+\Delta}$

Your view is that not all companies will default at the same time.
You don't want to buy protection on all, you buy protection on only 1 to default (the 1st one).

Important: what is the correlation of times to default

$\Rightarrow$ based on correlation, you can price this.

ex: $Y \sim \mathcal{N}(0,1)$

$Z \sim \mathcal{N}(0,1)$

$\mathrm{corr}(Z,Y) = \rho = \frac{1}{2}$

we can generate Y, Z / $\rho = \frac{1}{2}$

$$\boxed{\mathrm{corr}(Z,Y) = \frac{\mathrm{cov}(Z,Y)}{\sigma_Y \sigma_Z} \in [-1,1]}$$

rem: if $Y \perp\!\!\!\perp Z$ (independent)

$\Rightarrow \mathrm{cov}(Y,Z) = E[YZ] - E[Y]E[Z] = 0.$

$\Rightarrow \mathrm{corr}(Z,Y) = 0.$

However $\mathrm{corr}(Y,Z) = 0 \not\Rightarrow Y \perp\!\!\!\perp Z.$

$$\boxed{\text{So: } \begin{array}{l} Y \perp\!\!\!\perp Z \Rightarrow \rho = 0. \\ \rho = 0 \not\Rightarrow Y \perp\!\!\!\perp Z. \end{array}}$$

construction: Let (X_1, X_2, X_3) iid $\mathcal{N}(0,1)$

$$\begin{cases} Z = X_1 \\ Y = \rho X_1 + \sqrt{1-\rho^2}\, X_2 \end{cases} \Rightarrow \mathrm{corr}(Z,Y) = \rho$$

rem: Suppose $Z \sim \exp(1)$

$F(z) = 1 - e^{-z}$ have correlation matrix.

$F(y) = 1 - e^{-y}$

$$\begin{array}{c} Z \\ Y \\ W \end{array} \begin{pmatrix} & \rho_{zy} & \rho_{zw} \\ & & \\ & & \end{pmatrix}$$

The copula method assumes that the underlyings are Gaussian! not difficult.