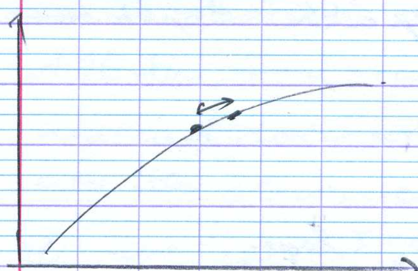
Lectures IV:

Modelling - term structure of interest rates.



$P(0,T) = e^{-\int_0^T r(t)dt}$.

$P(t,T)$ = price of zero-coupon bond

$R(t,T)$ = the rate starting at time t and maturity at T

$f(t,T,S)$ = forward rate at time t.

$\lim_{S \to T} f(t,T,S) = f(t,t) = r(t)$ = short rate.

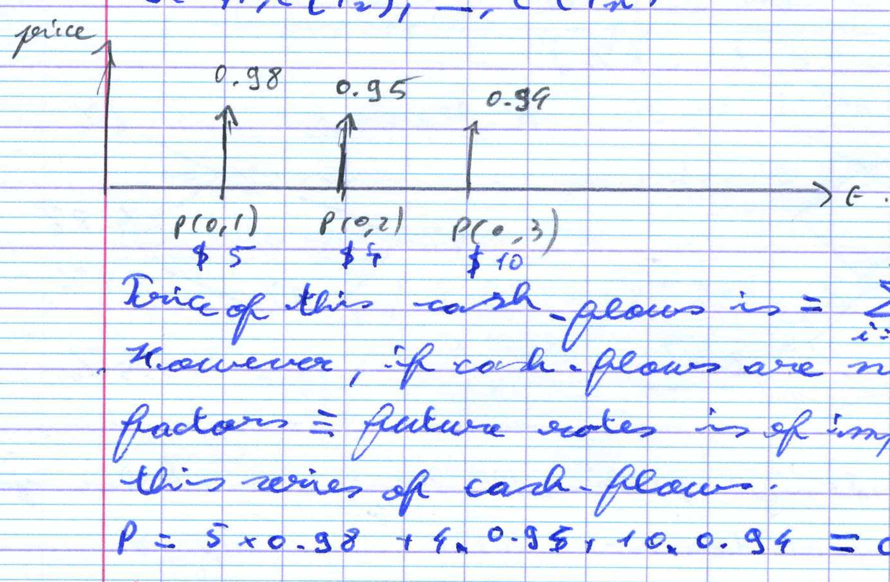
$0.98 = \frac{1}{1+r} = e^{-r} = \frac{1}{(1+\frac{r}{2})^2}$

- Why is term structure of interest rates important?

Let $P(t,T_i)$ = the observed price of 0-coupon bond.

Consider cash flows at time T_i for more:

$C(T_1), C(T_2), \dots, C(T_n)$



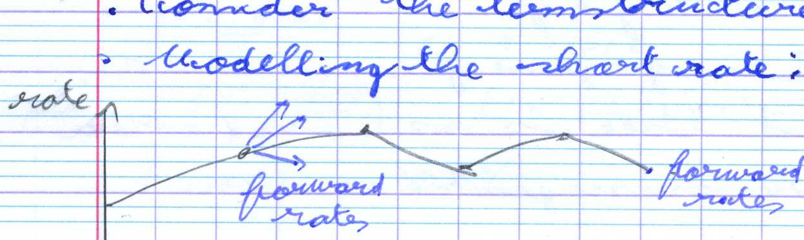
Price of this cash-flows is $= \sum_{i=1}^{n} C(T_i)\,P(0,T_i)$

- However, if cash-flows are not certain, the future factors ≡ future rates is of importance in pricing this series of cash-flows.

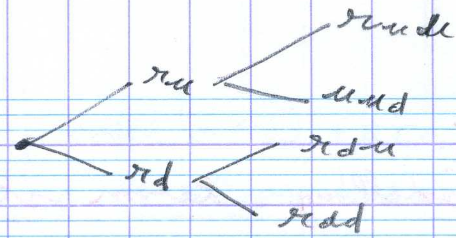
$P = 5 \times 0.98 + 4 \times 0.95 + 10 \times 0.94$ = C.F. are certain!!

Simple Model.

- Consider the term structure of interest rates today
- Modelling the short rate: discrete model.

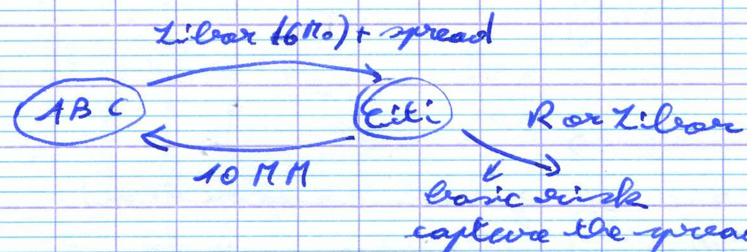


Binomial tree:



→ non-recombinant or path-dependent.

ex: interest rate cap.



LIBOR = London Interbank offered rate.

LIBID = London Interbank Bid rate.

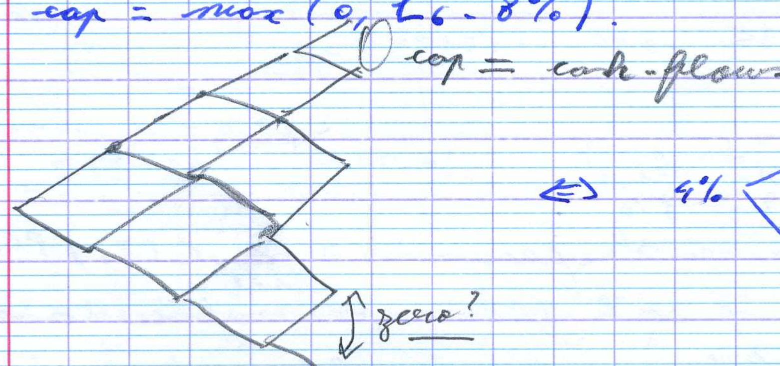
usually LIBOR + ($\frac{1}{8}$ of 1%) = LIBID.

prime rate = lend to good customers.

ex: Interest rate cap with strike of 8% = $\max(0, L_6 - 8\%)$

Libor(6)	6%	8%	10%
cap =	0	0	2

cap = $\max(0, L_6 - 8\%)$.



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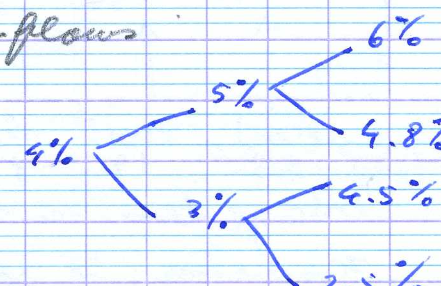


- path-dependency: rate will depend on whether you go first up or down.
- recombinant trees or binomial.

$$\neq \begin{cases} (x+y)^2 = x^2 + 2xy + y^2 \\ (x+y)^2 = x^2 + xy + yx + y^2 \end{cases}$$

A recombinant tree is as: $r_{ud} = r_{du}$.

They are more manageable.

- In this model, we assume: $p(r_u) = p(r_d) = \frac{1}{2}$
- How much should be r_u & r_d be?

→ It should "calibrate" to market.

All the instruments with observed prices are valued correctly: the value based on the tree are the observed price.

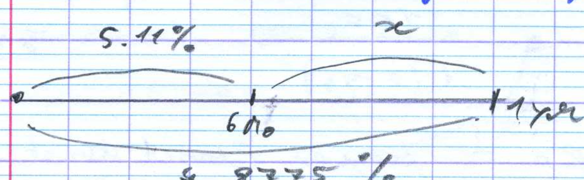
① we calibrate the par yield (yields of bonds that mature in period 1, period 2, ...)

$$P(1) = \frac{100 y_1 + 100}{1+y_1}$$

$P(2)$ = has a coupon such that price of this bond is 100.

ex: 5.11% : yield of a par bond maturing in 6Mo

→ What is the yield of a par bond maturing in 1yr?



1 yr rate = 4.8775 %.

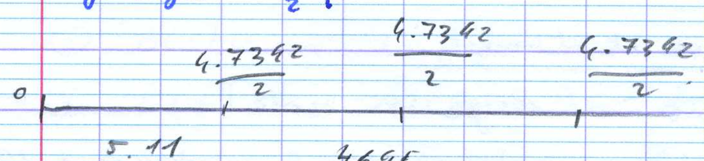
$$\Rightarrow \frac{1}{1.048775} = \frac{1}{1+\frac{0.511}{2}} + \frac{1}{1+\frac{x}{2}}$$

$\Rightarrow$ $x = 4.6956\,\%$.

To get numbers you don't have you do:

① interpolation:

$$y_{1.5yr} = \frac{1}{2}(4.8775 + 4.591) = 4.7342.$$



$$\frac{\frac{4.7342}{2}}{1+\frac{0.0511}{2}} + \frac{\frac{4.7342}{2}}{\left(1+\frac{0.04695}{2}\right)\left(1+\frac{0.0511}{2}\right)} + \frac{\frac{4.7342}{2}}{\left(1+\frac{x}{2}\right)\left(1+\frac{0.04695}{2}\right)\left(1+\frac{0.05}{2}\right)}$$

$$= \frac{\frac{5.11}{2}}{\left(1+\frac{0.0511}{2}\right)} + \frac{4.695/2}{\left(1+\frac{0.0511}{2}\right)\left(1+\frac{0.04665}{2}\right)} + \frac{x}{\left(1+\frac{0.0511}{2}\right)\left(1+\frac{0.04695}{2}\right)(1+x)}$$

Assume $x \simeq 4\%$.

Now: 2-year swap rate = 4.591

Indifferent between receiving this	$\frac{4.591}{2}$	$\frac{4.591}{2}$	$\frac{4.591}{2}$	$\frac{4.591}{2}$
Fixed cashflow and:	$\frac{5.11}{2}$	$\frac{4.6456}{2}$	$= \frac{y}{2}$	$\frac{x}{2}$

Indifferent:

$$\frac{4.591/2}{(1+\frac{0.0511}{2})} \qquad \frac{4.591/2}{(1+\frac{0.0511}{2})(1+\frac{0.04591}{2})} \qquad \frac{\frac{4.591}{2}}{(1+\frac{0.511}{2})(1+\frac{0.0456}{2})(1+\frac{y}{2})}$$

$$\frac{4.591/2}{\text{" — "}(1+\frac{x}{2})}$$

• What is the yield of a 0-coupon bond that matures in 2 periods?

5.11 % | 4.695 %

y_2

$$\frac{100}{(1+\frac{0.0511}{2})(1+\frac{0.0465}{2})} = \frac{100}{(1+y_2)^2} \Rightarrow \underline{y_2 =}$$

What is the yield of a 0-coupon bond that matures in 3 periods.

5.11 % | 4.695 % | 4 % | 100.

$$\frac{100}{(1+\frac{0.511}{2})(1+\frac{0.04695}{2})(1+\frac{0.04}{2})} = \frac{100}{(1+y_3)^3} \Rightarrow \underline{y_3 =}$$

. Suppose we are at time t:

$\Delta r_t = r_{t+\Delta t} - r_t$ = change in rate.

There are various models that represent this change

$$\Delta r_t = r_{t+\Delta t} - r_t = \underbrace{\mu(t)\Delta t}_{\text{drift}} + \underbrace{\sigma(t)\sqrt{\Delta t}}_{\text{volatility}}$$

In this model we assume $\mu(t) = 0$.

Strictly speaking, no matter where we start it, we use this model after sufficient downward movements: the rate can become negative.

Therefore, rather than considering Δr_t we consider $\Delta \ln r$

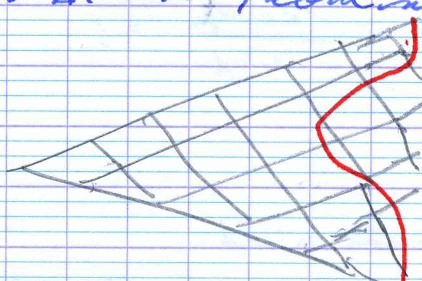
$\Rightarrow \quad \Delta \ln r_t = \ln r_{t+\Delta t} - \ln r_t = \mu(t)\Delta t + \sigma(t)\sqrt{\Delta t}$.

we assume $\begin{cases} \mu(t) = 0. \\ \sigma(t) = cte \text{ (volatility constant, independent of } t) \end{cases}$

$$\begin{aligned} \Delta \ln r_t &= \ln r_{t+\Delta t} - \ln r_t \\ &= \sigma\sqrt{\Delta t} \\ &= \ln\left(\frac{r_{t+\Delta t}}{r_t}\right) \\ &= \ln\left(\frac{r_{t+\Delta t} - r_t}{r_t} + 1\right) \\ &\simeq \frac{r_{t+\Delta t} - r_t}{r_t} \qquad \left(\text{since } \ln(1+x) \underset{x\to 0}{\simeq} x\right) \end{aligned}$$

Therefore, we model return instead of rate $\Rightarrow$ this ensures that rates will always be positive.

As $\Delta t \to 0$, continuous mvts:



$\rightarrow$ Brownian Motion.

Note: A Brownian motion $\{B(t), t \geq 0\}$ is a stochastic process such that:

① $B(0) = 0$

② $B(t)$ has independent and stationary increments

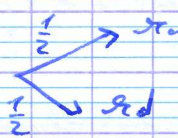
③ $B(t) \sim N(0, t)$ = Normal law, mean 0, Var = t

. $X(t) = \mu t + \sigma B(t)$ with $B(t)$ a standard Brownian motion is a Brownian motion with drift μ and variance $\sigma^2 t$.

. $Y(t) = e^{X(t)} = e^{\mu t + \sigma B(t)}$ is called a geometric Brownian Motion.

$\ln Y(t) = \mu t + \sigma B(t)$.

What should be r_u & r_d / it is compatible with market rates and volatility (calibration).

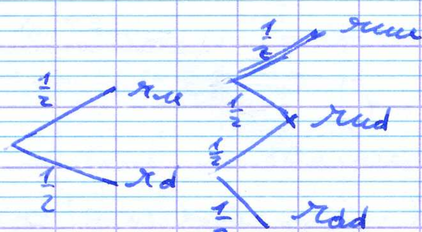


$$\Delta \ln r_1 = \ln r_{1+\Delta t} - \ln r_1 = \sigma \sqrt{\Delta t} \qquad (\mu(t)=0)$$

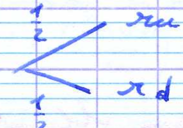
$$\begin{cases} \ln r_u - \ln r_1 = \sigma\sqrt{\Delta t} & \text{w.p. } \frac{1}{2} \\ \ln r_d - \ln r_1 = -\sigma\sqrt{\Delta t} & \text{w.p. } \frac{1}{2} \end{cases}$$

$$\ln \frac{r_u}{r_d} = 2\sigma\sqrt{\Delta t} \Rightarrow \boxed{\frac{r_u}{r_d} = e^{2\sigma\sqrt{\Delta t}}}$$

$$r_u = r_d e^{2\sigma\sqrt{\Delta t}}$$



The market should price the 0-coupon bond correctly with yield y_2.



$$P_u = \frac{100+y_2}{1+r_u} = \frac{1}{2}$$

$$P_d = \frac{100+y_2}{1+r_d} = \frac{1}{2}$$

$E[\text{price}] = \frac{1}{2} P_u + \frac{1}{2} P_d$.

0-coupon.

same maturity = same yield.

$$\text{price} = \frac{100}{(1+y_2)^2} = 100 = \frac{y_2}{1+y_2} + \frac{100+y_2}{(1+y_2)^2}$$

$$\boxed{\frac{1}{2}\left(\frac{100+y_2}{1+r_d e^{2\sigma\sqrt{\Delta t}}}\right) + \frac{1}{2}\left(\frac{100+y_2}{1+r_d}\right) = 100}$$

<HELP> for explanation. 9 Mo P141 Muni **FWCV**

FORWARD CURVE ANALYSIS
US Dollar

1wk 1.5 Yr

BASE CURVE DEFAULTS - BGN
Curve Dated: 3/19/07
Settlement Date: 3/21/07
Coupon/Spot: C
Bid/Ask/Mid: B
FMC # or SWDF # 23

1 GO Graph

2 GO Update Curve

3 GO Forwards Analysis

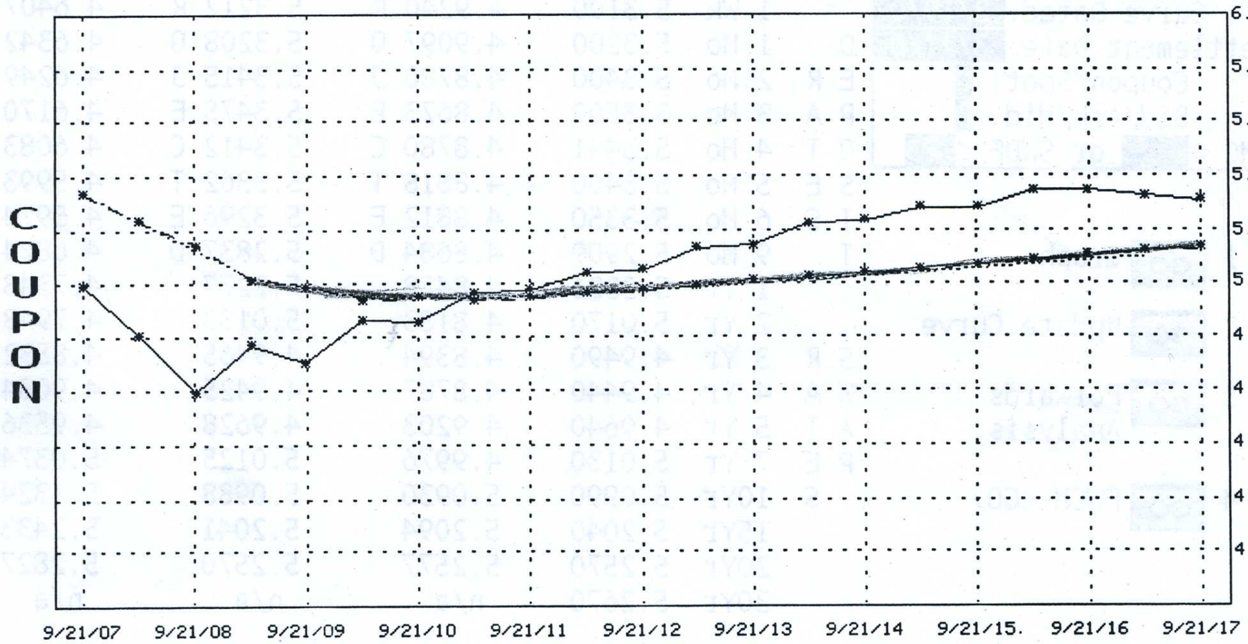
4 GO FWCM <GO>

		TERM	YIELD	12/21/07	P	3/26/07	P	9/22/08	P
		1 Wk	5.3100	4.9240	R	5.3212	R	4.6407	R
D		1 Mo	5.3200	4.9097	O	5.3208	O	4.6342	O
E	R	2 Mo	5.3400	4.8780	J	5.3415	J	4.6249	J
P	A	3 Mo	5.3500	4.8673	E	5.3475	E	4.6170	E
O	T	4 Mo	5.3441	4.8780	C	5.3412	C	4.6083	C
S	E	5 Mo	5.3400	4.8818	T	5.3362	T	4.5993	T
I	S	6 Mo	5.3350	4.8812	E	5.3296	E	4.5914	E
T		9 Mo	5.2909	4.8684	D	5.2837	D	4.6824	D
		1 Yr	5.2331	4.8488		5.2277		4.7348	
		2 Yr	5.0170	4.8183		5.0133		4.7948	
S	R	3 Yr	4.9490	4.8394		4.9465		4.8532	
W	A	4 Yr	4.9440	4.8787		4.9425		4.9084	
A	T	5 Yr	4.9640	4.9203		4.9628		4.9536	
P	E	7 Yr	5.0130	4.9976		5.0125		5.0374	
	S	10Yr	5.0990	5.0930		5.0988		5.1324	
		15Yr	5.2040	5.2094		5.2041		5.2433	
		20Yr	5.2570	5.2577		5.2570		5.2827	
		30Yr	5.2670	n/a		n/a		n/a	

<HELP> for explanation. P141 Muni **FWCV**

IMPLIED FORWARDS CURVE
US Dollar

6-Mo Forwards 6-Mo Intervals Date 9/21/07 Points 20 Page 1/2



Forwards Curve Overlay ☒Spot Curve ☒Coupon Curve

Citi

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Exchange Loan (*ExL*)

ExL is a defensive investment for leveraged loan investors with cash to invest, but are cautious given the recent volatility and uncertainty in the loan markets. Investors in ExL can enjoy an attractive return on their investment while limiting their downside credit risk.

Structure

- For illustration, consider a leveraged loan of Company XYZ with a coupon of LIBOR + [275] that matures in five years and is trading at 96. Investor ABC will participates at par in this loan using the ***ExL*** structure.
- At any time during the life of the trade, Investor ABC can either (i) convert the participation to an assignment subject to a pre-determined conversion grid; or (ii) exchange the participation for a five-year term loan to Citi which will pay Libor – 250. Investor ABC has the right to exercise this option even if Company XYZ suffers a credit event.
- The conversion grid[1] for converting the participation into an assignment is shown below. It reflects the pull-to-par nature of loans. Should the loan be converted to an assignment, the ***ExL*** trade will terminate:

Year	Price
1	96.50
2	97.25
3	98.00
4	98.75
5	99.50

- Investor ABC is protected from Company XYZ's default risk as long as the participation is not converted into an assignment. Should the credit of Company XYZ deteriorate, Investor ABC can replace the loan with one made to Citi that matures in five years from the time of the exchange and pays LIBOR-250.
- If the loan of Company XYZ is repaid in full prior to the maturity of ExL, the trade terminates.

[1] The pricing grid shown is for illustrative purposes only. The pricing grid will be determined on a name-by-name basis and may be subject to change.

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Comparative Analysis: Investing in ExL vs. Buying a Loan

Following is a short comparative analysis of participating in the ***ExL*** structure versus an outright purchase of the loan. For ease of argument we have ignored discounting and have rounded numbers.

For illustration purposes, suppose a fully funded term loan B of Company XYZ is currently trading at the price of 96 with a coupon of Libor + 275 bps. At the price of 96, a buyer of this loan is essentially earning an equivalent interest rate of approximately Libor + 350. The market price of this credit risk is therefore 350 bps. The investor enjoys a 75 bps spread premium but will have exposure to credit risk of Company XYZ. Should Company XYZ default on its loan, the investor's potential loan losses would be 1- recovery rate.

Suppose instead of buying the loan, the investor participates at par in Company XYZ through ***ExL*** structure. By foregoing the additional 75 bps premium, the investor has eliminated its exposure to the credit risk of Company XYZ. Should Company XYZ default, the investor can replace the defaulted loan with one made to Citi. Furthermore, if the leveraged loan market rebounds in the next six months and Company XYZ's term loan B trades at par, then the investor can exercise its option to convert the participation to an assignment at the price of 96.50 and sell the loan in the market at par, thereby monetizing the difference. The ***ExL*** structure will under-perform the alternative of purchasing the loan if the loan continues in its lackluster performance without a major rally or credit event. Consequently, the ***ExL*** structure is suited to investors who believe in a few years the credit market will either improve dramatically or suffer from major credit events. As

Advantages

- Protection against default
- Monetization of Liquidity
- Opportunity to take advantage of major market rally