# Optimal debt restructuring and lending policy in a monetary union*

Keshav Dogra[†]
Columbia University

November 24, 2014

**Abstract**

I present a theoretical framework to understand sovereign debt crises in a monetary union and the optimal policy response to these crises. The risk of default encourages indebted countries to pay down their short term debt, depressing consumption demand throughout the union. This fall in demand can cause the monetary union to hit the zero lower bound on nominal interest rates, leading to a union-wide recession. I evaluate three policies to prevent such a recession: debt relief, which writes off a portion of short term debt; lending policy, which allows indebted countries to issue new debt at above-market prices; and debt postponement, which converts short into long term debt. I show that if countries can be prevented from retrading in secondary markets after debt restructuring, all three policies are equivalent, and are welfare improving. If retrading is possible, lending policy and debt postponement are superior to debt relief.

**JEL classification**: E44, E62, F34, F41, H63

**Keywords**: Debt relief, sovereign debt crisis, zero lower bound, monetary union, maturity structure

*I want to thank Ricardo Reis for his advice and support throughout this project. All remaining errors are mine.

[†]Department of Economics, Columbia University, 1022 International Affairs Building, 420 West 118th Street, New York City, NY 10027. Email: kd2338@columbia.edu. Personal website: http://www.columbia.edu/~kd2338.

# 1 Introduction

Europe seems set for a lost decade of low growth and high unemployment, driven in large part by public and private deleveraging. During the recession, highly indebted periphery countries lost access to financial markets, forcing them to pursue fiscal austerity and write down debt in order to reduce sovereign risk premia. Public deleveraging, combined with the private deleveraging associated with the global financial crisis, caused a slump in demand and a continent-wide recession, which conventional monetary policy seems unable to avert. A natural policy response to such crises is to prevent indebted countries from deleveraging in order to reduce their spreads, either by restructuring sovereign debt (as in the Greek restructuring of 2012) or by purchasing, or commiting to purchase, sovereign debt (as in the ECB's Securities Markets Programme and Outright Monetary Transactions). However, such policies are controversial and the theoretical justification for them remains unclear.

I present a theoretical framework to understand the optimal policy response to episodes of international debt deleveraging in a monetary union. I build a model of a monetary union with nominal rigidities and defaultable debt, in which monetary policy is potentially constrained by the zero lower bound. The monetary union is a closed system with two groups of countries, borrowers (who initially have outstanding debt), and savers (who own the borrowers' debt). At date 1, it becomes common knowledge that at date 2, some borrower countries will have the option to default on their debt. Borrowers roll over their new debt by issuing new debt, internalizing the effect of their borrowing decision on their bond price (as in Eaton and Gersovitz [1981]). Consequently, borrowers have an incentive to reduce their consumption and reduce their debt, in order to raise the price at which they can issue their remaining debt. To maintain full employment, the monetary authority cuts interest rate to raise demand in creditor countries. However, they are limited by the zero lower bound (ZLB). When the ZLB binds, the monetary union enters a recession, and output falls below potential.

I characterize constrained efficient allocations in this economy, subject to the frictions imposed by the zero lower bound and default. I then ask whether optimal allocations can be implemented with three policies. The first policy I consider is debt relief, which writes off a portion of a country's short term debt. The second is lending policy, in which the union-wide authority offers to buy sovereign debt at above-market prices. This encompasses a range of policies, such as IMF crisis lending, the ECB's Securities Markets Programme (which involved directly purchases of sovereign debt) and the Outright Monetary Transactions (which involved a commitment to purchase sovereign debt). Finally, I consider debt postponement, which converts short term debt into long term debt.

First I consider an economy with no home bias, no long-term debt, and perfectly rigid prices. In this baseline economy, a transfer from creditor to debtor countries is Pareto improving. While transfers directly reduce creditors' income, they indirectly increase income throughout the monetary union, provided that borrowers spend the whole transfer, boosting aggregate demand. On net, creditors are no worse off, and borrowers are strictly better off. If borrowers are prevented from trading in bond markets after they receive a transfer, all three policies described above - debt relief, lending policy, and debt postponement - are equivalent, and implement constrained efficient allocations.

However, these policies are not equivalent if borrowers are permitted to 'retrade' after receiving a transfer. When retrading is permitted, borrowers will use some of their debt relief to issue less debt, rather than increasing consumption today. A much larger debt relief program is required to restore full employment, and such a program is not Pareto improving - it benefits borrowers, but makes savers strictly worse off. Lending policy, however, can still implement constrained efficient allocations. When the monetary authority buys bonds at below market prices, this encourages borrowers to issue more debt, inducing them to spend their transfer on date 1 consumption, rather than saving it for date 2. Intuitively, lending policy lowers borrowers' relative price of consumption at date 1, encouraging them to spend more at date 1 when their spending has a high social value.

Next I allow for long-term debt. If borrowers have some long-term debt outstanding, they over-issue new debt, diluting the value of their existing obligations. In normal times, when the ZLB does not bind, this incentive to over-issue debt renders a competitive equilibrium with long-term debt. But when the ZLB binds, borrowers with only short-term debt typically *under-issue* new debt, because they fail to internalize that by borrowing and spending, they boost union-wide aggregate demand. Optimal policy can use these two incentives to balance each other out. When the ZLB binds, converting short-term debt into long-term debt induces borrowers to dilute this long-term debt and issue the efficient amount of new debt. A recent literature has emphasized the benefits of long-term debt in insuring against risk and preventing self-fulfilling crises, and has studied how sovereigns trade off these benefits against the cost associated with debt dilution. I show that, in a liquidity trap, the 'cost' of debt dilution can actually be a benefit.

Transfers from creditors to debtors are Pareto improving at the ZLB because debtors spend the transfer, in part, on goods sold by creditor countries, increasing their income. One might worry that this result is not robust to the presence of home bias. If debtor countries spend most of the transfer on domestic goods and services, it would appear that creditor countries will no longer be better off. To address this concern, I allow for home bias. I show that the concern turns out to be unfounded: transfers are Pareto improving, even with home bias. If borrowers spend most of the transfer on their own goods and services, this increases their domestic income, which also increases their demand for foreign goods. Ultimately, budget constraints imply that a country must spend the whole of any transfer on buying either goods or assets from abroad. In fact, home bias increases the scope for Pareto improving policy: even when the ZLB does not bind, the competitive equilibrium is inefficient, and lending policy is Pareto improving.

A common argument against debt restructuring is that it gives countries an incentive to overborrow ex ante, knowing that they will be bailed out. To address this concern, I augment the model to include an ex ante stage in which countries decide how much to borrow and lend, taking into account that their debt may be restructured in the event of recession. Once the possibility of ex ante overborrowing is taken into account, it is necessary to combine ex post lending or debt restructuring policies with macroprudential capital controls or limits on borrowing ex ante. However, there remains a role for ex post lending and debt restructuring policies. In particular, some combination of ex ante debt limits and ex post lending policy or debt restructuring is always ex ante Pareto improving.

The rest of the paper is structured as follows. Next I discuss the related literature. Section 2 presents the model. Section 3 shows that the equilibrium is inefficient when the ZLB binds, characterizes constrained efficient allocations, and shows how they can be implemented, in a baseline economy with no home bias, rigid prices, and short term debt. Section 4 extends this to allow for long-term debt. Section 5 allows for home bias. Section 6 discusses whether ex post debt restructuring is efficient ex ante, given that it may encourage overborrowing. Section 7 concludes.

### 1.1 Related literature

My paper explores how debt restructuring and lending policy can be used to correct a macroeconomic externality. As such, it is related to a wide literature on debt restructuring and lending policy, which mostly considers different motiviations for these policies. It is also related to the recent literature on macroeconomic externalities, which largely considers other policy instruments which might correct these externalities.

The theoretical literature has emphasized three reasons why debt restructuring may be desirable. First, collective action problems (Wright [2012]), which can take many forms. Debt relief is a public good for creditors: if one creditor offers debt relief, the value of other creditors' claims increases. Holdout creditors have an incentive to delay agreeing to restructuring, in the hope that other creditors will settle first (Pitchford and Wright [2012]). A second strand of the literature (e.g. Krugman [1989]) emphasized debt overhang: writing down some debt may benefit creditors, if this increases the probability that the remaining debt will be repaid. This argument motivated 'market-based' debt reduction schemes, in which the debtor country buys back its own debt. These schemes were soon criticized by Bulow and Rogoff [1988, 1991] on the grounds that they mostly benefit creditors. Finally, in models with multiple equilibria, debt relief may prevent self-fulfilling crises (Cole and Kehoe [2000]). Most closely related to my paper, Roch and Uhlig [2012] show that a bailout guarantee can select the 'good' equilibrium in such a model. Relative to this whole literature, my contribution is to consider a different motivation for debt restructuring, namely to correct a macroeconomic externality.

More generally, my paper draws on the theoretical literature on sovereign debt (Eaton and Gersovitz [1981] is the seminal contribution; Aguiar and Amador [In Progress] provide a recent survey). In particular, recent contributions discuss the role of maturity. Aguiar and Amador [2014] show that indebted sovereigns should write down their short-term debt, but not their long-term debt: writedowns of long-term debt are Pareto-improving, but cannot be implemented at equilibrium prices. Arellano and Ramanarayanan [2012] discuss the tradeoff between short and long term debt. Long term debt hedges against fluctuations in interest rate spreads, while short term debt provides better incentives to repay. Hatchondo et al. [2014] show that the inefficiency associatied with debt dilution accounts for the bulk of default risk, and discuss how to design debt contracts that avoid dilution. Hatchondo et al. [2013] present a model in which voluntary debt exchanges can be Pareto improving for creditors and borrowers. Again, relative to this literature, I draw on standard models of defaultable debt to analyse how restructuring and lending policy can correct for a macroeconomic externality.

Another literature studies macroeconomic externalities associated with incomplete markets and/or fixed prices, and characterizes the macroprudential policies which correct for these externalities. Farhi and Werning [2013] provide a general theory of macroeconomic externalities. Farhi and Werning [2012] show that private insurance is inefficiently low for countries in a currency union, even if markets are complete. Individuals do not internalize that when they receive higher transfers, they increase their consumption of the nontradeable goods, which is desirable when employment is inefficiently low. The efficient allocation can be implemented with transfers within a fiscal union. However, transfers are not strictly necessary, as individual governments internalize the externality, and can implement the efficient allocation by trading in complete markets. In contrast, I study an economy with an *international* externality: sovereigns do not internalize that their borrowing decision affect demand in other markets, and supranational policy is necessary to implement efficient allocations. I also consider lending policies and debt restructuring, rather than a fiscal union.

Motivated by the European recession, a recent literature has considered sovereign debt crises in a monetary union and the role of policy. Most similar to my paper, Fornaro [2012] presents a model in which debt relief is Pareto improving in a monetary union when the ZLB binds. In his model, indebted countries face a shock to their borrowing constraint, and are forced to deleverage. Since policy presumably cannot circumvent the borrowing constraint, there is no scope in his model for the policies I consider, such as lending policy and debt rescheduling. Relative to his paper, my contribution is to consider alternative policies - debt relief, official lending, and debt rescheduling - and characterize optimal policy. One motivation for considering official lending and debt rescheduling is that these policies are more common (and arguably more politically feasible) than principal writedowns.[1] Forni and Pisani [2013] assess the effects of sovereign debt restructuring in a monetary union by simulating a medium-scale DSGE model. They assume that restructuring increases the spread faced by the sovereign, and this increase is fully transmitted to domestic households. I consider a relatively stylized model, but endogenize sovereign risk spreads, and analytically characterize optimal debt restructuring policy.

[1]Trebesch et al. [2012] survey sovereign debt restructurings, and show that outright face value reductions are not common: most restructurings were pure rescheduling deals. In the European crisis, ECB lending policies have allowed indebted countries to borrow at below-market interest rates (Krishnamurthy et al. [2014]).

## 2 A model of a currency union with defaultable debt

In this section I present a model which embeds defaultable debt, as in Eaton and Gersovitz [1981], into a standard model of a currency union with nominal rigidities, drawing closely on Gali and Monacelli [2008].

The currency union is a closed system consisting of a continuum of small open economies indexed by $i \in [0,1]$. Each economy is measure zero. Time is discrete, $t = 1, 2, ....$ Countries with $i \in [0, 1/2)$ are type $S$ ('savers'); countries with $i \in [1/2, 1]$ are type $B$ ('borrowers'). These types differ ony in their initial level of debt.

### 2.1 Households

Each economy contains a representative household with preferences

$$\sum_{t=1}^{\infty} \beta^{t-1} u(c_t^i - (1-\beta)\chi^i \delta_t^i)$$

where $u' > 0, u'' < 0$. $\delta_i = 1$ if country $i$ has defaulted on or before date $t$, and $(1-\beta)\chi^i$ is country $i$'s cost of default. I describe how default works below.

$c_t^i$ is a consumption index defined by

$$c_t^i = \frac{(c_{H,t}^i)^{1-\alpha}(c_{F,t}^i)^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}$$

where $c_{H,t}^i$ is an index of $i$'s consumption of goods produced at home, and $c_{F,t}^i$ is an index of $i$'s consumption of foreign goods. $\alpha$ measures the economy's openness: if $\alpha = 1$, there is no home bias in consumption. These consumption indices are defined as follows:

$$c_{H,t}^i = \left(\int_0^1 c_{H,t}^i(j)^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}j\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$c_{F,t}^i = \exp \int_0^1 \log c_{f,t}^i \, \mathrm{d}f$$

$$c_{f,t}^i = \left(\int_0^1 c_{f,t}^i(j)^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}j\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$\varepsilon > 1$ is the elasticity of substitution between varieties produced within any given country.

Households do not themselves participate in financial markets. They receive wages, profits from the monopolistically competitive firms, and lump sum transfers (or taxes) from their governments, who borrow and lend in financial markets on their behalf.[2] The household's budget constraint is

$$\int_0^1 p_t^i(j) c_{H,t}^i(j) \, \mathrm{d}j + \int_0^1 \int_0^1 p_t^f(j) c_{f,t}^i(j) \, \mathrm{d}j \, \mathrm{d}f \leq W_t^i + T_t^i$$

[2] Another interpretation of the model is that households participate in financial markets, and governments impose capital controls to induce households to make borrowing decisions that maximize domestic welfare (Na et al. [2014]).

where $W_t^i$ denotes the nominal wage, and we combine profits and transfers into $T_t^i$. Each household inelastically supplies a single unit of labor.

As is standard, the household's optimization problem yields the demand functions

$$c_{H,t}^i(j) = \left(\frac{p_t^i(j)}{p_t^i}\right)^{-\varepsilon} c_{H,t}^i$$
$$c_{f,t}^i(j) = \left(\frac{p_t^f(j)}{p_t^f}\right)^{-\varepsilon} c_{f,t}^i$$

for all $i, f, j \in [0,1]$, where we denote country $i$'s domestic PPI by

$$p_t^i = \left(\int_0^1 p_t^i(j)^{1-\varepsilon}\,\mathrm{d}j\right)^{\frac{1}{1-\varepsilon}}$$

Given that the law of one price holds, the price index for the bundle of goods imported from country $f$ is identical to that country's domestic PPI:

$$p_t^f = \left(\int_0^1 p_t^f(j)^{1-\varepsilon}\,\mathrm{d}j\right)^{\frac{1}{1-\varepsilon}}$$

As is standard, these demand functions satisfy $\int_0^1 p_t^i(j)c_{H,t}^i(j)\,\mathrm{d}j = p_t^i c_{H,t}^i$, $\int_0^1 p_t^f(j)c_{f,t}^i(j)\,\mathrm{d}j = p_t^f c_{f,t}^i$.

Each household spends the same amount on products produced by each foreign country, so we have $p_t^f c_{f,t}^i = p_t^* c_{F,t}^i$, where $p_t^* = \exp \int_0^1 p_t^f\,\mathrm{d}f$ is the union-wide price index, and (for each country) the price of imported goods.

Finally, $p_{C,t}^i = (p_t^i)^{1-\alpha}(p_t^*)^{\alpha}$ is the CPI in country $i$, and $i$'s optimal allocation of expenditure between domestic and imported goods is

$$p_t^i c_{H,t}^i = (1-\alpha)p_{C,t}^i c_t^i,\ p_t^* c_{F,t}^i = \alpha p_{C,t}^i c_t^i$$

### 2.2 Firms

Each country contains an index of monopolistically competitive firms indexed by $j \in [0,1]$. Each firm combines labor and domestically produced intermediate inputs to produce output using the concave, constant returns to scale technology

$$x_t^i(j) = A_t^i m_t^i(j)^{\phi} n_t^i(j)^{1-\phi}$$

where $\phi \in (0,1)$ and $A_t^i$ is a country-specific technology shock. The index of intermediate inputs, $m_t^i(j)$, is defined by

$$m_t^i(j) = \left(\int_0^1 m_t^i(j,k)^{\frac{\varepsilon-1}{\varepsilon}}\,\mathrm{d}k\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $m_t^i(j,k)$ denotes the quantity of intermediate goods used by firm $j$ in country $i$, and produced by firm $k$ in country $i$.

Again, the firm's cost-minimization problem yields the standard demand function

$$m_t^i(j,k) = \left(\frac{p_t^i(k)}{p_t^i}\right)^{-\varepsilon} m_t^i(j)$$

Let firm $j$'s nominal total cost function be

$$S\left(\frac{x_t^i(j)}{A_t^i}, W_t^i, p_t^i\right) = \frac{x_t^i(j)}{A_t^i}\frac{(p_t^i)^\phi (W_t^i)^{1-\phi}}{\phi^\phi(1-\phi)^{1-\phi}}$$

Nominal marginal cost is

$$\frac{1}{A_t^i}\frac{(p_t^i)^\phi (W_t^i)^{1-\phi}}{\phi^\phi(1-\phi)^{1-\phi}}$$

In symmetric equilibrium, each firm will employ one worker. Wages will be

$$W_t^i = p_t^i \frac{1-\phi}{\phi}\left(\frac{x_t^i}{A_t^i}\right)^{1/\phi}$$

Thus nominal marginal cost will be

$$p_t^i \frac{(x_t^i)^{\frac{1-\phi}{\phi}}}{\phi(A_t^i)^{1/\phi}}$$

Each firm $j$ faces demand from three sets of customers. First, domestic consumers, with demand

$$c_{H,t}^i(j) = \left(\frac{p_t^i(j)}{p_t^i}\right)^{-\varepsilon} c_{H,t}^i$$

Second, foreign consumers in country $f$, with demand

$$c_{i,t}^f(j) = \left(\frac{p_t^i(j)}{p_t^i}\right)^{-\varepsilon} c_{i,t}^f$$

Third, domestic firm $k$, with demand

$$m_t^i(k,j) = \left(\frac{p_t^i(j)}{p_t^i}\right)^{-\varepsilon} m_t^i(k)$$

Thus the firm faces total demand

$$x_t^i(j) = X_t^i\left(\frac{p_t^i(j)}{p_t^i}\right)^{-\varepsilon}$$

where

$$X_t^i = c_{H,t}^i + \int_0^1 c_{i,t}^f \,\mathrm{d}f + \int_0^1 m_t^i(k)\,\mathrm{d}k$$

### 2.3 Price setting

Firms face quadratic costs of price adjustment as in Rotemberg [1982]. Firm $j$ in country $i$ solves

$$\max \sum_{t=1}^{\infty} Q_t^i \left\{ p_t^i(j) x_t^i(j) - (1-\tau) S\left(\frac{x_t^i(j)}{A_t^i}, W_t^i, p_t^i\right) - \frac{\varphi}{2}\left(\frac{p_t^i(j)}{p_{t-1}^i(j)} - 1\right)^2 X_t^i \right\}$$
$$\text{s.t. } x_t^i(j) = X_t^i \left(\frac{p_t^i(j)}{p_t^i}\right)^{-\varepsilon}$$

where $\tau = 1/\varepsilon$ is a subsidy that corrects the distortion induced by monopolistic competition, $Q_t^i$ is the firm's nominal stochastic discount factor, with $Q_1^i = 1$.[3] Taking first order conditions and assuming a symmetric equilibrium with $p_t^i(j) = p_t^i$ yields

$$\varphi \pi_t^i(\pi_t^i - 1) = (\varepsilon - 1)(MC_t^i - 1) + \varphi Q_{t,t+1}^i \pi_{t+1}^i \frac{X_{t+1}^i}{X_t^i} \pi_{t+1}^i (\pi_{t+1}^i - 1)$$

where we define inflation$\pi_t^i = \dfrac{p_t^i}{p_{t-1}^i}$ and real marginal cost

$$MC_t^i = \frac{S_1\left(\frac{x_t^i(j)}{A_t^i}, W_t^i, p_t^i\right)}{p_t^i A_t^i}$$

In any symmetric equilibrium, each firm employs 1 worker, and we have

$$MC_t^i = \frac{(x_t^i)^{\frac{1-\phi}{\phi}}}{\phi (A_t^i)^{1/\phi}}$$

Finally, in equilibrium we have $x_t^i = X_t^i$. So the aggregate supply equations become

$$\varphi \pi_t^i(\pi_t^i - 1) = (\varepsilon - 1)\left(\frac{(x_t^i)^{\frac{1-\phi}{\phi}}}{\phi (A_t^i)^{1/\phi}} - 1\right) + \varphi Q_{t,t+1}^i \pi_t^i \frac{x_{t+1}^i}{x_t^i} \pi_{t+1}^i (\pi_{t+1}^i - 1)$$

### 2.4 Goods market clearing

Within each country $i$, each firm produces the same amount of gross output $x_t^i$, hires 1 worker, and uses the same amount of intermediate goods $m_{f,t}^i$ from each country $f$ (which is itself an

[3] I defer for now the question of who owns these firms; given the assumptions that will be made about monetary policy, this will not affect equilibrium in any way.

aggregate, containing the same amount of the produce of each country $f$ firm).

$$x_t^i = c_{H,t}^i + \int c_{i,t}^f \, \mathrm{d}i + m_t^i$$
$$x_t^i = (1-\alpha)\left(\frac{p_t^*}{p_t^i}\right)^{\alpha} c_t^i + \alpha \frac{p_t^*}{p_t^i} \int \left(\frac{p_t^f}{p_t^*}\right) c_t^f \, \mathrm{d}i + m_t^i$$

In equilibrium, $m_t^i = \left(\frac{x_t^i}{A_t^i}\right)^{1/\phi}$. So the complete set of equilibrium conditions are

$$x_t^i = (1-\alpha)\left(\frac{p_t^*}{p_t^i}\right)^{\alpha} c_t^i + \alpha \frac{p_t^*}{p_t^i} \int \left(\frac{p_t^f}{p_t^*}\right) c_t^f \, \mathrm{d}i + \left(\frac{x_t^i}{A_t^i}\right)^{1/\phi}, i \in [0,1], t = 1, 2, ...$$
$$\varphi \pi_t^i(\pi_t^i - 1) = (\varepsilon - 1)\left(\frac{(x_t^i)^{\frac{1-\phi}{\phi}}}{\phi (A_t^i)^{1/\phi}} - 1\right) + \varphi Q_{t,t+1}^i \pi_{t+1}^i \frac{x_{t+1}^i}{x_t^i} \pi_{t+1}^i (\pi_{t+1}^i - 1), t = 1, 2, ...$$

where as before, we define $\pi_t^i = \frac{p_t^i}{p_{t-1}^i}$, $p_t^* = \exp \int_0^1 \ln p_t^i \, \mathrm{d}i$.

**Proposition 2.1.** *1. If $\alpha = 1$, $A_t^i = A_t \forall i, t$, then $\pi_t^i = \pi_t, x_t^i = x_t, \forall i, t$.*

*2. If $\alpha = 1$, $A_t^i = A_t \forall i, t$, $\pi_t = 1, \forall t \geq 1$, then any $\{c_t^i\}$ is an equilibrium, provided that*

$$\int c_1^i \, \mathrm{d}i \leq y^*$$
$$\int c_t^i \, \mathrm{d}i = y^*, t \geq 2$$

*where $y_t^* = \arg\max_x x - \left(\frac{x}{A_t}\right)^{1/\phi}$.*

*3. If $\alpha < 1$, $A_t^i = A_t$ and $\varphi = \infty$ (prices are perfectly fixed), then any $\{c_t^i, x_t^i\}$ is an equilibrium, provided that*

$$y_t^i = (1-\alpha) c_t^i + \alpha \int y_t^f \, \mathrm{d}f \leq y^*$$

*where $y_t^i = x_t^i - \left(\frac{x_t^i}{A}\right)^{1/\phi}$.*

## 2.5 Government, default, bond pricing, and monetary policy

Next, I describe how governments borrow, lend and default.

Governments seek to maximize the welfare of their representative household. They can issue two securities, a one period bond, which obliges the issuer to repay 1 unit of output next period, and a perpetuity, which obliges the issuer to repay $1 - \beta$ units of output in each future period. I assume that a government cannot simultaneously issue debt and hold assets. At date 2, and only at date 2, a country with outstanding debt has the option to default on its debt. At the beginning of period 2, each country learns its utility cost of default, $\chi^i$. Each country's output cost $\chi^i$ is identically and independently drawn from a distribution $F(\chi)$. As described above, if a country defaults, it pays a utility cost which is equivalent to losing $\chi^i$ units of consumption in each period. In this economy, since there are no other shocks at date 2, whether a country defaults will depend solely on the level of $\chi^i$ relative to its external debt. This default cost shock is a simple way to capture the fact that international investors face some uncertainty about whether a sovereign will default, even if they know its external debt position and other fundamentals.[4] The shock that causes a recession at date 1 is an increase in this uncertainty, which increases default risk and credit spreads.[5]

I now describe when a country defaults.

**Lemma 2.2.** *At date 2, after default, in any equilibrium with $\pi_t = 0, t \geq 2$, the economy enters a steady state. A country which did not default with short term debt $d^i_{S,2}$ and long term debt $d^i_{L,2}$ consumes $c^i_t = y^* - (1-\beta)d^i_2$ in every period $t \geq 2$, where we define $d^i_2 = d^i_{S,2} + d^i_{L,2}$. Countries obtain utility*

$$V(-d^i_2) = \frac{u(y^* - (1-\beta)d^i_2)}{1-\beta}$$

*A country which defaulted and has default cost $\chi^i$ obtains utility*

$$V(-\chi^i) = \frac{u(y^* - (1-\beta)\chi^i)}{1-\beta}$$

*Country $i$ will default if $d^i_2 \geq \chi^i$.*

It follows from this lemma that borrowers will be indifferent at date 1 between having $d_2$ long term bonds outstanding at the end of date 1, and having $d_2$ short term bonds outstanding at the end of date 1 and rolling them over each period. Without loss of generality, I restrict attention to equilibria in which borrowers only have long term debt outstanding at the end of date 2, and to save on notation I let $d^i_2 = d^i_{L,2}$. The probability (as of date 1) that a country with debt $d^i_2$ will default at date 2 is $F(d^i_2)$; the probability that it will repay is $p(d^i_2) := 1 - F(d^i_2)$.

Having described equilibrium at date 2, given an amount of debt $d^i_2$ issued at date 2, I now describe the price that borrower government obtains for this debt. Suppose government a$i$ government starts date 1 owing $\bar{d}^i_1 - \bar{d}^i_2$ short term debt and $\bar{d}^i_2$ long term debt, so the total amount it must repay at date 1 is $\bar{d}^i_1$. We will see that if the government ends period 1 owing $d^i_2$, it can

[4]In standard models of defaultable international debt, default is driven by income shocks (Arellano [2008]). I follow a number of recent contributions which employ shocks to the utility cost of default as a tractable alternative (Roch and Uhlig [2012], Aguiar and Amador [2014]).

[5]One way to think about this model is that the fundamental shock that causes a recession is a *risk shock*, as in Christiano et al. [2014]: an increase in idiosyncratic uncertainty about the cost of default. The shock can also be interpreted as a credit spread shock, as in Curdia and Woodford [2010], although here credit spreads are derived explicitly from a model of defaultable debt.

sell its debt at price $Q(d_2^i) = p(d_2^i)\dfrac{Q^{rf}}{1-\beta}$, where $Q^{rf}$ is the price of a risk free bond, and which depends endogenously on $d_2^i$. The government internalizes that its bond price depends on its own borrowing decision.

In equilibrium, some debtor countries will default and some will not, but the fraction of countries who will default is known at time 1. Financial intermediaries hold defaultable short and long term debt issued by debtor countries, and issue short and long term bonds to creditor countries. The sole function of the financial intermediaries is to pool idiosyncratic country risk. Again, without loss of generality I assume that creditor countries only buy long term debt at date 2. Finally, savers also trade a risk free bond in zero net supply.

At the start of date 1, borrowers owe $\bar{d}_1 > 0$ at date 1 and $\bar{d}_2 \geq 0$ at date 2. Savers are initially owed $\bar{d}_1$ at date 1 and $\bar{d}_2$ at date 2. They can lend to borrowers, or sell back some of their bond holdings.

A borrower government's budget constraints are

$$\begin{aligned} Q(d_2^i)(d_2^i - \bar{d}_2) + T_1^i &= \bar{d}_1 \\ T_2^i &= d_2^i \end{aligned}$$

A saver government's budget constraints are

$$\begin{aligned} \bar{d}_1 + T_1^i &= Q_1(d_2^i - \bar{d}_2) + Q_1^{rf} a_2^i \\ T_2^i + p(d_2^i)d_2^i + a_2^i &= 0 \end{aligned}$$

Finally, monetary policy ensures that inflation is zero, except when constrained by the zero lower bound, $Q^{rf} \leq 1$. That is, we have

$$Q_t^{rf} \geq 1, \pi_t \leq 1, (1 - Q_t^{rf})(\pi_t - 1) = 0$$

## 2.6 Equilibrium

I now define equilibrium in the baseline economy with no home bias ($\alpha = 1$), zero inflation after date 1 ($\pi_t = 1, t \geq 2$), no productivity shocks ($A_t^i = A$) and short term debt ($\bar{d}_2 = 0$).

**Definition 2.3.** *An equilibrium in the baseline economy is a collection $c_1^S, c_1^B, d_2, a_2, Q_1, Q^{rf}, y_1$ and a bond pricing function $Q(\cdot)$ such that, given the initial debt level $\bar{d}_1$:*

1. *$c_1^S, d_2$ solve the saver country's problem:*

$$\begin{aligned} &\max_{c_1^S, d_2} u(c_1^S) + \beta V(a_2 + p(d_2)d_2) \\ &s.t.\ c_1^S + Q_1 d_2 + Q^{rf} a_2 = y_1 + \bar{d}_1 \end{aligned}$$

2. $c_1^B, d_2$ *solve the borrower country's problem:*

$$\max_{c_1^B, d_2} u(c_1^B) + \beta \left[ \int_0^{d_2} V(-\chi)\, \mathrm{d}F(\chi) + p(d)V(-d_2) \right]$$
$$s.t.\ c_1^B + \bar{d}_1 = y_1 + Q(d_2)d_2$$

3. *The bond pricing function is* $Q(d) = p(d)Q^{rf}$, *with* $Q(d_2) = Q_1$.

4. *The goods market clears:*
$$c_1^S + c_1^B = 2y_1$$

5. *The risk-free bond market clears:* $a_2 = 0$.

6. $Q^{rf} \leq 1$, $y_1 \leq y^*$, *with at least one strict equality.*

# 3 Liquidity traps and optimal lending policy in the baseline economy

In this section, I consider a baseline economy with no home bias and no initial long-term debt. First I show that the equilibrium without policy is generally inefficient, due to a macroeconomic externality. When borrower countries have too much short term debt, their attempt to deleverage causes a union-wide recession. I then characterize constrained efficient allocations, and discuss how they can be implemented with debt restructuring and lending policies. Optimal allocations require a transfer to borrower countries, which can be implemented through outright debt relief, lending policy, or converting short term debt into long term debt. If borrowers can be prevented from retrading in secondary markets, these policies are equivalent; if retrading is possible, debt relief does not implement all constrained efficient allocations, whereas lending policy does.

## 3.1 International deleveraging and liquidity traps

I now describe equilibrium, given an initial level of debt $\bar{d}_1 > 0$, and assuming no home bias ($\alpha = 1$) and no initial long term debt ($\bar{d}_2 = 0$). I show that in equilibrium, the risk of default at date 2 increases the spreads faced by borrower countries. Borrowers attempt to pay down their short term debt to reduce these spreads, reducing demand throughout the monetary union. The central bank reduces interest rates to keep output at its efficient level, whenever this is not prevented by the zero lower bound on nominal interest rates. When borrowers' external debt is sufficiently high, the zero lower bound binds, and output is below the efficient level throughout the monetary union.

I also make the following technical assumptions:

**Assumption 3.1.** *Either* $u(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$ *with* $\sigma > 1$, *or* $u(c) = \ln c$.

**Assumption 3.2.** $\gamma(d) := \dfrac{f(d)d}{1-F(d)}$ *is nondecreasing in d.*

**Assumption 3.3.** *$u'(2y^*) < \beta u'(y^* + (1-\beta)p(d^*)d^*)$ where $d^* := \arg\max_d p(d)d$.*

This assumption ensures that the ZLB will bind if the borrowers have enough external debt.

Borrowers attempt to deleverage, reducing their consumption to pay off debt and reduce their spreads. To see why, note that borrowers' Euler equation is

$$u'(c_1^B)[Q(d_2) + Q'(d_2)d_2] = \beta P(d_2)u'(c_2^B) \tag{1}$$

On the left hand side, a borrower's effective marginal price of debt - the funds it receives if it issues another bond - is $Q(d_2) + Q'(d_2)d_2$. This has two components. First, if the country issues another bond, it receives $Q(d_2)$, the price of the bond. Second, issuing another bond increases the probability of default, and reduces the value of the other bonds the country is issuing by $Q'(d_2)d_2 < 0$. On the right hand side, the cost of issuing another bond (lower utility in the steady state) is weighted by the probability that the borrower actually repays, $P(d_2)$.

A saver county's Euler equation is

$$Q(d_2)u'(c_1^S) = \beta P(d_2)u'(c_2^S) \tag{2}$$

Dividing (1) by (2) and rearranging, we have

$$\frac{u'(c_1^B)}{\beta u'(c_2^B)}\left[1 + \frac{Q'(d_2)d_2}{Q(d_2)}\right] = \frac{u'(c_1^S)}{\beta u'(c_2^S)}$$

$\frac{Q'(d_2)d_2}{Q(d_2)}$ is negative, so this expression means that borrowers reduce their consumption over time, relative to savers. Again, borrowers internalize that if their consumption is too low (their debt is too high) at date 2, this makes them very likely to default, and reduces the price they an obtain for their bonds at date 1. They therefore have an incentive to pay back their debt. This captures the stylized fact that a global financial shock caused a compression in current account balances and a decline in gross capital flows (Lane and Milesi-Ferretti [2012]).

If debtor countries reduce their consumption at date 1, then in order maintain efficient output and zero inflation, the monetary authority must cut interest rates to induce creditor countries to consume more. The more debt the borrowers must pay back, the more interest rates must fall (risk free bond prices must rise) to maintain full employment. Eventually, if debt is too large, the monetary authority would need a negative interest rate to maintain efficient output. This is not possible, because of the zero lower bound. So output will fall below potential output, and the monetary union will enter a recession at date 1. The following proposition formalizes this.

**Proposition 3.4.** *There exists $\bar{d}_1^*$ such that:*

1. *If $\bar{d}_1 < \bar{d}_1^*$, then $Q^{rf} < 1$ and $y_1 = y^*$. $Q^{rf}$ is increasing in $\bar{d}_1$.*

2. *If $\bar{d}_1 = \bar{d}_1^*$, then $Q^{rf} = 1$ and $y_1 = y^*$.*

3. *If $\bar{d}_1 > \bar{d}_1^*$, then $Q^{rf} = 1$ and $y_1 < y^*$. $y_1$ is decreasing in $\bar{d}_1$.*

$c_1^S$ *is increasing in* $\bar{d}_1$. $c_1^B$ *is decreasing in* $\bar{d}_1$.

*Proof.* See Appendix. □

Figure 1 shows a numerical example, with $\beta = 0.9, \sigma = 1, y^* = 1, F(d) = d$. The figure plots each country's consumption, the union-wide level of output, and the risk free bond price, as functions of borrowers' initial level of external debt, $\bar{d}_1$. Going from left to right, as debt increases, borrower countries reduce their consumption in order to pay down debt. The risk free interest rate falls - i.e. the bond price rises - in order to induce savers to increase their consumption, keeping $y_1 = y^*$. Once $\bar{d}_1 = \bar{d}_1^*$ (indicated by the black vertical line), the lower bound on interest rates binds - $Q^{rf} = 1$ - and output can fall below potential output. In this region, the higher the borrowers' initial level of external debt, the lower is output.

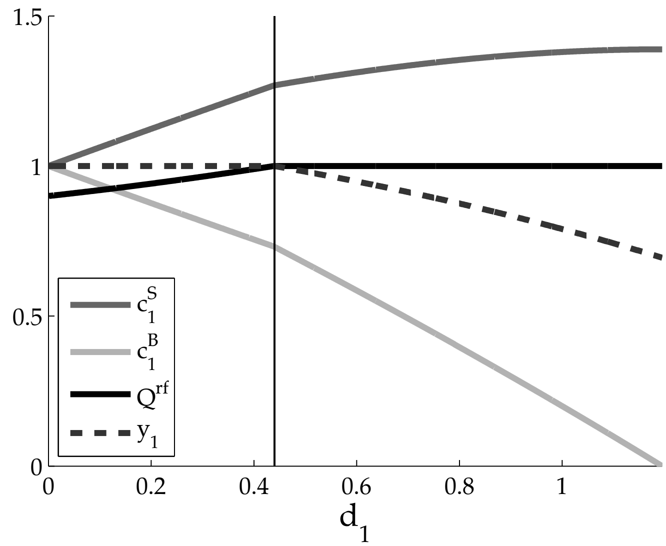


Figure 1: Equilibrium in the baseline economy

This result suggests that the outcome is inefficient. Collectively, all countries could produce and consume more, while still satisfying resource constraints. But each individual saver country prefers to save at a zero real interest rate, and each borrower country prefers to write down its debt in order to reduce its spreads. Individual governments do not internalize that their borrowing and lending decisions affect aggregate demand and output in other countries. This suggests that there is some scope for a Pareto improvement.

### 3.2 Constrained efficient allocations

I now characterize optimal policy, by considering a social planner's problem. The planner maximizes borrowers' utility, subject to three constraints: she must give the savers at least a certain level of utility, date 1 consumption cannot be greater than the full employment level of output, and - crucially - the savers' Euler equation must be satisfied with a non-negative risk free rate. That is, the planner solves

$$\max_{c_1^S, c_1^B, d_2} u(c_1^B) + \beta \left[ \int_0^d V(-\chi)\, \mathrm{d}F(\chi) + p(d)V(-d) \right] \tag{SP}$$

$$\text{s.t. } u(c_1^S) + \beta V(p(d_2)d_2) \geq U_S \tag{US}$$

$$c_1^S + c_1^B \leq 2y^* \tag{RC}$$

$$u'(c_1^S) \geq \beta u'(y^* + (1-\beta)p(d_2)d_2) \tag{ZLB}$$

There are two ways to interpret the zero lower bound constraint (ZLB). One interpretation is that the union-wide authority cannot prevent governments from lending to other governments, or holding risk free bonds. An alternative interpretation is that neither the union-wide authority nor governments can prevent their citizens from saving at a zero interest rate (should they choose to do so). In either case, (ZLB) must hold.

Without loss of generality, we focus on allocations in which $U_S \geq \dfrac{u(y^*)}{1-\beta}$; otherwise, type $S$ countries would be borrowers and type $B$ countries would be savers. Efficient allocations are solutions to (SP). The following proposition characterizes efficient allocations.

**Proposition 3.5.** *1. In every efficient allocation, there is full employment:* $c_1^S + c_1^B = 2y^*$

2. *There exists* $U_S^* > \dfrac{u(y^*)}{1-\beta}$ *such that the ZLB binds if* $U_S \geq U_S^*$.

3. $c_1^S$ *and* $d_2$ *are increasing in* $U_S$; $c_1^B$ *is decreasing in* $U_S$.

4. *The largest* $U_S$ *for which a solution to this program exists is*

$$\bar{U} := u(u'^{-1}(\beta u'(y^* + (1-\beta)p(d^*)d^*))) + \beta V(p(d^*)d^*)$$

*where* $d^* := \arg\max_d p(d)d$.

*Proof.* (1.) Putting Lagrange multipliers of $\nu, \lambda, \mu$ on the constraints, the first order conditions are

$$\nu u'(c_1^S) - \lambda + \mu u''(c_1^S) = 0$$

$$u'(c_1^B) - \lambda = 0$$

$$\nu\beta V'(-p(d)d)[p'(d)d + p(d)] - \beta p(d)V'(-d) - \mu\beta u''(y^* + p(d)d)[p'(d)d + p(d)] = 0$$

Since $u' > 0$, we must have $\lambda > 0$: every efficient allocation has full employment at date 1. □

There are a range of Pareto efficient allocations, indexed by savers' utility $U_S$. As the utility promised to savers increases, the planner finds it optimal to give savers higher consumption at both dates 1 and 2. However, it is still optimal to make borrowers deleverage, consuming less at date 1 than at date 2. Further, the amount of deleveraging increases in $U_S$, as the planner gives higher and higher date 1 consumption to savers. From the savers' perspective, this means they must tolerate a larger and larger fall in consumption between dates 1 and 2. When the planner is required to deliver a sufficiently high utility to savers - $U_S > U_S^*$ - she would like to give the savers such high date 1 consumption, and such a sharp fall in consumption between dates 1 and 2, that it violates the zero lower bound. It is still possible to increase the savers' utility beyond this point, but it is no longer possible to increase the amount of deleveraging. Instead, if the planner wants to increase $c_1^S$, she must also increase $d_2$ by more than she would if the ZLB was not a constraint, leading to a higher fraction of defaults than would otherwise be the case. This is still better for the saver, as long as $d_2 < d^*$. Once $d_2 = d^*$, debt is so high that requiring the borrowers to promise to repay more debt would actually decrease the amount received by savers, which cannot be Pareto optimal. Figure 2 provides a numerical example.

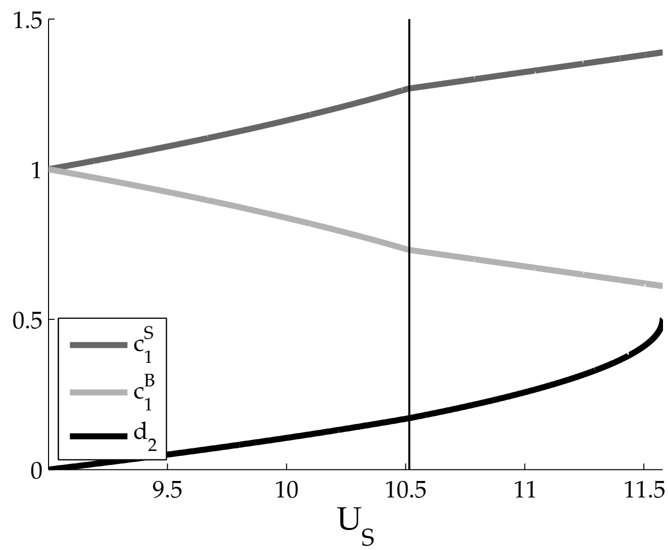


Figure 2: Constrained efficient allocations

Note that it is always efficient to have full employment at date 1. If savers' consumption is constrained by the zero lower bound, the planner can increase borrowers' consumption. Since we have seen that the equilibrium without policy has $y_t < y^*$ when the ZLB binds, the following Proposition is immediate.

**Proposition 3.6.** *In the baseline economy, if the ZLB does not bind, then the equilibrium is Pareto efficient. If the ZLB binds, the equilibrium is Pareto inefficient.*

### 3.3 Implementation without retrading: an equivalence result

Next, I discuss how optimal allocations can be implemented. I consider three policies. First, writedowns of short term debt. Second, lending policy, in which the union-wide authority buys the debt of borrower countries directly, possibly offering a higher price for this debt than borrowers could have obtained in the private market. Third, debt postponement, in which borrowers' short term debt is converted into long term debt.

The crucial question is whether it is possible to prevent countries from retrading after a debt restructuring agreement. To make things concrete, consider the Pareto efficient allocation which gives savers the same utility as in the equilibrium without policy. It would appear that this allocation can be implemented with debt relief for the borrowers at date 1. If borrowers maintained the same level of date 2 debt, debt relief would increase their date 1 consumption, stimulating aggregate income and thus compensating saver countries for the writedown of their assets. However, if borrowers can retrade after receiving debt relief, since they are now richer at date 1 they would like to reduce their date 2 borrowing. This means that even more debt relief is required to increase their date 1 consumption enough to restore full employment. Moreover, this will not be Pareto improving: savers are strictly worse off than in the equilibrium without policy, as they consume the same amount at date 1, and less at date 2. But clearly, if it is possible to prevent retrading, there is no such problem.

In this example, if retrading is possible, it will be necessary to combine debt relief with a subsidy to borrowing, encouraging borrowers to consume more at date 1 (where their consumption has a high social marginal utility) and write down less of their debt. This can be interpreted as a bond price support program, in which the union-wide authorities purchase borrowers' debt at a guaranteed price which is lower than the market price of debt.

First, I assume it is possible to prevent retrading. I make the extreme assumption that the union-wide authority can prevent saver and borrower countries from interacting in the bond market. The union-wide authority issues risk-free debt to saver countries, and offers to buy a fixed amount of debt $d_2$, at a fixed price $Q_1$, from the borrowers. The union-wide authority can also impose taxes $T_1^S, T_1^B$ on borrowers and savers at date 1. These taxes will typically be positive for savers, and negative for borrowers. Finally, I allow for the union wide authority to postpone the borrowers' debt, by giving them a positive amount of date 2 debt outstanding ($\bar{d}_2 > 0$) and compensating them with a transfer at date 1 ($T_1^B < 0$).

**Definition 3.7.** *An equilibrium without retrading in the baseline economy is a collection $c_1^S, c_1^B, a_2, Q^{rf}, y_1$ such that, given the initial debt level $\bar{d}_1$ and policy $d_2, \bar{d}_2, T_1^S, T_1^B, Q_1$:*

1. *$c_1^S, a_2$ solve the saver country's problem:*

$$\max_{c_1^S, a_2} u(c_1^S) + \beta V(a_2)$$
$$s.t.\ c_1^S + Q^{rf} a_2 = y_1 + \bar{d}_1 - T_1^S$$

2. *$c_1^B, d_2$ satisfy the borrower country's budget constraint:*

$$c_1^B + \bar{d}_1 = y_1 + Q_1(d_2 - \bar{d}_2) - T_1^B$$

3. *The government budget constraint is satisfied:*

$$Q_1 d_2 = Q^{rf} a_2 + T_1^S + T_1^B$$

4. *The goods market clears:*

$$c_1^S + c_1^B = 2y_1$$

5. *The risk-free bond market clears:*

$$a_2 = p(d_2) d_2$$

6. $Q^{rf} \leq 1$, $y_1 \leq y^*$, *with at least one strict equality.*

Given this definition, I show that debt relief, lending policy, and postponement are equivalent policies when retrading is prevented.

**Proposition 3.8.** *Any optimal allocation $c_1^S, c_1^B, d_2$ can be implemented as an equilibrium without retrading in three ways:*

1. *With debt relief ($T_1^B < 0$) and fair market prices ($Q_1 = Q^{rf} p(d_2)$)*

2. *With a subsidized price for debt ($Q_1 > Q^{rf} p(d_2)$) and no transfer to savers ($T_1^B = 0$)*

3. *With debt postponement ($-T_1^B = \bar{d}_2 > 0$), and fair market prices for debt ($Q_1 = Q^{rf} p(d_2)$).*

*These policies are related as follows:*

$$-T_1^B = (Q_1 - Q^{rf} p(d_2)) d_2 = \bar{d}_2 - Q^{rf} p(d_2) \bar{d}_2$$

*Proof.* Take any optimal $c_1^B, d_2$. To prove 1., let

$$T_1^B = y^* + Q^{rf} p(d_2) d_2 - \bar{d}_1 - c_1^B$$

To prove 2., choose $Q_1$ so that

$$c_1^B = y^* + Q_1 d_2 - \bar{d}_1$$

To prove 3., choose $\bar{d}_2$ so that

$$c_1^B = y^* + Q^{rf} p(d_2)(d_2 - \bar{d}_2) - \bar{d}_1 + \bar{d}_2$$

Comparing these three equations, we get the relation between the three policies stated above. □

Intuitively, in any optimal allocation, borrowers receive some amount at date 1, and are required to pay some amount at date 2. One way to implement this is to give borrowers a lump sum transfer at date 1, and require them to issue a certain amount of new debt (at market prices). An alternative way is to buy their debt at above-market prices. The difference between the actual price and the fair market price is an implicit transfer to borrowers, and plays exactly the same role as an explicit transfer (debt relief). A third way to implement this transfer is to turn short term debt into long term debt. Long term debt has a lower market value because of the possibility of default, so this postponement also acts as a transfer to borrowers.[6]

In this precise sense, debt relief, lending policy and debt postponement are all equivalent policies when it is possible to prevent retrading. Note that even when the ZLB binds, there are a range of optimal allocations, indexed by $d_2$. Allocations with higher $d_2$, higher $c_1^S$ and lower $c_1^B$ are better for savers and worse for borrowers. There are also a set of Pareto improving policies, relative to any equilibrium in which the ZLB binds. The Pareto improving allocation most favorable to borrowers keeps their debt level $d_2$, and the savers' consumption $c_1^S$, the same as in the equilibrium without policy.

### 3.4 Implementation with retrading: debt relief

The economy without retrading provides a useful benchmark result: debt relief, lending policy and debt postponement are different ways of providing essentially the same transfer to indebted countries. However, this result is arguably of little practical relevance. It is rare for creditors or international agencies to prevent a country from issuing *less* debt than the creditors require, and it is unclear how this could be enforced.[7] In the remainder of this paper, I assume that borrowers can freely decide how much debt to issue at date 1.

I now ask whether two of the policies considered so far - debt relief and bond price support programs - are still optimal when retrading is possible.[8] First I consider debt relief. We do not need a new equilibrium concept to think about debt relief with retrading. Instead, we can simply allow the union-wide authority to choose borrowers' initial level of debt, $\bar{d}_1$. The following result is immediate, given Propositions 3.4 and 3.6.

**Proposition 3.9.** *When retrading is permitted:*

1. *Debt relief only implements optimal allocations with $U_S \leq U_S^*$. These allocations can be implemented by writing down debt to a level below $\bar{d}_1^*$.*

2. *Debt relief is not Pareto improving. It makes savers worse off and makes borrowers better off, relative to an equilibrium with $\bar{d}_1 > \bar{d}_1^*$.*

*Proof.* The first part is immediate, since equilibria without policy are only optimal when $\bar{d}_1 < \bar{d}_1^*$. To prove the second part, note that $c_1^S$ and $p(d_2)d_2$ are increasing in $\bar{d}_1$. So reducing $\bar{d}_1$ strictly reduces borrowers' utility. □

[6] This is reminiscent of the well known result that governments can smooth risk, replicating the complete markets allocation, by issuing debt of different maturities. (Angeletos [2002], Buera and Nicolini [2004]), although here long-term debt effects a transfer to indebted countries, rather than insuring against risk. A concern often raised in this literature is that the gross positions required to replicate complete markets may be implausibly large (Buera and Nicolini [2004]). This concern may apply here too.

[7] While official lending often comes with 'conditionalities', these usually require that the recipient country's debt must be *reduced* going forward, not that it should not be reduced too much.

[8] I discuss debt postponement in Section 4 below.

Debt relief, together with no lending policy (i.e. a bond price function which merely replicates market prices, $\tilde{Q} = Q$) can implement efficient allocations in which the ZLB does not bind. The government can simply write off part of $\bar{d}_1$ until the remaining debt, $\bar{d}_1 + T_1^B$ (where $T_1^B < 0$) is less than $\bar{d}_1^*$. We already know that the competitive equilibrium in this case is efficient. But it is not a Pareto improvement on the equilibrium without policy. Borrowers are better off, but savers are strictly worse off. Because borrowers can retrade after receiving debt relief, since they are now richer at date 1 they would like to reduce their date 2 borrowing. This means that even more debt relief is required to increase their date 1 consumption enough to restore full employment. Moreover, this will not be Pareto improving: savers are strictly worse off than in the equilibrium without policy, as they consume the same amount at date 1, and less at date 2.

Figures 3 and 4 illustrate. Figure 3 shows how debt relief brings about a Pareto improvement when retrading is prevented. The black curves are borrowers' indifference curves, representing their preferences over date 1 consumption, $c_1^B$, and date 2 debt, $d_2$. The gray shaded area represents borrowers' budget set. If a borrower country takes out no debt, it consumes its income, $y_1$, minus its outstanding debt, $\bar{d}_1$. As the country issues more debt, it obtained more resources at date 1. But the price it can obtain for this debt decreases as debt increases, so consumption is a concave function of debt issued. Finally, the gray dashed line shows the combinations of $c_1^B, d_2$ that satisfy the (ZLB) and (RC), i.e. that satisfy

$$u'(2y^* - c_1^B) = \beta u'(y^* + p(d_2)d_2)$$

Without policy, $c_1^B, d_2$ lie below the ZLB curve, because output is below potential output. If the union-wide authority gives a transfer $T$ to borrower countries (for example, by writing off debt), that shifts the borrower's budget set up, until a point on the ZLB curve is in the budget set.

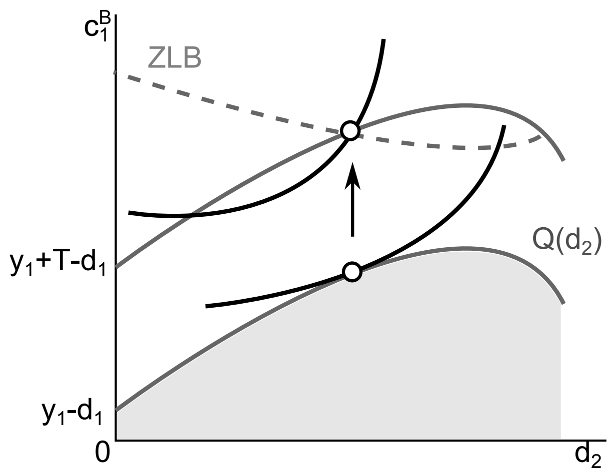


Figure 3: Debt relief without retrading

However, Figure 4 shows that this point is not optimal given the new budget set. Even if borrowers receive a large enough transfer that it is *possible* for them to consume enough to restore the efficient level of output, they would not find it optimal to do so. Instead, they would prefer to use some of the transfer to issue less new debt $d_2$, reducing their borrowing costs. The resulting equilibrium will not have full employment (because consumption is below the ZLB curve) and it will not be a Pareto improvement on the equilibrium without policy (because $d_2$ has fallen, and saver countries receive less in the steady state). In order for debt relief to restore full employment, it would be necessary to make an even larger transfer, reducing $d_2$ further. Again, this will not be a Pareto improvement on the status quo: saver countries will be worse off.

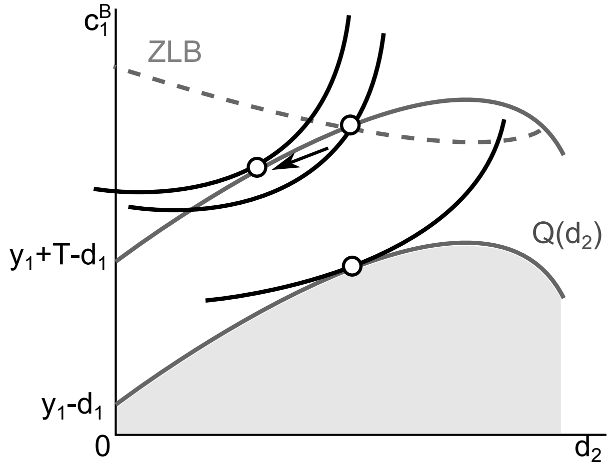


Figure 4: Debt relief with retrading

Another way to interpret this result is as follows. Borrower countries have a higher marginal propensity to consume than saver countries, because they do not have perfect access to capital markets. However, they are not completely liquidity constrained, so their MPC is strictly less than 1. A transfer to borrowers increases their spending, increasing aggregate demand and raising savers' income. But it does not raise savers' income one for one, so their income does not rise enough to compensate them for the fall in the value of their assets.

### 3.5 Implementation with retrading: lending policy

I maintain the assumption that retrading is possible, but now focus on the second policy considered above: lending policy, or a bond price support program. Again, to simplify matters I assume that the union-wide authority directly finances borrower governments, and prevents saver and borrower countries from interacting in bond markets in any other way. The union-wide authority offers a bond pricing function $\tilde{Q}(d)$, which need not be the same as the market bond pricing function $Q(d)$. These loans are financed by issuing risk-free debt to saver countries with price $Q^{rf}$, and through lump sum taxes on savers and borrowers, $T_1^S, T_1^B$. (The tax on borrowers may be negative, i.e. it may be a subsidy.)

This definition of lending policy is intended to capture certain features of the ECB lending programs during the European crisis, which included both direct purchases of government debt (the Securities Markets Programme), commitments to purchase government bonds (Outright Monetary Transactions), and long term loans to banks, which could use these loans to purchase government debt (the Long-Term Refinancing Operations). The explicit motivation for these policies was that they would reduce sovereign risk, which would have beneficial macroeconomic spillovers, and there is some evidence for this proposition (Krishnamurthy et al. [2014]). More generally, Lane and Milesi-Ferretti [2012] find that official lending (IMF and EU loans, but mainly ECB liquidity funds) compensated for the exit of private capital flows from deficit countries with a pegged exchange rate, during the global financial crisis.

**Definition 3.10.** *An equilibrium with lending policy in the baseline economy is a collection $c_1^S, c_1^B, d_2, Q^{rf}, y_1$ such that, given the initial debt level $\bar{d}_1$ and given a bond pricing function $\tilde{Q}(\cdot)$*

1. *$c_1^S, a_2$ solve the saver country's problem:*

$$\max_{c_1^S, a_2, d_2} u(c_1^S) + \beta u(a_2)$$
$$s.t.\ c_1^S + Q^{rf} a_2 = y_1 + \bar{d}_1 - T_1^S$$

2. *$c_1^B, d_2$ solve the borrower country's problem:*

$$\max_{c_1^B, d_2} u(c_1^B) + \beta \left[ \int_0^d V(y^* - \chi)\, \mathrm{d}F(\chi) + p(d)V(-d) \right]$$
$$s.t.\ c_1^S + \bar{d}_1 = y_1 + Q(d_2)d_2 - T_1^B$$

3. *The government budget constraint is satisfied:*

$$Q(d_2)d_2 = Q^{rf} a_2 + T_1^S + T_1^B$$

4. *The goods market clears:*

$$c_1^S + c_1^B = 2y_1$$

5. *The risk-free bond market clears:*

$$a_2 = p(d_2)d_2$$

6. *$Q^{rf} \leq 1$, $y_1 \leq y^*$, with at least one strict equality.*

The following proposition states that equilibria with lending policy implement efficient allocations.

**Proposition 3.11.** *Any efficient allocation can be implemented as an equilibrium with lending policy.*

Intuitively, bond pricing functions sketch out a nonlinear budget constraint for borrowers. By changing the slope of this budget constraint, we can induce borrower countries to accept any feasible allocation. Figure 5 illustrates. The union-wide authority offers a new bond price schedule, $Q^*(d_2)$, which gives borrower countries a higher average and (crucially) marginal price

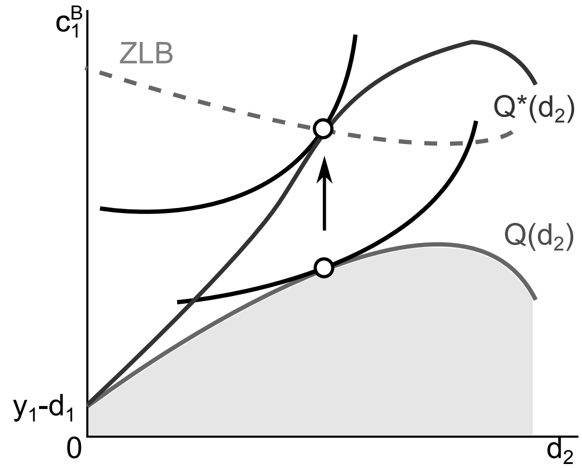


Figure 5: Lending policy

for their debt. This induces borrowers not to reduce their debt below $d_2$, and so savers are no worse off than in the equilibrium without policy, so this lending policy is Pareto improving.

Why can lending policy implement all optimal allocations, while debt relief cannot? Both policies can be used to engineer a transfer to borrowers, as proposition 3.8 states. The key difference is that lending policy can also affect the marginal price of debt, while debt relief cannot. By increasing the marginal price of debt, lending policy encourages borrowers to spend more of their wealth at date 1, boosting aggregate demand.

### 3.6 Optimal lending policy at the ZLB

Having shown that some lending policy implements optimal allocations, I now discuss what kind of lending policy does so.

We already know that when the ZLB binds in equilibrium, the equilibrium without policy is not efficient. The following proposition describes how lending policies implement efficient allocations when the ZLB binds.

**Proposition 3.12.** *When $U_S > U_S^*$:*

1. *The solution to the planner's problem cannot be implemented as an equilibrium without policy.*

2. *The solution to the planner's problem can be implemented as an equilibrium with a lending policy. The marginal price of debt must be higher than in the equilibrium without policy: that is,*

$$\tilde{Q}(d_2) + \tilde{Q}'(d_2)d_2 > Q(d_2) + Q'(d_2)d_2$$

3. *When the ZLB binds in equilibrium, there always exists an equilibrium with lending policy which is Pareto superior to the equilibrium without policy.*

4. *Given $\bar{d}_1$, efficient allocations with higher $U_S$ have higher $a_2$ and lower $T_1^S$.*

*Proof.* (4.) When the ZLB binds, $u'(y^* + \bar{d}_1 - T_1^S - a_2) = \beta u'(y^* + (1-\beta)a_2)$. Allocations which are better for savers have higher $a_2$ and higher $c_1^S = y^* + \bar{d}_1 - T_1^S - a_2$, which means they must have lower $T_1^S$. □

The first part of the proposition follows from the result above that equilibria without policy are inefficient when the ZLB binds. Since there are some efficient allocations in which the ZLB binds, clearly these allocations cannot be implemented as an equilibrium without policy. The second part of the proposition states that lending policy can implement allocations in which the ZLB binds. Furthermore, lending policy must offer borrowers a higher marginal price of debt than in the equilibrium without policy. To see why, return to Figure 5, and note that the slope of the new bond pricing function is higher than the slope of the old function at the same level of debt. The third part of the proposition states that in particular, there are some equilibria with lending policy which are Pareto improving relative to a competitive equilibrium with a binding ZLB and underemployment of resources. The last part of this proposition states that allocations which are relatively favorable for savers involve less debt relief and more lending.

Debt relief, together with no lending policy (i.e. a bond price function which merely replicates market prices, $\tilde{Q} = Q$) can implement efficient allocations in which the ZLB does not bind. The government can simply write off part of $\bar{d}_1$ until the remaining debt, $\bar{d}_1 + T_1^B$ (where $T_1^B < 0$) is less than $\bar{d}_1^*$. We already know that the competitive equilibrium in this case is efficient. But it is not a Pareto improvement on the equilibrium without policy. Borrowers are better off, but savers are strictly worse off.

# 4 Long term debt and debt postponement

In this section I consider equilibria in which borrower countries have some outstanding long-term debt, $\bar{d}_2 > 0$. This is of interest for two reasons. First, long-term debt introduces a new inefficiency, independent of the zero lower bound: borrowers have an incentive to over-issue new debt, to dilute existing debt. Second, in order to analyze debt postponement policy, which converts short term debt into long term debt, we need to characterize equilibria with long term debt.

## 4.1 Equilibrium with long term debt

I now define equilibrium with long-term debt, in the standard way. I let $d_2$ denote the total face value of borrowers' obligations to savers at the start of date 2. New debt issued at date 1 is $d_2 - \bar{d}_2$. The probability of default, and the endogenous bond price, only depend on $d_2$.

**Definition 4.1.** *An equilibrium in the economy with long term debt is a collection* $c_1^S, c_1^B, d_2, a_2, Q_1, Q^{rf}, y_1$ *and a bond pricing function* $Q(\cdot)$ *such that, given initial debt levels* $\bar{d}_1, \bar{d}_2$*:*

1. $c_1^S, d_2$ *solve the saver country's problem:*

$$\max_{c_1^S, d_2} u(c_1^S) + \beta V(a_2 + p(d_2)d_2)$$
$$s.t.\ c_1^S + Q_1(d_2 - \bar{d}_2) + Q^{rf} a_2 = y_1 + \bar{d}_1$$

2. $c_1^B, d_2$ *solve the borrower country's problem:*

$$\max_{c_1^B, d_2} u(c_1^B) + \beta \left[ \int_0^d V(-\chi)\, \mathrm{d}F(\chi) + p(d)V(-d) \right]$$
$$s.t.\ c_1^S + \bar{d}_1 = y_1 + Q(d_2)(d_2 - \bar{d}_2)$$

3. *The bond pricing function is* $Q(d) = p(d)Q^{rf}$, *with* $Q(d_2) = Q_1$.

4. *The goods market clears:*
$$c_1^S + c_1^B = 2y_1$$

5. *The risk-free bond market clears:* $a_2 = 0$.

6. $Q^{rf} \leq 1$, $y_1 \leq y^*$, *with at least one strict equality.*

The following proposition characterizes equilibrium.

**Proposition 4.2.** *For any* $\bar{d}_2$, *there exists* $\bar{d}_1^*(\bar{d}_2)$ *such that:*

1. *If* $\bar{d}_1 < \bar{d}_1^*$, *then* $Q^{rf} < 1$ *and* $y_1 = y^*$. $Q^{rf}$ *is increasing in* $\bar{d}_1$.

2. *If* $\bar{d}_1 = \bar{d}_1^*$, *then* $Q^{rf} = 1$ *and* $y_1 = y^*$.

3. *If* $\bar{d}_1 > \bar{d}_1^*$, *then* $Q^{rf} = 1$ *and* $y_1 < y^*$. $y_1$ *is decreasing in* $\bar{d}_1$.

$c_1^S$ *is increasing in* $\bar{d}_1$. $c_1^B$ *is decreasing in* $\bar{d}_1$.

As in the economy with only short term debt, borrower countries have an incentive to write down their short term debt at date 1. If their debt is sufficiently large, this depresses aggregate demand by so much that the market clearing risk free rate of interest is negative, and the monetary union enters a recession.

## 4.2 Debt dilution and inefficiency

The following proposition states that with outstanding long-term debt, even if the ZLB does not bind, the equilibrium is inefficient. This is for the standard reason that borrowers have an incentive to dilute long-term debt: issuing more debt reduces the value of their outstanding obligations.

**Proposition 4.3.** *In the economy with long term debt* ($\bar{d}_2 > 0$), *if the ZLB does not bind, then the equilibrium is Pareto inefficient.*

When the ZLB does not bind, borrowers issue too much new debt in order to dilute the value of their existing debt. It can be Pareto improving to coordinate buy-backs of long-term debt, but this cannot be implemented at market prices (Aguiar and Amador [2014]). It follows that a policy of replacing long term debt with short term debt is Pareto improving.

### 4.3 Postponement

Postponement is an important feature of debt restructurings in practice. Trebesch et al. [2012] find that out of 186 debt exchanges with foreign private creditors since 1950, 57 involved a cut in face value, while 129 were pure debt reschedulings, involving only a lengthening of maturities. Recently, the IMF has proposed modifying its lending framework to give a greater role for 're-profiling', as an attractive alternative to outright debt forgiveness. Reprofiling was also proposed as a solution to the Greek debt crisis in 2011.

Recall that when the ZLB binds, borrower countries without long-term debt typically issue too little new debt, and reduce their consumption too much, because they do not internalize the effect of their consumption on union-wide aggregate demand. This suggests that when the ZLB binds, it may, perversely, be efficient for the borrowers to have long-term debt. As we will see, equilibria with a binding ZLB and long-term debt are only constrained efficient in a knife-edge case, when the dilution incentive to overborrow exactly outweighs the macroeconomic externality to underspend. However, optimal policy can use this idea to implement constrained efficient allocations. Suppose borrowers have only short-term debt, and the ZLB binds. The union-wide authority can postpone a portion of this short term debt, converting it into long-term debt. If the amount to be converted is chosen correctly, this implements an efficient allocation, as the following Proposition states.

**Proposition 4.4.** *Take any solution of the social planner's problem when $U_S > U_S^*$. It can be implemented as an equilibrium with long term debt for some $\bar{d}_1, \bar{d}_2 > 0$.*

Another reason to favor long-term debt is that it is less vulnerable to self-fulfilling crises (Cole and Kehoe [2000]). This factor is absent here: the model has a unique equilibrium. Yet another reason is that long-term debt helps hedge shocks (Angeletos [2002], Buera and Nicolini [2004]). This factor is also absent in the model so far, since there is no aggregate risk.

## 5 Home bias

Debt restructuring and lending policies which transfer resources from creditor to debtor countries can be Pareto improving only because debtors spend the transfer, in part, on goods sold by creditor countries, increasing their income. One might worry that if debtor countries spend most of the transfer on domestic goods and services, creditor countries will no longer be better off. To address this concern, I consider an economy with home bias ($\alpha < 1$) but, for tractability, assume perfectly rigid prices ($\varphi = \infty$) and no long-term debt ($\bar{d}_2 = 0$). I show that the central result from the baseline economy goes through: transfers from creditors to debtors are still Pareto improving in a liquidity trap.

### 5.1 Equilibrium with home bias

With home bias and rigid prices, different countries will have different levels of income as well as different consumption. Recall that the market clearing condition with fixed prices and home bias is

$$y_t^i = (1-\alpha)c_t^i + \alpha \int y_t^f \, \mathrm{d}f \leq y^*$$

Households in country $i$ spend a fraction $(1-\alpha)$ of their total consumption expenditures on domestically produced goods. Since prices are constant and equal to unity, this means that the quantity of domestic goods they consume is a fixed proportion of their total consumption. Households in other countries spend a fraction $\alpha$ of their total consumption (equivalently, of their income) on country $i$'s goods. If $\alpha = 1$, there is no home bias and every country's output is the same. If $\alpha < 1$, countries with lower domestic consumption will experience lower output.

When output is below potential in country $i$, the social value of higher consumption (from country $i$'s perspective) is higher than the private value. Suppose that country $i$ receives a larger transfer from abroad (e.g. because it borrows more). Its citizens feel richer, and (since prices are fixed) increase consumption of both domestic and foreign goods. Since output is demand constrained, the increase in their consumption of domestic goods increases their income, making them better off and leading to a second round effect on domestic spending. Farhi and Werning [2012] explore these within-country externalities at great length, and show that there is a role for government intervention in insurance markets, to correct the discrepancy between the private and national value of transfers. Since my goal is to study between-country externalities, I abstract from within-country externalities by assuming that the government borrows and lends on behalf of its citizens, internalizing the effect of its decisions on domestic output.[9]

To characterize equilibrium, start with date 2. Resource constraints are

$$\begin{aligned} y_2^S &= (1-\alpha)c_2^S + \alpha\bar{y}_2 \leq y^* \\ y_2^B &= (1-\alpha)c_2^B + \alpha\bar{y}_2 \leq y^* \\ y_2^D(\chi) &= (1-\alpha)c_2^D(\chi) + \alpha\bar{y}_2 \leq y^* \\ \bar{y}_2 &= \frac{1}{2}y_2^S + \frac{p(d_2)}{2}y_2^B + \frac{1}{2}\int_0^{d_2} y_2^D(\chi)\,\mathrm{d}F(\chi) \end{aligned}$$

where $y_2^D(\chi)$, $c_2^D(\chi)$ denote the income and consumption of a defaulting country with default

[9] Again, another interpretation of the model is that households participate in financial markets, and governments impose capital controls along the lines described in Farhi and Werning [2012] to induce households to make borrowing decisions that maximize domestic welfare. As noted above, even without home bias, if households participate in financial markets, it would be necessary for national governments to impose capital controls to correct the externalities associated with default and bond pricing, as described in Na et al. [2014].

cost $\chi$. We also have the budget constraints:

$$\begin{aligned} c_2^S &= y_2^S + p(d_2)d_2 \\ c_2^B &= y_2^B - d_2 \\ c_2^D(\chi) &= y_2^D(\chi) \end{aligned}$$

This implies that $c_2^D(\chi) = y_2^D(\chi) = \bar{y}_2, \forall \chi$.

I assume monetary policy does the best it can, which is to set $y_2^S = y^*$. This means that

$$\begin{aligned} \bar{y}_2 &= y^* - \frac{1-\alpha}{\alpha} p(d_2)d_2 \\ y_2^B &= y^* - \frac{1-\alpha}{\alpha}[1 + p(d_2)]d_2 \\ c_2^B &= y^* - \frac{1-\alpha}{\alpha}[1 + p(d_2)]d_2 - d_2 \end{aligned}$$

A borrower will be indifferent between repaying and defaulting when

$$\begin{aligned} c_2^B &= c_2^D - \chi \\ y^* - \frac{1-\alpha}{\alpha}[1 + p(d_2)]d_2 - d_2 &= y^* - \frac{1-\alpha}{\alpha} p(d_2)d_2 - \chi \\ \frac{d_2}{\alpha} &= \chi \end{aligned}$$

The probability that a country repays debt $d_2$ is $p(d_2) = Pr(\chi > d_2/\alpha) = 1 - F\left(\frac{d_2}{\alpha}\right)$, which is increasing in $\alpha$. With home bias and sticky prices, governments are more likely to default, because they internalize that repaying their debt would lead to a domestic recession.

Now consider equilibrium at date 1.

$$\begin{aligned} y_1^S &= (1-\alpha)c_1^S + \alpha \bar{y}_1 \leq y^* \\ y_1^B &= (1-\alpha)c_1^B + \alpha \bar{y}_1 \leq y^* \\ \bar{y}_1 &= \frac{y_1^S + y_1^B}{2} \\ c_1^S &= y_1^S + \bar{d}_1 - Q(d_2)d_2 \\ c_1^B &= y_1^B - \bar{d}_1 + Q(d_2)d_2 \end{aligned}$$

Solving for all variables as a function of $y_1^S$,

$$\begin{aligned}\bar{y}_1 &= y_1^S - \frac{1-\alpha}{\alpha}[\bar{d}_1 - Q(d_2)d_2] \\ c_1^S &= y_1^S + \bar{d}_1 - Q(d_2)d_2 \\ y_1^B &= y_1^S - 2\frac{1-\alpha}{\alpha}[\bar{d}_1 - Q(d_2)d_2] \\ c_1^B &= y_1^S - \frac{2-\alpha}{\alpha}[\bar{d}_1 - Q(d_2)d_2]\end{aligned}$$

When the zero lower bound is slack, monetary policy sets $y_1^S = y^*$. But note that with home bias, even when the ZLB is slack, borrower countries experience a recession.

Governments internalize the effect of their borrowing and lending decisions on domestic output. For example, saver country governments perceive that they face the constraints

$$\begin{aligned}c_1^S &= y_1^S + \bar{d}_1 - Q(d_2)d_2 \\ y_1^S &= (1-\alpha)c_1^S + \alpha\bar{y}_1\end{aligned}$$

and take $\bar{y}_1$, not $y_1^S$, as given. So they effectively face the constraint

$$c_1^S = \bar{y}_1 + \frac{\bar{d}_1 - Q(d_2)d_2}{\alpha}$$

Similarly, the remaining constraints are

$$\begin{aligned}c_2^S &= \bar{y}_2 + \frac{p(d_2)d_2}{\alpha} \\ c_1^B &= \bar{y}_1 - \frac{\bar{d}_1 - Q(d_2)d_2}{\alpha} \\ c_2^B &= \bar{y}_2 - \frac{d_2}{\alpha}\end{aligned}$$

**Definition 5.1.** *An equilibrium in the economy with home bias is a collection $c_1^S, c_1^B, d_2, a_2, Q_1, Q^{rf}, y_1$ and a bond pricing function $Q(\cdot)$ such that, given the initial debt level $\bar{d}_1$:*

1. *$c_1^S, d_2$ solve the saver country's problem:*

$$\max_{c_1^S, d_2} u(c_1^S) + \beta V\left(\frac{\bar{y}_2 - y^*}{1-\beta} + a_2 + \frac{p(d_2)d_2}{\alpha}\right)$$
$$s.t.\ c_1^S + \frac{Q_1 d_2 + Q^{rf} a_2}{\alpha} = \bar{y}_1 + \frac{\bar{d}_1}{\alpha}$$

2. $c_1^B, d_2$ *solve the borrower country's problem:*

$$\max_{c_1^B, d_2} u(c_1^B) + \beta \left[ \int_0^{d/\alpha} V(\frac{\bar{y}_2 - y^*}{1-\beta} - \chi) \, \mathrm{d}F(\chi) + p(d) V\left(\frac{\bar{y}_2 - y^*}{1-\beta} - \frac{d}{\alpha}\right)\right]$$

$$s.t. \ c_1^S + \frac{\bar{d}_1}{\alpha} = y_1 + \frac{Q(d_2)d_2}{\alpha}$$

3. *The bond pricing function is* $Q(d) = p(d)Q^{rf}$, *with* $Q(d_2) = Q_1$.

4. *The goods markets clear:*

$$y_1^S = (1-\alpha)c_1^S + \alpha\bar{y}_1$$
$$y_1^B = (1-\alpha)c_1^B + \alpha\bar{y}_1$$
$$\bar{y}_1 = \frac{y_1^S + y_1^B}{2}$$
$$\bar{y}_2 = y^* - \frac{1-\alpha}{\alpha} p(d_2)d_2$$

5. *The risk-free bond market clears:* $a_2 = 0$.

6. $Q^{rf} \leq 1$, $y_1^S \leq y^*$, *with at least one strict equality.*

I now characterize the equilibrium. Equilibria have the same structure as before: if debt is sufficiently high, the ZLB binds. But, as we have seen, output is always below potential in borrower countries, even if the ZLB does not bind.

**Proposition 5.2.** *There exists* $\bar{d}_1^*$ *such that:*

1. *If* $\bar{d}_1 < \bar{d}_1^*$, *then* $Q^{rf} < 1$ *and* $y_1^S = y^*$. $Q^{rf}(\bar{d}_1)$ *is increasing in* $\bar{d}_1$.

2. *If* $\bar{d}_1 = \bar{d}_1^*$, *then* $Q^{rf} = 1$ *and* $y_1^S = y^*$.

3. *If* $\bar{d}_1 > \bar{d}_1^*$, *then* $Q^{rf} = 1$ *and* $y_1^S < y^*$. $y^S(\bar{d}_1)$ *is decreasing in* $\bar{d}_1$.

$c_1^S$ *is increasing in* $\bar{d}_1$. $\bar{y}_1, y_1^B, \bar{y}_2, y_1^B$ *and* $c_1^B$ *are decreasing in* $\bar{d}_1$.

### 5.2 Efficient allocations

I now characterize efficient allocations by solving a social planner's problem. It is convenient to define $d = d_2/\alpha$. The planner solves

$$\max u(c_1^B) + \beta \left[ \int_0^d V(-(1-\alpha)p(d)d - \chi)\, \mathrm{d}F(\chi) + p(d)V\left(-(1-\alpha)p(d)d - d\right) \right]$$

$$\begin{aligned} \text{s.t. } u(c_1^S) + \beta V(\alpha p(d)d) &\geq U_S \\ u'(c_1^S) &\geq \beta u'(y^* + (1-\beta)\alpha p(d)d) \\ \left(1 - \frac{\alpha}{2}\right) c_1^S + \frac{\alpha}{2} c_1^B &\leq y^* \\ \left(1 - \frac{\alpha}{2}\right) c_1^B + \frac{\alpha}{2} c_1^S &\leq y^* \end{aligned}$$

As before, without loss of generality we assume $U_S > \dfrac{u(y^*)}{1-\beta}$. The following Proposition characterizes efficient allocations. As before, it is never efficient for the zero lower bound to constrain output at date 1.

**Proposition 5.3.** *1. In every efficient allocation, $y_1^S = y^*$.*

*2. There exists $U_S^* > (1+\beta)u(y^*)$ such that the ZLB binds if $U_S \geq U_S^*$.*

*3. $c_1^S$ and $d_2$ are increasing in $U_S$; $c_1^B$ is decreasing in $U_S$.*

### 5.3 Home bias and inefficiency

**Proposition 5.4.** *Any equilibrium in the economy with home bias is constrained inefficient.*

*Proof.* When the ZLB does not bind, a necessary condition for optimality is

$$\frac{u'(c_2^S)}{u'(c_1^S)} = \frac{u'(c_2^B)}{u'(c_1^B)} \frac{1}{1-\gamma(d)} + \Omega$$

where

$$\Omega = \frac{1}{\phi'(d)} \frac{1-\alpha}{2-\alpha} \left[ \int_0^d \frac{u'(c_2^D(\chi)) - u'(c_2^B)}{u'(c_1^B)}\, \mathrm{d}F(\chi) + p(d) \frac{u'(c_2^B)}{u'(c_1^B)} [\phi'(d) - 1] \right] < 0$$

But in equilibrium, we have

$$\frac{u'(c_2^S)}{u'(c_1^S)} = \frac{u'(c_2^B)}{u'(c_1^B)} \frac{1}{1-\gamma(d)}$$

So the equilibrium allocation cannot be a solution to the planner's problem.

When the ZLB binds in equilibrium, $y_1^S < y^*$ and so (by the above Proposition) the allocation is not constrained efficient. □

With home bias, equilibrium is typically inefficient, even when the ZLB does not bind. Borrowers deleverage too rapidly. When deciding how much new debt to issue, borrower governments trade off the benefit of debt - higher consumption at date 1 - against the cost - lower consumption at date 2, if they recive a high default cost $\chi$ and have to repay the debt. They internalize the fact that higher consumption at date 1 boosts their own domestic output, and lower consumption at date 2 reduces their own domestic output. But they do not internalize that their consumption affects demand and output in other borrower countries. Those other countries benefit from higher consumption at date 1, but lose out from lower consumption at date 2. But the benefits outweigh the costs, because for every dollar of debt issued, less than one dollar will be repaid.

Another way to see this is that in this economy, an individual country's decision to default increases aggregate demand, because defaulting countries consume more than countries which repay their debt. In equilibrium, there are an inefficiently low number of defaults - at least from the perspective of borrower countries as a class. Creditors are hurt by default, but this is already priced into the cost of debt. So borrowers as a whole could strike a Pareto-improving deal with creditors where they take on more debt, reducing the price of debt to compensate creditors for their losses, and making borrowers better off throught the aggregate demand externalities from defaulting countries' higher consumption. In practice, there may be negative externalities associated with default (for example, the effect on the banking system in creditor countries) which are not priced into government debt. In this case, unsurprisingly, debt and default might be too high in equilibrium. I abstract from such externalities here, in order to focus on the Keynesian rationale for debt restructuring.

To summarize, transfers from creditors to debtors are Pareto improving at the ZLB because debtors spend the transfer, in part, on goods sold by creditor countries, increasing their income - even if debtor countries spend most of the transfer on domestic goods and services. If borrowers spend most of the transfer on their own goods and services, this increases their domestic income, which also increases their demand for foreign goods. Ultimately, budget constraints imply that a country must spend the whole of any transfer on buying either goods or assets from abroad.

As in the baseline economy, debt writedowns and lending policy are equivalent when retrading is prevented, but lending policy is preferable when retrading is possible.

**Proposition 5.5.** *In the economy with home bias, the solution to the planner's problem can be implemented as an equilibrium with lending policy. The marginal price of debt must be higher than in the equilibrium without policy. Given $\bar{d}_1$, efficient allocations with higher $U_S$ have higher $a_2$ and lower $T_1^S$.*

## 6 Ex ante policy and overborrowing

A common argument against debt restructuring is that it gives countries an incentive to overborrow, knowing that they will be bailed out. To address this argument, I augment the model to include an ex ante stage in which countries decide how much to borrow and lend, taking into account that the union-wide authority may offer bailouts or debt restructuring ex post.

## 6.1 Ex ante overborrowing

I now endogenize date 1 debt, $\bar{d}_1$, in the baseline economy with no home bias and rigid prices. At date 0, countries initially have no debt, and have preferences

$$U(c_0^i, \theta_i) + E_0[\beta u(c_1^i) + \beta^2 u(c_2^i)]$$

where $U_c > 0, U_{cc} < 0$. $\theta_i$ measures country $i$'s demand for date 0 consumption, with $U_{c\theta} > 0$. $\theta_B > \theta_S = 1$: borrowers are more impatient and have a more urgent need for consumption at date 0. While I model $\theta$ as a preference or discount factor shock, it can easily be reinterpreted in terms of income, so that type $B$ countries borrow because they have temporarily low income at date 0: simply let $U(c, \theta) = u(c - \theta)$. Crucially, I assume that $\theta_i$ is private information: the union-wide authority (and the fictitious social planner) cannot directly observe a country's need for date 0 consumption. Instead, they must infer $\theta_i$ by observing a country's debt levels.

At date 1, with probability $\pi$, it becomes common knowledge that countries can default at date 2, with default costs $\chi$ drawn from $F(\chi)$. The equilibrium, conditional on the endogenously chosen levels of debt, is as above. With probability $1 - \pi$, it becomes common knowledge that countries cannot default at date 2 (equivalently, $\chi = \infty$ with probability 1).

For now, I assume countries can only trade a short term bond. If the crisis does not occur at date 1, their budget constraints at dates $t = 0, 1, ..$ are

$$\begin{aligned} c_t^S &= y_t + d_t - Q_t^{rf} d_{t+1} \\ c_t^B &= y_t - d_t + Q_t^{rf} d_{t+1} \\ d_0 &= 0 \end{aligned}$$

In the non-crisis state, $Q_1^{rf} = \beta$, and indebted countries smooth debt repayments: $c_t^S = c_1^S, c_t^B = c_1^B, \forall t \geq 1$.

The following proposition states that if borrower countries are impatient enough, relative to savers, or if the probability of crisis $\pi$ is low enough, they choose $d_1 > \bar{d}_1^*$, triggering the ZLB in the crisis state.

**Proposition 6.1.** *For any $\pi$, there exists an increasing function $\theta_{ZLB}(\pi)$ such that $d_1 > \bar{d}_1^*$ if $\theta_{ZLB}(\pi)$.*

This is essentially identical to the result of Korinek and Simsek [2014], in a similar model. Intuitively, countries want to take on more debt the more impatient they are; they also take on more debt if it is less likely that they will be forced to deleverage.

### 6.2 Ex ante constrained efficient allocations under full information

To characterize ex ante efficient allocations, I again consider a social planner's problem. For now, I assume $\pi = 1$.

It is convenient to write the planner's problem in recursive form. Given that borrowers receive date 1 utility $V_B$, let $V_S(V_B)$ be the maximum date 1 utility that can be given to savers. I restrict attention to the case where $V_S(V_B) > (1+\beta)u(y^*) > V_B$: it is optimal to promise savers more utility than borrowers at date 1, because savers are more patient.

$$V_S(V_B) = \max u(c_1^S) + \beta V(p(d_2)d_2) \tag{3}$$

$$\text{s.t. } u(c_1^B) + \beta \left[ \int_0^d V(-\chi)\,\mathrm{d}F(\chi) + p(d)V(-d) \right] = V_B \tag{VB}$$

$$c_1^S + c_1^B \leq 2y^* \tag{RC1}$$

$$u'(c_1^S) \geq \beta u'(y^* + (1-\beta)p(d_2)d_2) \tag{ZLB}$$

The following proposition characterizes the solutions to this date 1 Pareto problem.

**Proposition 6.2.** *$V_S(V_B)$ is convex and weakly decreasing. There exist $V_B^{UE} < V_B^{ZLB} < (1+\beta)u(y^*)$ such that:*

1. *If $V_B < V_B^{UE}$, (ZLB) binds, (RC1) is slack, $V_S'(V_B) = 0$*
2. *If $V_B \in (V_B^{UE}, V_B^{ZLB})$, (ZLB) and (RC1) both bind, $-\frac{u'(c_1^S)}{u'(c_1^B)} < V_S'(V_B) < 0$*
3. *If $V_B > V_B^{ZLB}$, (ZLB) is slack, (RC1) binds, and $V_S'(V_B) = -\frac{u'(c_1^S)}{u'(c_1^B)} < 0$*

The planner solves

$$\max \alpha[U(c_0^S, \theta_S) + \beta V_S(V_B)] + (1-\alpha)[U(c_0^B, \theta_B) + \beta V_B] \tag{4}$$

$$\text{s.t. } c_0^S + c_0^B \leq 2y^* \tag{5}$$

I now characterize constrained efficient allocations under full information.

**Proposition 6.3.** *Any ex ante constrained efficient allocation is ex post efficient. Define the wedge between borrowers' and savers' Euler equations:*

$$\tau = \frac{U_c(c_0^B, \theta_B)}{\beta u'(c_0^B)} - \frac{U_c(c_0^S, \theta_S)}{\beta u'(c_0^S)}$$

*If the ZLB binds at date 1, $\tau > 0$. If the ZLB is slack, $\tau = 0$.*

If the union-wide authority knows which countries need to borrow and which do not, it is never ex ante optimal to commit to ex post inefficient outcomes, such as a recession.

## 6.3 Ex post efficient bailouts are not ex ante Pareto improving

It follows from the previous proposition that the ex post efficient policies discussed above - debt relief, lending policy, and postponement - are not ex ante Pareto improving, relative to an equilibrium without policy.

Consider the following experiment. Suppose it is common knowledge that, whatever level of debt $\bar{d}_1$ borrower countries take out at date 0, at date 1 the union-wide authority will implement an ex post constrained efficient allocation $c_1^S, c_1^B, d_2$. This allocation may depend on the aggregate debt taken out by borrower countries $\bar{d}_1$, but individual borrower (saver) countries correctly perceive that their transfer (tax) does not depend on their own level of debt. In equilibrium, since there is no default at date 1, borrowers and savers face the same interest rate between dates 0 and 1, and we have $\tau = 0$. This is not constrained efficient.

To implement constrained efficient allocations, it is necessary to impose a macroprudential tax or limit on borrowing. Country $i$'s budget constraint at date 0 becomes

$$c_0^i = y_0 + Q_t^{rf} d_1 - T_0(d_1, \theta_i)$$

where $T_0(d_1, \theta_i)$ is a nonlinear tax schedule that depends on a country's borrowing. This nests two simple special cases. First, borrowers can receive a tax on debt and a compensating lump sum transfer: $T_0(d, \theta_B) = -\bar{T}(\theta_B) + \tau(\theta_B) d$. Second, the union-wide authority can impose a hard limit on borrowing above a certain level, together with a compensating transfer: $T_0(d, \theta_B) = -\bar{T}(\theta_B)$ if $d \leq d^*$, $T_0(d, \theta_B) = \infty$ if $d > d^*$. The tax on borrowing (alternatively, the borrowing constraint) prevent borrower countries from taking out more debt, in anticipation of the transfers they will receive at date 1.

## 6.4 Ex ante efficient allocations under private information

To characterize ex ante efficient allocations, I again consider a social planner's problem. In addition to the ZLB and resource constraints considered above, the planner faces incentive compatibility constraints, which state that no country's allocation can be so generous that another country prefers that allocation to its own. For now, I assume $\pi = 1$.

Again, we write the planner's problem in recursive form. The planner solves

$$\max \alpha [U(c_0^S, \theta_S) + \beta V_S(V_B)] + (1 - \alpha)[U(c_0^B, \theta_B) + \beta V_B] \tag{6}$$

$$\text{s.t. } c_0^S + c_0^B \leq 2y^* \tag{7}$$

$$U(c_0^S, \theta_S) + \beta V_S(V_B) \geq U(c_0^B, \theta_S) + \beta V_B \tag{ICS}$$

$$U(c_0^B, \theta_B) + \beta V_B \geq U(c_0^S, \theta_B) + \beta V_S(V_B) \tag{ICB}$$

I now characterize constrained efficient allocations.

**Proposition 6.4.** *1.* (7) *always binds.*

2. *There exist $\alpha_{ICS}, \alpha_{ICB}$ such that $0 < \alpha_{ICS} < \alpha_{ICB} < 1$,* (ICS) *binds if $\alpha < \alpha_{ICS}$, and* (ICB) *binds if $\alpha > \alpha_{ICB}$.*

3. *$V_S'(V_B) < 0$ (that is, the allocation is ex post Pareto efficient) unless* (ICS) *binds. In this case, we may have $V_S'(V_B) = 0$ (that is, the allocation may be ex post Pareto inefficient).*

When (ICS) binds, so saver countries are tempted to mimic borrowers, it may be optimal to have a recession at date 1, even though this is ex post inefficient. This gives the borrowers lower date 1 consumption; but in return, they can enjoy higher date 0 consumption without inducing savers to mimic them.

Constrained efficient allocations under private information can be implemented with a combination of macroprudential taxes or limits on borrowing ex ante, and debt restructuring ex post. Unlike under full information, however, taxes can no longer be targeted at different countries directly. Country $i$'s budget constraint at date 0 becomes

$$c_0^i = y_0 + Q_t^{rf} d_1 - T_0(d_1)$$

where $T_0(d_1)$ is a nonlinear tax schedule that now depends on only a country's borrowing, not on its type $\theta_i$.

## 7 Conclusion

In an international liquidity trap, sovereign debt restructuring and lending policy can be Pareto improving because they support aggregate demand in highly indebted countries, which in turn supports output and incomes in these countries' trading partners. In order to obtain a Pareto improvement, optimal policy combines a transfer to indebted countries with a reduction in these countries' effective borrowing rates, inducing them to spend this transfer on imports, benefiting their trading partners, instead of buying back their debt. Lending policy and debt restructuring are superior to outright debt relief, since they combine a transfer with a reduction in effective borrowing rates.

A large existing literature has explored alternative motivations for sovereign debt restructuring and lending policy, in particular debt overhang, and the need to prevent self-fulfilling crises. In this paper I analyzed a model without these features, in order to focus on the Keynesian rationale for debt restructuring and lending policy. There may be important interactions between self-fulfilling crises and the macroeconomic externality considered in this paper. The risk of self-fulfilling crises in the future would encourage indebted countries to deleverage even more today. Equally, as indebted countries have lower income today because of the recession, they are more vulnerable to such crises. One direction for future research is to analyze this interaction between these channels and the aggregate demand channel explored in this paper.

Another direction is to compare debt policies to alternative monetary and fiscal policies. Higher inflation could potentially boost aggregate demand in two ways. First higher inflation dilutes the value of nominal debt and indirectly provides debt relief. Second, higher expected inflation decreases real interest rates, stimulating spending throughout the monetary union. However, these benefits must be traded off against the usual distortions associated with inflation. Finally, conventional fiscal stimulus - particularly in surplus economies - could potentially increase demand, and might involve less risk of moral hazard than debt relief. In future versions of this paper I will consider these policies.

## References


Mark Aguiar and Manuel Amador. Take the short route: How to repay and restructure sovereign debt with multiple maturities. 2014.

Mark Aguiar and Manuel Amador. *Sovereign Debt*. Elsevier, In Progress.

George-Marios Angeletos. Fiscal Policy With Noncontingent Debt And The Optimal Maturity Structure. *The Quarterly Journal of Economics, MIT Press*, 117(3):1105–1131, August 2002.

Cristina Arellano. Default risk and income fluctuations in emerging economies. *American Economic Review*, 98(3):690–712, 2008.

Cristina Arellano and Ananth Ramanarayanan. Default and the maturity structure in sovereign bonds. *Journal of Political Economy*, 120(2):187–232, 2012.

Francisco Buera and Juan Pablo Nicolini. Optimal maturity of government debt without state contingent bonds. *Journal of Monetary Economics, Elsevier*, 51(3):531–554, April 2004.

Jeremy Bulow and Kenneth Rogoff. The Buyback Boondoggle. *Brookings Papers on Economic Activity*, 19(2):675–704, 1988.

Jeremy Bulow and Kenneth Rogoff. Sovereign Debt Repurchases: No Cure for Overhang. *The Quarterly Journal of Economics*, 106(4):1219–35, November 1991.

Lawrence Christiano, Roberto Motto, and Massimo Rostagno. Risk shocks. *American Economic Review*, 104(1):27–65, 2014.

Harold L. Cole and Timothy J. Kehoe. Self-fulfilling debt crises. *The Review of Economic Studies*, 67(1):pp. 91–116, 2000.

Vasco Curdia and Michael Woodford. Credit spreads and monetary policy. *Journal of Money, Credit and Banking*, 42(s1):3–35, 09 2010.

Jonathan Eaton and Mark Gersovitz. Debt with potential repudiation: Theoretical and empirical analysis. *Review of Economic Studies*, 48(2):289–309, April 1981.

Emmanuel Farhi and Iván Werning. Fiscal Unions. NBER Working Papers 18280, National Bureau of Economic Research, Inc, August 2012.

Emmanuel Farhi and Iván Werning. A theory of macroprudential policies in the presence of nominal rigidities. Working Paper 19313, National Bureau of Economic Research, August 2013.

Luca Fornaro. International debt deleveraging. 2012.

Lorenzo Forni and Massimiliano Pisani. Macroeconomic Effects of Sovereign Restructuring in a Monetary Union: A Model-based Approach. IMF Working Papers 13/269, International Monetary Fund, December 2013.

Jordi Gali and Tommaso Monacelli. Optimal monetary and fiscal policy in a currency union. *Journal of International Economics*, 76(1):116–132, September 2008.

Juan Carlos Hatchondo, Leonardo Martinez, and César Sosa Padilla. Voluntary sovereign debt exchanges. *Journal of Monetary Economics*, 2013.

Juan Carlos Hatchondo, Leonardo Martinez, and César Sosa Padilla. Debt dilution and sovereign default risk. Working paper, 2014.

Anton Korinek and Alp Simsek. Liquidity trap and excessive leverage. Technical report, March 2014.

Arvind Krishnamurthy, Stefan Nagel, and Annette Vissing-Jorgensen. ECB policies involving government bond purchases: Impact and channels. Working paper, August 2014.

Paul R. Krugman. Financing vs. Forgiving a Debt Overhang. NBER Working Papers 2486, National Bureau of Economic Research, Inc, November 1989.

Philip R. Lane and Gian Maria Milesi-Ferretti. External adjustment and the global crisis. *Journal of International Economics*, 88(2):252 – 265, 2012.

Seunghoon Na, Stephanie Schmitt-Grohé, Martin Uríbe, and Vivian Z. Yue. A model of the twin Ds: Optimal default and devaluation. Working Paper 20314, National Bureau of Economic Research, July 2014.

Rohan Pitchford and Mark L. J. Wright. Holdouts in Sovereign Debt Restructuring: A Theory of Negotiation in a Weak Contractual Environment. *Review of Economic Studies*, 79(2):812–837, 2012.

Francisco Roch and Harald Uhlig. The dynamics of sovereign debt crises and bailouts. *draft, University of Chicago*, 2012.

Julio J Rotemberg. Monopolistic Price Adjustment and Aggregate Output. *Review of Economic Studies*, 49(4):517–31, October 1982.

Christoph Trebesch, Mr Michael G Papaioannou, and Mr Udaibir S Das. *Sovereign Debt Restructurings 1950-2010: Literature Survey, Data, and Stylized Facts (EPub)*. Number 12-203. Andrews McMeel Publishing, 2012.

Mark L. J. Wright. Sovereign debt restructuring: Problems and prospects. *Harvard Business Law Review*, 2(1):153–198, 2012.