Iterative detection for V-BLAST MIMO communication systems based on expectation maximisation algorithm

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By applying the expectation maximisation algorithm to the maximum likelihood detection of layered space-time codes, the conditional log-likelihood of a single layer is iteratively maximised, rather than maximising the intractable likelihood function of all layers. Computer simulations demonstrate the efficiency of the proposed detection scheme.

System description: Consider the system of N transmit and M receive antennas. The single data stream in the input is demultiplexed into N substreams, and each substream is modulated independently then transmitted over a rich-scattering wireless channel to M antennas. Each antenna receives signals transmitted from the entire N transmit antennas. The transmission is performed by burst of length l. We adopt a quasi-static approximation of the fading channel, i.e. the channel remains unchanged during each burst. For simplicity of presentation, we consider a BPSK modulation for each layer. The received signal at each instant time can be written as

$$\mathbf{y} = \mathbf{Hb} + \mathbf{z} \tag{1}$$

where $\mathbf{z} = [z_1 z_2 \cdots z_M]^T$ is assumed to be an symmetric i.i.d. complex Gaussian noise vector with zero mean and unit variance, $\mathbf{b} = [b_1 b_2 \cdots b_N]^T$ is the transmitted signal where $\mathbf{b} \in 2\{-1, 1\}^N$, $\mathbf{y} = [y_1 y_2 \cdots y_M]^T$ is the received signal, and $H_{M\times N}$ is the channel matrix. In rich-scattering environments, the elements of the channel matrix can be modelled as symmetric i.i.d. complex Gaussian random variables with zero mean and variance equal to γ/N. From the above definitions, it is clear that γ represents the average SNR at each receive antenna.

The maximisation of the conditional probability density of the received vector in (1) is equivalent to the following nonlinear optimisation problem:

$$\hat{\mathbf{b}} = \arg \max_{\mathbf{b}\in\{-1,1\}^N} [2y^H \mathbf{Hb} - \mathbf{b}^H \mathbf{H}^H \mathbf{Hb}] \tag{2}$$

Since the optimisation in (2) is performed over $\{-1, 1\}^N$, its computational complexity is exponentially in N and thus becomes prohibitive for even a moderate number of layers. By using the EM algorithm, we decompose the N-dimensional maximisation problem in (2) into N one-dimensional maximisation problems. First, we define the complete data set $\mathbf{x_k}$, as required by the EM algorithm, as follows:

$$\mathbf{x_k} = \mathbf{h_k} b_k + \mathbf{z_k} \tag{3}$$

where $\mathbf{x_k}$ is an M element column vector, b_k is the symbol bit of the kth layer, $\mathbf{z_k}$ is the complex M-dimensional AWGN, the components of which are statistically independent with identical power σ_k^2, and $\mathbf{h_k}$ is the kth column of $\mathbf{H}$. From (1) and (3), we have

$$\sum_{k=1}^{N} \sigma_k^2 = \sigma^2 \tag{4}$$

and

$$\mathbf{y} = \sum_{k=1}^{N} \mathbf{x_k} \tag{5}$$

where $\sigma^2 = 1$. In [1] it is proven that the maximisation of the log-likelihood function

$$L(\mathbf{b}) = \frac{-1}{2}(\mathbf{y} - \mathbf{Hb})^H(\mathbf{y} - \mathbf{Hb}) \tag{6}$$

is equivalent to the maximisation of the function $U(\mathbf{b}, \acute{\mathbf{b}})$ over $\mathbf{b}$, where $U(\mathbf{b}, \acute{\mathbf{b}})$ is:

$$U(\mathbf{b}, \acute{\mathbf{b}}) = E[\log(f_{\mathbf{x}}(\mathbf{x}; \mathbf{b}))|\mathbf{y}, \mathbf{b} = \acute{\mathbf{b}}] \tag{7}$$

and

$$f_{\mathbf{x}}(\mathbf{x}; \mathbf{b}) = \frac{1}{(2\pi)^{N/2}\Pi_{k=1}^{N}\sigma_k} \times \exp\left(\sum_{k=1}^{N} \frac{-1}{2\sigma_k^2}(\mathbf{x_k} - \mathbf{h_k} b_k)^H(\mathbf{x_k} - \mathbf{h_k} b_k)\right) \tag{8}$$

In (7–8), $\mathbf{x} = [\mathbf{x}_1^T\ \mathbf{x}_2^T\ \mathbf{x}_3^T \cdots \mathbf{x_N}^T]^T$ and $\acute{\mathbf{b}}$ is an estimate of $\mathbf{b}$. The iterative structure of the detection algorithm is straightforward. To decrease the probability of error, in nth iteration, $U(\mathbf{b}, \mathbf{b}^{(n)})$ is maximised over $\mathbf{b}$ to obtain a new estimate of $\mathbf{b}^{(n+1)}$. After each iteration, the likelihood function increases [1]. Now an analytical solution to the maximisation of $U(\mathbf{b}, \mathbf{b}^{(n)})$ is derived to reduce the N-dimensional maximisation problem into N one-dimensional maximisation problems. We start by expanding the log-likelihood function

$$\log_e f_{\mathbf{x}}(\mathbf{x}; \mathbf{b}) = \sum_{k=1}^{N} \frac{-1}{2\sigma_k^2}(\mathbf{x_k}^H\mathbf{x_k} + b_k^2 \mathbf{h}_k^H \mathbf{h}_k - b_k \mathbf{h}_k^H \mathbf{x_k} - b_k \mathbf{x_k}^H \mathbf{h_k}) + \mathrm{C} \tag{9}$$

where C is a constant. We define $\mathbf{A} = [b_1 \mathbf{h}_1^T/\sigma_1^2\ b_2 \mathbf{h}_2^T/\sigma_2^2 \ldots b_N \mathbf{h}_N^T/\sigma_N^2]^T$ and since $b_k^2 = 1$, (9) can be simplified to

$$\log_e f_{\mathbf{x}}(\mathbf{x}; \mathbf{b}) = g(\mathbf{x}) + 2Re(\mathbf{A}^H\mathbf{x}) \tag{10}$$

Ignoring the first term which has no effect on the maximisation process, and substituting (10) into (7), we will have

$$U(\mathbf{b}, \mathbf{b}^{(n)}) = Re(\mathbf{A}^H E[\mathbf{x}|\mathbf{y}, \mathbf{b}^{(n)}]) \tag{11}$$

It can be easily shown that

$$E[\mathbf{x}|\mathbf{y}, \mathbf{b}^{(n)}] = \frac{1}{\sigma^2}[\sigma_1^2(\mathbf{y} - \mathbf{Hb}^{(n)})^T \sigma_2^2(\mathbf{y} - \mathbf{Hb}^{(n)})^T \ldots \sigma_N^2(\mathbf{y} - \mathbf{Hb}^{(n)})^T]^T + [b_1^{(n)}\mathbf{h}_1^T b_2^{(n)}\mathbf{h}_2^T \ldots b_N^{(n)}\mathbf{h_N}^T]^T \tag{12}$$

Hence,

$$U(\mathbf{b}, \mathbf{b}^{(n)}) = \sum_{k=1}^{N} Re\left(b_k \frac{\mathbf{h_k}^H}{\sigma_k^2}\left[b_k^{(n)}\mathbf{h_k} + \frac{\sigma_k^2}{\sigma^2}(\mathbf{y} - \mathbf{Hb^{(n)}})\right]\right) \tag{13}$$

Obviously, to maximise the whole sum, it is sufficient to maximise each term in the sum, separately. Since b_k's can only take the values $+1$ and -1, the following decisions on b_k's maximise (13):

$$b_k^{(n+1)} = sign\left\{Re\left(b_k^{(n)}\mathbf{h_k}^H\mathbf{h_k} + \frac{\sigma_k^2}{\sigma^2}\mathbf{h_k}^H(\mathbf{y} - \mathbf{Hb^{(n)}})\right)\right\} \tag{14}$$

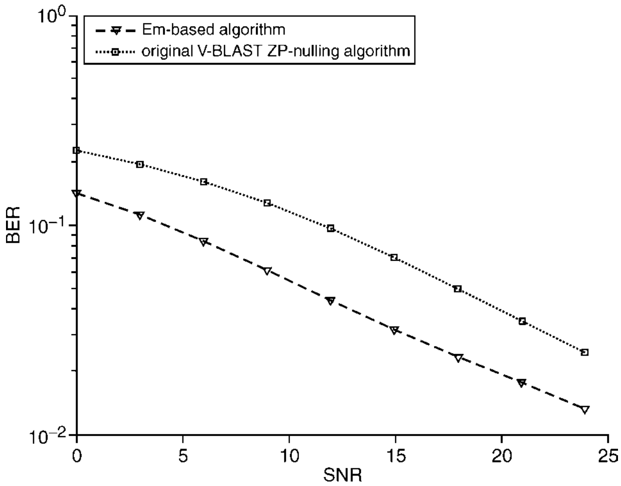

Fig. 1 *Plot of average BER against SNR at each receive antenna*

Simulation results: We have evaluated the performance of the proposed algorithm by simulation. We considered $N = 8$, $M = 8$. Also, we assumed a block fading model where the channel matrix remains constant within each burst, but independently changes from burst to burst. Fig. 1 compares the performance of our algorithm with that of the nulling and cancelling with optimal ordering. As can be realised, our proposed EM-based algorithm substantially outperforms the original nulling and cancelling with optimal ordering scheme proposed in [2], over the whole SNR range.

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