Bailout Stigma

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Abstract

Financially distressed firms may be reluctant to accept government bailouts for fear that it may signal the weakness of their balance sheets and inhibit their future financing. We study such bailout stigma via a model in which a firm must finance projects by selling legacy assets. The value of the asset is informed privately by the firm, and this results in inefficient trading of the asset and insufficient project financing. Although the adverse selection problem creates a scope for government intervention, accepting a bailout can signal the toxicity of the asset for a firm, and this worsens the adverse selection for the firm in the subsequent trading of its remaining asset. We find multiple equilibrium responses to a government bailout. Bailout terms that would otherwise be acceptable may be refused due to the stigma. Even terms that are so generous as to be acceptable for firms with non-toxic assets may result in low take-up; nevertheless, such a policy could work by allowing a firm to improve its market perception by refusing the bailout. Bailout that leads to immediate market activation is welfare dominated by an equilibrium in which no such market activation occurs. A secret bailout which conceals the identity of its recipient can mitigate the stigma and can implement the (constrained) efficient outcome.

Keywords: Adverse selection, bailout stigma, secret bailout

1 Introduction

History is fraught with financial crises and large-scale government interventions, the latter often involving a highly visible, significant wealth transfer from taxpayers to banks and their creditors.

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For example, Veronesi and Zingales (2010) estimate that, in the initial transactions with nine largest banks under the Trouble Asset Relief Program (TARP), the US government paid $125 billion for assets - preferred stock and warrant - worth $86 - 109 billion. The benefits of such interventions are hard to measure since they depend on an unobservable counterfactual that would have played out in the absence of such interventions. Two recent papers have portrayed a plausible counterfactual in the form of market freeze and provided theoretical arguments for when government interventions may improve welfare (Philippon and Skreta, 2012; Tirole, 2012). The essence of the argument is that the government can jump-start the market when severe adverse selection leads to market freeze. By cleaning up bad assets or ‘dregs skimming’ through public bailouts, the government can improve market confidence, thereby galvanizing transactions in healthier assets. But this argument is based on a one-shot game and, therefore, cannot address the dynamic impact of public bailouts. The dynamic consideration is important since the costs and benefits of bailout depend on who participate and who hold out. Those that participate in the program may signal the market of their vulnerability and hence may experience higher borrowing costs following the bailout. This may warrant compensating those that participate in the bailout at the terms much more favorable than those in the static setting. As a result, the costs of public bailout can be much larger when such reputational concerns are taken into account.

Several observations from the TARP suggest that the fear of stigma can be significant. First, the transparency provision in the TARP required all transactions be publicly announced within two days of execution, thereby putting the information on TARP participants in public domain. Second, some firms were reluctant to participate in the program with a view to not being stigmatized. For example, although GM and Chrysler received the rescue loans under the Auto Industry Program in the TARP, Ford refused it with a view to ‘legitimately portraying itself as the healthiest of Detroit’s automakers’ (“A risk for Ford in shunning bailout, and possibly a reward”, The New York Times, December 19, 2008). Third, there was an element of forced participation during the early stage of the program. At the now-famous meeting held on October 13, 2008, Henry Paulson, then Secretary of the Treasury, compelled the CEOs of nine largest banks to be the initial participants in the TARP (“Eight days: the battle to save the American

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1The Congressional Budget Office (2012) estimates the overall cost of the TARP at around $32 billion, the largest part of which stems from assistance to AIG and the automotive industry while capital injections to financial institutions are estimated to have yielded a net gain.

2Such reluctance to receive government offers of recapitalization was also noted during the Japanese banking crisis of the 1990s (Corbett and Mitchell, 2000; Hoshi and Kashyap, 2010), with which the subprime mortgage crisis in the US shares a lot in common. The fear of stigma also underlies some US banks’ shunning of the Federal Reserve’s discount window facility (Persistiani, 1998; Furfine, 2001; Armantier et al., 2011; Ennis and Weinberg, 2013).
financial system”, *The New Yorker*, September 21, 2009). Fourth, participants in the TARP were eager to exit the program early, often naming stigma as the main reason. For instance, Signature Bank of New York was one of the first to repay its TARP debt of $120 million. Its chairman, Scott A. Shay, said, “We don’t want to be touched by the stigma attached to firms that had taken money.” ("Four small banks are the first to pay back TARP funds”, *The New York Times*, April 1, 2009). It is also well known that Jamie Dimon, CEO of JP Morgan Chase, wanted to exit the TARP to avoid the stigma (“Dimon says he’s eager to repay ‘Scarlet Letter’ TARP”, *Bloomberg*, April 16, 2010).³

In view of the above observations, the aim of this paper is to examine how stigma affects the efficacy of government bailout. Our model is a two-period extension of Tirole (2012) in the most parsimonious way. There is a continuum of firms, each endowed with one unit of asset in each period. For each firm, the quality of asset in both periods is identical, which is the firm’s private information. In each period, an investment opportunity with positive NPV arrives for each firm. But firms are liquidity-constrained and their projects are unpledgeable so that they need to sell their assets to finance the investment. Firms’ first-period actions - whether they sell their assets, to whom, and at what terms - are observed publicly. Tirole’s main insight is that, when the market for asset trade freezes due to severe adverse selection, government purchase of low-quality assets at favorable terms, hence a bailout, can rejuvenate the market by improving the market’s belief on the quality of remaining assets. But in our two-period model, the market’s belief is updated not only on the cross section of firms within each period but also across the two periods since the firm’s first-period action is observed publicly. When the financing need arises again in the second period, the market’s offer is based on its revised belief.

In this setup, stigma can be defined as an adverse effect on the market’s second-period offer to a firm when it took a certain action rather than the alternatives in the first period. In the absence of government intervention, stigma can be attached to those firms that choose to sell in the first period if the firms with low-quality assets are more likely to sell. We call this the early sales stigma. The adverse effect of the early sales stigma is measured as the difference in the market’s second-period offers to those who sold in the first period and those who did not. Compared to the one-shot game, firms are more likely to delay their asset sales in the first period for fear of the early sales stigma, which renders market freeze more likely. This further justifies the case for government bailouts in the first period. Government bailouts introduce another type of stigma, however. Granted that firms with low-quality assets participate in the bailout program in the first period, the market’s offer in the second period would be strictly lower for

³Of course, the fear of stigma is not the only reason for an early exit. Wilson and Wu (2012) find that early exit by banks is also related to CEO pay, bank size, capital, and other financial conditions.
those who participate in the bailout than those who held out. This difference in the market’s second-period offers is the adverse effect of stigma from accepting the bailout offer in the first period. We call this the bailout stigma.\(^4\) It affects the firm’s first-period participation decision, hence how likely the bailout program is successful in rejuvenating the market.

An immediate implication of stigma is that even troubled firms may be reluctant to accept the government’s offer for fear of its adverse effect in the second period. A flip side is the reputational gain for those that refuse to accept the bailout. An important way in which the bailout helps the firms is by creating this opportunity to boost reputation by refusing to accept the bailout offer. In fact this is a very important lesson that can be learned from our analysis of bailout in a dynamic context, which has not been well appreciated in the literature or policy analysis. But for such reputation building to be possible, there must be some firms accepting the bailout offer. This can increase the cost of bailout beyond what can be effective in the static game. To attract participation, the government will be compelled to offer a very attractive term, often strictly more favorable than the terms prevailing in the market. We summarize below our main findings.

Without government intervention, market freeze is more likely in the first period compared to the one-shot game because of the early sales stigma. Namely, the set of firms choosing to sell their assets in the first period is a subset of those that sell in the one-shot game. Thus the dynamic adverse selection renders government intervention a further rationale compared to the static game. With public bailout, firms now have three alternatives in the first period: they can accept the government’s offer; they can sell assets to the market; or they hold on to their assets. Due to the endogeneity of belief, there are multiple equilibria, which can be largely grouped into three types. First, if the bailout stigma is expected to be severe, firms with high-quality assets refuse to accept a bailout, which further aggravates the stigma. As a result, the bailout offer, even if strictly better than what is offered in the market, need not have any effect in that no firms accept the government’s offer. Thus bailout has no effect in this case even though it can rejuvenate the market in the static setting. Second, bailouts can crowd out the market in the first period and could result in strictly lower welfare. The multiplicity of equilibria of this type implies that the worst equilibrium is discontinuous in the bailout term, suggesting that there is a minimum term strictly higher than the market’s offer that the government will be compelled to offer to rejuvenate the market. Third, even when the market is rejuvenated, the outcome may not be desirable in the sense that there is another equilibrium with no market rejuvenation that is more desirable from welfare perspectives. Such equilibria are possible when the bailout

\(^4\)Throughout the paper, we use the term ‘stigma’ to refer to both types. But the exact reference should be clear from the context.
stigma is so severe that the immediate market rejuvenation becomes too costly. Thus the dregs skimming role of bailout emphasized by Tirole takes a very different nuance, and dregs skimming may not even happen in the dynamic context.

Given that the adverse effect of the bailout stigma stems from the transparency of the bailout program, we next discuss whether secrecy can mitigate the bailout stigma and hence encourage participation. Specifically, we consider the case where the identity of participants in the government program is not revealed to the market.\footnote{One may question whether secret bailouts of this kind can be implemented without political influence or cronyism. Legislation is one way to tackle such problems. For example, a special act such as the Emergency Economic Stabilization Act can explicitly incorporate a clause that guarantees the disclosure of information at the end of the program and criminal liabilities for those who are found to have been involved in any wrongdoing.} In this case, the market observes only those that sell assets to the market in the first period. Thus secrecy pools together those accepting the bailout and those holding on to their assets in the first period. It can thus encourage participation in the bailout as well as sales to the market. The latter is in part accomplished by ‘punishing’ those that do not sell their assets to the market since they will be then pooled with those participating in the bailout. The effect is ambiguous, however, since it undermines the important channel of providing the firms with the opportunity to boost their reputation by refusing to accept the bailout, which is now weakened due to the pooling. While the precise effect of secrecy depends on the different types of equilibria as in the case under transparency, our general finding is that secret bailouts lead to more participation in the first period but result in less asset trade in the second period.

Finally we discuss the welfare implications of various bailout policies and the design of optimal bailout policy. We cast the problem in the mechanism design framework. By focusing on deterministic mechanisms in which the government intervenes only in the first date, we show that the optimal bailout mechanism has a cutoff structure: firms with low-quality assets sell in both dates; those with intermediate-quality assets sell only in the first date; and those with high-quality assets do not sell in either dates. Moreover the optimal policy is implemented by a secret bailout that does not rejuvenate the market immediately. This is because, when the market is not rejuvenated immediately, secrecy eliminates the adverse effect of bailout stigma while holdout firms with high-quality assets are less adversely affected than when some firms choose to sell to the market in date 1. In addition, we can Pareto-rank different types of equilibria that arise under different disclosure rules. Specifically, we show that secret bailouts dominate transparent bailouts when the market is not immediately rejuvenated but the dominance relation is reversed when the market is immediately rejuvenated.

The rest of the paper is organized as follows. Section 2 contains a review of the related
studies. In Section 3, we briefly reproduce Tirole’s results in our benchmark model. Our two-period model is analyzed in Section 4 without government intervention, and in Section 5 with government intervention. Section 6 discusses the effect of bailouts under secrecy. Section 7 provided the analysis of optimal policy design while Section 8 concludes the paper. All proofs are relegated to the appendix.

2 Related Literature

The broad theme of this paper is related to an extensive literature on the benefits and costs of government intervention in distressed banks. But relatively fewer papers study the optimal form of public bailout. In a model with adverse selection, Philippon and Skreta (2012) show that optimal interventions involve use of debt instruments. With additional moral hazard but limits on pledgeable income, Tirole (2012) justifies asset purchases. When there is debt overhang due to lack of capital, Philippon and Schnabl (2013) find that optimal interventions take the form of capital injection in exchange for preferred stock and warrants. Since these studies rely on static models, however, they cannot address the reputational concern in accepting a government bailout, which is the main focus of this paper.

Although not directly related to government bailouts, several studies document evidence of banks’ reluctance to borrow from the Federal Reserve. Peristiani (1998) provides early evidence that banks were reluctant to borrow from the Federal Reserve’s discount window even at a rate below the Federal Reserve target rate, and the reluctance grew when overall health of the banking sector deteriorated. Furfine (2001) finds similar evidence from the Federal Reserve’s Special Lending Facility that operated during 1999-2000. Such reluctance is interpreted to be due to the fear of stigma. Armantier et al. (2011) provide more recent evidence from the 2007-2008 financial crisis utilizing the Federal Reserve’s Term Auction Facility bid data and estimate the cost of stigma and its effect. Defining the bid over the discount window rate as the discount window stigma, they find the average stigma of 0.37 and that the stigma is more likely for banks with greater funding needs and under more volatile market conditions. They also report the real effects of stigma in the form of higher interbank borrowing rates and lower stock prices.

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6The primary rationale for intervention is to prevent contagion of bank runs whether it stems from depositor panic (Diamond and Dybvig, 1983), contractual linkages in bank lending (Allen and Gale, 2000), or aggregate liquidity shortages (Diamond and Rajan, 2005). The costs of anticipated bailouts due to the time-inconsistency of policy are discussed, among others, in Stern and Feldman (2004).

7During the US subprime crisis, the EESA initially granted the Secretary of the Treasury authority to purchase or insure troubled assets owned by financial institutions. But the Capital Purchase Program under the TARP switched to capital injection against preferred stock and warrants.
following the visit to the discount window. These empirical findings are formalized in Ennis and Weinberg (2013), who construct a model where illiquid banks hold assets with long maturity but face short-term liquidity needs that can be met through borrowing from either liquid banks (interbank lending) or the central bank’s discount window. When the loan is due, repayment can be made by selling assets in the competitive market. They identify conditions for an equilibrium where banks with high quality assets use interbank lending and those with low quality assets use the discount window. Such signals are reflected in the subsequent pricing of their assets. Our paper is also concerned with stigma but the channels of stigma formation and policy-related issues such as different disclosure rules are quite distinct.

Our paper is in the same vein as the literature on dynamic adverse selection that asks how dynamic trading in the market for durable goods can mitigate or completely resolve the lemons problem (Inderst and Müller, 2002; Janssen and Roy, 2002; Hendel et al., 2005; Moreno and Wooders, 2010). The key insight from these studies is that dynamic trading generates sorting opportunities, which are not available under the static market setting. With unrestricted trading in secondary markets, Hendel et al. (2005) show that buyers can sort themselves into different quality goods as long as the vintage of a good - the number of times the good has been traded in the past - is observable. In this case, the vintage of a good is a perfect signal of its quality and is used to sort buyers with different valuations into different vintages, resolving the lemons problem. For sellers, the sorting opportunity can be a delay in sales as in Janssen and Roy (2002) in the Walrasian setting, in Inderst and Müller (2002) with costly search, and in Moreno and Wooders (2010) with random matching. Such a delay signals high quality. For example, Janssen and Roy show that the distribution of quality is partitioned in equilibrium such that higher quality goods are traded at later dates at higher prices. But each seller has only one opportunity to trade in these studies. Thus delay is the only available signal and the adverse effect from early sales is irrelevant. Our two-period model offers a richer setting in which early sales, participation in the bailout and refusal to accept the bailout offer all have distinct reputational effects.

Finally, our discussions on bailouts under secrecy are related to the studies on dynamic adverse selection that have a specific focus on the role of information. The main issue here is what type of information becomes available over time and how it affects efficiency in trading. Some recent contributions are Hörner and Vieille (2009), Daley and Green (2012), Fuchs et al.

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8To be precise, one of the key building blocks in Ennis and Weinberg (2013) is a matching friction in interbank lending, which makes illiquid banks with high quality assets also use the discount window if they are not matched with liquid banks. Thus the matching friction prevents full separation in equilibrium.
(2012), and Kim (2012). In a model of bilateral bargaining over an infinite horizon, Hörner and Vieille (2009) show that with public offers (transparency), there is a bargaining impasse: an offer is accepted with a positive probability only in the first round and, if it is rejected, no further offers are made that can be accepted. This is due to the seller’s inability to commit not to solicit future offers. With private offers (secrecy), trade occurs with probability one and welfare is more likely to be higher than under public offers. Fuchs et al. (2012) also obtain the results that show secrecy weakly dominates transparency. This begs a question of whether transparency generally hurts efficiency in the dynamic setting. Kim (2012) takes up this question and shows that the type of information matters. Specifically he shows that a coarser information structure - time-on-the-market - enhances efficiency but a finer information structure - number-of-previous-matches - reduces efficiency compared to the case of secrecy. In Daley and Green (2012), exogenous news about the quality of a seller’s good is revealed over time in addition to the seller’s past behavior. They show that the quality of news has a non-monotonic effect on efficiency in that better quality news can increase or decrease efficiency depending on the market’s initial belief. Once again, each seller has only one trading opportunity in these studies. Thus the reputational effect of information is only one-dimensional. That is, although past rejections can boost reputation, acceptance ends the game. In contrast, the seller has three distinct signalling opportunities in our model, i.e., early sales, acceptance of bailout offer, refusal to accept the bailout offer. Due to the multiple channels of signaling, secrecy dominates transparency only in the case of complete pooling in the first period; otherwise, transparency dominates secrecy.

3 Benchmark Model: Tirole (2012)

We first consider Tirole’s one-shot model as a benchmark. In this model, a continuum of firms seeks to finance a project which requires a fund of $I > 0$ but would yield the return $R$. We assume that the project is socially valuable, so that the “net” return $S = R - I$ is strictly positive. There are investors willing to fund the project competitively. But limited pledgeability of the project inhibits direct financing. Instead, the firm can only fund the project by selling the legacy asset it owns. The value of the asset $\theta$ is privately known to the firm. One interpretation is that the firm owns a bundle of assets some of which are toxic, but exactly how much of it is toxic is unknown to the market. For convenience, we call a firm with a legacy asset $\theta$ a type-$\theta$ firm. Each

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9 Others include dynamic extensions of Spence’s signaling model with public offers (Nödeke and van Damme, 1990), private offers (Swinkels, 1999), and private offers with additional public information such as grade (Kremer and Skrzypacz, 2007).

10 In other words, the gross revenue generated by the project is $S + I$. The current notation turns out to be convenient.
firm’s type distributed from \([0, 1]\) according to a prior distribution \(F\) which has strictly positive density \(f\) in its interior.

Throughout we assume that \(F\) satisfies log-concavity. For some results, we shall assume a slightly stronger condition that \(f\) itself is log-concave. For a later purpose, it is useful to define a truncated conditional expectation \(\phi(a, b) \equiv \mathbb{E}[\theta | a \leq \theta \leq b]\), given a truncation \([a, b]\). Note that \(\phi(a, b)\) is continuous and increasing in \(a\) and \(b\). Assume also \(I \leq \mathbb{E}[\theta]\), so that on average the asset sale should finance the project. This informational asymmetry constitutes the adverse selection problem that will limit the firm’s ability to sell the asset to finance its project. After the nature draws the firm’s type \(\theta\), investors simultaneously offer prices at which the legacy asset the firm holds is bought, if they so choose. If any offer is made, the firm decides whether to accept an offer.

Without any government intervention, this market exhibits the standard lemons problem. Firms with lower types have a stronger incentive to sell than those with higher types, so an equilibrium is characterized by a cutoff type \(\theta_0\) such that the firm sells to the market if and only if \(\theta < \theta_0\). Investor competition means that if there is any trade, the price must equal the average value of the types \(\mathbb{E}[\theta | \theta < \theta_0]\) sell to the market. Assuming that the sale will generate enough revenue to fund the project (a condition yet to be checked), a firm will earn

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R + \mathbb{E}[\theta | \theta < \theta_0] - I = S + \mathbb{E}[\theta | \theta < \theta_0] = S + \phi(0, \theta_0)
\]

from the sale. The cutoff type must find this payoff to be no less than \(\theta_0\), the payoff from holding the asset to its maturity, and must be indifferent if \(\theta_0 < 1\). Log concavity of \(F\) implies that there is a unique \(\theta_0\) satisfying this condition. While the dependence of \(\theta_0\) on \(S\) will be often suppressed, when it is relevant, we shall write \(\theta_0(S)\). Since \(\theta_0(S)\) is increasing in \(S\), we can define \(\overline{S}_0 := 1 - \mathbb{E}[\theta]\) and \(\underline{S}_0 < \overline{S}_0\) such that \(\mathbb{E}[\theta | \theta < \theta_0(\overline{S}_0)] = I\). The following characterization then holds: the market freezes completely if \(S < \underline{S}_0\); the market freezes partially with \(\theta_0(S) \in (0, 1)\), if \(S \in (\underline{S}_0, \overline{S}_0)\), and the market is fully active if \(S > \overline{S}_0\). In the first two cases, adverse selection is severe enough to inhibit financing of the efficient project. The central insight of Tirole (2012) is that the government may play a crucial role of remedying the adverse selection and the associated market failure.

Suppose the government offers to purchase the legacy asset at some price \(p_g\), before the private market opens. If \(p_g \geq 1 - S\), then all types of the firm will sell to the government. Suppose offer is not as favorable but is still more favorable than the market; i.e., \(p_g \in (\max\{I, p_0\}, 1 - S)\),

\[11\] Log concavity of \(F\) implies that \(\partial \phi(a, b)/\partial a, \partial \phi(a, b)/\partial b \leq 1\), a property we shall use throughout (Bagnoli and Bergstrom, 2005)
where $p_0 := \phi(0, \theta_0(S))$ is the market clearing price of the asset without government intervention if $\theta_0(S) > 0$ (i.e., if the asset trades), or else $p_0 := 0$. Such an intervention will not just subsidize funding for the investment, but it could also boost the capacity of the market to fund the project. In particular, unless the government offer is so good to attract all types, the market must always be active.\(^{12}\)

While the types of the firm selling to the government or the private market may be indeterminate, Tirole proposes a mild and reasonable refinement whereby the private market may shut down with a vanishingly small probability. This refinement gives rise to a sorting of types such that the government collects the worst types of the asset, and the private market buys higher quality assets.\(^{13}\) Hence, the equilibrium is characterized by two cutoffs $0 \leq \theta_g \leq \hat{\theta}_0 \leq 1$ such that the types $\theta < \theta_g$ sell to the government, the types $\theta \in (\theta_g, \hat{\theta}_0)$ sell to the private market, and the types $\theta > \hat{\theta}_0$ do not sell at all.

In equilibrium, if $\theta_g < \hat{\theta}_0$, then the investors must break even, so the asset will sell at price $p_1 = \phi(\theta_g, \hat{\theta}_0)$. Further, $\hat{\theta}_0$ must be the highest type willing to sell at that price. That is, $\hat{\theta}_0 = \xi(\theta_g)$, where $\xi(\theta) := \inf\{\theta' | \theta' \leq \phi(\theta, \theta') + S\}$. If both bailout and private market are active in equilibrium, we must have $p_1 = p_g$, or else the lower-price offer will not be accepted. It follows that

$$p_g = \phi(\theta_g, \xi(\theta_g)).$$

The critical cutoff type $\hat{\theta}_g$ satisfying (4) is well defined.\(^{14}\) Moreover, $\hat{\theta}_g > 0$ since $p_g > \max\{I, p_0\}$.\(^{15}\) This means that $\xi(\theta_g) > \xi(0) = \theta_0(S)$, so the bailout supports more trade and financing.

**Proposition 1.** (Tirole, 2012) If the government offers to purchase the legacy asset at price $p_g \in (\max\{I, p_0\}, 1 - S)$, then types $\theta < \hat{\theta}_g$ sell to the government and types $\theta \in (\hat{\theta}_g, \xi(\hat{\theta}_g))$ sell to the market at the same price $p = p_g$, where $\hat{\theta}_g$ is given by (4). Any offer $p_g > \max\{I, p_0\}$ increases the volume of trade and financing whenever $S < \bar{S}_0$.

Suppose $F$ is uniform on $[0, 1]$, and $I \in (0, \frac{1}{2})$. Then, $\underline{S}_0 = I$ and $\bar{S}_0 = 1/2$. Hence,\(^{12}\) If the market were not active, the firm will only sell to the government if and only if $\theta \leq \theta_g$ for some $\theta_g \in (0, 1)$.

It is then optimal for investors to offer price $\theta_g + \varepsilon$, for some small $\varepsilon > 0$, and attract types $[\theta_g, S + \theta_g + \varepsilon]$ which clearly yield strictly positive profit if $\varepsilon$ is sufficiently small.

\(^{13}\) Formally, if the private market collapses with probability $\varepsilon > 0$, a firm will sell to the market (as opposed to the government) if and only if

$$(1 - \varepsilon)(p + S) + \varepsilon \theta \geq p_g + S \iff \theta \geq [p_g + S - (1 - \varepsilon)(p + S)]/\varepsilon,$$

where $p$ is the price that prevails at the market.

\(^{14}\) This is the consequence of the log concavity assumption... Elaborate....

\(^{15}\) If $\theta_g = 0$, then since $\xi(0) = \theta_0(S)$, $\phi(\theta_g, \xi(\theta_g)) = \phi(0, \theta_0(S)) = p_0$, so we have a contradiction.
without government intervention, the market freezes if \( S < I \), the market is partially active with types \( \theta < 2S = \theta_0(S) \) selling at price \( p = S \). If \( S \geq \frac{1}{2} \), the market is fully active. A government purchase of the legacy asset at \( p_g \geq \min\{\max\{I, S\}, 1 - S\} \) leads to the firm with types \( \theta < p_g - S \) selling to the government (so the government incurs losses from the sale) and the firm with type \( \theta \in (p_g - S, p_g + S) \) selling to the market. Since \( p_g + S > 2S = \theta_0(S) \), the intervention improves trade and financing. As illustrated in the example and formally stated in Proposition 1, even though the government purchases the asset at the same price as the market, it ends up attracting the worst types of the asset. Such a “dregs skimming” role played by the bailout improves the market perception of the remaining types, thus enabling them to obtain funding from the market to a greater extent than would be possible without the intervention.

One thing that is missing about this benchmark model is any reputation consideration for the firms receiving the bailout. As motivated at length in the introduction, a decision to receive a government bailout signals possible weakness of the firm’s balance sheet, which may engender significant cost in the future financing. We therefore turn to a dynamic model of adverse selection and government bailout.

4 A Model of Dynamic Adverse Selection

We consider a two period extension of Tirole’s adverse selection model. The firm and competitive investors live for two dates \( t = 1, 2 \). The firm owns two units of legacy assets of the same quality, and it can sell a unit at each date to fund a project that arrives in each of the two dates. As before, each project requires \( I \) for investment and returns \( S \) net of investment cost. The firm can sell only one unit of its asset at a time,\(^{16}\) and the return from \( t = 1 \) investment project is arrives after \( t = 2 \) and cannot be used to fund the investment project arriving in date \( t = 2 \). While this model is stylized, it introduces reputational considerations facing the firm in a way that is simple and allows for clear comparison with Tirole (2012). As will be seen, the main feature of this model is the inference that the market makes on the firm’s type from its behavior in date 1. Obviously, the inference is irrelevant in the one-shot model, but it now clearly affects the terms of trade in date 2. Ultimately, our main focus will be how this reputational concern feeds back into the firm’s decision to accept government bailout in \( t = 1 \), but reputational concern also affects the firm’s market behavior in \( t = 1 \) even without the government intervention.

We thus begin by considering the game without intervention. The game proceeds formally

\(^{16}\) Any equilibrium that involve firms selling more than one unit at the first date can be implemented as an equilibrium in which firms sell one unit in each date. It can be supported if the market observes the number of units each firms sells, since it can attach an unfavorable belief for those selling two units.
as follows. In $t = 1$, investors make offers to purchase a unit of the asset from the firm in a Bertrand fashion. Each firm then decides whether to accept an offer. The term of the trade (but nothing else) is observed by all players. In case of a sale at price $p_1 \geq I$, the selling firm invests in the project. In $t = 2$, investors offer to purchase the second asset, and again the firm decides on the offer and invest in the second project in case of receiving at least $I$ from the sale. We study perfect Bayesian equilibria of this game in which the firm discount $t = 2$ payoff by an arbitrarily small amount. Formally, we focus on an equilibrium of the game without discounting that is nevertheless obtained as a limit of the equilibria of the games with discounting as the discount factor approaches to 1. The purpose of this refinement is, in a way much like Tirole (2012), to ensure a single crossing property necessary for a sharp prediction. Throughout, we invoke this refinement along with the one in which the market collapses with arbitrarily small probability. We call a Perfect Bayesian equilibrium with these two refinements simply an “equilibrium.” As we now characterize, any such equilibrium has a cutoff structure:

**Lemma 1.** In equilibrium, there exist two cutoffs $\theta_1 \leq \theta_2$ such that the firm sells its asset in $t = 1$ if and only if $\theta < \theta_1$ and sells its asset in $t = 2$ if and only if $\theta < \theta_2$. If $\theta_1 \in (0, 1)$, then $\theta_2 > \theta_1$.

A typical equilibrium looks as described below.

Lemma 1 states that types below some cutoff $\theta_1$ sell in $t = 1$, and those above the cutoff do not. It also states that all those that sell in $t = 1$ also sell in $t = 2$, and that if $\theta_1 < 1$, then a positive measure of firms with $\theta$’s above $\theta_1$ (those firms that did not sell in $t = 1$) do sell in $t = 2$. Since the market forms a correct belief about the types of assets sold in each date, and since investors compete in Bertrand fashion, the price of the asset sold in each period must be equal to the average quality of the asset sold in each date for a given observable history. In particular, the price of the asset sold in date $t = 1$ must satisfy

$$p_1 = \mathbb{E}[\theta | \theta \leq \theta_1] = \phi(0, \theta_1).$$
At date 2, the investors offer prices that depend on date \( t = 1 \) behavior of the firm. Let \( p_1^1 \) and \( p_0^0 \) denote respectively the prices that are offered to the firms that sold at \( t=1 \) and those that did not. As noted, since all the firms that sell in \( t = 1 \) also sell in \( t = 2 \), their price must also equal \( p_1^1 = p_1 = \phi(0, \theta_1) \). Finally, the firms that do not sell in \( t=1 \) are offered price \( p_0^0 = \phi(\theta_1, \theta_2) \). As before, the cutoff \( \theta_2 \) must equal \( \xi(\theta_1) \), so we must have \( p_0^0 = \phi(\theta_1, \xi(\theta_1)) \).

Observe that \( p_0^0 > p_1 \). That is, those firms that sell their assets early suffer from a “stigma.” And their stigma continues in the second period, being subject to the same low price. By contrast, those types that refuse to sell in \( t = 1 \) boost their reputation and enjoy a high price.

Given the characterization so far, the equilibrium is pinned down by the cutoff type \( \theta_1 \). This is done by studying its incentive. When a type \( \theta \)-firm sells its asset in \( t = 1 \), it earns payoff of

\[
\Psi(\theta; S) = p_1 + S - \theta - (p_0^0 - p_1^1) \\
= \phi(0, \theta) + S - \theta - (\phi(\theta, \xi(\theta)) - \phi(0, \theta)).
\]

The first term is the one-period payoff from selling the asset, and this term captures the standard adverse selection problem; i.e., the price is given by the mean quality below the cutoff. The second term is new here and captures the reputational loss from selling in \( t = 1 \); i.e., the early sale results in the opportunity to sell at a higher price enjoyed by the firm who refuse to sell in \( t = 1 \).

The cutoff \( \theta_1 \) must be determined as follows. If \( \theta_1 \in (0,1) \) in equilibrium, then the cutoff type must be indifferent to selling early, so we must have \( \Psi(\theta_1; S) = 0 \). Equilibria with non-interior cutoffs need not be characterized by \( \Psi(\theta; S) \), since the payoff comparison depends on out-of-equilibrium beliefs. Nevertheless, for ease of exposition, we focus on equilibria such that \( \theta_1 = 0 \) implies \( \Psi(0; S) \leq 0 \) and \( \theta_1 = 1 \) implies \( \Psi(1; S) \geq 0 \). One can think of this as a restriction on out-of-equilibrium beliefs that ensures continuity of the equilibria in \( S \). Our primary motivation for invoking this restriction is conciseness of exposition. As will be seen, we shall be more thorough in our characterization in the case of bailout. We will comment on these extra equilibria, whenever they become relevant for interpretation of the effects of bailout in our model.

To further simplify the analysis and exposition, we assume:

**Assumption 1.** (i) \( \Psi(\theta; S) \) is strictly decreasing in \( \theta \) for each \( S > 0 \); (ii) If \( \Psi(0; S) \geq 0 \), then \( \Psi(0; S') > 0 \) for \( S' > S \).

The first assumption, broadly satisfied by the standard distribution functions, ensures that
an interior cutoff is uniquely characterized. The second is also satisfied by standard distributions, and facilitates a simple classification of equilibria with respect to the value of $S$. Equilibrium characterization without these assumptions are more cumbersome without adding much insight.\footnote{In fact, many distributions widely accepted in economic analysis satisfy these assumptions, such as truncated normal distributions on the interval $[0,1]$, beta distributions with various values of the shape parameters, and the uniform distribution on $[0,1]$.}

We are now in a position to provide an equilibrium characterization.

**Proposition 2.**  
1. We have $\theta_1 \leq \theta_0 \leq \theta_2$, with strict inequalities if the cutoff in the one shot model satisfies $\theta_0 \in (0,1)$. That is, the equilibrium trade is lower in $t = 1$ and higher in $t = 2$ than it is in the static model.

2. There exist $\mathcal{S}^* > \mathcal{S}_0$ and $\overline{\mathcal{S}}^* > \max\{\mathcal{S}^*, \mathcal{S}_0\}$ such that $t = 1$ market is fully active if $S \geq \overline{\mathcal{S}}^*$ but suffers from partial freeze if $S \in (\mathcal{S}^*, \overline{\mathcal{S}}^*)$ and full freeze if $S < \mathcal{S}_0$.

The differences in the results relative to Proposition 1 are explained by the reputational concern present in our dynamic adverse selection model. Selling in $t = 1$ not only suffers from the standard adverse selection but also from dynamic reputational loss. This stigma explains the reduced trade in $t = 1$. The early sale stigma also explains the increased range of $S$’s for which the market $t = 1$ freezes, as reported Proposition 2-2. Meanwhile, the flip side of the reputational loss from selling is the reputational gain the firm would achieve by “refusing to sell” in $t = 1$. This reputation gain leads to a better term and thus mitigates adverse selection in $t = 2$.

The equilibrium of the two-period model can be again illustrated via our uniform distribution example with $I \in (0,1/2)$. In this example, we get $\mathcal{S}^* = I + \frac{1}{2}$ and $\overline{\mathcal{S}}^* = 1$. Observe $\mathcal{S}^* > \mathcal{S}_0 = I$ and $\overline{\mathcal{S}}^* > \overline{\mathcal{S}}_0 = 1/2$. Further, for $S \in [I + \frac{1}{2}, 1)$, then $\theta_0 = 2S \in (\theta_1, \theta_2)$, which also confirms that $\theta_0 = 2S \in (\theta_1, \theta_2)$. Figure 1 compares the size of the trading volume in date 1 in the static and dynamic models against $S$. Clearly, for $S \leq \overline{\mathcal{S}}^*$, there is a potential role for government bailout. We now turn to this issue.

## 5 Dynamic Adverse Selection and Bailout Stigma

In this section, we study government bailout of the firms via purchase of their assets. Specifically, the government offers to buy one unit of the legacy asset at price $p_g > 0$ from each firm before the private market is open in $t = 1$. After the government purchase offer is made and a firm decides on the offer, the investors may offer to purchase the asset as well. If a firm sells its asset at price greater than or equal to $I$, it finances the project and collects the surplus of $S$. Whether
the firm sold its asset in the first period, to which party it sold, and at what terms are all publicly observed.

Date 2 game proceeds as in the previous section with the investors making offers to purchase the remaining asset, and the firm deciding on that offer. As before, the primary purpose of this second period sale is to create a reputational consideration for the firm; the terms of the second-period sale depends on the belief formed on the firm based on its first-period action. We assume that the government participates in the bailout only in the first period. This is realistic since governments are reluctant to extend bailouts on the permanent basis. Further, our goal is to study the reputational consequence of taking the government bailout, and this can be studied most effectively when no government bailout is available in the second period.

We first begin by characterizing the structure of equilibrium. To facilitate the analysis, we invoke the assumptions that the sale to private investors is subject to an arbitrarily small probability of collapsing and that the players discount the second period payoffs by an arbitrarily small amount. Formally, we consider a game with \((\varepsilon, \delta)\) such that the sale of an asset to a private investor fails with probability \(\varepsilon\) and the players discount the second period payoffs with factor \(\delta\), and focus on the limit of the (Perfect Bayesian) equilibrium outcome of that game as \(\varepsilon \to 0\) and \(\delta \to 1\). These features ensure single crossing properties and thus allow for sharp characterization of equilibria, which are shown to have the following structure.

**Lemma 2.** There are cutoff types \(0 \leq \theta_g \leq \theta_1 \leq \theta_g^0 \leq 1\) such that in the first period, the types \(\theta \in (0, \theta_g)\) sell to the government, the types \(\theta \in (\theta_g, \theta_1)\) sell to the market, the types \(\theta \in (\theta_1, \theta_g^0)\)
sell to the government, and the types \( \theta \in (\theta^0_g, 1) \) do not sell. If \( \theta_g < \theta_1 \), then \( \theta_g > 0 \), and all types in \([0, \theta_1)\) sell to the market in the second period, but the types \( \theta \in (\theta_1, \theta^0_g) \) do not sell its asset in the second period. If \( \theta^0_g < 1 \), then there exists \( \theta_2 \in (\theta^0_g, 1] \) such that all types in \((\theta^0_g, \theta_2)\) sell to the market in the second period.

The lemma suggests that the equilibria are of the following three types: (i) no effect equilibrium in which \( \theta_g = 0 \); (ii) no immediate market revival in which \( \theta_g = \theta_1 \) and (iii) immediate market revival in which \( \theta_g < \theta_1 \). Here we shall describe the qualitative features of the equilibria and the condition for their occurrence in the uniform distribution case with \( S \in [I + \frac{1}{2}, 1) \) and \( I \leq 1/2 \). The characterization of the equilibria for the general case will be provided in the appendix.

5.1 “No Effect” Equilibria

It is possible for government bailout to have no effect in equilibrium. Not surprisingly, the outcome of such an equilibrium is precisely the same as that in Proposition 2. Clearly, the no effect equilibrium arises if the government offer is sufficiently low, for instance if \( p_g \) is less than \( p^*_1 \) the date 1 market price without government intervention. More interestingly, the no effect equilibrium may arise even when the government offers a strictly better term than the market. The reason for this is the bailout stigma.

To see how bailout stigma can support such an equilibrium, recall the equilibrium without government intervention, and suppose that the equilibrium involved active date 1 market with price \( p^*_1 \) and date 2 market with price \( p^*_2 \) for those refusing to sell in date 2. Since the latter types receive a favorable perception, \( p^*_2 = \mathbb{E}[\theta | \theta_1 \leq \theta \leq \theta_2] > \mathbb{E}[\theta | \theta \leq \theta_1] = p^*_1 \).

Suppose now the government intervenes by offering to purchase in date 1 at price \( p_g \) that is strictly greater than \( p^*_1 \) but less than \( p^*_2 \). There is an equilibrium in which such an offer is rejected by all types of the firm. To see this, consider any type \( \theta \). If it rejects the government offer, as suggested by the equilibrium strategy, then its payoff is no less than

\[ \theta + p^*_2 + S \]  

since the type has an option not to sell to the market and receives the price of \( p^*_2 \) for its asset at date 2.

Suppose next the firm deviates and accepts the bailout offer \( p_g \), but as a consequence is subject to the extreme belief of having worst asset \( \theta = 0 \). Then, even though the firm will
finance its project at \( t = 1 \), due to the stigma, the firm will never profitably sell its asset in date 2. Hence, the payoff for the firm from accepting the bailout is

\[ \theta + p_g + S. \] (4)

Comparing (4) with (3) reveals that the firm will never accept the bailout under the extreme stigma.

**Proposition 3.** If the bailout term has \( p_g \leq p_2^* \) (the date 2 price for the date 1 holdouts), then there is an equilibrium in which the government bailout is never accepted and thus has no effect.

### 5.2 Equilibria without Immediate Market Revival

Of particular interest is an equilibrium in which the government bailout offer is accepted by some types of the firm. Such an equilibrium exists for any \( p_g \geq I \) in our model as well, but such an equilibrium need not involve an active private market in date 1. In Tirole’s one-shot model, any equilibrium with an active bailout also sees an active private market. By contrast, in the current model there always exists an equilibrium in which the government bailout is not accompanied by rejuvenation of private market in date 1. Such an equilibrium may arise even when the private market is active absent government intervention. In that case, the government bailout “crowds out” private market in date 1. Depending on the types of the firm accepting government rescue and its date 2 consequence, there are three different types of equilibria, labeled G1, G2 and G3, which are depicted below. These figures display the types of the firm selling to the government (green/light shaded) and to the private market (blue/dark-shaded) in date \( t = 1 \) (first line) and date \( t = 2 \) (second line).

In these equilibria, the private market is not active (or is not rejuvenated) in date 1, due to the (out-of-equilibrium) belief that any offer private investors may make would attract the types of firm that make them unable to break even. Such a belief need not be extreme. Since the government typically loses money on the bailout, a purchase would not be profitable for private investors even if slightly better types of firm than those accepting the government bailout are attracted to their offers. Indeed, our formal analysis focuses on out-of-equilibrium beliefs in which the private investors attract strictly better types than the government bailout whenever their offers are (at least weakly) more favorable than the government bailout.

The three different types are differentiated by the degree of bailout stigma—i.e., the degree to which the firm types receiving the bailout suffers from an unfavorable belief formed on the remaining asset it holds. The belief directly impacts the terms of its sale in date 2.
Figure 2 – Equilibrium Actions without Market Revival
Importantly, this effect feeds back into the date 1 incentive by the firm to participate in the bailout: the worsened terms of asset sale in date 2 caused by the stigma discourage participation in bailout by the firm with good assets, which aggravates the stigma further. In G1 equilibrium, the bailout stigma is so severe that the date \( t = 2 \) market collapses for the types accepting the bailout. This happens if the average type of the bailed-out firm \( E[\theta | \theta \leq \theta_g] \) is less than \( I \). Obviously, the bailout firm will not be able to finance in date 2, and foregoes the surplus of \( S \).

In G2 equilibrium, the types of the bailed-out firm are favorable enough to support date 2 asset sale and the financing of the project. G3 equilibrium is similar to G2, except that not all types of the firm accepting bailout choose to sell in the date 2, due to the standard adverse selection problem.

Which types of equilibria arise depend on the terms of the government bailout \( p_g \), for G2 and G3 arise for higher range of \( p_g \) than G1, but the presence of the feedback loop noted implies that the endogenous belief plays an important role as well for the selection of an equilibrium. To appreciate this point, it is worth studying the incentive tradeoffs a firm faces in the G1 and G2 equilibria. It is clear G2 is never a possibility unless \( S > S_0 \), the minimal surplus for the market to form in a one-shot model. So we assume this. We study the incentive a marginal type \( \theta_g \) of firm faces in each type of equilibrium. In each equilibrium, the marginal type is indifferent between accepting the bailout at \( p_g \) and not accepting it. In the latter case, the firm realizes \( \theta_g \) in date 1 but gets to improve its reputation and is able to improve its sale price for the remaining asset to \( E[\theta | \theta_g < \theta < \xi(\theta_g)] \) and can further finance its date 2 project, so its payoff is

\[
\Pi_{\text{no bailout}}(\theta_g) := \theta_g + E[\theta | \theta_g < \theta < \xi(\theta_g)] + S,
\]

in each of these two equilibria. The payoff the firm receives from accepting the bailout differs, depending on the severity of the bailout stigma. If the stigma is severe enough for the date 2 market to collapse for the bailout firm, then

\[
\Pi_{\text{bailout}}^{G1}(\theta_g) := p_g + S + \theta_g.
\]

By contrast, suppose the belief is that a market will develop, meaning that \( p_2^g = E[\theta | \theta < \theta_g] \geq I \). Then, the payoff from accepting the bailout is

\[
\Pi_{\text{bailout}}^{G2}(\theta_g) := p_g + S + E[\theta | \theta < \theta_g] + S.
\]

Suppose we have G1 equilibrium such that \( \theta_g^{G1} \) that satisfies \( \Pi_{\text{bailout}}^{G1}(\theta_g) = \Pi_{\text{no bailout}}(\theta_g) \) and \( E[\theta | \theta < \theta_g^{G1}] < I \), validating the pessimistic prophecy of market collapse. Suppose at the
<table>
<thead>
<tr>
<th>Equilibrium Types</th>
<th>Equilibrium Cutoffs ($\theta_2 = 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta_g$</td>
</tr>
<tr>
<td>NE $p_g &lt; S$</td>
<td>0</td>
</tr>
<tr>
<td>G1 $p_g \in \left[\frac{1}{2}, I + \frac{1}{2}\right]$</td>
<td>$2p_g - 1$</td>
</tr>
<tr>
<td>G2 $p_g \in [I, \frac{3}{2} - S]$</td>
<td>$p_g + S - \frac{1}{2}$</td>
</tr>
<tr>
<td>G3 No Existence</td>
<td></td>
</tr>
<tr>
<td>M1 $p_g \in \left[5I + 2S - 1, \frac{1}{2}(3 - S)\right]$</td>
<td>$\frac{5}{3}(p_g - (2S - 1))$</td>
</tr>
<tr>
<td></td>
<td>$2p_g - 2$</td>
</tr>
<tr>
<td>M2 $p_g \in \left[\max{S + 2I, \frac{1}{3}(2 + S)}, 1\right]$</td>
<td>$p_g - S$</td>
</tr>
</tbody>
</table>

Table 1 – The marginal types in the equilibria with intervention in the uniform distribution.

The same time that $E[\theta|\theta < \theta_g^G] \approx I$. Then, the given policy $p_g$ can admit G2 equilibrium as well. The reason is that, given $S > S_0$, the optimistic belief about date 2 sale and financing means that the firm faces a discretely higher benefit from accepting the bailout. Formally, when $S > S_0$, $\Pi_{bailout}^G(\theta_g) > \Pi_{bailout}(\theta_g)$, whenever $E[\theta|\theta < \theta_g] \approx I$. Since the payoff from non-acceptance is continuous, this means the new marginal type $\theta_g^{G2}$ satisfying $\Pi_{bailout}^G(\theta_g) = \Pi_{no \, bailout}(\theta_g)$ will satisfy $E[\theta|\theta < \theta_g^G] \geq I$, which now validates the optimistic belief! The intuition is the feedback loop between the stigma and incentive for accepting bailout. This point suggests that multiple equilibria exist for a range of bailout terms $p_g$. As can be seen from Table 1, given $S \in [I + \frac{1}{2}, 1)$, both G1 and G2 arise for $p_g \in [I, I + \frac{1}{2})$.

Does bailout improve the trading of the asset? Bailout stigma reduces the incentive for a firm to participate in bailout and thus reduces its effectiveness in enabling financing. This is certainly true in comparison with the one shot, particularly since bailout need not enlist the private investors to purchase asset from the firm. Not every aspect of the stigma is bad, however. The stigma associated with bailout means an opportunity for the firm to improve its reputation, by “refusing to accept the bailout.” To see this, suppose $S < S^*$, so that without government bailout firm can never sell its asset and finance its project in date 1. In that case, a government bailout at $p_g \geq I$ can lead to a lower tail $[0, \theta_g]$ of the firm accepting the bailout. By “not accepting” the bailout, the firm can signal the value of the asset to be $\theta > \theta_g$. This improves the terms of trade and the chance of financing in $t = 2$. Hence, any such offer $p_g \geq I$ improves the trading of the asset (and the financing by the firm). Rather paradoxically, therefore, reputational loss form some types (bailout takers) means reputational gain for others (the non-takers). From this perspective, bailout stigma by itself does not eliminate the rationale for bailout.

Matters are more complicated if $S > S^*$, however. In that case, a market is active even without the government bailout. Hence, bailout crowds out private market, so the benefit of bailout is unclear. Endogenous belief as well as the selection of equilibrium matters. In
particular, whether G1 or G2/G3 or NE emerges determines the value of bailout. Figure 3 depicts the benefit under alternative equilibria for this case of uniform distribution. The figure depicts the highest type of the firm that sells its asset and thus finances its project in $t = 1$. This marginal type is a sufficient statistic of the volume of the overall trade, since the higher the marginal type in $t = 1$, the higher is the marginal type in date $t = 2$. Details of the alternative equilibria are also described in Table 1. As it shows, an offer at $p_g$ that is higher than the equilibrium price without intervention, may have no effect (in case of NE), deteriorate welfare (in case of G1 equilibrium) and may improve welfare (in case of G2 equilibrium).

The results for the general model are described as follows.

**Proposition 4.** Suppose the government offers to purchase the asset at $p_g \geq \max\{I, p_1^*\}$, where $p_1^*$ is the equilibrium price of the asset without government intervention. (We adopt a convention that $p_1^* \equiv 0$ if there is no equilibrium with active $t = 1$ market, i.e., if $S < S^*$.

1. If $S < S^*$, then the bailout leads to a G1 equilibrium with $\theta_g > 0$ and $\theta_2 > \theta_1$, so the total trade and financing improves with bailout.
2. If $S \geq S^*$, then the bailout admits a G1 equilibrium in which $\theta^G_1 < \theta^*_1$ if $p_g < p^G_1$, for some $p^G_1 > p_1$, and a G2 equilibrium in which $\theta^G_2 > \theta^*_1$ and $\theta_2 > \theta^*_2$. Bailout deteriorates welfare in the former equilibrium and improves welfare in the latter equilibrium.

3. If $S > S_0$, then there exists a range of $p_g$'s for which multiple types of bailout equilibria exist. The welfare-worst equilibrium—selection of an equilibrium that supports the smallest trade—rises discontinuously at some $p_g$. 

The proposition suggests that a government bailout at some attractive term can help overcome the dynamic adverse selection problem, despite the fact that bailout entails stigma and that the market-financing at date 1 may not be revived or even crowded out. The mechanism for the improvement comes in part from the funding made available by the government for the date 1 investment. But also important is the indirect effect; the bailout provides an opportunity for the firm to improve its reputation by refusing to accept the bailout. The improved reputation can enable the funding from the market in date 2 that would otherwise not be available. This paradoxical benefit is often not well-appreciated. Commentators regard widespread reluctance among firms to receive the bailout as a policy failure. Our result shows, however, the policy may be helping even such firms indirectly by improving their reputation. Of course, there will not be any reputation-boosting if all firms refuse government bailout. For the policy to be effective, it is thus important for “some” types to accept the bailout. And this latter—getting some types to accept the bailout—may prove to be costly for the policy maker, for that requires compensating those firms for the stigma they are subject to.

In this regard, the discontinuity of the welfare-worst equilibrium in Proposition 4-3 is particularly relevant for policy makers. Given the multiplicity of equilibria, the policy maker may ensure that the intervention improves (or at least does not worsen) the situation in the worst case scenario. From this perspective, the discontinuous rise in welfare suggests that the government must offer a sufficiently attractive term—a “minimum” bailout term—to be assured of having some positive effect. This minimum term is the price that is attractive even for a firm with a relatively good asset, so that the firm receiving the bailout at that term is not in danger of a severe stigma. In particular, it is strictly higher than the term that would prevail in the market without intervention. The presence of the minimum bailout term seems consistent with the experiences from the recent financial crisis. Even amid liquidity dry-ups, firms were reluctant to accept bailout for fear of reputational loss, and to convince them to accept the bailout governments had to offer bailout terms that are more favorable than are available in the markets.
5.3 Equilibria with Immediate Market Revival and Their Cost

An important role of government bailout highlighted by Tirole (2012) was to activate the private market. As was seen in Section 2, a government bailout takes the most toxic kind of asset out of the system, and thus improves the market perception of the assets and brings confidence to private investors seeking to buy the assets. We argue here this “dregs skimming” role of government bailout can be fundamentally curtailed by the bailout stigma. First, the immediate revival of market worsens the bailout stigma, which raises the cost of the bailout for the government, meaning that the bailout must involve a premium over a market price of asset to be acceptable. Second, the bailout need not take out the worst types of the asset, and weakens its ability to boost market confidence. Most importantly, the stigma price of the market revival is so strong that market revival becomes socially undesirable: namely, for any equilibrium in which the date 1 market is active, welfare can be improved by switching to an equilibrium in which no market is revived (i.e., G2 or G3). To appreciate these points, it is worth studying the equilibria in which the bailout revives a date 1 market.

There may be two different types of such equilibria, the first of which is M1 depicted below. This equilibrium is at first glance similar to that studied by Tirole (2012): The government bailout is accepted by the worst types of the firm, leaving the remaining types to be served by the market at a term better than what they would get otherwise. A closer inspection shows, however, the high price market revival imposes for the government offering the bailout and for the welfare. The fact that the government bailout attracts the worse types than the market means the types accepting the bailout will be subject to a severe stigma in date 2 market. For there to be incentives to accept the bailout, the bailout term $p_g$ by the government must overcompensate the stigma loss. Specifically, if $\theta_g$ is the marginal type of the firm receiving the bailout and $\theta_1$ is the marginal type receiving the market offer, then $p_g$ must be such that

$$p_g + E[\theta|\theta < \theta_g] = 2E[\theta|\theta_g \leq \theta \leq \theta_1],$$

(5)
where the LHS is the payoff a firm will receive by accepting the bailout, and the RHS is payoff a firm will receive by selling to the market, the break-even price it receives on two dates. Unlike Tirole (2012), for the bailout to be acceptable, its term must include the premium over market \( p_g - E[\theta|\theta_g \leq \theta \leq \theta_1] \) that compensates for the stigma loss \( E[\theta|\theta_g \leq \theta \leq \theta_1] - E[\theta|\theta < \theta_g] \). This means that the cost of supporting any given trade is much higher than in the one-shot model.

Next, the extreme stigma associated with bailout may entail an equilibrium in which the firm simply abandons the second-date financing. Since the types of the firm following such a strategy have better types than those selling to the market in date 1, bailout does not skim dregs in such an equilibrium labeled M2. As seen in Figure 5, M2 equilibrium involves a non-continuous set of firm types accepting the bailout.

The extreme bailout stigma associated with market revival raises the cost of bailout itself. In fact, the cost of market revival is so significant as to defeat its purpose—i.e., it becomes socially undesirable. Specifically, for a given bailout term \( p_g \) that supports a market revival equilibrium, there is another equilibrium which does not revive the market immediately but dominates that market revival equilibrium in trade volume. This can be seen in the uniform example. As seen in Figure 3 as well as Table 1, the same bailout term supports more trade when it does not revive market (under G2 equilibrium) than when it revives the market (M2 equilibrium).

This point can be argued more intuitively. Suppose for a heuristic argument the marginal type of the firm selling its asset in date 1 is the same under both equilibria and equal to \( \hat{\theta} \). Such a type must be indifferent between the option of not selling its asset in date 1 (in which case it sells in date 2 at a higher price \( p_2^0 = E[\theta|\theta > \hat{\theta}] \)) and the option of selling either to the market (in case of M1) or to the government (in case of G2). The former option is the same in both equilibrium, given the same marginal type. The latter option is different. In the case of G2 equilibrium, it is \( p_g + E[\theta|\theta \leq \hat{\theta}] \) but in the case of M1 equilibrium, (5) means that the payoff is \( p_g + E[\theta|\theta \leq \theta_g^{M1}] \). Since the bailout stigma means that \( \theta_g^{M1} < \hat{\theta} = \theta_g^{G2} = \theta_g^{M1} \), the same marginal type would have strictly lower incentive to sell (to the market) in M1 than to sell to the government.
the government in the G2 equilibrium. This reasoning suggests that the marginal type of sale is lower in the M1 equilibrium.

The following results are obtained from the general model.

**Proposition 5.** Suppose a government bailout at price $p_g$ is accepted with positive probability and triggers an immediate market revival. Then, the following holds.

1. The bailout term involves a premium over market terms; i.e., the asset price at the private market is strictly less than the government purchase offer.

2. The volume of asset sale in both dates is weakly greater than without any bailout.

3. There is another equilibrium without date 1 market revival that supports a greater trade and financing and thus higher welfare, than the equilibrium with market revival.

6 A Secret Bailout

In the previous section, we show that the *bailout stigma* leads to low take-up by the firms even with very generous bailout terms. Participation in bailout signals toxicity of the legacy assets than those held by non-participants, which creates a reputational cost in subsequent asset trading for financing the future project, which further discourages those types from receiving the government offer. This *bailout stigma* may undermine the role of the bailout as ways to alleviate the lemons problem in the private market.

One may claim that such a bailout stigma can be eliminated if the bailout is implemented anonymously – identities of the bailout recipients are concealed. In this section, we study whether this “secret bailout” can achieve high take-up as intended its impact on the equilibrium outcome. Specifically, I assume that investors in either date $t = 1, 2$ cannot observe whether firms accept the bailout offer $p_g$, while it is observable whether they sell their assets in $t = 1$. The welfare effect of this identity concealment is not straightforward *a priori*. On the one hand, this *secret bailout* can improve efficacy of the bailout because the signaling effect of bailout participation arises from its observability to the subsequent investors. On the other hand, it may be socially undesirable because the firms may miss an opportunity to signal high quality of their assets by declining the government offer.

Given a government offer $p_g$, there are possibly multiple equilibria – with or without immediate market revival in $t = 1$ – like in the previous analysis under *transparency*. We study how the secret bailout impacts on each case of the equilibrium outcomes.
6.1 The Equilibrium without Immediate Market Revival

First consider an equilibrium where the private market in $t = 1$ is not active. In this equilibrium, investors in $t = 2$ cannot have any additional information about types of the assets from the past. Since the private market freezes in $t = 1$, the only way to finance for the surplus $S$ is to accept the government offer $p_g$. Under secrecy, such a participation decision is not observable to investors in the subsequent market, thus they cannot distinguish the bailout recipients from others who refuse to accept the government offer.

As a result, there is no dynamic consideration of participating in the bailout program, so the firms’ participation in bailout only depends on its payoff in the current period. Put differently, the firm with a type $\theta$ accepts the government offer $p_g$ if and only if

$$\theta \leq p_g + S.$$

In date 2, no additional information is conveyed, hence a decision to trade the asset is made in the same way as the one-shot game. For a price offer $E[\theta | \theta \leq \theta_0]$, a firm sells its asset $\theta$ if and only if

$$\theta \leq E[\theta | \theta \leq \theta_0] + S,$$

where $\theta_0$ is the marginal type of trading the asset in the one-shot benchmark. Investors in $t = 2$ break even by making an offer $E[\theta | \theta \leq \theta_0]$.

In equilibrium, the government can encourage take-up of the bailout by hiding identities of participants. In fact, the government can achieve the maximum participation of the firms in the bailout: they accept the bailout offer $p_g$ if and only if its total gains are higher than value of the assets they hold ($\theta \leq p_g + S$). The maximum take-up of the bailout also implies that no stigma is imposed on the bailout recipients, thus the surplus of asset trading in $t = 1$ given the bailout terms $p_g$ is also maximized. The resulting equilibrium outcome is depicted in Figure 6.

However, the high take-up may not well lead to improvement in the total welfare because the bailout cannot alleviate the adverse selection problem in the subsequent period. In date 1,
firms with assets of good quality have no option to signal their types to the investors in the subsequent market. If the private market has a severe adverse selection problem – i.e. if $\theta_0$ is relatively low in comparison to $p_g + S$, those firms holding high quality assets would rather not trade in the date-2 market as well. Such an ineffectiveness of the bailout on the lemons problem can undermine the gains from trading in date 2.

If the private market is immediately revived responding to the bailout, those firms holding high-quality assets may have an opportunity to improve market perception by refusing to trade in date 1. Under secrecy, however, such an opportunity for building a good reputation may not exist for they cannot distinguish themselves from bailout participants. In the following analysis, we investigate availability of such an option for building a good reputation in the equilibrium with immediate market revival under secrecy.

### 6.2 The Equilibrium with Immediate Market Revival and its Cost

Consider an equilibrium where the private market in $t = 1$ is revived, given a bailout offer $p_g$. In this equilibrium, the investors in $t = 2$ can observe whether firms trade their assets with the date-1 investors. If no trade occurs in the date-1 private market, the subsequent investors in $t = 2$ believe that those firms either accept the bailout or hold their assets, but cannot distinguish the bailout recipients from others.

It is not clear a priori how such an indistinguishability affects the equilibrium outcome. On the one hand, bailout participants may be able to avoid *stigma* by being pooled with the other firms who own high quality assets and refuse to trade, which possibly leads to higher take-up than under transparency. On the other hand, the firms holding high quality assets are possibly discouraged from delaying their trade decision due to this pooling effect. The following lemma shows that the latter effect dominates the former one, hence there is no available option for the firms to improve market perception by delaying their asset trading.

**Lemma 3.** Under secrecy, there is no equilibrium with immediate market revival where a positive measure of firms sell their assets only in $t = 2$.

Lemma 3 has an important implication: the *bailout stigma* prevails even under secrecy when the market is reactivated immediately in date 1. From the observation that no asset trade in date 1 the subsequent investors infer that the available asset types are either the worst ones – bailout participants – or the best ones – non-participants. However, it is unprofitable for them to make a high price offer to attract high-typed assets held by the non-participants, for such an
offer attracts the worst types as well, which reduces the expected revenue. In equilibrium, only the worst-typed assets held by the bailout participants are traded in \( t = 2 \) after no trade occurs in \( t = 1 \), so the \textit{bailout stigma} implicitly remains.

When the private market is active in \( t = 1 \), the equilibrium outcome is characterized with three cutoffs \(- \theta_g, \theta_1, \text{ and } \theta_0^g\) as depicted in Figure 7. This equilibrium is characterized in the similar way to the analogous equilibrium under transparency: (i) \( \theta \leq \theta_g \) accepts the government offer \( p_g \) in \( t = 1 \), and sells the remaining asset in \( t = 2 \); (ii) \( \theta \in (\theta_g, \theta_1] \) sells its assets in both dates; (iii) \( \theta \in (\theta_1, \theta_0^g] \) sells its asset only once in \( t = 1 \); and (iv) \( \theta > \theta_0^g \) does not sell its asset in either date. First, \( \theta_g \) is the marginal type to accept the government offer in \( t = 1 \), hence all types below \( \theta_g \) are indifferent in their decision to participate in the bailout:

\[
p_g + \mathbb{E}[\theta | \theta \leq \theta_g] = 2\mathbb{E}[\theta | \theta_g < \theta \leq \theta_1].
\]

Second, \( \theta_1 \) is the marginal type to be indifferent between selling two units and only one unit. In equilibrium, the firms can sell one unit of their assets by accepting the government offer \( p_g \) in \( t = 1 \) by Lemma 3:

\[
2\mathbb{E}[\theta | \theta_g < \theta \leq \theta_1] + 2S = \theta_1 + p_g + S.
\]

Finally, \( \theta_0^g \) is the marginal type who sells only one unit of their assets to the government in \( t = 1 \): \( \theta_0^g = p_g + S \).

The immediate market revival aggravates the total welfare because the \textit{bailout stigma} does not disappear under secrecy. Consider a government offer \( p_g \) which admits the multiple equilibria with and without the date-1 market revival. In the equilibrium with immediate market revival, the bailout terms are required to compensate its recipients for a tougher funding condition they face in the subsequent period. Moreover, there is no opportunity to improve future trading terms by refusing to trade in the current period. In sum, the bailout offer \( p_g \) boosts the total gains from trade by a greater degree in the equilibrium without the date-1 market revival than it would in the other equilibrium.

**Proposition 6.** If a government offer \( p_g \) can characterize both equilibria with and without immediate market revival under secrecy, the equilibrium without immediate market revival welfare-dominates the other.
7 Welfare and Optimal Policy Design

The preceding analysis has examined how bailout stigma affects the outcomes arising from a variety of bailout scenarios. We now turn to their welfare implications and the design of optimal bailout policy. As will be seen, both welfare and policy analyses are greatly facilitated by recasting the bailout problem in a mechanism design framework. This latter framework would allow for a general class of bailout policies that the government may employ. For instance, our framework allows for a menu of bailout packages that differ in the bailout terms and disclosure options, possibly revealing the identities of firms choosing a package but concealing the identities of the firms choosing a different package.

The two important restrictions we shall impose are that (1) the government never offers a random package and never rations a firm on the offered package and that (2) the government offers no bailout and never intervenes the market in date 2. The second assumption is justified by the empirical fact—as well as being consistent with the treatment so far—that the bailout is often confined to a limited duration (modeled in our paper by date 1). The first assumption is also realistic, consistent with the practices of the government. Its main purpose is to focus on deterministic mechanisms. We doubt that random mechanisms are ever optimal but allowing for random mechanisms makes it intractable to succinctly characterize the second restriction.

To this end, we appeal to the revelation principle, and view the government as proposing a mechanism that specifies the quantity \( q(\theta) \in Q := \{0, 1, 2\} \) of the asset the firms sell and the transfer \( t(\theta) \in \mathbb{R} \) the firms receive, as a function of their reported types \( \theta \). The restriction to deterministic allocation stems from the restriction (1) above. The resulting map \( (q, t) : [0, 1] \to Q \times \mathbb{R} \) then describes a mechanism or equivalently an outcome that may arise in an equilibrium under a policy treatment. Since the only reason for selling an asset for a firm is to finance the project and enjoy the surplus, it is without loss to restrict attention to mechanisms in which \( t(\theta) \geq I \). For any mechanism \( M = (q, t) \in \mathcal{M} \), if a firm with type \( \theta \) reports \( \tilde{\theta} \), it gets the payoff

\[ p^2_g = E[\theta | \theta \leq \theta_g] \quad p_1 = E[\theta | \theta_g < \theta \leq \theta_1] \quad p_g + p^2_g = 2p_1 \]

Figure 7 – The equilibrium behavior with market revival under a secret bailout.
of
\[ U^M(\tilde{\theta}|\theta) := t(\tilde{\theta}) + \theta(2 - q(\tilde{\theta})) + Sq(\tilde{\theta}), \]

since each unit of asset sold enables the financing of a unit of project with net surplus \( S \) and the remaining unsold units \((2 - q(\tilde{\theta}))\) yields the value \( \theta \) to the firm. Of course, for any outcome \( M = (q,t) \in \mathcal{M} \) to be consistent with equilibrium, it must be incentive compatible:

\[ u^M(\theta) := U^M(\theta|\theta) \geq U^M(\tilde{\theta}|\theta), \quad \forall \theta, \tilde{\theta}, \quad (IC') \]

Next, each firm has the option of not participating and enjoying the payoff realized from its asset. In other words,

\[ u^M(\theta) \geq 2\theta, \quad \forall \theta, \tilde{\theta}. \quad (IR) \]

To analyze the social welfare associated a given outcome \((q,t) \in \mathcal{M}\), we must explicitly consider the cost associated with the government deficit. In keeping with the standard approach as well as Tirole (2012), we assume that a dollar deficit entails a deadweight loss of \( \lambda > 0 \). The social welfare from \( M = (q,t) \in \mathcal{M} \) is given by:

\[ W(M) := \int_0^1 \left[ u^M(\theta) + \theta q(\theta) - t(\theta) - \lambda(t(\theta) - \theta q(\theta)) \right] f(\theta)d\theta, \]

where the first term is the surplus accruing to the firms, the next two terms \( \theta q(\theta) - t(\theta) \) aggregate the surplus the government and the investors enjoy, and finally the last terms \( \lambda(t(\theta) - \theta q(\theta)) \) account for the deadweight loss associated with deficit the government runs (and thus must finance through distortionary measures). Note that investors must break even in any equilibrium, so the government may need to bear net deficit to support asset trade.

In the sequel, we are interested in the case in which absent government bailout not all types of firm can sell to competitive investors. Due to the cost of public fund \( (\lambda > 0) \), the government will not wish to offer strict rents to the highest type. That is, \((IR)\) is binding for firm with \( \theta = 1 \) a firm with type \( \theta = 1 \) enjoys the payoff of 2 if it fails to sell the asset or even if it sells the asset, it will never receive the payment greater than maximum possible value. As will be seen, given these two conditions, \((IR)\) will hold for all other types of firms. Finally, the government in our model offers bailout only in the first date, and never intervenes in date 2 markets. This constraint, and as well as the availability of investors willing to purchase firms’ assets competitively, must limit the set of possible outcomes. We let \( \mathcal{M} \) denote the set of all mechanisms/outcomes that satisfy all these properties. The next lemma provides a characterization on how the features of our model and the basic feasibility limit the set of implementable allocations:
Lemma 4. (i) If \( M = (q, t) \in \mathcal{M} \), then \( q(\cdot) \) is nonincreasing, and \( q(\theta) \geq 1 \) for all \( \theta \leq \theta_0 \) and \( q(\theta) \leq 1 \) for all \( \theta > \theta_0 \), where \( \theta_0 \) is the highest type that sells asset in one-shot model without government bailout.

(ii) [Revenue Equivalence] If \( M = (q, t) \) and \( M' = (q', t') \) both in \( \mathcal{M} \) have \( q = q' \), then \( W(M) = W(M') \). In other words, an equilibrium allocation pins down the welfare, expressed as follows:

\[
\int_0^1 \left[ J(\theta)q(\theta) - 2\lambda + 2 \left( (1 + \lambda)\theta + \lambda \frac{F(\theta)}{f(\theta)} \right) \right] f(\theta) d\theta, \tag{6}
\]

where

\[
J(\theta) := (1 + \lambda)S - \lambda \frac{F(\theta)}{f(\theta)}.
\]

(iii) Consider two possible equilibria, labeled A and B, (possibly associated with different levels of \( p_g \) or by different disclosure policies) such that equilibrium \( i = A, B \) induces trade volume \( q_i(\cdot) \) across the two dates. Suppose

\[
\int_0^1 q_A(\theta)f(\theta)d\theta = \int_0^1 q_B(\theta)f(\theta)d\theta
\]

but there exists \( \tilde{\theta} \in (0, 1) \) such that \( q_A(\theta) \geq q_B(\theta) \) for \( \theta \leq \tilde{\theta} \) and \( q_A(\theta) \leq q_B(\theta) \) for \( \theta \geq \tilde{\theta} \). Then, equilibrium A yields higher welfare than equilibrium B, strictly so if \( q_A(\theta) \neq q_B(\theta) \) for a positive measure of \( \theta \)'s.

Part (i) of Lemma 4 characterizes the set of possible allocations that are consistent with incentive compatibility and the government’s laissez faire approach for date 2. In particular, it implies that no firm with type greater than the one-shot threshold can sell in both dates and no firm with type less than the one-shot threshold will fail to sell even a unit. While this characterization involves a nontrivial restriction on the set of possible allocations, but it is worth emphasizing that it allows for a very general set of bailout policies that the government may employ in terms of the bailout terms and disclosure options. For instance, the government may offer a menu of bailout packages with varying degrees of disclosure. Formally, the government may offer a menu of packages \( \{(p_{ig}, \gamma^i)\}_{i \in I} \), where \( I \) is an arbitrary index set, such that a set of firms choosing package \( i \) is allowed to sell its asset in date 1 at price \( p_{ig}^i \) and their identities are revealed with probability \( \gamma^i \in \{0, 1\} \). One simple example is that the government offers a

\[\text{Footnote 18: For instance, the government induce a set of types given by a subdistribution } F^i \text{ of } F, \text{ where } 0 \leq F^i(\theta) \leq F(\theta) \text{ for all } \theta, \text{ and both } F^i \text{ and } 1 - F^i \text{ are nondecreasing.}\]

31
menu of two packages \{(p_1, 1), (p_2, 0)\} so that those firms that pick the first package can sell its asset in date 1 at price \(p_1\), and this is revealed to the market, and those that choose the second package can sell its asset in date 1 at price \(p_2\), and the identities of these firms are concealed from the market. Our framework will encompass all such possibilities; in short, we shall allow for arbitrary bailout and disclosure policies the government may employ.

Parts (i) and (ii) allow us to formulate the optimal bailout mechanism as a solution to the following program:

\[
[P] \max_{q:[0,1] \to Q} \int_0^1 \left[ J(\theta)q(\theta) - 2\lambda + 2 \left( (1 + \lambda)\theta + \lambda \frac{F(\theta)}{f(\theta)} \right) \right] f(\theta)d\theta
\]

subject to

- \(q(\cdot)\) is nondecreasing;
- \(q(\theta) \geq 1\) if \(\theta < \theta_0\) and \(q(\theta) \leq 1\) if \(\theta > \theta_0\).

We are now ready to characterize an the optimal bailout mechanism. Toward this end, note first that the virtual value \(J(\theta)\) is decreasing in \(\theta\), given the log-concavity we assume (Assumption 1). We can thus define a cutoff type

\[
\hat{\theta}^* := \sup \left\{ \theta \in [0,1] \left| (1 + \lambda)S \geq \lambda \frac{F(\theta)}{f(\theta)} \right. \right\}.
\]

**Theorem 1.** The optimal bailout mechanism has

\[
q^*(\theta) = \begin{cases} 
2 & \text{if } \theta \leq \min\{\hat{\theta}^*, \theta_0\} \\
1 & \text{if } \theta \in (\min\{\hat{\theta}^*, \theta_0\}, \max\{\hat{\theta}^*, \theta_0\}] \\
0 & \text{if } \theta > \max\{\hat{\theta}^*, \theta_0\}.
\end{cases}
\]

The optimal policy is implemented by a secret bailout policy in date 1 with \(p_g = \hat{\theta}^* - S\) via an equilibrium that involves no date 1 market activation.

Part (iii) of Lemma 4 facilitates comparisons of alternative equilibria studied in the previous section. To economize on description, we say that an equilibrium of type \(A\) **dominates in welfare** an equilibrium of type \(B\), if for any equilibrium with property \(B\) that arises under any bailout term \(p_g\), there is an equilibrium with property \(A\) arising from some bailout term \(p'_g\) which yields a weakly higher welfare, and strictly higher welfare in some instance.

**Proposition 7.** The equilibria are compared as follows.
(i) Given a transparent bailout policy, an equilibrium with no market activation dominates in welfare an equilibrium with market activation.

(ii) Given a secret bailout policy, an equilibrium with no market activation dominates in welfare an equilibrium with market activation.

(iii) An equilibrium with no market activation under secrecy dominates in welfare an equilibrium with no market activation under transparency.

(iv) An equilibrium with market activation under transparency dominates in welfare an equilibrium with market activation under secrecy.

The proposition suggests that transparency is better for any bailout policy that seeks to activate market immediately, but the secrecy works better for a policy that does not activate the market. But the latter policy dominates the former in welfare, and implements the (constrained) optimal policy identified in Theorem 1.

References


A Appendix: Proofs

Proof of Lemma 1. First, define the following notations:

i. $p_1$: a price offer made by date-1 investors;

ii. $p_2^n$: a price offer made by date-2 investors if a firm sells $n \in \{0,1\}$ unit of its asset in date 1.

Given an arbitrary discount rate $\delta \in (0,1)$, if a firm sells two units of its asset for two periods, the total payoff is

$$p_1 + S + \delta(p_2^1 + S).$$

If it does not sell in date 1 but sells its asset in date 2, the payoff is equal to

$$\theta + \delta(p_2^0 + S).$$

This firm prefers to hold its asset in $t = 1$ if and only if

$$\theta \geq p_1 + S - \delta(p_2^0 - p_2^1) := \theta_1.$$

In $t = 1$, investors rationally infer that only $\theta \leq \theta_1$ are tradable in the market, thus $p_1 < \theta_1$, whereas $p_2^0 \geq \theta_1$. Suppose that a firm with $\theta \leq \theta_1$ sells in $t = 1$ but does not sell in $t = 2$ in equilibrium. Its payoff is then equal to

$$p_1 + S + \delta\theta.$$

If this firm deviates and sells only in $t = 2$, its total payoff is

$$\theta + \delta(p_2^0 + S).$$

Hence it is optimal for this firm not to sell in $t = 1$ if and only if

$$p_1 + S + \delta\theta \leq \theta + \delta(p_2^0 + S),$$

which implies

$$\theta \geq S + \frac{p_1 - \delta p_2^0}{1 - \delta} \geq S + \frac{p_1 - \delta\theta_1}{1 - \delta} \to -\infty \text{ as } \delta \to 1.$$
Thus every firm with $\theta \leq \theta_1$ sells two units over two periods in equilibrium.

Also, the date-1 holdouts sell their assets in $t = 2$ if and only if

$$\theta \geq p_2^0 + S := \theta_2.$$ 

In equilibrium, $\theta_1 \leq \theta_2$.

Finally, suppose that $\theta_1 < 1$ but $\theta_1 = \theta_2$ in equilibrium. If an investor in $t = 2$ offers $p'_2 = \theta_1$ to the firms who do not sell in $t = 2$, there is a $\varepsilon > 0$ such that $\theta \in [\theta_1, \theta_1 + \varepsilon]$ sell to this investor because $\theta_1 + S > \theta_1$, a contradiction. Q.E.D.

Proof of Proposition 2. To prove the first claim, consider the date-1 cutoff $\theta_1$. It is determined by the following indifference condition

$$\theta_1 \leq 2\phi(0, \theta_1) - \phi(\theta_1, \xi(\theta_1)) + S,$$

From this indifference condition, we have

$$\theta_1 \leq \phi(0, \theta_1) + (\phi(0, \theta_1) - \phi(\theta_1, \xi(\theta_1))) + S \leq \phi(0, \theta_1) + S,$$

hence $\theta_1 \leq \theta_0$ since $\theta_0 := \sup\{\theta : \theta \leq \phi(0, \theta_0) + S\}$. The strict inequality holds when $\theta_1 < 1$. Next, consider the date-2 cutoff $\theta_2 \equiv \xi(\theta_1)$, which is pinned down by the indifference condition

$$\theta_2 \leq \phi(\theta_1, \xi(\theta_1)) + S.$$ 

Since $\theta_1 \geq 0$, we have

$$\theta_2 = \xi(\theta_1) \geq \xi(0) = \theta_0,$$

where inequality strictly holds for $\theta_0 < 1$ and $\theta_1 > 0$.

To prove the second claim, recall that $\theta_1$ is pinned down by the following condition:

$$\theta_1 \leq 2\phi(0, \theta_1) - \phi(\theta_1, \xi(\theta_1)) + S,$$

where inequality strictly holds for interior cases. By Assumption 1, $\theta_1 - 2\phi(0, \theta_1) + \phi(\theta_1, \xi(\theta_1))$ is increasing with $\theta_1$, so $\theta_1$ is uniquely found and increasing with $S$. Therefore, there are thresholds $S^*$ and $\bar{S}^*$ such that $\theta_1 = 1$ if $S \geq \bar{S}^*$ but $\theta_1 = 0$ i.e. $\phi(0, \theta_1) < 1$ if $S < S^*$. Q.E.D.

Proof of Lemma 2. Assume that the private market randomly collapses with an arbitrary small probability $\varepsilon > 0$ shortly after the government offer is made. If a firm with a type $\theta$ accepts the
government offer \(p_g\), its total expected payoff is
\[
p_g + S + (1 - \varepsilon) \max\{\theta, p^g_2 + S\} + \varepsilon \theta,
\]
where \(p^g_2\) is the price offer to the bailout recipients in date 2. If the firm declines the government offer while it plans to sell to the date-1 market if active, its total expected payoff is
\[
(1 - \varepsilon)(2p_1 + 2S) + \varepsilon 2\theta.
\]
If the firm plans to sell to the date-2 market only after refusing the government offer, its total expected payoff is
\[
(1 - \varepsilon)(\theta + \max\{\theta, p^0_2 + S\}) + \varepsilon \theta.
\]
Suppose \(\theta \leq p^g_2 + S\). Then the firm sells to the government instead of to the date-1 market if and only if
\[
p_g + S + p^g_2 + S \geq (1 - \varepsilon)(2p_1 + 2S) + \varepsilon 2\theta,
\]
\[\iff 2\varepsilon \theta \leq (1 - \varepsilon)(2p_1 + 2S) - (p_g + p^g_2 + 2S).
\]
Hence, there is a cutoff \(\theta_g \leq \theta_1\) such that the firm with \(\theta \leq \theta_g\) sells to the government instead of the date-1 market in \(t = 1\). Next suppose \(\theta > p^g_2 + S\). Then the firm sells to the government in \(t = 1\) if and only if
\[
p_g + S + \theta \geq (1 - \varepsilon)(2p_1 + 2S) + \varepsilon \theta,
\]
\[\iff (1 - 2\varepsilon)\theta \geq (1 - \varepsilon)(2p_1 + 2S) - (p_g + S).
\]
For a sufficiently small \(\varepsilon > 0\), the firm sells to the government in \(t = 1\) if \(\theta > \theta_1\) as well. Moreover, the firm sells to the market in \(t = 2\) instead of selling to the government in \(t = 1\) if and only if
\[
p_g + S + \theta \leq (1 - \varepsilon)(\theta + \max\{\theta, p^0_2 + S\}) + \varepsilon \theta,
\]
\[\iff \varepsilon \theta + (1 - \varepsilon) \max\{\theta, p^0_2 + S\} \leq p_g + S.
\]
Hence, there is a cutoff \(\theta^0_g \geq \theta_1\) such that (i) if \(\theta \in [\theta_1, \theta^0_g]\), the firm sells to the government in \(t = 1\) while does not sell to the date-2 market; and (ii) if \(\theta > \theta^0_g\), the firm does not sell to either party in \(t = 1\).

\textit{Q.E.D.}

\textit{Proof of Proposition 3.} Consider that investors believe \(\theta_g = 0\) on the equilibrium path. If a firm
with an arbitrary type \( \theta \in [0, 1] \) receives the bailout terms \( p_g \), his payoff is equal to \( p_g + S + \theta \). Since \( p^*_g > p_g \), this firm deviates and holds out his asset until date 2 for \( p^*_g + S + \theta \geq p_g + S + \theta \). If \( \theta \leq \theta_1 \), it is not optimal for these firms to deviate and receive the bailout since
\[
2p^*_1 + 2S \geq \theta_1 + p^*_2 + S \geq \theta_1 + p_g + S \geq \theta + p_g + S.
\]
If \( \theta > \theta_1 \), then it is optimal for them not to deviate as well since \( p^*_2 \geq p_g \). Therefore, firm’s optimality conditions hold.

Given the firms’ equilibrium behavior characterized by the cutoffs \( \theta_1 \) and \( \theta_2 \), the price offers \( p^*_1 \) and \( p^*_2 \) break even on the equilibrium path, and we can appropriately set an out-of-equilibrium belief that any offer higher than these prices is not profitable. \( Q.E.D. \)

**Proof of Proposition 4.** First, consider \( S < S^* \). In this case, the equilibrium without bailout is characterized by the cutoffs \( \theta_1 = 0 \) and \( \theta_2 = \xi(\theta_1) = \xi(0) = \theta_0 \) and the associated equilibrium price in \( t = 2 \) is \( p^*_2 = \phi(0, \theta_2) = \phi(0, \theta_0) \). For some bailout terms \( p_g > p^*_2 \), the G1 equilibrium without market revival in \( t = 1 \) arises, where the date-2 price for the holdouts is \( p_2 = p_g = \phi(\theta_g, \xi(\theta_g)) \). In this equilibrium, the total trading volume in this equilibrium is \( F(\xi(\theta_g)) > F(\xi(0)) = F(\theta_2) \).

Second, consider \( S \geq S^* \). If G1 equilibrium arises, the marginal type for receiving bailout \( \theta^*_g \) has to be low to an extent that \( \phi(0, \theta^*_g) < I \leq \phi(0, \theta^*_1) \implies \theta^*_g < \theta^*_1 \). The total trading volume in G1 equilibrium \( F(\xi(\theta^*_g)) \) is strictly lower than that in NE equilibrium:
\[
F(\xi(\theta^*_g)) < F(\xi(\theta^*_1)) + F(\theta^*_1).
\]

Since \( \theta^*_g \) is increasing with \( p_g \), there is a lower bound \( p^*_g \) such that G1 equilibrium no longer arises from \( p_g \) if \( p_g > p^*_g \): \( \phi(0, \theta^*_g) \geq I \) for all \( p_g > p^*_g \). For this range of \( p_g \), the bailout recipients can trade in the date-2 market as well in equilibrium, i.e. only G2 and G3 equilibria can arise. In G2 equilibrium, the marginal type for receiving the bailout \( \theta^*_g \) is pinned down by the following indifference condition:
\[
\theta^*_g + \phi(\theta^*_g, \xi(\theta^*_g)) + S = p_g + \phi(0, \theta^*_g) + 2S,
\]
which can be rewritten
\[
\theta^*_g = \phi(0, \theta^*_g) + \phi(\theta^*_g, \xi(\theta^*_g)) = p_g + S.
\]
Since the left hand side is increasing with \( \theta_g^{G2} \), this cutoff is increasing with \( p_g \). The total trading volume in this equilibrium is \( F(\theta_g^{G2}) + F(\xi(\theta_g^{G2})) \), increasing with \( p_g \). Therefore, the total trading volume in G2 equilibrium is larger than the equilibrium without bailout if and only if \( p_g \) is sufficiently high.

**Q.E.D.**

**Proof of Proposition 5.** In the equilibrium with immediate market revival, firms are indifferent in their asset trading:

\[
2p_1 + 2S = p_g + p_2^g + 2S,
\]

which can be rewritten

\[
p_g = 2p_1 - p_2^g
\]

\[
= 2\phi(\theta_M, \theta_1^M) - \phi(0, \theta_M)
\]

\[
= \phi(\theta_M, \theta_1^M) + (\phi(\theta_g, \theta_1^M) - \phi(0, \theta_M))
\]

\[
> \phi(\theta_M, \theta_1^M).
\]

Next, consider a bailout offer \( p_g \) triggers both equilibrium with and without market revival in \( t = 1 \). I compare all possible equilibrium outcomes without immediate market revival to those with immediate market revival to show non-market revival in \( t = 1 \) welfare dominates.

First, consider G2 and M1. The total trading volumes in those equilibria are \( F(\theta_g^{G2}) + F(\xi(\theta_g^{G2})) \) and \( F(\theta_1^{M1}) + F(\xi(\theta_1^{M1})) \), respectively, so it suffices to show \( \theta_g^{G2} \geq \theta_1^{M1} \). Suppose to the contrary that \( \theta_g^{G2} < \theta_1^{M1} \). In G2 equilibrium, the indifference condition for receiving the bailout is

\[
p_g + S + \phi(0, \theta_g^{G2}) = \theta_g^{G2} + \phi(\theta_g^{G2}, \xi(\theta_g^{G2})),
\]

while the associated indifference condition in M1 equilibrium is

\[
p_g + S + \phi(0, \theta_1^{M1}) = \theta_1^{M1} + \phi(\theta_1^{M1}, \xi(\theta_1^{M1})).
\]

Since the right hand side is increasing with \( \theta_1^{M1} \), \( \theta_g^{G2} < \theta_1^{M1} \) means \( \theta_g^{G2} < \theta_1^{M1} \). However, this implies

\[
\theta_1^{M1} + \phi(\theta_1^{M1}, \xi(\theta_1^{M1})) + S > p_g + \phi(0, \theta_1^{M1}) + 2S,
\]

so the marginal type \( \theta_g^{M1} \) deviates and hold out his asset in \( t = 1 \) in M1 equilibrium, a contradiction.

Second, consider G2 and M2. Suppose to the contrary that M2 equilibrium yields higher total trading volume than G2 equilibrium. Define \( p_2^{M2} \) and \( p_2^{G2} \) as the price offers in \( t = 2 \) for
the date-1 holdouts in M2 and G2 equilibrium, respectively. Since \( p_g = p_{M2}^G < p_{G2}^G \), we have \( \theta_{G2}^G > \theta_{M2}^M \). For the trading volume in M2 equilibrium to outnumber that in G2 equilibrium, it is necessary \( \theta_{M2}^M > \theta_{G2}^G \). This implies

\[
\theta_{M2}^M + \phi(\theta_{M2}^M, \xi(\theta_{M2}^M)) + S > p_g + \phi(0, \theta_{M1}^M) + 2S,
\]

where the equality holds for \( \theta_{G2}^G \). It can be rewritten

\[
\theta_{M2}^M + p_{M2}^M + S > \theta_{G2}^G + \phi(0, \theta_{G1}^G) + 2S,
\]

hence the marginal type \( \theta_{M2}^M \) deviates and does not sell in \( t = 1 \), a contradiction.

Third, consider G3 and M1. The date-2 price for the holdouts in each equilibrium will be \( p_{G3}^G = \phi(\theta_0^G, \xi(\theta_0^G)) \) and \( p_{M1}^M = \phi(\theta_{M1}^M, \xi(\theta_{M1}^M)) \) respectively. Since \( \theta_0^G > \theta_0^G = \theta_{G3}^G > \theta_{M1}^M \), the total trading volume in G3 is greater than that in M1.

Finally, consider G3 and M2. Again, \( p_{G3}^G = p_g = p_{M2}^M \), while \( \theta_0^G = \theta_{G3}^G > \theta_{M2}^M \), which implies the total trading volume in G3 is higher than M2.

**Proof of Lemma 3.** First define a break-even price \( \hat{p}_2 \) for which investors purchase the asset \( \theta \in [0, \theta_g] \cup [\theta_1, \theta_2] \):

\[
\hat{p}_2(\theta_g, \theta_1, \theta_2) = \frac{1}{F(\theta_2) - F(\theta_1) + F(\theta_g)} \left( \int_0^{\theta_g} \theta dF(\theta) + \int_{\theta_1}^{\theta_2} \theta dF(\theta) \right),
\]

(7)

Before we prove Lemma 3, we show the following properties on \( \hat{p}_2 \).

**Lemma A.1.** If \( F(\theta) \), the distribution function of \( \theta \), is log-concave, then \( \partial \hat{p}_2(\theta_g, \theta_1, \theta_2)/\partial \theta_2 < 1 \) for all \( \theta_2 > \theta_1 \).

**Proof.** Define \( \tilde{F}(\cdot) \) as \( \tilde{F}(\theta) = F(\theta) - (F(\theta_1) - F(\theta_g)) \) for all \( \theta > \theta_1 \). If \( F \) is log-concave, then \( \tilde{F} \) is so log-concave in \( \theta > \theta_1 \): this is because

\[
\frac{\partial \log \tilde{F}(\theta)}{\partial \theta} = \frac{f(\theta)}{F(\theta)} \frac{F(\theta)}{F'(\theta)} = \frac{f(\theta)}{F'(\theta)} \frac{F'(\theta)}{F(\theta) - (F(\theta_1) - F(\theta_g))} = \frac{f(\theta)}{F(\theta)} \left( 1 + \frac{(F(\theta_1) - F(\theta_g))}{F(\theta) - (F(\theta_1) - F(\theta_g))} \right),
\]

where each term in the last expression is decreasing in \( \theta > \theta_1 \). From partial derivative of \( p_2 \) in
with respect to $\theta_2$, we have
\[
\frac{\partial \hat{p}_2}{\partial \theta_2} = -\frac{f(\theta_2)}{(F(\theta_2) - F(\theta_1) + F(\theta_g))^2} \int_0^{\theta_g} u dF(u) + \frac{\partial}{\partial \theta_2} \left( \frac{1}{F(\theta_2) - F(\theta_1) + F(\theta_g)} \int_{\theta_1}^{\theta_2} u dF(u) \right).
\]

Therefore, it suffices to show the following:
\[
\frac{\partial}{\partial \theta_2} \left( \frac{1}{F(\theta_2) - F(\theta_1) + F(\theta_g)} \int_{\theta_1}^{\theta_2} \theta dF(\theta) \right) \leq 1.
\]

To show this, define the following map $\delta : (\theta_1, 1] \to \mathbb{R}$ such that
\[
\delta(\theta) := \theta - \frac{1}{F(\theta) - F(\theta_1) + F(\theta_g)} \int_{\theta_1}^{\theta} u dF(u).
\]

Integrating by part, we can show $\delta$ is increasing in $\theta > \theta_1$:
\[
\delta(\theta_2) = \theta_2 - \frac{1}{F(\theta_2) - F(\theta_1) + F(\theta_g)} \int_{\theta_1}^{\theta_2} s dF
\]
\[
= -\frac{F(\theta_g)}{F(\theta_2)} \theta_1 + \frac{1}{F(\theta_2)} \int_{\theta_1}^{\theta_2} \tilde{F}(u) du,
\]

where both terms in the last expression are decreasing since $\tilde{F}$ is log concave for all $\theta_2 > \theta_1$, which completes proof. Q.E.D.

Lemma A.2. $\partial \hat{p}_2 / \partial \theta_g < 0$ if and only if $\theta_g < \hat{p}_2(\theta_g, \theta_1, \theta_2)$.

Proof. $\partial \hat{p}_2 / \partial \theta_g$ is written as
\[
\frac{\partial \hat{p}_2}{\partial \theta_g} = \frac{f(\theta_g)}{(F(\theta_2) - F(\theta_1) + F(\theta_g))^2} (\theta_g - \hat{p}_2(\theta_g, \theta_1, \theta_2)).
\]

Since $\frac{f(\theta_g)}{(F(\theta_2) - F(\theta_1) + F(\theta_g))^2} > 0$, the sign of $\partial \hat{p}_2 / \partial \theta_g$ is same as that of $(\theta_g - \hat{p}_2(\theta_g, \theta_1, \theta_2))$. Q.E.D.

To show Lemma 3, suppose to the contrary that an equilibrium with a government offer $p_g$ activates the private market in both dates:

1. $\theta \in [0, \theta_g]$ participate in bailout in date 1 and sell to the market at price $p_2$ in date 2,
ii. \( \theta \in (\theta_g, \theta_1] \) sell their assets to the market in each date,

iii. \( \theta \in (\theta_1, \theta_2] \) sell only at price \( p_2 \) in date 2.

Let \( p_t \) be an asset price offered to a firm who begins selling its asset in \( t = 1,2 \). First assume the cutoff \( \theta_2 \) is interior. If \( \theta = \theta_1 \), it is indifferent selling its asset in \( t = 1 \) or keeping it for the next trading in \( t = 2 \):

\[
2p_1 + 2S = \theta_1 + p_2 + S.
\]

In date 1, the firm \( \theta = \theta_1 \) has to be indifferent selling its asset to either the government or the private investor:

\[
p_g + p_2 = 2p_1.
\]

Therefore, the cutoff \( \theta_1 \) is fixed by the government offer \( p_g \):

\[
\theta_1 = p_g + S.
\]

After no trade in the private market in \( t = 1 \), the investors in \( t = 2 \) believe that \( \theta \in [0, \theta_g] \cup (\theta_1, 1] \). Given the date-1 cutoffs \( \theta_g \) and \( \theta_1 \), the marginal type for trading in \( t = 2 \) is determined by the following indifference condition:

\[
\theta_2 = \sup_{\theta > \theta_1} \{ \theta \leq \hat{p}_2(\theta_g, \theta_1, \theta) + S \},
\]

where \( \hat{p}_2(\theta_g, \theta_1, \theta) \) is defined by (7). By Lemma A.1, \( \theta_2 \) is uniquely determined. Note that \( \hat{p}_2(\theta_g, \theta_1, \theta_2) \) is continuous and increasing with \( \theta_2 \).

If \( \theta_2 > \theta_1 \), the break-even price \( p_2 \) has to be greater than the government offer \( p_g \), or else the firm would rather sell to the government in \( t = 1 \). Hence, we have \( p_g < p_1 = \phi(\theta_g, \theta_1) < p_2 = \hat{p}_2(\theta_g, \theta_1, \theta_2) \). Moreover \( \hat{p}_2(\theta_g, \theta_1, \theta_2) \) is the conditional expected value of \( \theta \in [0, \theta_g] \cup (\theta_1, \theta_2] \), thus \( \phi(\theta_g, \theta_1) < \hat{p}_2(\theta_g, \theta_1, \theta_2) \), which implies

\[
\phi(0, \theta_2) < \hat{p}_2(\theta_g, \theta_1, \theta_2) \implies \phi(0, \theta_2) + S < \hat{p}(\theta_g, \theta_1, \theta_2) + S = \theta_2.
\]

Therefore, \( \theta_0 < \theta_2 \) in this equilibrium, where \( \theta_0 = \phi(0, \theta_0) + S \).

Moreover, \( \phi(\theta_g, \theta_1) < \phi(0, \theta_0) \): otherwise, if \( \phi(\theta_g, \theta_1) \geq \phi(0, \theta_0) \), we have \( \theta_0 = \phi(0, \theta_0) + S > \hat{p}_2(\theta_g, \theta_1, \theta_0) + S \), which leads to \( \theta_0 > \theta_2 \), a contradiction. This also implies

\[
\theta_g < \phi(\theta_g, \theta_1) < \phi(0, \theta_0) < \phi(0, \theta_2) < \hat{p}_2(\theta_g, \theta_1, \theta_2),
\]
hence $\hat{p}_2(\theta_g, \theta_1, \theta_2)$ is decreasing with $\theta_g$ by Lemma A.2. Let $\theta_g$ satisfy $\phi(0, \theta_g) = p_g$. Take an arbitrarily small $\varepsilon > 0$ such that the corresponding date-2 cutoff with the marginal type $\theta_g + \varepsilon$ selling to the government is

$$\theta_2 = \hat{p}_2(\theta_g + \varepsilon, \theta_1, \theta_2) + S < \phi(0, \theta_0) + S.$$ 

This implies $\theta_0 > \theta_2$, hence there is no equilibrium with $\theta_g + \varepsilon$. In fact, there is no cutoff $\theta_g > 0$ for participating in the bailout to sustain this equilibrium since any possible break-even price $\hat{p}_2(\theta_g, \theta_1, \theta_2)$ is decreasing with $\theta_g$, a contradiction. 

*Q.E.D.*

**Proof of Proposition 6.** First, consider the equilibrium without immediate market revival. The total trading volume in this equilibrium is equal to

$$p_g + S + \theta_0.$$ 

In the equilibrium with the date-1 market revival, the total trading volume is equal to

$$p_g + S + \theta_1,$$ 

where

$$\theta_1 = 2E[\theta|\theta_g < \theta \leq \theta_1] - p_g + S$$

$$= E[\theta|\theta \leq \theta_g] + S < \theta_0.$$ 

The last inequality follows from the fact $\theta_g < \theta_1$. Therefore, the total trading volume is higher in the equilibrium without the date-1 market revival. 

*Q.E.D.*

**Proof of Lemma 4.** We begin with the proof of part (i). As is standard (so we do not offer a proof), the monotonicity of $q$ follows from (IC). The second part of the characterization follows from the feature of the model that the government offers a bailout only in the first date. To prove $q(\theta) \leq 1$ for all $\theta > \theta_0$, suppose to the contrary that $q(\theta) = 2$ for some $\theta > \theta_0$. Letting $\theta' := \sup\{\theta|q(\theta) = 2\}$, we have $\theta' > \theta_0$ by monotonicity of $q$. Since all types below $\theta'$ sell their assets in both dates, they must be indifferent to all offers accepted by types $\theta < \theta'$. Let $\theta'' \in \arg\min_{\theta \leq \theta'} t_2(\theta)$, where $t_i(\theta)$ is the transfer firm with type $\theta$ gets in date $i = 1, 2$. [What if “min” is not well defined?] Then, the equilibrium payoff for type $\theta''$ must equal

$$t_1(\theta') + t_2(\theta') + 2S = t_1(\theta'') + t_2(\theta'') + 2S,$$ 

(11)
or else either type will deviate. Since \( \theta'' \) is offered the lowest price, it cannot exceed the uninformed equilibrium offer; i.e.,
\[
t_2(\theta'') \leq \phi(0, \theta_0). \tag{12}
\]

(This is intuitive but I find it difficult to prove. It is intuitive since \( \phi(0, \theta_0) \) is the price investors offer to firms when firms’ types are not revealed at all, and \( t_2(\theta'') \) is the lowest price any type is offered in any given equilibrium.) Suppose type \( \theta' \) chooses the package for type \( \theta'' \) in date 1 and chooses not to sell in date 2. Then, its payoff will be
\[
t_1(\theta'') + S + \theta'.
\]
Since this payoff cannot exceed the payoff from trading in both dates,
\[
\theta' \leq t_2(\theta'') + S.
\]

It then follows from (12) that
\[
\theta_0 < \theta' \leq \phi(0, \theta_0) + S,
\]
which is a contradiction to the definition of \( \theta_0 \). We thus conclude that \( q(\theta) \leq 1 \) for all \( \theta > \theta_0 \).

Next suppose \( q(\theta) < 1 \) for some \( \theta < \theta_0 \). This means that all types \( \theta \) and higher do not sell their assets (or receive bailout) in each date. This means that the payoffs of the type must be \( 2\theta \). But by not selling any asset (either to private investors or to the government), it can at least attract an offer of \( \phi(0, \theta_0) \) in date 2, so we must have in equilibrium
\[
\theta_0 > \theta \geq \phi(0, \theta_0),
\]
which again is a contradiction.

We next prove part (ii). First, recall
\[
t(\theta) = u^M(\theta) - \theta(2 - q(\bar{\theta})) - S q(\bar{\theta}). \tag{13}
\]
Next, the envelope theorem applied to \( JC \) along with \( u^M(1) = 2 \) gives
\[
u^M(\theta) = u^M(1) - \int_\theta^{1} (2 - q(s)) ds = 2 - \int_\theta^{1} (2 - q(s)) ds. \tag{14}
\]
Substituting (13) and (14) into the welfare and integrating by parts gives:

\[ W(M) = \int_0^1 \left[ u^M(\theta) + (1 + \lambda)\theta q(\theta) - (1 + \lambda)t(\theta) \right] f(\theta)d\theta. \]

\[ = \int_0^1 \left[ u^M(\theta) + (1 + \lambda)\theta q(\theta) - (1 + \lambda)(u^M(\theta) - \theta(2 - q(\theta)) - S(q(\theta))) \right] f(\theta)d\theta \]

\[ = \int_0^1 \left[ (1 + \lambda)S(q(\theta)) - \lambda u^M(\theta) + 2(1 + \lambda)\theta \right] f(\theta)d\theta \]

\[ = \int_0^1 \left[ (1 + \lambda)S - \lambda \left( \frac{F(\theta)}{f(\theta)} \right) \right] q(\theta) - 2\lambda \left( (1 + \lambda)\theta + \lambda \left( \frac{F(\theta)}{f(\theta)} \right) \right) f(\theta)d\theta. \]

The expression in (6) is therefore derived. Revenue equivalence follows also from the observation that the welfare depends only on the allocation rule \( q \).

We now prove part (iii). The welfare difference between the two equilibria is

\[ W_A - W_B = \int_0^1 J(\theta)[q_A(\theta) - q_B(\theta)]f(\theta)d\theta \]

\[ = \int_0^{\tilde{\theta}} J(\theta)[q_A(\theta) - q_B(\theta)]f(\theta)d\theta + \int_{\tilde{\theta}}^1 J(\theta)[q_A(\theta) - q_B(\theta)]f(\theta)d\theta \]

\[ > \int_0^{\tilde{\theta}} J(\tilde{\theta})[q_A(\theta) - q_B(\theta)]f(\theta)d\theta + \int_{\tilde{\theta}}^1 J(\theta)[q_A(\theta) - q_B(\theta)]f(\theta)d\theta \]

\[ = J(\tilde{\theta}) \int_0^1 (q_A(\theta) - q_B(\theta))f(\theta)d\theta = 0, \]

where the inequality follows from the fact that \( q_A(\theta) \geq q_B(\theta) \) for \( \theta \leq \tilde{\theta} \) and \( q_A(\theta) \leq q_B(\theta) \) for \( \theta \geq \tilde{\theta} \) and that \( J \) is decreasing. The inequality must be strict if \( q_A(\theta) \) and \( q_B(\theta) \) differ on a positive measure of \( \theta \)'s, since \( J \) is strictly decreasing. \( Q.E.D. \)

**Proof of Theorem 1.** Let \( q \) be an arbitrary feasible allocation rule satisfying the constraints of \([P]\). Then,

\[ W(q^*) - W(q) \]

\[ = \int_0^1 J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta \]

\[ = \int_0^{\min\{\hat{\theta}^*, \theta_0\}} J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta + \int_{\min\{\hat{\theta}^*, \theta_0\}}^{\max\{\hat{\theta}^*, \theta_0\}} J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta \]

\[ + \int_{\max\{\hat{\theta}^*, \theta_0\}}^1 J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta. \]
The first integral is nonnegative since $J(\theta) \geq 0$ and $q(\theta) \leq 2 = q^*(\theta)$ for $\theta < \min\{\hat{\theta}^*, \theta_0\} \leq \hat{\theta}^*$. The last integral is also nonnegative since $J(\theta) \leq 0$ and $q(\theta) \geq 0 = q^*(\theta)$ for $\theta > \max\{\hat{\theta}^*, \theta_0\} \geq \hat{\theta}^*$. Finally, the non negativity of the middle integral can be seen as follows. Suppose first $\hat{\theta}^* < \theta_0$. Then, for any $\theta \in (\min\{\hat{\theta}^*, \theta_0\}, \max\{\hat{\theta}^*, \theta_0\}] = (\hat{\theta}^*, \theta_0]$, $J(\theta) \leq 0$ and $q(\theta) \geq 1 = q^*(\theta)$, so the middle integral is nonnegative. Suppose next $\hat{\theta}^* < \theta_0$. Then, for $\theta \in (\min\{\hat{\theta}^*, \theta_0\}, \max\{\hat{\theta}^*, \theta_0\}] = (\theta_0, \hat{\theta}^*]$, $J(\theta) \geq 0$ and $q(\theta) \leq 1 = q^*(\theta)$, so the middle integral is nonnegative. Since all three integrals are nonnegative, the allocation rule $q^*$ is optimal.

The last statement follows from the Proposition 6.  

Q.E.D.