Bailouts in the Presence of Uncertainty and Moral Hazard*

Keeyoung Rhee†
Columbia University

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Abstract

When a firm is financially distressed, it is often uncertain whether the distress stems from an unfolding economic crisis or excessive risk-taking by the firm. I analyze how these uncertainties as well as a government’s desire to control future moral hazard influence a bailout decision. To this end, I develop a two-period model in which the government privately receives a signal on the unknown state of the economy. In this model, bailing out a distressed firm influences the belief about the state held by another firm in the later period, yielding two conflicting effects. First, the bailout indicates an increased chance that the economy is in crisis, which discourages the later firm from risk taking. Second, it signifies an increased likelihood of future bailout, which encourages risk taking. When the prior probability of crisis is low, the latter effect dominates. Hence, the government takes a tougher stance, bailing out less frequently than it would without the long-term consideration. When the prior probability of crisis is high, the former effect dominates. Therefore, the government takes an “alarmist” stance, bailing out more frequently than it would without the long-term consideration.

Keywords: Bailout, moral hazard, uncertainty, inference problem, alarmist stance

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†kr2379@columbia.edu.
1 Introduction

When policymakers are faced with a decision about bailing out an illiquid firm, they are often uncertain about the source of the illiquidity—whether it arises from the excessive risk-taking by the firm or from an unfolding crisis. If illiquidity is a result of the firm’s mismanagement of financial risks, an ideal policy may be to hold the illiquid firm accountable: letting it default can discipline subsequent market participants to manage their financial risks prudentially. However, if illiquidity is an indication of an unfolding economic crisis, bailing out an illiquid firm may be a right decision lest the risk spread to other firm and create a catastrophic impact on the financial market.

Anecdotal evidence underlines such a policy dilemma faced by the policymakers. For instance, when Lehman Brothers was in illiquidity distress in September 2008, the U.S. government decided not to bail it out for the following reasons: first, policymakers assessed that the financial system was sufficiently stable to weather the bankruptcy of Lehman\(^1\); second, the policymakers were strongly concerned about the impact that a series of previous bailouts would have on the moral hazard of market participants in the future.\(^2\) Days after the Lehman bankruptcy, the liquidity position of AIG, along with the rest of the economic outlook, turned quickly dire, raising a question about whether the U.S. regulators might have misjudged the consequence of Lehman’s failure.\(^3\)

The purpose of this paper is to study the implications of the uncertainties—the unknown state of the economy and the unobserved risk-taking behavior of a distressed firm. Specifically, I ask: \((i)\) how these uncertainties shape the government’s bailout decision, given its desire to control moral hazard in the future; and \((ii)\) how the bailout decision impacts the risk-taking behavior of current and future firms.

To address these questions, I develop a two-period model of a government that faces a series of short-run firms subject to a liquidity shock (called illiquidity throughout the paper). At the beginning of the game, nature draws a persistent “crisis” or “non-crisis” state of the economy unknown to the players. A firm in each of the two periods chooses a type of the project to run, “safe” or “risky,” and this choice is unobserved by the other players. Before the project matures, the firm can be hit with illiquidity, in which case it

\(^1\)The former Fed chairman Ben Bernanke testified before the U.S. Senate Committee on Banking, Housing, and Urban Affairs on September 23, 2008: “…But the troubles at Lehman had been well known for some time, and investors clearly recognized . . . that the failure of the firm was a significant possibility. Thus, we judged that investors and counterparties had had time to take precautionary measures.” See http://www.federalreserve.gov/newsevents/testimony/bernanke20080923a1.htm for more detail.

\(^2\)During the meeting of the Fed’s FOMC on September 16, 2008, some members stated that “letting Lehman fail was the only way to provide credibility to the assertion that no firm was ‘too big to fail’.” See The Financial Crisis Inquiry Report for more detail.

\(^3\)Greenspan (2010) also documented that policymakers have failed to predict the length and depth of bubbles in financial markets with a probability of approximately 30 percents.
needs to be bailed out by the government to pursue its project. The probability of illiquidity depends on the underlying state of the economy as well as a type of the project the firm chooses. The government prefers to bail out an illiquid firm only if it strongly believes that the economy is in crisis. The firm prefers the risky project only if the probability of crisis is low and the probability to be bailed out is high, whereas the government wants the firm to choose the safe project regardless of the economic state.

I first study how the uncertainties themselves produce the strategic interplay between the government and the firm in the one-period benchmark. In this benchmark, two interesting results arise. First, the government randomizes in a bailout decision when the prior probability of crisis is low, a result of an inference problem that the government faces in the event of illiquidity. If there were a pure-strategy equilibrium in which the government surely bails out an illiquid firm, the firm would definitely choose the risky project. Knowing this, the government would infer that the illiquidity stems from excessive risk-taking and therefore refuses to bail out the firm. Hence, the bailout cannot characterize a pure-strategy equilibrium. Likewise, there cannot be a pure-strategy equilibrium in which the government refuses to bail out an illiquid firm. Were there such an equilibrium, the firm would certainly choose the safe project. Knowing this, the government would infer that the illiquidity results from the economic crisis, thereby certainly bailing out the firm. Consequently, there is a unique mixed-strategy equilibrium. In this equilibrium, the government bails out the firm in distress with a probability that induces the firm to be indifferent in its project selection; the firm chooses the safe project with a probability that makes the government indifferent in its bailout decision. The randomization result also captures the dilemma that the government often faces in bailout decisions; no matter how low the prior probability of crisis is, the government is induced to be unsure about what an ideal policy is for a distressed firm.

Second, the firm’s risk-taking behavior exhibits non-monotonicity with respect to the prior belief about the state. For the low range of the prior probability of crisis in which the government randomizes in its bailout decision, the firm randomizes in choosing the safe project with a probability that makes the government indifferent in its bailout decision. All else being equal, if the prior probability of crisis increases, the government is more inclined to bail out the firm in case it is distressed. Thus, the firm’s risk-taking increases with the prior probability of crisis. However, when the prior probability of crisis is sufficiently high, the firm’s optimal choice is the safe project even with high likelihoods of the bailout. Consequently, it takes low liquidity risk if the economic crisis is either unlikely or very likely, while the firm takes excessive risk in the intermediate range of the prior probability of crisis.

In the two-period model, we can analyze how a bailout decision impacts future moral hazard and how this impact shapes a dynamic incentive for the government to bail out a
distressed firm. To this end, I assume that the government privately receives an informative signal on the underlying state before making a decision on bailing out an illiquid firm. With the private signal, a bailout decision made in the current period conveys additional information about the underlying state of the economy to another firm in the later period. Specifically, bailing out a distressed firm indicates an increased likelihood of crisis.

Interestingly, bailing out an illiquid firm has two conflicting effects on the moral hazard of the later firm. On the one hand, the current bailout signifies a high probability of crisis to the later firm. This tends to discourage risk taking by the later firm, holding the government’s response to illiquidity constant. On the other hand, the current bailout also indicates an increased likelihood that the later firm will be bailed out in the case it is illiquid. This effect encourages the later firm to increase its risk-taking.

Which of these two effects dominates depends on the prior belief about the underlying state. When the prior probability of crisis is low, the latter effect dominates. As a result, the later firm takes more liquidity risk if it observes bailout in the current period than it would otherwise do. However, when the prior probability of crisis is high, the former effect dominates; the bailout indicates an increased possibility that the economy is in crisis. Therefore, the later firm reduces its risk exposure after observing the bailout.

Knowing these effects, the government utilizes its bailout decision as a way to manipulate the later firm’s beliefs and thereby control its moral hazard. When the prior probability that the economy is in crisis is low, the current bailout induces the later firm to increase its risk-taking. Therefore, the government takes a tougher stance, bailing out a currently illiquid firm less frequently than it would without the long-term consideration of its bailout decision. When the prior probability of crisis is high, the current bailout warns the later firm of a strong possibility of crisis, which induces the firm to lower its risk exposure. Therefore, the government takes an “alarmist” stance, bailing out a currently illiquid firm more frequently than it would without the long-term consideration.

This main result gives new insights into the dynamic incentives that government bailouts involve. Contrary to the received wisdom, the alarmist approach can reduce risk taking by firms because bailout can serve as a credible warning about the impending crisis. This new result is also consistent with the phenomenon observed in the wake of the recent financial crisis. After the government-assisted purchase of Bear Stearns by JP Morgan in March 2008, four then-major investment banks took efforts to reduce their portions of short-term financing. Specifically, between the period from March to May in 2008 and the period from July to August in 2008, the ratio of the overnight and open repo financing to the total repo financing decreased: Lehman’s ratio fell from 45% to 40%; Merrill Lynch from 46% to 43%;

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4One relevant example is the “stress test” in which the policymakers assess the ability of financial institutions to withstand a large-scale macroeconomic shock. Goldstein and Leitner (2013) study an optimal disclosure policy of the stress-test result privately known to the policymaker.
Morgan Stanley from 70% to 55%; and Goldman Sachs from 18% to 10.\footnote{For more detail, see The Financial Crisis Inquiry Report, pp. 296 - 297.}

2 Related Literature

The setting of this paper builds on Holmström and Tirole (1998), in which a firm is subject an interim shock that requires refinancing from the outside liquidity suppliers. In this work, the liquidity shock occurs before the firm chooses its effort, and thus, the liquidity shock contains no additional information on the moral hazard of the firm. In my model, the liquidity shock arrives after the firm chooses the riskiness of the project. Furthermore, the distribution of the liquidity shock depends on the underlying state as well as the risk-taking behavior by a firm. However, a government, the only liquidity supplier in the model, does not know the true state of the economy or risk-taking by the firm. When the liquidity shock occurs, the government faces an inference problem related to the source of this shock, which creates the strategic interplay between the government and the firm.

The most relevant work for this paper is Nosal and Ordoñez (2013). In this study, a government observes a string of distressed firms over time but does not know whether those firms are distressed as a result of an unfolding crisis. The government delays its decision on intervention until it is fully convinced that the economy is in crisis. This paper yields a similar result that the government bails out a distressed firm less likely than it would without any long-term consideration. However, such a decreased likelihood of bailout is not motivated by the government’s willingness to learn more about the underlying state. Instead, the government uses its refusal to bail out a distressed firm as a way of controlling future moral hazard. Furthermore, a bailout also indicates an increased likelihood that the economy is in crisis. If this effect dominates, the government is willing to bail out a distressed firm in an attempt to alert the other firms to a strong possibility of the economic crisis, as opposed to Nosal and Ordoñez (2013).

A large body of literature addresses the time-inconsistency problem inherent in bailout policies. Many papers (for example, see Mailath and Mester (1994) and Acharya and Yorulmazer (2007)) argue that the ex-post optimal bailout decision creates more risk-taking by the firms, which reduces the ex-ante welfare. To address this problem, recent papers propose ex-ante solutions: the ex-ante regulation to limit risk-taking by firms (Farhi and Tirole, 2012; Chari and Kehoe, 2013); or the ex-ante design of bailout terms (Bianchi, 2012). This paper also focuses on an incentive problem caused by government bailouts, but does not involve the time-inconsistency problem. Instead, this paper studies how the ex-post bailout decision influences moral hazard in subsequent periods. The response of future firms to a current bailout creates dynamic incentives for a bailout decision.
Some papers also emphasize a role of bailouts as a way to resolve the coordination failure among debtholders. For example, Cheng and Milbradt (2011) argue that expecting a bailout reduces a chance that creditors refuse to roll over their short-term lending. Therefore, a borrowing firm does not choose the socially undesirable project for “gambling for resurrection”. This paper also discusses a bailout as a way to control moral hazard, but such a result does not arise from instilling the creditors’ confidence. Instead, a bailout is used to warn a future firm of an impending economic crisis.

A recently growing body of literature studies an intervention structure to rejuvenate the asset markets with adverse selection (for example, see Philippon and Skreta (2012) and Tirole (2012)). One strand of this literature focuses on how a seller’s decision on trading influences market perception about quality of its asset. Fuchs and Skrzypacz (2013) argues that banning future asset trading can improve efficiency by eliminating the opportunity for the seller to boost its reputation by refusing to trade. Che, Choe and Rhee (2014) also study implications of the stigma – the bailout recipients have a tough financing condition in the future. They find concealing the identity of a bailout recipient may mitigate the stigma. My paper also studies the dynamic incentives that the government bailouts incorporate. However, it studies the impacts on moral hazard of firms over time instead of adverse selection.

The rest of the paper is organized as follows. Section 3 introduces the model. In Section 4, I analyze the equilibrium strategies in a one-period benchmark model. Section 5 analyzes the equilibrium behavior in a two-period model without private information on the state of the world. Section 6 discusses a dynamic incentive for bailing out an illiquid firm and its influence on a bailout decision, and Section 7 concludes the paper.

3 The Model: Investment with Uncertainty

Consider a two-period economy \((t = 1, 2)\) with a series of short-lived firms that undertake a financial project subject to an interim liquidity shock. At the beginning of the game, nature draws an underlying state of the world \(\omega \in \{n, c\}\) that fully persists for two periods. In \(t = 1\), a firm chooses a type of financial project in which to invest. Before the project fully matures, the firm can be randomly hit with an interim liquidity shock (or illiquidity). To continue the project, the firm must be bailed out by the government. The government decides whether to bail out in the event of the liquidity shock, and the project returns at the end of the period. In \(t = 2\), the period-1 firm dies, and a new firm arrives in the economy and plays the same investment game with the government.

In this model, there are information frictions. First, neither the government nor a firm knows the true state \(\omega \in \{n, c\}\) drawn at the beginning of the game. Instead, they share a
common prior belief $p_0 \in (0, 1)$:

$$p_0 := \mathbb{P}(\omega = c).$$

Second, the government cannot observe which type of project the firm chooses in each period.

There are two different types of projects $\rho \in \{s, r\}$: a safe project (named “$s$”) and a risky project (named “$r$”). In each period, a firm can be hit with a liquidity shock with a probability $q_\rho^\omega \in [0, 1]$, which depends on the underlying state $\omega \in \{n, c\}$ as well as the project type $\rho \in \{s, r\}$. Specifically, I assume the following:

$$0 = q_n^s < q_c^s \leq q_n^r < q_c^r.$$  \hspace{1cm} (1)

In other words, the likelihood that the liquidity shock occurs increases if (i) either the firm chooses the risky project or (ii) the economy is in crisis.

I assume the following state-dependent preferences. First, bailing out an illiquid firm is socially desirable only when the government is strongly convinced that the economy is in crisis. The firm prefers the risky project if the likelihood of $\omega = c$ is low but the bailout is highly expected; otherwise, the firm prefers the safe project. However, the government wants the firm to choose the safe project regardless of its belief about $\omega$.

To justify these preferences, I build a payoff structure as follows. Each type of project $\rho \in \{s, r\}$ yields a return $V_\rho$ to a firm in each period and social welfare $W_\rho$ to the government if the firm is not hit with the liquidity shock. If the liquidity shock occurs, the firm’s payoff depends on the government’s decision about a bailout: if the firm is bailed out, then it pursues its project but partially recoups the return $v_\rho < V_\rho$ only; otherwise, the firm defaults and gets zero return from its project. Bailing out a distressed firm gives a positive social surplus $B$ if $\omega = c$ but it only gives a negative social surplus $-L$ if $\omega = n$. I assume that illiquidity deteriorates the social welfare; for every $\rho \in \{s, r\}$, $W_\rho > B$.

This payoff structure can be endogenized by standard models in the literature. First, after being bailed out, a distressed firm can retrieve only a portion of the return because an outside liquidity supplier asks all pledgeable income of the project (Holmström and Tirole, 1998). Second, bailing out a distressed firm can create a positive externality at the expense of distortionary wealth transfer, as indicated in the public finance literature. Particularly, the benefit from bailouts can be substantial when the economy is severely exposed to counterparty risks, and therefore, the bankruptcy of an individual firm quickly spreads to other firms via contagion (Allen and Gale, 2000; Allen, Babus and Carletti, 2012). I use the reduced-form payoff structure because endogenizing these considerations does not add insights into the main results.

To justify the assumptions on the preferences structurally, I derive the conditions on the likelihoods of illiquidity $\{q_\rho^\omega\}_{\rho \in \{s, r\}}, \omega \in \{n, c\}$. First, consider that the firm chooses the project
type $\rho \in \{s, r\}$ in the state $\omega \in \{n, c\}$. If the firm expects to be bailed out in the case it is illiquid, the expected return is equal to

$$V^\omega_\rho := (1 - q^\omega_\rho)V_\rho + q^\omega_\rho v_\rho.$$ 

If the firm expects not to be bailed out, then the expected return is equal to

$$V^\omega_\rho := (1 - q^\omega_\rho)V_\rho.$$ 

The firm’s preference on the project selection can be built on the following conditions. First, we assume

$$V^n_r \geq V^n_s.$$  \hspace{1cm} (2)

Under this assumption, the firm chooses the risky project if crisis is unlikely while bailout is likely. Second, we assume that the firm prefers the safe project if the bailout is less likely. To support this assumption, I assume the following:

$$V^\omega_r \leq V^\omega_s \quad \forall \omega \in \{n, c\}$$  \hspace{1cm} (3)

and

$$V^c_r \leq V^c_s.$$  \hspace{1cm} (4)

Under these assumptions, the firm chooses the safe project if either crisis is very likely or the bailout is unlikely. A condition to satisfy (2) is that the likelihood of illiquidity under the risky project $q^n_r$ is relatively low in $\omega = n$:

$$q^n_r \leq \frac{V_r - V^n_s}{V_r - v_r}.$$  \hspace{1cm} (5)

In (3), the firm chooses the safe project if the government is unlikely to bail out the firm in distress. To formulate this preference, $q^n_r$ should not be too low:

$$q^n_r \geq \frac{V_r - V^n_s}{V_r}.$$  \hspace{1cm} (6)

We can build (4) with an assumption that a likelihood of illiquidity under the risky project $q^c_r$ is sufficiently high in $\omega = c$:

$$q^c_r \geq \frac{V_r - V^c_s}{V_r - v_r}.$$  \hspace{1cm} (7)
Table 1 – Payoff structures with respect to the underlying state $\omega \in \{n, c\}$.

<table>
<thead>
<tr>
<th>Firms</th>
<th>Bailouts</th>
<th>No Bailouts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe</td>
<td>$\nabla^n_s, -L$</td>
<td>$\nabla^n_s, 0$</td>
</tr>
<tr>
<td>Risky</td>
<td>$\nabla^r_n, -L$</td>
<td>$\nabla^n_r, 0$</td>
</tr>
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In state $\omega = n$.

With similar assumptions on $q^\omega_r$, we can justify the assumption that the safe project is ex ante efficient. For $\omega = n$, this condition is equivalent to $W_s \geq (1 - q^n_s)W_r$, which can be rewritten as

$$q^n_r \geq \frac{W_r - W_s}{W_r}.$$  \hspace{1cm} (8)

For $\omega = c$, the condition is equivalent to $(1 - q^c_s)W_s + q^c_sB \geq (1 - q^c_c)W_r + q^c_rB$, or rewritten as follows:

$$q^c_r \geq \frac{W_r - (1 - q^c_s)W_s + q^c_sB}{W_r - B}.$$  \hspace{1cm} (9)

From the government’s point of view, the risky project exposes the economy to liquidity shock too often regardless of the underlying state of the economy. However, the firm believes that the risky project exposes the firm to liquidity shock too often only if it is strongly convinced that the economy is in crisis, thereby taking high risk if crisis is unlikely but bailout is likely. Throughout this paper, I assume that all the conditions from (5) to (9) hold.

**Remark 1.** Should there be no uncertainty about the underlying state $\omega \in \{n, c\}$, the firm will choose the safe project in equilibrium. First, if $\omega = n$, the government prefers not to bail out the firm in the event of illiquidity. Conditional on no bailout, the firm will not choose the risky project by (3). Second, if $\omega = c$, the government surely bails out an illiquid firm because it creates ex post social surplus $B > 0$. Nonetheless, the firm still refrains from choosing the risky project by (4) because $\nabla^c_r \leq \nabla^s_c$.

### 4 Benchmark: the One-Period Model

Before studying the two-period model, I first study a benchmark one-period model. In this model, nature draws the state of the economy $\omega \in \{n, c\}$; a firm chooses a type of project; and in the event of illiquidity, the government decides whether to bail out an illiquid
firm. In this benchmark, I analyze how the uncertainties about the state of the economy can influence the strategic interaction between the bailout decision by the government and project selection made by the firm, even without the dynamic incentive involved in a bailout decision.

To this end, I characterize the equilibrium with respect to the prior belief about the state $p_0 \in (0.1)$. Define $\sigma \in [0, 1]$ as the firm’s (mixed) strategy of choosing the safe project and $\beta \in [0, 1]$ as the government’s bailout strategy, respectively. If illiquidity occurs, the government is faced with an inference problem on the source of the illiquidity, associated with the dual uncertainties. Given the choice probability of the safe project $\sigma$, the government updates the belief about the state in accordance with Bayes:

$$p_1(p_0, \sigma) = \frac{p_0(\sigma q_s^c + (1 - \sigma)q_c^c)}{p_0(\sigma q_s^c + (1 - \sigma)q_c^c) + (1 - p_0)(1 - \sigma)q_r^c}.$$  \hspace{1cm} (10)$$

The posterior belief $p_1$ increases in both $p_0$ and $\sigma$, which can be interpreted as follows: the more likely it is that the firm chooses the safe project, the more likely it is that the government will infer that the illiquidity arises as a result of an the economy-wide crisis. For short, I hereafter abbreviate $p_1(p_0, \sigma)$ to $p_1$.

The government is willing to bail out an illiquid firm only if $p_1$ is sufficiently high, because the bailout is socially optimal in state $\omega = c$. Specifically, the government bails out the illiquid firm if and only if

$$p_1 B - (1 - p_1)L \geq 0.$$  \hspace{1cm} (11)$$

When the game begins with a low prior $p_0$, the safe project has to be chosen with a necessarily high probability to induce the bailout. When the prior belief $p_0$ is sufficiently high, the government will certainly bail out the distressed firm even with zero probability that the safe project is chosen.

**Lemma 1.** There exists a cutoff $p_g^* \in [0, 1]$ such that $p_1 B - (1 - p_1)L \geq 0$ with $\sigma = 0$ for all $p_0 \geq p_g^*$.

The underlying state also influences the firm’s optimal choice of the project. From the firm’s preferences on risk-taking, recall that the firm prefers the risky project if it expects to be bailed out in the event of illiquidity:

$$V_r^n \geq V_s^n.$$  \hspace{1cm} (2)$$

Recall also that the firm prefers the safe project even with a high likelihood of bailout:

$$V_r^c \leq V_s^c.$$  \hspace{1cm} (4)$$
Therefore, the firm prefers the safe project if its belief $p_0$ exceeds a certain threshold $p_f^*$, which is determined as follows:

$$p_0 V_s^c + (1 - p_0) V_r^m \geq p_0 V_r^c + (1 - p_0) V_r^m,$$

$$\iff p_0 \geq p_f^* \equiv \frac{V_r^m - V_s^m}{(V_s^c - V_r^c) + (V_r^m - V_s^m)}.$$

If the firm is strongly convinced that the economy is in crisis, it does not prefer risk-taking even if the bailout is highly anticipated.

**Lemma 2.** When $\beta = 1$, there is a threshold $p_f^*$ such that the firm chooses the safe project for every $p_0 > p_f^*$.

Using these cutoffs $\{p_g^*, p_f^*\}$, I characterize the equilibrium strategies $\{\sigma^*, \beta^*\}$ with respect to the prior belief $p_0$. First, consider a high range of $p_0 \in (p_f^*, 1]$. For all priori in this range, the firm surely chooses the safe project ($\sigma^* = 1$) for any given probability of bailout $\beta \in [0, 1]$ by Lemma 2. Given $\sigma^* = 1$, illiquidity surely stems from the economic crisis $p_1 = 1$, and therefore, the government certainly bails out the firm when it suffers from illiquidity ($\beta^* = 1$). Therefore, $\beta^* = 1$ and $\sigma^* = 1$ characterize the equilibrium.

Second, consider an intermediate range of $p_0 \in [p_g^*, p_f^*]$. For all priori in this range, the government surely bails out the firm in liquidity distress ($\beta^* = 1$) for every $\sigma \geq 0$ by Lemma 1. Given that the bailout occurs with probability 1, the firm’s optimal selection is the risky project ($\sigma^* = 0$) by Lemma 2. This strategy profile thus characterizes the equilibrium.

Finally, consider a low range of $p_0 < p_g^*$. For all priori in this range, there exists a unique mixed-strategy equilibrium. To see this, suppose that the firm chooses the safe project ($\sigma = 1$) in equilibrium. When illiquidity hits the firm, $p_1 = 1$, and therefore, the government certainly bails out the firm ($\beta = 1$). Given $\beta = 1$, the firm’s optimal choice is the risky project ($\sigma = 0$) because $p_0 < p_f^*$. The government then rationally infers $\sigma = 0$ and then refuses to bail out this firm in case it is illiquid ($\beta = 0$) because $p_0 < p_g^*$. Given $\beta = 0$, the firm’s optimal selection is then the safe project ($\beta = 1$), so this loop goes back to the beginning.

In the mixed-strategy equilibrium, the government’s indifference pins down the posterior $p_1$, which in turn pins down the firm’s equilibrium strategy $\sigma^*$:

$$p_1(p_0, \sigma) B - (1 - p_1(p_0, \sigma)) L = 0. \quad (12)$$
Likewise, the firm’s indifference pins down the government’s equilibrium bailout strategy:

\[
(1 - p_0 q^c_s) V_s + p_0 q^c_s \beta^* v_s = (1 - (p_0 q^c_r + (1 - p_0) q^n_r)) V_r + (p_0 q^c_r + (1 - p_0) q^n_r) \beta^* v_r.
\]

\[
\Rightarrow \beta^* = \frac{(1 - p_0 q^c_s) V_s - (1 - (p_0 q^c_r + (1 - p_0) q^n_r)) V_r}{(p_0 q^c_r + (1 - p_0) q^n_r) v_r - p_0 q^c_s v_s}.
\]

(13)

Figure 1 – The equilibrium strategies in the one-period benchmark with uncertainties.

In sum, the equilibrium is characterized as the following proposition:

**Proposition 1.** In the one-period economy with a common prior belief \( p_0 \in (0,1) \), the equilibrium is characterized as a cutoff structure \( \{p^*_g, p^*_f\} \):

i. For \( p_0 > p^*_f \), \( \beta^* = 1 \) and \( \sigma^* = 1 \): the firm chooses the safe project, and the government surely bails out when the firm is distressed by illiquidity.

ii. For \( p_0 \in [p^*_g, p^*_f] \), \( \beta^* = 1 \) and \( \sigma^* = 0 \): the firm chooses the risky project, and the government surely bails out after the liquidity shock arrives.

iii. For \( p_0 < p^*_g \), \( \beta^*, \sigma^* \in (0,1) \): the firm randomizes in its project choice, and the government also randomizes in its bailout decision.

**Remark 2.** The intermediate region \( [p^*_g, p^*_f] \) exists if the benefit \( B > 0 \) from bailing out an illiquid firm at a time of the economic crisis is relatively high. Formally, \( p^*_g \leq p^*_f \) if and only if

\[
\frac{1}{1 + \frac{q^c_r B}{q^c_L}} < \frac{V^m_r - V^m_s}{(V^c_r - V^c_s) + (V^m_r - V^m_s)}.
\]

If not, then the intermediate region does not exist.

Several interesting implications arise from the static benchmark. First, for the prior beliefs in a low range \( p_0 < p^*_g \), the inference problem induces the government to randomize.
in bailing out an illiquid firm. Without the inference problem, no such randomization result would arise. If the project selection is observed, the bailout decision must be a pure strategy: if the risky project is chosen, the government never bails out an illiquid firm; otherwise, the government surely bails out. Likewise, if the government knows the true state of the economy, there is no randomization in the bailout decision: if \( \omega = n \), the government always refuses to bail out an illiquid firm; otherwise, the government surely bails out. The randomization in a bailout decision also represents a policy dilemma that the government often faces in its decision about bailing out a distressed firm. Once \( p_0 < p_g^* \), the posterior belief \( p_1 \) updated after illiquidity always induces the government to be unsure whether it is right to bail out an illiquid firm or not.

Second, the firm’s equilibrium risk-taking behavior exhibits non-monotonicity with respect to the prior probability of crisis. For the low range of the prior belief \( p_0 < p_g^* \), the firm chooses the safe project with a probability that makes the government indifferent in bailing out the firm in case illiquidity occurs. In this range, holding the choice probability of the safe project constant, the government strictly prefer to bail out the firm in liquidity distress if the prior belief \( p_0 \) increases. To induce the government to be indifferent, the equilibrium choice probability of the safe project \( \sigma^* \) decreases with \( p_0 \). For the intermediate range of the prior belief \( p_0 \in [p_g^*, p_f^*] \), the firm chooses the risky project because the equilibrium probability of being bailed out is sufficiently high in this range. However, for the high range of the prior belief \( p_0 > p_f^* \), the firm’s optimal selection is the safe project. In sum, the safe project is chosen with a high probability if crisis is either unlikely or very likely.

This non-monotonicity of the firm’s risk taking can be also seen as “signal jamming.” Intuitively, the firm takes advantage of the government’s inability to extract the firm’s risk-taking behavior from the underlying state of economic crisis in the event of the liquidity shock. The firm makes use of the signal jamming effectively when the inference problem on the source of illiquidity becomes severe, i.e., the prior belief \( p_0 \) is in the intermediate range. If the prior belief \( p_0 \) is close to zero, the firm cannot fully exploit the signal jamming for its risk-taking. However, the incentive to utilize the signal jamming no longer arises from a high prior \( p_0 > p_f^* \).

### 5 The Two-Period Model without Private Signal

In practice, an economic crisis is not a transient event; rather, it persists over time, and therefore, a policymaker takes into account the long-term impact of bailing out a distressed firm on future moral hazard. To capture this long-term consideration of a bailout policy, I study a two-period model in which a government is a long-run player that considers not only the immediate benefit of bailing out a distressed firm but also its effect on moral hazard in
subsequent periods.

In this section, I first study a two-period model \((t = 1, 2)\) without private information on the state – neither firms nor the government have additional information on the state, and they observe all financial events. Recall that each firm in \(t = 1, 2\) is short-lived, so the government is the only player that considers the long-term effect of its bailout decision in \(t = 1\). Finally, recall that the underlying state \(\omega \in \{n, c\}\) is drawn that fully persists for two periods.

Surprisingly, the government’s bailout decision in \(t = 1\) has no impact on moral hazard of the subsequent firm in \(t = 2\) because the bailout decision does not contain any additional information to the subsequent firm. In fact, the arrival of illiquidity in \(t = 1\) is the only financial event that provides new information on the state to the subsequent firm in \(t = 2\), which yields \(p_1\) as its beginning-of-period-2 belief shared by the government. Since there is no long-term effect of the bailout, the government and the firm in \(t = 1\) behave in the same way as they would in the static benchmark.

Formally, the equilibrium is characterized in the following way. In \(t = 1\), the firm in \(t = 1\) takes liquidity risk according to \(\sigma^*\) in Proposition 1. After the liquidity shock in \(t = 1\), a firm in \(t = 2\) has the belief \(p_1\), shared by the government. Their equilibrium strategies are analogous to (1) but based on the updated belief \(p_1\) as their prior belief in \(t = 2\).

For the low range \(p_0 \leq p_g^*\), the period-1 firm’s equilibrium strategy \(\sigma^*\) induces the indifference in the bailout decision with the posterior belief \(p_1\):

\[
p_1B - (1 - p_1)L = 0.
\]

Correspondingly, the government randomizes in the bailout decision with probability \(\beta^*\) as in (13). If the liquidity shock arrives again in \(t = 2\), the resulting posterior belief – denoted by \(p_2\) – is sufficiently high to induce the bailout unconditionally:

\[
p_2B - (1 - p_2)L > 0.
\]

With a high likelihood \(p_0 > p_g^*\), the updated belief \(p_1\) satisfies \(p_1B - (1 - p_1)L > 0\) for all \(p_0 > p_g^*\), so it is also sufficiently high such that the government bails out the period-2 firm with probability 1. The period-2 firm’s risk-taking behavior is \(\sigma^*(p_1)\).

If the game begins with a low prior belief \(p_0 < p_g^*\), the government can be in a “bailout trap” – the bailout probability discreetly increases over time. In \(t = 1\), the government bails out an illiquid firm with probability \(\beta^*(p_0) < 1\). The updated belief \(p_1\) after the liquidity shock makes the government indifferent: \(p_1B - (1 - p_1)L = 0\). This implies that the government will surely bail out an illiquid firm in \(t = 2\) \((\beta^*(p_1) = 1)\).

Furthermore, such a no-impact result also has a policy implication. If policymakers
do not have additional knowledge about the underlying state of the economy, market participants adjust their risk exposure based only on the past event of illiquidity, not on the bailout decision that comes after illiquidity.

6 The Two-Period Model with the Private Signal

In this section, I assume that the government privately receives an informative signal on the underlying state $\omega \in \{n, c\}$ before it makes a decision on bailing out a distressed firm. With this assumption on information structure, I can capture the dynamic incentive involved in a bailout decision. A decision on the bailout in the first period reflects the government’s superior knowledge about the underlying state. Thus, the bailout influences the beliefs about the underlying state held by a later firm in the next period, thereby enabling the government to affect risk taking by this later firm.

The assumption of private information is consistent with the regulatory features observed in various financial sectors. Specifically, regulators are authorized to monitor risk management by registered firms. One relevant observation is the stress test conducted by the U.S. regulators in the wake of the recent financial crisis. In 2008 after the bailout of Bear Stearns, the Federal Reserve tested the ability of other major investment banks to manage a macroeconomic shock but kept the results internal.\(^6\)

Formally, I assume that in $t = 1$, the government privately receives a binary signal $\eta \in \{n, c\}$ that is positively correlated with the underlying state $\omega$:

$$\mathbb{P}(\eta = \omega | \omega) = \lambda \in (1/2, 1) \text{ for all } \omega \in \{n, c\}.$$ 

If illiquidity arrives in $t = 1$, the government’s private belief after the signal $\eta \in \{n, c\}$ is determined in accordance with Bayes: when the government receives $\eta = c$, its posterior belief – denoted by $p_1^c$ – is equal to

$$p_1^c := \frac{\lambda p_1}{\lambda p_1 + (1 - \lambda)(1 - p_1)}; \quad (14)$$

if the government receives $\eta = n$, its posterior belief – denoted by $p_1^n$ – is equal to

$$p_1^n := \frac{\lambda(1 - p_1)}{\lambda(1 - p_1) + (1 - \lambda)p_1}. \quad (15)$$

\(^6\)Despite the collaboration with the SEC for these stress tests, the Federal Reserve did not even share its analyses and conclusions. See The Financial Crisis Inquiry Report, pp. 298.
\( \eta \) is positively correlated with \( \omega \), so we have the following relationship:

\[
p_1^c > p_1 > p_1^n.
\]

The government’s economic outlook worsens after it receives \( \eta = c \).

Before I proceed with the equilibrium analysis, I introduce more notations. First, \( p_2^B \) and \( p_2^N \) denote the updated beliefs held by a firm in \( t = 2 \) after observing the current bailout (B) and no-bailout (N), respectively. Likewise, \( \sigma_B \) and \( \sigma_N \) denote the period-2 firm’s (mixed) strategies – its probabilities of choosing the safe project – in response to the current bailout decision in \( t = 1 \).

### 6.1 Bailout Strategies with Long-Term Consideration

In this section, I discuss how the government’s desire to control future moral hazard influences its bailout decision at present. To this end, I consider that the government discounts the period-2 social welfare by \( \delta \in [0, 1] \). If \( \delta > 0 \), then the government has a long-term consideration of its bailout decision. If \( \delta = 0 \), the government does not take into account the long-term effect of bailing out a distressed firm. I analyze the dynamic incentives for the bailout by comparing equilibrium outcomes in these two cases, \( \delta > 0 \) and \( \delta = 0 \).

Define \( W_2(\sigma, p) \) as the period-2 expected welfare when the period-2 firm chooses the safe project with probability \( \sigma \), evaluated by the government’s belief \( p \) before the game in \( t = 2 \) begins:

\[
W_2(\sigma, p) := \left[ \sigma (1 - pq_s^c)W_s + (1 - \sigma)(1 - (pq_s^c + (1 - p)q_r^n))W_r \right]
+ \max\{0, (\sigma pq_s^c + (1 - \sigma) pq_r^c)B - (1 - \sigma)(1 - p)q_r^nL\}.
\]  \hspace{1cm} (16)

The first term is the expected welfare the government obtains when the period-2 firm’s project will return in the absence of illiquidity. The second term is the expected welfare in the event of illiquidity in \( t = 2 \). Given \( \sigma \) and \( p \), the government decides on whether to bail out an illiquid firm in \( t = 2 \). The bailout occurs when it is expected to give an ex post social surplus \(- (\sigma pq_s^c + (1 - \sigma) pq_r^c)B - (1 - \sigma)(1 - p)q_r^nL > 0\). If not, the bailout does not occur, and the government receives zero social welfare when illiquidity arrives in \( t = 2 \).

From an ex ante perspective, the government want a firm to choose the safe project, i.e., \( W_2(\sigma, p) \) is increasing with \( \sigma \) for any \( p \). First, the liquidity shock is unlikely to occur if the later firm chooses the risky project. Second, even in the event of illiquidity, the government believes that illiquidity of a firm arises from economic crisis. Thus, the expected social surplus of bailout also increases as it becomes more likely that the later firm chooses the safe project.

When illiquidity hits a firm in \( t = 1 \), the government’s willingness to bail out the illiquid
firm depends on the signal realization $\eta \in \{n, c\}$. If the government bails out this illiquid firm, the long-term expected welfare is

$$p_1^n B - (1 - p_1^n) L + \delta W_2(\sigma_B, p_1^n).$$

(17)

If the government decides not to bail out, the expected social welfare is

$$0 + \delta W_2(\sigma_N, p_1^n).$$

(18)

Therefore, the government with the signal $\eta \in \{n, c\}$ bails out an illiquid firm in $t = 1$ if and only if

$$p_1^n B - (1 - p_1^n) L \geq \delta (W_2(\sigma_N, p_1^n) - W_2(\sigma_B, p_1^n)).$$

(19)

Because $W_2(\sigma, p)$ is increasing with $\sigma$, the right-hand side is positive if and only if $\sigma_N > \sigma_B$.

As in the one-period benchmark, I characterize the equilibrium with respect to the prior belief $p_0$. For exposition, I assume the following:

**Assumption 1.** $p_1^g B - (1 - p_1^g) L \geq 0$.

This assumption implies $p_1^g < p_1^s$. In the range of the belief $p \in (p_1^g, p_1^f)$, the firm takes excessive liquidity risk by the highest degree ($\sigma = 0$) conditional on bailout with certainty ($\beta = 1$). Even without this assumption, the resulting equilibrium outcome belongs to a boundary case of our main result.

The equilibrium bailout strategy in $t = 1$ varies with the prior belief about the underlying state $p_0$: for a low range of $p_0$, the government bails out an illiquid firm only if it receives $\eta = c$; for a high range of $p_0$, the government bails out the even after $\eta = n$. With a low prior belief $p_0$, the government cannot have an incentive to bail out an illiquid firm unless it receives the signal $\eta = c$. The government’s indifference in its bailout decision after $\eta = c$ pins down the period-1 firm’s equilibrium mixed strategy. With a high prior belief $p_0$, the government is strongly willing to bail out the illiquid firm after $\eta = c$. In equilibrium, the government’s indifference in its bailout decision after $\eta = n$ pins down the period-1 mixed strategy in equilibrium.

**Proposition 2.** There is a cutoff $p_0^* \in [0, 1]$ such that

1. For $p_0 \leq p_0^*$, an equilibrium with $\beta_c^* \in (0, 1]$ and $\beta_n^* = 0$ arises.
2. For $p_0 > p_0^*$, an equilibrium with $\beta_c^* = 1$ and $\beta_n^* \in (0, 1]$ arises.

Proposition 2 has an important feature; bailing out a distressed firm in $t = 1$ indicates that the government is more likely to hold the posterior belief $p_1^c$ than $p_1^n$ because $\beta_c^* \geq \beta_n^*$ for every $p_0$. In this respect, a bailout has two opposite effects on future moral hazard. On
The government has an incentive not to bail out a firm after $\eta = c$. On the other hand, it indicates an increased chance that the economy is in crisis that discourages the later firm from excessive risk-taking. On the other hand, the government indicates an increased likelihood of future bailout that encourages the later firm to increase its risk taking.

Which of these two conflicting effects dominates determines the government’s stance on bailing out a distressed firm. If the bailout induces a later firm in the next period to increase its risk taking ($\sigma_B < \sigma_N$), then the government has an incentive not to bail out a distressed firm in the current period. In equilibrium, the government is less likely to bail out than it would be without any consideration of future impact of its decision. Otherwise, if the bailout discourages the later firm from taking liquidity risk ($\sigma_B > \sigma_N$), the government has an incentive to bail out a distressed firm. In equilibrium, the government is more likely to bail out than it would be without the long-term consideration.

In the following analysis, I show that which of these two effects dominates varies with the prior belief $p_0$. To this end, I investigate the sign of $\sigma_N - \sigma_B$ in the equilibria characterized in Proposition 2. For the low range of the prior beliefs ($p_0 < p_s^{**}$), the equilibrium with $\beta_c^{**} > 0$ and $\beta_n^{**} = 0$ arises. For the low range of the prior beliefs ($p_0 \geq p_s^{**}$), the equilibrium with $\beta_c^{**} = 1$ and $\beta_n^{**} > 0$ arises.

First, consider a low prior belief $p_0 \leq p_s^{**}$. Given the low prior belief, the posterior belief after $\eta = c$ is likely to be lower than the threshold $p_f^{**}$, and therefore, the bailout may not be able to induce the later firm to reduce its risk exposure. The following lemma shows that the current bailout creates future moral hazard in the equilibrium arising from $p_0 \leq p_s^{**}$.

**Lemma 3.** $\sigma_N \geq \sigma_B$ in the equilibrium with $\beta_c^{**} \in (0, 1)$ and $\beta_n^{**} = 0$.

Knowing this effect of the bailout, the government is less willing to bail out an illiquid firm in $t = 1$ than it would be in the static economy. In equilibrium with $\delta > 0$, the government takes a tougher stance in equilibrium, bailing out less frequently than with
δ = 0. The dynamic incentive for a bailout decision also influences the period-1 firm’s risk-taking behavior. Compared to δ = 0, the period-1 firm decreases its risk taking in equilibrium.

To see the government’s tough stance on a bailout decision more clearly, take a prior belief \( p_0 \) such that the equilibrium with randomization in the bailout in \( t = 1 \) only after \( \eta = c \) arises in both cases \( \delta > 0 \) and \( \delta = 0 \). Let \( \beta^*_\eta \) denote the government’s equilibrium bailout strategy with full discount on the period-2 welfare \( \delta = 0 \); let \( \sigma^{**}_0 \) denote the equilibrium mixed strategy to choose the safe project in \( t = 1 \). With \( \delta > 0 \), the indifference of the government with \( \eta = c \) pins down the period-1 firm’s equilibrium strategy \( \sigma^{**} \):

\[
p_1^c B - (1 - p_1^c)L = \delta [W_2(\sigma_N, p_1^c) - W_2(\sigma_B, p_1^c)].
\]

Likewise, the firm’s indifference in its project choice also pins down \( \beta^*_c \), given \( \beta^{**}_n = 0 \):

\[
(\lambda p_1(\sigma^{**}, p_0) + (1 - \lambda)(1 - p_1(\sigma^{**}, p_0)))\beta^*_c = \bar{\beta}(p_0),
\]

where \( \bar{\beta} \) is the bailout probability pinned down by the firm’s indifference.\(^7\) With the full discount on the period-2 welfare \( \delta = 0 \), the equilibrium strategies \( \beta^*_c, \beta^*_n, \) and \( \sigma^{**}_0 \) – are also pinned down by their counterparts’ indifference conditions: \( \sigma^{**}_0 \) is pinned down by

\[
p_1^c B - (1 - p_1^c)L = 0,
\]

and \( \beta^*_c > 0 \) (given \( \beta^{**}_n = 0 \)) is pinned down by

\[
(\lambda p_1(\sigma^{**}_0, p_0) + (1 - \lambda)(1 - p_1(\sigma^{**}_0, p_0)))\beta^*_c = \bar{\beta}(p_0).
\]

Because \( \sigma_N \geq \sigma_B \) by Lemma 3, we have \( \sigma^{**} \geq \sigma^{**}_0 \). Additionally, we have \( \beta^{**}_c \leq \beta^*_c \) because \( p_1(\sigma, p) \) is increasing with \( \sigma \).

**Proposition 3.** There is a cutoff \( p^{**}_B \in [0, 1] \) such that for all \( p_0 \leq p^{**}_B \),

(i) With \( \delta > 0 \), the government is less likely to bail out an illiquid firm in \( t = 1 \) than it would be with \( \delta = 0 \).

(ii) With \( \delta > 0 \), the period-1 firm is more likely to choose the safe project than the government would be to decide whether to bail out with \( \delta = 0 \).

The government’s tough stance on the bailout may have a positive welfare effect in the

\(^7\)More precisely, \( \bar{\beta} \) induces the indifference of the firm in its project choice:

\[(1 - p_0 q^c_s)\bar{v}_s + p_0 q^c_s \bar{\beta} \bar{v}_s = (1 - (p_0 q^c_r + (1 - p_0) q^n_r))\bar{v}_r + (p_0 q^c_r + (1 - p_0) q^n_r)\bar{z}_r.\]

Since the firm is more likely to be hit by the liquidity shock with a higher \( p_0 \), \( \bar{\beta} \) is an increasing function of \( p_0 \).
The government has an incentive to bail out a firm after $\eta = n$. A decreased chance of receiving a bailout discourages the period-1 firm from risk taking, yielding $\sigma_0^* < \sigma^{**}$ as shown in Proposition 3. In other words, the government’s desire to control future moral hazard restrains the firm in $t = 1$ from taking excessive liquidity risk.

This welfare result coincides with Nosal and Ordoñez (2013), whereas it differs in the motive of the low frequency of bailouts. In their work, the government is willing to learn more by observing incidents of financial distress until it is strongly convinced that the economy is in crisis. Such a delayed intervention motivated by its desire to learn serves as a strategic restraint. However, in this model, the government’s reluctance to bail out a distressed firm stems from its desire to control future moral hazard; the bailout encourages a later firm to increase risk taking.

Next, consider a high prior belief ($p_0 \geq p_0^{**}$) from which the equilibrium with $\beta_c^{**} = 1$ and $\beta_n^{**} > 0$ arises. With a high prior $p_0$, the posterior belief after $\eta = c$ can be greater than the cutoff $p_f^*$. If $p_c^f > p_f^*$, the later firm in $t = 2$ may reduce its risk exposure after observing the bailout in $t = 1$. The following lemma claims that the bailout decreases future moral hazard in equilibrium.

**Lemma 4.** $\sigma_N \leq \sigma_B$ in the equilibrium with $\beta_c^{**} = 1$ and $\beta_n^{**} \in (0, 1)$.

In the equilibrium, the government takes an *alarmist* stance, bailing out more frequently than it would with $\delta = 0$. By Lemma 4, the bailout in $t = 1$ serves as an alert to the later firm in $t = 2$ of the economy-wide crisis. The government wants to utilize this positive effect of bailing out a distressed firm, creating an incentive to bail out in $t = 1$. Regarding this dynamic incentive for a bailout decision, the period-1 firm increases its risk taking compared to the case with $\delta = 0$.

To see the *alarmist* approach more clearly, take a prior belief $p_0 > p_0^{**}$ for which the government randomizes in bailing out a distressed firm with a positive probability after
\( \eta = n \) in both cases \( \delta = 0 \) and \( \delta > 0 \). With \( \delta > 0 \), the period-1 firm’s equilibrium strategy \( \sigma^{**} \) is pinned down by the indifference for the government with \( \eta = n \) in its bailout:

\[
p_1^n B - (1 - p_1^n) L = \delta [W_2(\sigma_N, p_1^n) - W_2(\sigma_B, p_1^n)].
\]

Likewise, the government’s equilibrium bailout strategy \( \beta^{**}_n > 0 \) is also pinned down by the firm’s indifference in its project choice, given \( \beta^{**}_c = 1 \):

\[
(\lambda p_1 + (1 - \lambda)(1 - p_1)) + (\lambda(1 - p_1) + (1 - \lambda)p_1)\beta^{**}_n = \bar{\beta}(p_0),
\]

where \( \bar{\beta} \) is the bailout probability which pins down the firm’s indifference with given prior belief \( p_0 \). Similarly, for \( \delta = 0 \), the period-1 equilibrium strategies – \( \beta^{*}_c, \beta^{*}_n, \) and \( \sigma^{**}_0 \) – are pinned down by their counterparts’ indifference conditions: \( \sigma^{**}_0 \) is pinned down by

\[
p_1^n B - (1 - p_1^n) L = 0.
\]

Also, \( \beta^{*}_n > 0 \) is pinned down by the firm’s indifference, given \( \beta^{*}_c = 1 \):

\[
(\lambda p_1 + (1 - \lambda)(1 - p_1)) + (\lambda(1 - p_1) + (1 - \lambda)p_1)\beta^{**}_n = \bar{\beta}(p_0),
\]

\( \sigma_N \leq \sigma_B \) by Lemma 4, so we have \( \sigma^{**} \leq \sigma^{**}_0 \). In addition, we have \( \beta^{**}_n \geq \beta^{*}_n \) because \( p_1 \equiv p_1(\sigma, p) \) is increasing with \( \sigma \).

**Proposition 4** (An “Alarmist” Approach). There is a cutoff \( p^{**}_A \in [0, 1] \) such that for all \( p_0 \geq p^{**}_A \),

(i) With \( \delta > 0 \), the government is more likely to bail out the illiquid firm in \( t = 1 \) than it would be with \( \delta = 0 \).

(ii) With \( \delta > 0 \), the period-1 firm is less likely to choose the safe project than the government would be to decide whether to bail out with \( \delta = 0 \).

**Proposition 4** suggests a new role of government bailouts: bailouts can credibly alert an impending economic crisis to future firms, thereby disciplining their liquidity management. Although this new role of bailouts appears to be in conflict with the conventional wisdom that bailouts exacerbate future moral hazard, it is consistent with the observations in the wake of the recent financial crisis. First, as discussed earlier, the major investment banks reacted to the bailout of Bear Stearns by reducing their exposure to liquidity risks. Second, on the verge of the Bear Stearns crisis, the announcement of a liquidity injection program (Term Securities Lending Facility) signified a sign of the policymakers’ knowledge that this investment bank was in financial distress (Brunnermeier, 2009).

The **alarmist** approach to a bailout decision may worsen the moral hazard of the current
firm. With a high prior belief $p_0 \geq p_A^{**}$, a bailout induces a later firm in $t = 2$ to decrease its risk-taking. The dynamic incentive to control future moral hazard leads to an increased frequency of bailing out a distressed firm in $t = 1$ ($\beta_n^{**} > \beta_n^*$). An increased likelihood of receiving a bailout encourages the period-1 firm to increase its risk-taking ($\sigma^{**} < \sigma_0^*$).

Figure 2 graphically illustrates the main results in Proposition 3 and 4. For the low range of the prior belief $p_0 \leq p_B^{**}$, the government has a desire to control moral hazard and to signal its private belief $\eta = n (\beta_c^{**} < \beta_c^*)$ by not bailing out a distressed firm. The resulting unwillingness to bail out induces the period-1 firm to lower its risk taking comparing to the static economy ($\sigma^{**} > \sigma_0^*$). For the high range of the prior belief $p_0 \geq p_A^{**}$, the government uses the bailout as a warning of the economic crisis, thereby discouraging the later firm from its risk-taking ($\beta_n^{**} > \beta_n^*$). The increased chance of bailout induces the period-1 firm to take more liquidity risk than in the static economy ($\sigma^{**} < \sigma_0^*$).

7 Concluding Remarks

This paper provides a framework for analyzing how uncertainties about the underlying state of the economy influence the strategic interaction between a bailout decision by a government and risk taking by current and future firms. In the one-period benchmark, the inference problem on the source of financial distress of a firm produces interesting implications. First, the government randomizes in bailing out the firm in case it is distressed for a low range of the prior belief about the state, which reflects a dilemma that the policymakers often face in their decision on bailouts. Second, the firm takes the highest risk for an intermediate range of the prior belief about the state. Intuitively, the firm exploits the government’s inability to disengage risk taking by this firm from the state of economic crisis most effectively when the government suffers from the uncertainties most severely.

Using these results, I further study long-term effects of a bailout on future moral hazard and how these effects influence the government’s decision on bailing out a distressed firm. To this end, I consider a two-period model in which the government possesses private information about the underlying state. In this setting, a bailout decision in the current period can affect the belief about the state held by a firm in the later period. Such a belief manipulation has two opposing effects. On the one hand, the bailout indicates an increased possibility of crisis, which tends to discourage the later firm from risk taking. On the other hand, the bailout also signals an increased chance of future bailout, which in turn encourages risk taking.

Which of these two effects dominates determines the government’s stance on bailout. When the prior probability that the economy is in crisis is low, the later firm responds to
The Equilibrium Investment Strategy in $t = 1$

The Equilibrium Bailout Strategy after $\eta = c$

The Equilibrium Bailout Strategy after $\eta = n$

Figure 2 – The period-1 equilibrium strategies in the two-period economy with respect to $p_1 \in (0, 1)$. The blue lines indicate the equilibrium strategies with $\delta > 0$ and the red lines depict the equilibrium strategies with $\delta = 0$. 

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the current bailout by increasing its risk taking. Regarding this effect, the government takes a tougher stance on bailout which leads to a decreased likelihood of bailing out a distressed firm. More importantly, when the prior probability of crisis is high, the bailout warns the later firm of a strong possibility of crisis, which reduces the later firm’s risk exposure. Therefore, the government takes an “alarmist” stance, bailing out a distressed firm more frequently than it would without the long-term consideration. This alarmist approach gives government bailouts a new role as a credible alert that controls future moral hazard.

References


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Appendix: Proofs in Section 4

Proof of Proposition 1. First consider $p_0 > p_f^*$. By Lemma 2, the firm’s equilibrium strategy is $\sigma^* = 1$ when the government’s bailout strategy is $\beta^* = 1$. The associated posterior belief after the liquidity shock is $p_1 = 1$, thus $\beta^* = 1$. This strategy profile constitutes the equilibrium.

Second, consider $p_0 \in (p_g^*, p_f^*)$. By Lemma 1, the government’s bailout strategy is $\beta^* = 1$ for all $\sigma \in [0, 1]$. Since $p_0 \leq p_f^*$, the firm’s optimal asset choice is the risky asset $\sigma^* = 0$, which constructs the equilibrium.

Lastly, consider $p_0 \leq p_g^*$. I show in the text that only the mixed strategy equilibrium arises for all $p_0 \leq p_g^*$. Therefore, the firm’s mixed strategy $\sigma^*$ supports the government indifference condition:

$$p_1 B - (1 - p_1) L = 0,$$

which yields

$$\sigma^* = \frac{\left(\frac{1 - p_0}{p_0} \times \frac{L}{B} q^n - q^c\right)}{q^c + \left(\frac{1 - p_0}{p_0} \times \frac{L}{B} q^n - q^c\right)}.$$

The government’s mixed strategy $\beta^*$ yields the firm’s indifference condition:

$$(1 - p_0 q^c) V_s + p_0 q^c \bar{v}_s = (1 - (p_0 q^c + (1 - p_0) q^n)) V_r + (p_0 q^c + (1 - p_0) q^n) \bar{v}_r.$$

This strategy profile constitutes the equilibrium: given $\beta^*(p_0)$, the government is indifferent between bailing out the illiquid firm and not, so randomization in the bailout with probability $\beta^*(p_0)$ is a best response; given $\beta^*(p_0)$, the firm is indifferent between the two assets, so the mixed strategy $\sigma^*(p_0)$ is a best response. Q.E.D.

Appendix: Proofs in Section 6

B.1 Proofs of Lemma 3 and 4

Proof of Lemma 3. Suppose to the contrary that $\sigma_N < \sigma_B$. The indifference condition for the government with $\eta = c$ is

$$p_1 c B - (1 - p_1 c) L = \delta(W_2(\sigma_N, p_1 c) - W_2(\sigma_B, p_1 c)) < 0.$$

On the left hand side, we have $p_1 c B - (1 - p_1 c) L < 0$ which implies $p_1 c < p_f^*$ by Assumption 1. On the right hand side, $\sigma_B$ can be either $\sigma^*(p_f^*)$ or $\sigma_f^*(p_2^N)$. Since $p_f^c$ is greater than $p_f^N$ and $p_1 c, \sigma_N < \sigma_B$ implies $p_1 c \geq p_f^*$, a contradiction. Q.E.D.
Proof of Lemma 4. Suppose to the contrary that \( \sigma_N > \sigma_B \). The indifference condition for the government with \( \eta = n \) is

\[
p^n_B(1 - p^n_1)L = \delta(W_2(\sigma_N, p^n_1) - W_2(\sigma_B, p^n_1)) > 0.
\]

On the left hand side, \( p^n_B(1 - p^n_1)L > 0 \), which implies \( p^n_1 > p^*_g \). Since \( \sigma_B \geq 0 \), \( \sigma_N = \sigma^*_f(p^n_1) = 1 \). However, \( \sigma_B = \sigma^*_f(p^B_1) \) with \( p^B_1 > p^n_1 \), so \( \sigma_B = 1 \), which is a contradiction because \( \sigma_N - \sigma_B = 1 - 1 = 0 \).

Q.E.D.

B.2 Proof of Proposition 2

I prove Proposition 2 by characterizing the equilibrium in the two-period economy with the private signal \( \eta \in \{n,c\} \).

Before I start the proof, recall the regularity conditions in Assumption 1:

\[
p^*_B(1 - p^*_1)L \geq 0.
\]

Also, recall from Lemma 3 and 4 that

a) \( \sigma_N \geq \sigma_B \) when \( \beta^{**}_c \in (0,1) \) and \( \beta^{**}_n = 0 \); and

b) \( \sigma_N \leq \sigma_B \) when \( \beta^{**}_c = 1 \) and \( \beta^{**}_n \in (0,1) \).

I first characterize the equilibrium when \( \sigma_N \neq \sigma_B \) for the two bailout strategy profiles above, and then generalizes the result.

First consider \( \beta^{**}_c > 0 \) and \( \beta^{**}_n = 0 \). After the bailout in \( t = 1 \), the period-2 firm believes \( p^B_2 = p^c_1 \) since \( \beta^{**}_n = 0 \), so we have \( \sigma_B = \sigma^*(p^c_1) \). Moreover, \( \sigma_N - \sigma_B > 0 \) implies \( p^n_B(1 - p^n_1)L > 0 \). This gives the following properties: \( \sigma^*(p^c_1) = 0 \) and \( \sigma_N = \sigma^*(p^n_1) \). We can rewrite the indifference condition for the government with \( \eta = c \) as follows:

\[
p^n_B(1 - p^n_1)L = \delta(W(\sigma^*(p^n_1), p^n_1) - W(\sigma^*(p^c_1), p^c_1)). \tag{B.1}
\]

Let \( \hat{\sigma}_c \) be the firm’s mixed strategy to pin down (B.1). Moreover, the bailout strategy pins down the indifference condition for the period-1 firm with its (mixed) strategy \( \hat{\sigma}_c \):

\[
(\lambda p_1(\hat{\sigma}_c, p_0) + (1 - \lambda)(1 - p_1(\hat{\sigma}_c, p_0))) \times \beta^{**}_c = \overline{\beta}(p_0), \tag{B.2}
\]

where \( \lambda p_1 + (1 - \lambda)(1 - p_1) \) is the probability that the period-1 firm believes \( \eta = c \) after the liquidity shock. Recall \( \overline{\beta}(p_0) \) is the bailout probability after the liquidity shock which induces the firm’s indifference condition before the liquidity shock arrives:

\[
(1 - p_0 q^c_s)\overline{V}_s + p_0 q^c_s \overline{\beta} v_s = (1 - (p_0 q^c_s + (1 - p_0) q^n_r))\overline{V}_r + (p_0 q^c_s + (1 - p_0) q^n_r) \overline{\beta} v_r.
\]
Also, \( \beta \) is increasing with \( p_1 \): the more likely the economy is in crisis, the more the firm prefers to choose the safe asset with the same bailout probability.

Next, consider the period-1 bailout strategy \( \beta^{**}_c = 1 \) and \( \beta^{**}_n \in (0, 1) \). The indifference condition for the government with \( \eta = n \) is then equal to

\[
p_1^n B - (1 - p_1^n) L = \delta (W_2(\sigma_N, p_1^n) - W_2(\sigma_B, p_1^n)).
\]

Since the bailout does not occur only after \( \eta = n \), we have \( \sigma_N < \sigma_B \) by Lemma 4, which implies \( p_1^c > p_2^B \geq p_1^f \). Let \( \hat{\sigma}_n \) be the period-1 firm’s mixed strategy to pin down the indifference condition (B.3). Moreover, the period-1 bailout strategy induces the period-1 firm’s indifference condition, given the firm chooses the safe asset with probability \( \hat{\sigma}_n \):

\[
(\lambda p_1(\hat{\sigma}_n, p_0) + (1 - \lambda)(1 - p_1(\hat{\sigma}_n, p_0))) \\
+ (\lambda(1 - p_1(\hat{\sigma}_n, p_0)) + (1 - \lambda)p_1(\hat{\sigma}_n, p_0)) \times \beta^{**}_n = \beta(p_0).
\]

The following lemma provides a useful property for equilibrium characterization:

**Lemma B1.** For all \( p_0 \in (0, 1) \), \( \hat{\sigma}_c < \hat{\sigma}_n \).

**Proof.** The posterior belief \( p_1^c \) associated with \( \hat{\sigma}_c \) gives \( \sigma^*(p_1^c) = 0 \), which implies \( p_1^c \leq p_1^f \). However, the period-1 firm’s strategy \( \hat{\sigma}_n \) gives \( p_1^c > p_2^B \geq p_1^f \), which yields

\[
p_1^c(\hat{\sigma}_c, p_0) \leq p_1^c(\hat{\sigma}_n, p_0).
\]

As a result, we have \( \hat{\sigma}_c < \hat{\sigma}_n \). \( Q.E.D. \)

This lemma can be generalized for the case \( \sigma_B = \sigma_N \).

By Lemma B1, we have the following observation:

\[
(\lambda p_1(\hat{\sigma}_c, p_0) + (1 - \lambda)(1 - p_1(\hat{\sigma}_c, p_0))) < (\lambda p_1(\hat{\sigma}_n, p_0) + (1 - \lambda)(1 - p_1(\hat{\sigma}_n, p_0)));
\]

which has the following implications:

(i) if an equilibrium with \( \beta^{**}_c > 0 \) and \( \beta^{**}_n = 0 \) arises from a prior \( p_0 \), no equilibrium with \( \beta^{**}_c = 1 \) and \( \beta^{**}_n > 0 \) arises from the same prior \( p_0 \); and

(ii) if an equilibrium with \( \beta^{**}_c = 1 \) and \( \beta^{**}_n > 0 \) arises from a prior \( p_0 \), no equilibrium with \( \beta^{**}_c > 0 \) and \( \beta^{**}_n = 0 \) arises from the same prior \( p_0 \).
Since $\bar{\beta}$ is increasing with $p_0$, the equilibrium with $\beta_{c}^{**} > 0$ and $\beta_{n}^{**} = 0$ arises from a relatively low $p_0$, and the equilibrium with $\beta_{c}^{**} = 1$ and $\beta_{n}^{**} > 0$ arises from a relatively high $p_0$. In the following subsections, I characterize the equilibria with respect to $p_0 \in (0, 1)$.

**B.2.1 Case 1: $\beta_{c}^{**} \in (0, 1)$ and $\beta_{n}^{**} = 0$.**

First consider a bailout strategy $\beta_{c}^{**} \in (0, 1)$ and $\beta_{n}^{**} = 0$. The indifference condition in bailout with the signal $\eta = n$ is equal to

$$p_{c}^B - (1 - p_{c}^B)B - \delta(W_2(\sigma^*(p_{c}^n), p_{c}^n) - W_2(\sigma^*(p_{c}^B), p_{c}^B)) = 0. \tag{B.1}$$

Also, Lemma B1 implies $p_{c}^n(\hat{\sigma}_c, p_0) < p_{c}^n(\hat{\sigma}_n, p_0)$, so we have

$$p_{c}^n(\hat{\sigma}_c, p_0)B - (1 - p_{c}^n(\hat{\sigma}_c, p_0))L < p_{c}^n(\hat{\sigma}_n, p_0)B - (1 - p_{c}^n(\hat{\sigma}_n, p_0))L < \delta(W_2(\sigma^*(p_{c}^n), p_{c}^n) - W_2(\sigma^*(p_{c}^B), p_{c}^B)).$$

Therefore, the government strictly prefers not to bail out an illiquid firm in $t = 1$ after $\eta = n$.

The equilibrium strategy profile is determined as follows: first, the period-1 firm’s equilibrium strategy pins down the indifference of the government with $\eta = c$ in (B.1). Hence $\sigma^{**} = \hat{\sigma}_c$. Second, the bailout strategy pins down the indifference in asset choice of the period-1 firm:

$$(\lambda p_1(\sigma^{**}, p_0) + (1 - \lambda)(1 - p_1(\sigma^{**}, p_0)))\beta_{c}^{**} = \bar{\beta}(p_0). \tag{B.2}$$

This strategy profile constructs the equilibrium.

**B.2.2 Case 2: $\beta_{c}^{**} = 1$ and $\beta_{n}^{**} = 0$**

Recall that $\hat{\sigma}_c$ decreases with $p_0$ and $\bar{\beta}$ increases with $p_0$. Consider the prior belief $p_0$ exceeds a upper bound of the range for the equilibrium in subsubsection B.2.1 to be sustained, but not sufficiently high to support $\beta_{n}^{**} > 0$. There are two possible equilibrium outcomes: the period-1 firm chooses the risky asset ($\sigma^{**} = 0$); or its asset choice is randomized ($\sigma^{**} \in (0, 1)$.)

In the first case, the equilibrium bailout strategy is conjectured as $\beta_{c}^{**} = 1$ and $\beta_{n}^{**} = 0$. When $\sigma^{**} = 0$ characterizes the equilibrium, the government prefers to bail out an illiquid firm with $\eta = c$ in $t = 1$ if and only if

$$p_{c}^B - (1 - p_{c}^B)L \geq \delta(W_2(\sigma^*(p_{c}^n), p_{c}^n) - W_2(\sigma^*(p_{c}^B), p_{c}^B)) \geq 0;$$

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but it would not bail out the firm with \( \eta = n \):

\[
p_1^n B - (1 - p_1^n) L \leq \delta(W_2(\sigma^*(p_1^n), p_1^n) - W_2(\sigma^*(p_1^c), p_1^n)) \leq 0.
\]

The period-1 firm prefers to choose the risky asset:

\[
(\lambda p_1(0, p_0) + (1 - \lambda)(1 - p_1(0, p_0))) \geq \beta(p_0).
\]

If a prior belief \( p_0 \) satisfies these conditions, then this equilibrium is characterized.

When \( \sigma^{**} \in (0, 1) \) characterizes the equilibrium, the government’s preference on a bailout is same to the previous case, while the probability has to pin down the firm’s indifference in its asset choice:

\[
(\lambda p_1(\sigma^{**}, p_0) + (1 - \lambda)(1 - p_1(\sigma^{**}, p_0))) = \beta(p_0).
\]

If such a mixed strategy \( \sigma^{**} \in (0, 1) \) exists, then we characterize the equilibrium.

**B.2.3 Case 3: \( \sigma^c_g = 1 \) and \( \sigma^n_g \in (0, 1) \)**

Consider a prior belief \( p_0 \) such that the firm’s strategy \( \sigma^{**} \) in the equilibrium in subsubsection B.2.2 yields \( p_1^c \geq p_1^f \). If \( p_1^n B - (1 - p_1^n) L \geq \delta(W_2(\sigma^*(p_1^n), p_1^n) - W_2(\sigma^*(p_1^c), p_1^n)) \) with these \( p_0 \) and \( \sigma^{**} \), then there is a mixed strategy of the period-1 firm \( \sigma^{**} = \hat{\sigma}_n \in (0, 1) \) which pins down the indifference in a bailout for the government with \( \eta = n \):

\[
p_1^n B - (1 - p_1^n) L = \delta(W_2(\sigma^*(p_1^n), p_1^n) - W_2(\sigma_B, p_1^n)). \tag{B.3}
\]

In addition, the government’s strategies \( \beta^{**}_c = 1 \) and \( \beta^{**}_n > 0 \) pin down the firm’s indifference in its asset choice:

\[
(\lambda p_1(\hat{\sigma}_n, p_0) + (1 - \lambda)(1 - p_1(\hat{\sigma}_n, p_0))) \\
+ (\lambda(1 - p_1(\hat{\sigma}_n, p_0)) + (1 - \lambda)p_1(\hat{\sigma}_n, p_0)) \times \beta^{**}_n = \beta(p_0).
\]

Finally, the period-2 firm’s updated belief after the bailout has to be sufficiently high: \( p_2^B \geq p_2^f \). If \( p_2^B > p_2^f \), then those period-1 strategies characterize the equilibrium with \( \sigma_B = 1 \). If not, find the period-2 firm’s mixed strategy \( \sigma_B \geq \sigma^*(p_1^B) \) such that

\[
p_1^n B - (1 - p_1^n) L = \delta(W_2(\sigma^*(p_1^n), p_1^n) - W_2(\sigma_B, p_1^n)),
\]

subject to \( p_2^B = p_2^f \) and \( \text{(B.4)} \). The associated mixed strategy profiles characterize the equilibrium.
B.2.4 Case 4: $\sigma^c_g = 1$ and $\sigma^n_g = 1$

Finally, there is no strategy profile to satisfy the equilibrium conditions in the previous equilibrium cases. Then there is an equilibrium characterized as follows: the government bails out with probability 1 ($\beta^c_{\eta^*} = \beta^n_{\eta^*} = 1$); and the period-1 firm chooses the risky asset ($\sigma^{**} = 0$) if $p_0 \leq p^*_f$ and the safe asset ($\sigma^{**} = 1$) otherwise if $p_0 > p^*_f$.

B.3 Proof of Proposition 3 and 4

Consider the government fully discounts the period-2 expected welfare, i.e. $\delta = 0$. Denote the government’s equilibrium bailout strategy with full discount by $\beta^\eta_{\eta^*}$ for $\eta \in \{n, c\}$. Also, denote the period-1 firm’s equilibrium strategy by $\sigma^*_0$. In this case, the government bails out an illiquid firm after the signal $\eta \in \{n, c\}$ if and only if

$$p^0_1 B - (1 - p^0_1) L \geq 0.$$  

Analogous to Lemma B1 and Proposition 2, we have the same equilibrium characterization result as with long-term consideration in a bailout ($\delta > 0$): for a low $p_0$, the equilibrium with $\beta^c_{\eta^*} > 0$, $\beta^n_{\eta^*} = 0$, and $\sigma^*_0 = 0$ arises; for a high $p_1$, the equilibrium with $\beta^c_{\eta^*} = 1$, $\beta^n_{\eta^*} > 0$, and $\sigma^*_0 > 0$ arises.

Proof of Proposition 3. Consider the period-1 firm’s strategy $\sigma^*_0$ which pins down the following indifference condition:

$$p_1(\sigma^*_0, p_0, \eta = c) B - (1 - p_1(\sigma^*_0, p_0, \eta = c)) L = 0.$$  

Since $\sigma^*_N \geq \sigma^*_B$ by Lemma 3, we have $\sigma^{**} \geq \sigma^*_0$ when $\sigma^{**}$ is the equilibrium strategy to pin down the analogous indifference condition in a bailout with $\delta > 0$, (B.1). Therefore, we have $p_1(\sigma^{**} p_0) \geq p_1(\sigma^*_0, p_0)$, which implies

a. If the equilibrium with $\beta^c_{\eta^*} \in (0, 1)$ and $\beta^n_{\eta^*} = 0$ arises from a prior belief $p_0$ with $\delta > 0$, the equilibrium bailout strategy with the same $p_0$ under $\delta = 0$ is $\beta^c_{\eta^*} \in (0, 1]$ and $\beta^n_{\eta^*} = 0$ with $\delta = 0$.

b. If the equilibrium with $\beta^c_{\eta^*} \in (0, 1)$ and $\beta^n_{\eta^*} = 0$ arises from a prior belief $p_0$ with $\delta = 0$, the equilibrium with with $\beta^c_{\eta^*} \in (0, 1)$ and $\beta^n_{\eta^*} = 0$ also arises from the same $p_1$ with $\delta > 0$.

Therefore, there is a cutoff $p^*_B$, denoted by

$$p^*_B = \arg\max\{p_0 \in (0, 1) : \sigma^{**} = 0 \text{ or } \beta^c_{\eta^*} = 1 \text{ with } \delta > 0\},$$

such that (i) $\beta^n_{\eta^*} = 0$; (ii) $\beta^c_{\eta^*} \leq \beta^c_{\eta^*}$; and (iii) $\sigma^{**} \geq \sigma^*_0$ for all $p_0 \leq p^*_B$. Q.E.D.
Proof of Proposition 4. Consider the period-1 firm’s strategy \( \sigma_0^{**} \) which pins down the following indifference condition:

\[
p_1(\sigma_0^{**}, p_0, \eta = n)B - (1 - p_1(\sigma_0^{**}, p_0, \eta = n))L = 0.
\]

Since \( \sigma_N \leq \sigma_B \) by Lemma 4, we have \( \sigma^{**} \leq \sigma_0^{**} \) when \( \sigma^{**} \) is the equilibrium strategy to pin down the analogous indifference condition in a bailout with \( \delta > 0 \), (B.1). Therefore, we have \( p_1(\sigma^{**}p_0) \leq p_1(\sigma_0^{**}, p_0) \), which implies the following: If the equilibrium with \( \beta_c^{**} = 1 \) and \( \beta_n^{**} > 0 \) arises from a prior belief \( p_0 \) with \( \delta = 0 \), the equilibrium with \( \beta_c^{**} = 1 \) and \( \beta_n^{**} > 0 \) also arises from the same \( p_0 \) with \( \delta > 0 \). Therefore, there is a cutoff \( p_A^{**} \), denoted by

\[
p_A^{**} = \arg \min \{ p_0 \in (0, 1) : \sigma^{**} = 0 \text{ or } \beta_c^{**} = 1 \text{ with } \delta > 0 \},
\]

such that (i) \( \beta_c^{**} = \beta_c^* = 1 \); (ii) \( \beta_n^{**} \geq \beta_n^* \); and (iii) \( \sigma^{**} \leq \sigma_0^{**} \) for all \( p_0 \leq p_A^{**} \). Q.E.D.