Solutions to HW10

-86 Expected Frequency is found by using the Poisson distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad \text{where } \lambda = [1(1) + 2(11) + \ldots + 7(10) + 8(9)]/75 = 4.907$$

Estimated mean = 4.907

Value	1	2	3	4	5	6	7	8
Observed Frequency	1	11	8	13	11	12	10	9
Expected Frequency	2.7214	6.6770	10.9213	13.3977	13.1485	10.7533	7.5381	4.6237

Since the first category has an expected frequency less than 3, combine it with the next category:

Value	1-2	3	4	5	6	7	8
Observed Frequency	12	8	13	11	12	10	9
Expected Frequency	9.3984	10.9213	13.3977	13.1485	10.7533	7.5381	4.6237

The degrees of freedom are k − p − 1 = 7 − 1 − 1 = 5

a) 1) The variable of interest is the form of the distribution for the number of flaws.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.01$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.01,5}^2 = 15.09$

7)

$$\chi_0^2 = \frac{(12 - 9.3984)^2}{9.3984} + \cdots + \frac{(9 - 4.6237)^2}{4.6237} = 6.955$$

8) Since 6.955 < 15.09 do not reject H_0. We are unable to reject the null hypothesis that the distribution of the number of flaws is Poisson.

b) P-value = 0.2237 (found using Minitab)

9-88 Estimated mean = 10.131

Value	5	6	8	9	10	11	12	13	14	15
Rel. Freq	0.067	0.067	0.100	0.133	0.200	0.133	0.133	0.067	0.033	0.067
Observed (Days)	2	2	3	4	6	4	4	2	1	2
Expected (Days)	1.0626	1.7942	3.2884	3.7016	3.7501	3.4538	2.9159	2.2724	1.6444	1.1106

Since there are several cells with expected frequencies less than 3, the revised table would be:

Value	5-8	9	10	11	12-15
Observed (Days)	7	4	6	4	9
Expected (Days)	6.1452	3.7016	3.7501	3.4538	7.9433

The degrees of freedom are k − p − 1 = 5 − 1 − 1 = 3

a) 1) The variable of interest is the form of the distribution for the number of calls arriving to a switchboard from noon to 1pm during business days.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.05$
5) The test statistic is

6) Reject H_0 if
7)

$$\chi_0^2 = \frac{(7-6.1452)^2}{6.1452} + \frac{(4-3.7016)^2}{3.7016} + \frac{(6-3.7501)^2}{3.7501} + \frac{(4-3.4538)^2}{3.4538} + \frac{(9-7.9433)^2}{7.9433} = 1.72$$

8) Since 1.72 < 7.81 do not reject H_0. We are unable to reject the null hypothesis that the distribution for the number of calls is Poisson.

b) The P-value is between 0.9 and 0.5 using Table IV. P-value = 0.6325 (found using Minitab)

9-91 Estimated mean = 49.6741 use Poisson distribution with λ=49.674
All expected frequencies are greater than 3.
The degrees of freedom are k − p − 1 = 26 − 1 − 1 = 24

a) 1) The variable of interest is the form of the distribution for the number of cars passing through the intersection.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,24}^2 = 36.42$

7) Estimated mean = 49.6741

$$\chi_0^2 = 769.57$$

8) Since 769.57 >>> 36.42, reject H_0. We can conclude that the distribution is not Poisson at $\alpha = 0.05$.
b) P-value = 0 (found using Minitab)

10-33 a) 1) The parameter of interest is the difference in blood cholesterol level, μ_d where d_i = Before − After.
2) H_0 :
3) H_1 :
4) $\alpha = 0.05$
5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

6) Reject the null hypothesis if $t_0 >$ where = 1.761

7) 26.867
19.04
15

$$t_0 = \frac{26.867}{19.04/\sqrt{15}} = 5.465$$

8) Because 5.465 > 1.761 reject the null and conclude the data support the claim that the mean difference in cholesterol levels is significantly less after fat diet and aerobic exercise program at the 0.05 level of significance.

P-value=P($t >$ 5.565) $\cong 0$

b) 95% confidence interval:

$$\bar{d} - t_{\alpha,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d$$

$$26.867 - 1.761\left(\frac{19.04}{\sqrt{15}}\right) \le \mu_d$$

$18.20 \le \mu_d$

Because the lower bound is positive, with 95% confidence the mean difference in blood cholesterol level is significantly less after the diet and aerobic exercise program.