Solutions to Homework 11

1.

15-1 a)

1. The parameter of interest is median of pH.
2. $H_0 : \tilde{\mu} = 7.0$
3 $H_1 : \tilde{\mu} \neq 7.0$
4. α=0.05
5. The test statistic is the observed number of plus differences or $r^+ = 8$.
6. We reject H_0 if the *P-value* corresponding to $r^+ = 8$ is less than or equal to $\alpha = 0.05$.
7. Using the binomial distribution with $n = 10$ and $p = 0.5$, *P-value* = $2P(R^+ \geq 8 \mid p = 0.5) = 0.1$
8. Conclusion: we cannot reject H_0. There is not enough evidence to reject the manufacturer's claim that the median of the pH is 7.0

b)

1. The parameter of interest is median of pH.
2. $H_0 : \tilde{\mu} = 7.0$
3 $H_1 : \tilde{\mu} \neq 7.0$
4. α=0.05
5. The test statistic is $z_0 = \dfrac{r^* - 0.5n}{0.5\sqrt{n}}$
6. We reject H_0 if $|Z_0|>1.96$ for α=0.05.
7. r*=8 and $z_0 = \dfrac{r^* - 0.5n}{0.5\sqrt{n}} = \dfrac{8 - 0.5(10)}{0.5\sqrt{10}} = 1.90$
8. Conclusion: we cannot reject H_0. There is not enough evidence to reject the manufacturer's claim that

the median of the pH is 7.0

P-value = $2[1 - P(|Z_0| < 1.90)] = 2(0.0287) = 0.0574$

15-2 a)

1. The parameter of interest is median titanium content.
2. $H_0 : \tilde{\mu} = 8.5$
3 $H_1 : \tilde{\mu} \neq 8.5$
4. α=0.05
5. The test statistic is the observed number of plus differences or $r^+ = 7$.
6. We reject H_0 if the *P-value* corresponding to $r^+ = 7$ is less than or equal to α=0.05.
7. Using the binomial distribution with n=10 and p=0.5, *P-value* = $2P(R^* \leq 7|p=0.5)=0.359$
8. Conclusion: we cannot reject H_0. There is not enough evidence to reject the manufacturer's claim that

the median of the titanium content is 8.5.

b)

1. Parameter of interest is the median titanium content
2. $H_0 : \tilde{\mu} = 8.5$
3. $H_1 : \tilde{\mu} \neq 8.5$
4. α=0.05
5. Test statistic is $z_0 = \dfrac{r^+ - 0.5n}{0.5\sqrt{n}}$
6. We reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$
7. Computation: $z_0 = \dfrac{7 - 0.5(20)}{0.5\sqrt{20}} = -1.34$
8. Conclusion, cannot reject H_0. There is not enough evidence to conclude that the median titanium content differs from 8.5. The *P-value* = 2*P(|Z| > 1.34) = 0.1802.

15-6 a)

1. Parameters of interest are the median caliper measurements
2. $H_0 : \tilde{\mu}_D = 0$
3. $H_1 : \tilde{\mu}_D \neq 0$
4. α=0.05
5. The test statistic is $r = min(r^+, r^-)$.
6. Because α =0.05 and $n = 12$, Appendix A, Table VIII gives the critical value of $r^*_{0.05} = 2$. We

reject

H_0 in favor of H_1 if $r \leq 2$.

7. There are four ties that are ignored so that $r^+ = 6$ and $r^- = 2$ and $r = min(6,2) = 2$
8. Conclusion, reject H_0. There is a significant difference in the median measurements of the two calipers at α = 0.05.

b)

1. Parameters of interest are the median caliper measurements
2. $H_0 : \tilde{\mu}_D = 0$
3. $H_1 : \tilde{\mu}_D \neq 0$
4. α=0.05
5. Test statistic $z_0 = \dfrac{r^+ - 0.5n}{0.5\sqrt{n}}$
6. We reject H_0 if the $|Z_0| > Z_{0.025} = 1.96$
7. Computation: $z_0 = \dfrac{6 - 0.5(8)}{0.5\sqrt{8}} = 1.41$
8. Conclusion, do not reject H_0. There is not a significant difference in the median measurements of the two calipers at α = 0.05.

The *P-value* = 2[1-P($|Z_0|$ < 1.41) = 0.159.

15-8 a)

1. Parameters of interest are the median drying times for the two formulations
2. $H_0 : \tilde{\mu}_D = 0$
3. $H_1 : \tilde{\mu}_D \neq 0$
4. α=0.01
5. The test statistic is $r = min(r^+, r^-)$.
6. Because α = 0.01 and n = 20, Appendix A, Table VIII gives the critical value of $r^*_{0.01} = 3$. We reject H_0 in favor of H_1 if $r \leq 3$.
7. There are two ties that are ignored so that $r^+ = 3$ and $r^- = 15$ and $r = min(3,15) = 3$
8. Conclusion, reject H_0. There is a significant difference in the drying times of the two formulations at α = 0.01.

b)

1. Parameters of interest are the median drying times of the two formulations
2. $H_0 : \tilde{\mu}_D = 0$
3. $H_1 : \tilde{\mu}_D \neq 0$
4. α=0.01
5. Test statistic $z_0 = \dfrac{r^+ - 0.5n}{0.5\sqrt{n}}$
6. We reject H_0 if the $|Z_0| > Z_{0.005} = 2.58$
7. Computation: $z_0 = \dfrac{3 - 0.5(18)}{0.5\sqrt{18}} = -2.83$
8. Conclusion, reject H_0 and conclude that there is a significant difference between the drying times for the two formulations at α = 0.01.

The *P-value* = 2[1-P($|Z_0|$ < 2.83) = 0.005.

c) *P*-value = 2P($R^- \leq 3$ | p = 0.5) = 0.0075. The exact *P*-value computed here agrees with the normal approximation in Exercise 15-11 in the sense that both calculations would lead to the rejection of H_0.

2. (a) Assuming that the value of _ is known, find the median μ, the value such that

P(X > μ) = 0.5.

SOLUTION:We must find the value of x such that $e^{-\lambda x}$ = 0.50. Taking logarithms,

etc., reduces this to x = $\ln 2 / \lambda$.

(b) Suppose that _ is not known, but instead it is known that the median is μ = 3.5.

Find λ .

SOLUTION: In this case, we must find the value of λ such that $e^{-\lambda \cdot (3.5)}$ = 0.50;

$\lambda = \ln 2/(3.5) = 0.1980.$

3.

11-1 a) $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

$$S_{xx} = 157.42 - \tfrac{43^2}{14} = 25.348571$$

$$S_{xy} = 1697.80 - \tfrac{43(572)}{14} = -59.057143$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{-59.057143}{25.348571} = -2.330$$

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x} = \tfrac{572}{14} - (-2.3298017)(\tfrac{43}{14}) = 48.013$$

$$SS_R = \hat{\beta}_1 S_{xy} = -2.3298017(-59.057143)$$
$$= 137.59$$

$$SS_E = S_{yy} - SS_R$$
$$= 159.71429 - 137.59143$$
$$= 22.123$$

$$\hat{\sigma}^2 = MS_E = \frac{SS_E}{n-2} = \frac{22.123}{12} = 1.8436$$

b) $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$

$$\hat{y} = 48.012962 - 2.3298017(4.3) = 37.99$$

c) $\hat{y} = 48.012962 - 2.3298017(3.7) = 39.39$

d) $e = y - \hat{y} = 46.1 - 39.39 = 6.71$

11-2 a) $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

$$S_{xx} = 143215.8 - \tfrac{1478^2}{20} = 33991.6$$

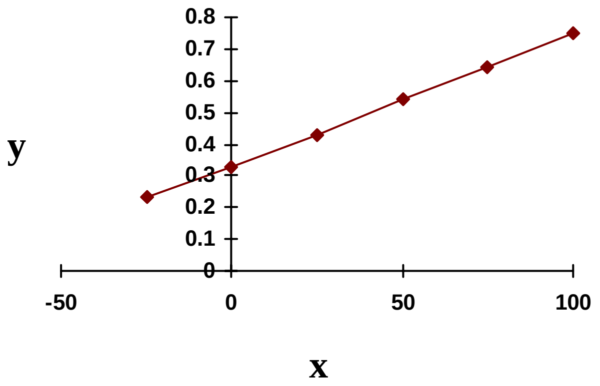
$$S_{xy} = 1083.67 - \tfrac{(1478)(12.75)}{20} = 141.445$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{141.445}{33991.6} = 0.00416$$

$$\hat{\beta}_0 = \tfrac{12.75}{20} - (0.0041617512)(\tfrac{1478}{20}) = 0.32999$$

$$\hat{y} = 0.32999 + 0.00416x$$

$$\hat{\sigma}^2 = MS_E = \frac{SS_E}{n-2} = \frac{0.143275}{18} = 0.00796$$



b) $\hat{y} = 0.32999 + 0.00416(85) = 0.6836$

c) $\hat{y} = 0.32999 + 0.00416(90) = 0.7044$

d) $\hat{\beta}_1 = 0.00416$

11-3 a) The regression equation is

Rating Pts = - 5.56 + 12.7 Yds per Att

Predictor	Coef	SE Coef	T	P
Constant	-5.558	9.159	-0.61	0.549
Yds per Att	12.652	1.243	10.18	0.000

S = 5.71252   R-Sq = 78.7%   R-Sq(adj) = 78.0%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	3378.5	3378.5	103.53	0.000
Residual Error	28	913.7	32.6		
Total	29	4292.2			

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$S_{xx} = 1627.847 - \frac{(219.55)^2}{30} = 21.106$$

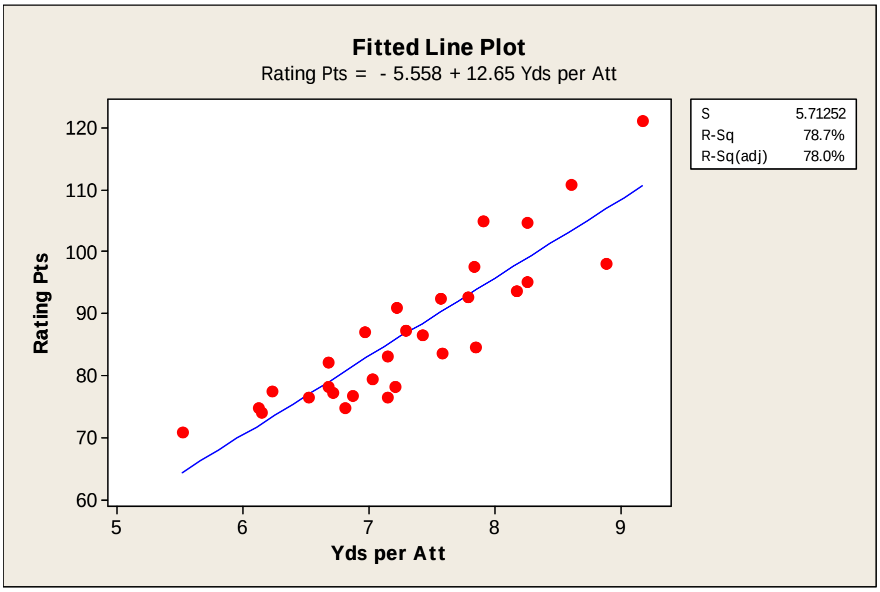
$$S_{xy} = 19375.21 - \frac{(219.55)(2611)}{30} = 267.037$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{267.037}{21.106} = 12.652$$

$$\hat{\beta}_0 = \frac{2611}{30} - (12.652)\left(\frac{219.55}{30}\right) = -5.56$$

$$\hat{y} = -5.56 + 12.652x$$

$$\hat{\sigma}^2 = MS_E = \frac{SS_E}{n-2} = \frac{913.7}{28} = 32.6$$



b) $\hat{y} = -5.56 + 12.652(7.5) = 89.33$

c) $-\hat{\beta}_1 = -12.652$

d) $\frac{1}{12.652} \times 10 = 0.79$

e) $\hat{y} = -5.56 + 12.652(7.21) = 85.66$

$$e = y - \hat{y}$$
$$= 78.1 - 85.66 = -7.56$$

11-6 a)

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The regression equation is
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MPG = 39.2 - 0.0402 Engine Replacement

Predictor	Coef	SE Coef	T	P
Constant	39.156	2.006	19.52	0.000
Engine Replacement	-0.040216	0.007671	-5.24	0.000

S = 3.74332   R-Sq = 59.1%   R-Sq(adj) = 57.0%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	385.18	385.18	27.49	0.000
Residual Error	19	266.24	14.01		
Total	20	651.41			

$\hat{\sigma}^2 = 14.01$

$\hat{y} = 39.2 - 0.0402x$

b) $\hat{y} = 39.2 - 0.0402(150) = 33.17$

c) $\hat{y} = 34.2956$

$e = y - \hat{y} = 41.3 - 34.2956 = 7.0044$

$\hat{\sigma}^2 = 14.01$

$\hat{y} = 39.2 - 0.0402x$

b) $\hat{y} = 39.2 - 0.0402(150) = 33.17$

c) $\hat{y} = 34.2956$

$e = y - \hat{y} = 41.3 - 34.2956 = 7.0044$

11-6 a)

The regression equation is

MPG = 39.2 - 0.0402 Engine Replacement

Predictor	Coef	SE Coef	T	P

Constant	39.156	2.006	19.52	0.000
Engine Replacement	-0.040216	0.007671	-5.24	0.000

S = 3.74332   R-Sq = 59.1%   R-Sq(adj) = 57.0%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	385.18	385.18	27.49	0.000
Residual Error	19	266.24	14.01		
Total	20	651.41			