# IEOR 3600: HMWK 3

1. *Polling:* Suppose during a Presidential election (only two candidates for simplicity), it has been determined (by extensive sampling) that $p = 55\%$ of all voters will vote for candidate $A$ and $q = 1 - p = 45\%$ for candidate $B$. If you randomly select 5 voters, what is the probability that exactly 3 will vote for candidate $A$?

2. (*Continuation:*) Suppose that apriori we do not know the proportion $p$ of voters who will vote for candidate $A$, but that when we randomly selected five voters, exactly 3 said they would vote for $A$. Find the value of $p$ that maximizes the probability of this event (e.g., of getting exactly 3 out of the 5 saying they would vote for $A$.)

3. You come down with a headache. Your physician hands you two identical looking pills, telling you to take one when you get home. Unbeknownst to you: Pill 1 is a real drug and is known to stop headaches 75% of cases, whereas pill 2 is merely a placebo and is known to stop headaches in only 10% of cases. You took one and your headache did not go away. What is the probability that you took the placebo?

4. Three coins are in a box. Two of them are fair, and one of them has $p = 3/4$ of landing $H$. A coin is randomly selected from the box, to be flipped.
   (a) The coin is flipped two times, both times landing $H$. What is the probability that the coin was fair?
   (b) Suppose the coin is flipped $n$ times, and all $n$ times landed $H$. Prove that the probability that the coin is fair converges to 0 as $n \to \infty$.

5. A company makes boxes of various sizes as follows: The length (L) is a rv with probability mass function $P(L = 2) = 0.3,\ P(L = 4) = 0.7$. The width (W) is a rv with probability mass function $P(W = 3) = 0.3,\ P(W = 4) = 0.4,\ P(W = 5) = 0.3$. The height (H) is a rv with probability mass function $P(H = 2) = 1/3,\ P(H = 3) = 1/3,\ P(H = 4) = 1/3$. Assume that these random variables are independent.
   (a) Find $E(W)$ and $Var(W)$.
   (b) What is the expected value of the volume of a box? What is the variance of the volume of the box?

6. Suppose that $N$ is a rv with a geometric distribution; $P(N = k) = (1 - p)^{k-1}p,\ k \geq 1$, where $0 < p < 1$ is the probability of success. Show that $P(N > n + k \mid N > n) = P(N > k)$ for any $n, k \geq 1$. Thus given $N > n$, the remainder $N - n$ itself has the same geometric $(p)$ distribution, independent of the value of $n$. This is known as the discrete *memoryless property* of the geometric distribution. For example, if $N$ denotes the lifetime (in days) of a light bulb, then this property would imply that: Given that the bulb has already lived beyond 365 days, the probability that it will live at least another 365 days is the same as for a new bulb (here we are using $k = n = 365$). (Clearly a typical bulb would not have such a lifetime distribution, why?.)

7. *From the Text:* Ch. 2: Pages 58-65, Exercises 122,161, 172. Ch. 3: Page 77, Exercise 47. Page 103, Exercises 109,110,111.