IEOR 3600: HMWK 4

1. Ch. 4, Page 118: Exercise 25, 27,30, Pages 120-121: Exercises 33, 38. Page 134: Exercise 65. Page 139: Exercises 76,77,78.

2. A district D within a state has a population of size $n = 1000$. Suppose that each voter will, independently, vote for candidate A (versus B) with probability $p = 0.51$. Use the normal approximation to the binomial distribution to compute the probability that A wins the popular vote of the district (e.g., receives more than 500 votes).

   Repeat for the case when $n = 10,000$.

3. Suppose that U has a uniform distribution over the continuous interval $(0, 1)$. For $x \in (0, 0.5)$ compute the conditional cumulative distribution function (cdf)
   $G(x) = P(U \leq x \mid U \leq 0.5)$. Show that $G(x)$ is the cdf of the uniform distribution over the smaller interval $(0, 0.5)$. Thus: Conditional on the event $U \leq 0.5$, the distribution of U is uniform $(0, 0.5)$.

   More generally prove that: If X is uniform (a, b), and $a < c < d < b$, then conditional on the event $X \in (c, d)$, the distribution of X is uniform (c, d): Compute, for $x \in (c, d)$, the conditional cumulative distribution function (cdf)
   $G(x) = P(X \leq x \mid X \in (c, d))$ and show that it is the cdf of the uniform distribution over the smaller interval (c, d).

4. Suppose that X has an exponential distribution at rate $\lambda = 2$; the probability density function is $f(x) = 2e^{-2x},\ x \geq 0$ and the cdf is $F(X \leq x) = 1 - e^{-2x},\ x \geq 0$. Find the cdf and density of the rv $Y = X^2$. Compute $E(Y)$ and $Var(Y)$ (Hint: $E(Y^2) = E(X^4)$.)