IEOR 3600: Solutions HMWK 8 (not from Text)

1. *Estimating the difference between two proportions:* It is thought that people in California are more likely to support a certain environmental law than people in New York. A random sample of 1000 Californians yields 375 of them that support the law (hence a proportion of $\overline{p}_1(n_1) = 0.375$), while a random sample of 750 New Yorkers yields 220 of them that support the law (hence a proportion of $\overline{p}_2(n_2) = 0.293$). Letting p_1 and p_2 denote the real (unknown) proportions, compute a 95% confidence interval for the difference $p_1 - p_2$. Is there a significant difference?

   SOLUTION:

   A 95% CI is given by

   $$\overline{p}_1(n_1) - \overline{p}_2(n_2) \pm (1.96)\hat{S}(n_1, n_2) = 0.044$$

   where

   $$\hat{S}(n_1, n_2) = \sqrt{\overline{p}_1(n_1)(1 - \overline{p}_1(n_1))/n_1 + \overline{p}_2(n_2)(1 - \overline{p}_2(n_2))/n_2} = 0.0226.$$

   $\overline{p}_1(n_1) - \overline{p}_2(n_2) = 0.082$, the CI is $0.082 \pm 0.044 = [0.038, 0.126]$. This interval is strictly above the origin meaning that it appears that $p_1 > p_2$ as was originally stated.

3. χ^2: A testing center for undergraduates says that the test scores on an entrance exam are normally distributed with mean $\mu = 74$ and variance $\sigma^2 = 8$. You wish to check wether $\sigma^2 = 8$ is realistic. So you go out and randomly sample 20 undergraduate students. We know that (due to the normality),

   $$\chi^2 = \frac{19s^2(20)}{\sigma^2}$$

   has a χ^2 distribution with 19 degrees of freedom. And thus if $\sigma^2 = 8$, then

   $$\chi^2 = \frac{19s^2(20)}{8}$$

   has a χ^2 distribution with 19 degrees of freedom. Your sample yields $s^2(20) = 20$. Using this sample, do you believe the claim that $\sigma^2 = 8$?

   SOLUTION: $\chi^2 = \frac{19s^2(20)}{8} = (19)(20)/8 = 47.5$. But $P(8.91 \leq \chi^2 \leq 32.85) = 0.95$, so since $\chi^2 > 32.85$ we reject the assertion that $\sigma^2 = 8$. (We would believe that $\sigma^2 > 8$, since then this would result in a smaller value for our observed χ^2. A calculation reveals that only for $\sigma^2 > 11.875$ would our observed χ^2 fall within the 95% interval $[8.91, 32.85]$; $\frac{19s^2(20)}{11.875} = 32.85$.

4. *Hypothesis testing for a proportion :* Two candidates A, B are running for office. B claims that the race is a dead heat, that is, that the true current proportion for each candidate is $p = 0.50$. Candidate A, however, disagrees, he thinks he is currently ahead. A pollster polls a randomly selected group of $n = 1000$ people and finds that 535 prefer A (hence $\hat{p}(n) = 0.535$). At the 95% significance level, using the null hypothesis $H_0 : p = 0.50$ versus the alternative $H_1 : p > 0.50$, would you reject the "dead heat" claim and conclude that A is ahead?

   SOLUTION: Under $p_0 = 0.50$, $\sqrt{p(1-p)} = 0.50$ and thus $Z = \sqrt{n}(\hat{p}(n) - 0.50)/\sqrt{p_0(1-p_0)} = \sqrt{10}(0.70) = 2.21$. Meanwhile, $P(Z > 1.645) = 0.05$; $z_{0.05} = 1.645$. Since $Z > 1.645$, we reject H_0 and accept H_1.