Solutions to HW9

9-42 a) 1) The parameter of interest is the true mean hole diameter, μ.

2) $H_0 : \mu = 1.50$

3) $H_1 : \mu \neq 1.50$

4) $\alpha = 0.01$

5) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$

7) $\bar{x} = 1.4975$, $\sigma = 0.01$

$$z_0 = \frac{1.4975 - 1.50}{0.01/\sqrt{25}} = -1.25$$

8) Since $-2.58 < -1.25 < 2.58$, do not reject the null hypothesis and conclude the true mean hole diameter is not significantly different from 1.5 in. at $\alpha = 0.01$.

b) p-value=$2(1-\Phi(|Z_0|))=2(1-\Phi(1.25))\cong 0.21$

c)

$$\beta = \Phi\left(z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

$$= \Phi\left(2.58 - \frac{(1.495-1.5)\sqrt{25}}{0.01}\right) - \Phi\left(-2.58 - \frac{(1.495-1.5)\sqrt{25}}{0.01}\right)$$

$= \Phi(5.08) - \Phi(-0.08) = 1 - .46812 = 0.53188$

power=$1-\beta=0.46812$.

d) Set $\beta = 1 - 0.90 = 0.10$

$$n = \frac{(z_{\alpha/2} + z_\beta)^2\sigma^2}{\delta^2} = \frac{(z_{0.005} + z_{0.10})^2\sigma^2}{(1.495-1.50)^2} \cong \frac{(2.58+1.29)^2(0.01)^2}{(-0.005)^2} = 59.908,$$

$n \cong 60$.

e) For $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$1.4975 - 2.58\left(\frac{0.01}{\sqrt{25}}\right) \le \mu \le 1.4975 + 2.58\left(\frac{0.01}{\sqrt{25}}\right)$$

$$1.4923 \le \mu \le 1.5027$$

The confidence interval constructed contains the value 1.5, thus the true mean hole diameter could possibly be 1.5 in. using a 99% level of confidence. Since a two-sided 99% confidence interval is equivalent to a two-sided hypothesis test at $\alpha = 0.01$, the conclusions necessarily must be consistent.

9-44 a) α=0.01, n=20, the critical values are

b) α=0.05, n=12, the critical values are

c) α=0.1, n=15, the critical values are

9-47 a) $2 * 0.025 \le p \le 2 * 0.05$ then $0.05 \le p \le 0.1$

b) $2 * 0.025 \le p \le 2 * 0.05$ then $0.05 \le p \le 0.1$

c) $2 * 0.25 \le p \le 2 * 0.4$ then $0.5 \le p \le 0.8$

9-48 a) $0.025 \le p \le 0.05$

b) $1 - 0.05 \le p \le 1 - 0.025$ then $0.95 \le p \le 0.975$

c) $0.25 \le p \le 0.4$

9-63 a) α=0.01, n=20, from table V we find the following critical values 6.84 and 38.58
b) α=0.05, n=12, from table V we find the following critical values 3.82 and 21.92
c) α=0.10, n=15, from table V we find the following critical values 6.57 and 23.68

9-64 a) α=0.01, n=20, from table V we find $\chi^2_{\alpha,n-1} = 36.19$

b) α=0.05, n=12, from table V we find $\chi^2_{\alpha,n-1} = 19.68$

c) α=0.10, n=15, from table V we find $\chi^2_{\alpha,n-1} = 21.06$

9-65 a) α=0.01, n=20, from table V we find $\chi^2_{1-\alpha,n-1}$ = 7.63

b) α=0.05, n=12, from table V we find $\chi^2_{1-\alpha,n-1}$ = 4.57

c) α=0.10, n=15, from table V we find $\chi^2_{1-\alpha,n-1}$ = 7.79

9-69

a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true standard deviation of performance time σ. However,

the answer can be found by performing a hypothesis test on σ^2.

2) H_0 : $\sigma^2 = .75^2$

3) H_1 : $\sigma^2 > .75^2$

4) $\alpha = 0.05$

5) =

6) Reject H_0 if $\chi^2_0 > \chi^2_{\alpha,n-1}$ where $\chi^2_{0.05,16} = 26.30$

7) n = 17, s = 0.09

$$= \frac{(n-1)s^2}{\sigma^2} = \frac{16(0.09)^2}{.75^2} = 0.23$$

8) Because 0.23 < 26.30 do not reject H_0 and conclude there is insufficient evidence to indicate the true variance of performance time content exceeds 0.75^2 at $\alpha = 0.05$.

P-value: Because=0.23 the *P*-value>0.995

b) The 95% one sided confidence interval given below, includes the value 0.75. Therefore, we are not be able to conclude that the standard deviation is greater than 0.75.

$$\frac{16(.09)^2}{26.3} \le \sigma^2$$

$$0.07 \le \sigma$$

9-71

a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true standard deviation of titanium percentage, σ. However, the answer can be found by performing a hypothesis test on σ^2.

2) H_0 : $\sigma^2 = (0.25)^2$

3) H_1 : $\sigma^2 \neq (0.25)^2$

4) $\alpha = 0.05$

5) =

6) Reject H_0 if where 32.36 or where 71.42

7) n = 51, s = 0.37

=

8) Since 109.52 > 71.42 we reject H_0 and conclude there is sufficient evidence to indicate the true standard deviation of titanium percentage is significantly different from 0.25 at $\alpha = 0.01$.

P-value: p/2<.005 then p<0.01

b) 95% confidence interval for σ:

First find the confidence interval for σ^2 :

For $\alpha = 0.05$ and n = 51,71.42 and32.36

$$\frac{50(0.37)^2}{71.42} \leq \sigma^2 \leq \frac{50(0.37)^2}{32.36}$$

$$0.096 \leq \sigma^2 \leq 0.2115$$

Taking the square root of the endpoints of this interval we obtain,

$$0.31 < \sigma < 0.46$$

Since 0.25 falls below the lower confidence bound we would conclude that the population standard deviation is not equal to 0.25.

9-81 a)

1) The parameter of interest is the true percentage of football helmets that contain flaws, p.

2) H_0 : p = 0.1

3) H_1 : p > 0.1

4) $\alpha = 0.01$

5) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 > z_\alpha$ where $z_\alpha = z_{0.01} = 2.33$

7) x = 16 n = 200 $\hat{p} = \frac{16}{200} = 0.08$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.08 - 0.10}{\sqrt{\frac{0.10(1-0.10)}{200}}} = -0.94$$

8) Because -0.94 < 2.33 do not reject the null hypothesis. There is not enough evidence that the proportion of football helmets with flaws exceeds 10%.

P-value = 1- Φ(-0.94) =0.8264

b) The 99% lower confidence interval:

$$\hat{p} - z_\alpha \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p$$

$$.08 - 2.33\sqrt{\frac{0.08(0.92)}{200}} \le p$$

$$0.035 \le p$$

Because $0.035 \le p = 0.1$ then we fail to reject the null hypothesis.