Section 4-4

4-25. $E(X) = \int_{-1}^{1} 1.5x^3 dx = 1.5 \frac{x^4}{4} \Big|_{-1}^{1} = 0$

$V(X) = \int_{-1}^{1} 1.5x^3 (x-0)^2 dx = 1.5 \int_{-1}^{1} x^4 dx$

$= 1.5 \frac{x^5}{5} \Big|_{-1}^{1} = 0.6$

4-27. $E(X) = \int_{1}^{\infty} x 2x^{-3} dx = -2x^{-1} \Big|_{1}^{\infty} = 2$

4-30. (a) $E(X) = \int_1^{70} x f(x) dx = \int_1^{70} \frac{70}{69x} dx = \frac{70}{69} \ln x \Big|_1^{70} = 4.3101$

$E(X^2) = \int_1^{70} x^2 f(x) dx = \int_1^{70} \frac{70}{69} dx = 70$

Var(X)= $E(X^2) - (EX)^2 = 70 - 18.5770 = 51.4230$

(b) 2.5*4.3101=10.7753

(c) $P(X > 50) = \int_{50}^{70} f(x) dx = 0.0058$

4-33. a) E(X) = (-1+1)/2 = 0,

$V(X) = \frac{(1-(-1))^2}{12} = 1/3, \ and \ \sigma_x = 0.577$

b) $P(-x < X < x) = \int_{-x}^{x} \frac{1}{2} dt = 0.5t \Big|_{-x}^{x} = 0.5(2x) = x$

Therefore, x should equal 0.90.

c) $F(x) = \begin{cases} 0, & x < -1 \\ 0.5x + 0.5, & -1 \le x < 1 \\ 1, & 1 \le x \end{cases}$

4-38. Let X denote the changed weight.
Var(X) = $4^2/12$
Stdev(X) = 1.1547

4-65. a) E(X) = 200(0.4) = 80, V(X) = 200(0.4)(0.6) = 48 and $\sigma_X = \sqrt{48}$.

Then, $P(X \le 70) \cong P\left(Z \le \frac{70.5 - 80}{\sqrt{48}}\right) = P(Z \le -1.37) = 0.08534$

b)

$$P(70 < X < 90) \cong P\left(\frac{70.5 - 80}{\sqrt{48}} < Z \le \frac{89.5 - 80}{\sqrt{48}}\right) = P(-1.37 < Z \le 1.37)$$
$$= 0.91466 - 0.08534 = 0.82931$$

c)

$$P(79.5 < X \le 80.5) \cong P\left(\frac{79.5 - 80}{\sqrt{48}} < Z \le \frac{80.5 - 80}{\sqrt{48}}\right) = P(-0.07217 < Z \le 0.07217)$$
$$= 0.0575$$

4-76. a) $P(X \le 0) = \int_0^0 \lambda e^{-\lambda x} dx = 0$

b) $P(X \ge 2) = \int_2^\infty 2e^{-2x} dx = -e^{-2x}\Big|_2^\infty = e^{-4} = 0.0183$

c) $P(X \le 1) = \int_0^1 2e^{-2x} dx = -e^{-2x} \Big|_0^1 = 1 - e^{-2} = 0.8647$

d) $P(1 < X < 2) = \int_1^2 2e^{-2x} dx = -e^{-2x} \Big|_1^2 = e^{-2} - e^{-4} = 0.1170$

e) $P(X \le x) = \int_0^x 2e^{-2t} dt = -e^{-2t} \Big|_0^x = 1 - e^{-2x} = 0.05$ and x = 0.0256

4-77. If E(X) = 10, then $\lambda = 0.1$.

a) $P(X > 10) = \int_{10}^{\infty} 0.1e^{-0.1x} dx = -e^{-0.1x} \Big|_{10}^{\infty} = e^{-1} = 0.3679$

b) $P(X > 20) = -e^{-0.1x} \Big|_{20}^{\infty} = e^{-2} = 0.1353$

c) $P(X < 30) = -e^{-0.1x} \Big|_0^{30} = 1 - e^{-3} = 0.9502$

d) $P(X < x) = \int_0^x 0.1e^{-0.1t} dt = -e^{-0.1t} \Big|_0^x = 1 - e^{-0.1x} = 0.95$ and x = 29.96.

4-78. (a) $P(X < 5) = 0.3935$

(b)

$$P(X < 15 \mid X > 10) = \frac{P(X < 15, \quad X > 10)}{P(X > 10)} = \frac{P(X < 15) - P(X < 10)}{1 - P(X < 10)} = \frac{0.1447}{0.3679} = 0.3933$$

(c) They are the same.