Solution to HW8

2.

9-38 a) 1) The parameter of interest is the true mean melting point, μ.

2) $H_0 : \mu = 155$

3) $H_1 : \mu \neq 155$

4) $\alpha = 0.01$

5)

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$

7)= 154.2, $\sigma = 1.5$

$$z_0 = \frac{154.2 - 155}{1.5/\sqrt{10}} = -1.69$$

8) Since –1.69 > -2.58, do not reject the null hypothesis and conclude there is not sufficient evidence to support the claim the mean melting point is not equal to 155 °F at $\alpha = 0.01$.

b) P-value = 2*P(Z <- 1.69) =2* 0.045514 = 0.091028

c)

$$\beta = \Phi\left(z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

$$= \Phi\left(2.58 - \frac{(155-150)\sqrt{10}}{1.5}\right) - \Phi\left(-2.58 - \frac{(155-150)\sqrt{10}}{1.5}\right)$$

$= \Phi(-7.96) - \Phi(-13.12) = 0 - 0 = 0$

d)

$$n = \frac{(z_{\alpha/2} + z_\beta)^2\sigma^2}{\delta^2} = \frac{(z_{0.005} + z_{0.10})^2\sigma^2}{(150-155)^2} = \frac{(2.58+1.29)^2(1.5)^2}{(5)^2} = 1.35,$$

$n \cong 2$.

9-39 a.) 1) The parameter of interest is the true mean battery life in hours, μ.

2) $H_0 : \mu = 40$

3) $H_1 : \mu > 40$

4) $\alpha = 0.05$

5)

6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$

7) $\bar{x} = 40.5$, $\sigma = 1.25$

$$z_0 = \frac{40.5 - 40}{1.25/\sqrt{10}} = 1.26$$

8) Since 1.26 < 1.65 do not reject H_0 and conclude the battery life is not significantly different greater than 40 at $\alpha = 0.05$.

b) P-value = $1 - \Phi(1.26) = 1 - 0.8962 = 0.1038$

c) $\beta = \Phi\left(z_{0.05} + \frac{40 - 42}{1.25/\sqrt{10}}\right)$

$= \Phi(1.65 + -5.06)$
$= \Phi(-3.41)$
$\cong 0.000325$

d) $$n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.05} + z_{0.10})^2 \sigma^2}{(40 - 44)^2} = \frac{(1.65 + 1.29)^2 (1.25)^2}{(4)^2} = 0.844, \quad n \cong 1$$

e) 95% Confidence Interval

$$\bar{x} + z_{0.05}\sigma / \sqrt{n} \le \mu$$
$$40.5 + 1.65(1.25) / \sqrt{10} \le \mu$$
$$39.85 \le \mu$$

The lower bound of the 90 % confidence interval must be greater than 40 to verify that the true mean exceeds 40 hours.

9-40 a)

1) The parameter of interest is the true mean tensile strength, μ.
2) $H_0 : \mu = 3500$
3) $H_1 : \mu \neq 3500$
4) $\alpha = 0.01$
5)
6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$
7) $\bar{x} = 3450$, $\sigma = 60$

$$z_0 = \frac{3450 - 3500}{60 / \sqrt{12}} = -2.89$$

8) Since --2.89 < –2.58, reject the null hypothesis and conclude the true mean tensile strength is significantly different from 3500 at $\alpha = 0.01$.

b) Smallest level of significance =
P-value = 2[1 –(2.89)]= 2[1 – .998074] = 0.004
The smallest level of significance at which we are willing to reject the null hypothesis is 0.004.

c) δ = 3470 – 3500 = -30

$$\beta = \Phi\left(z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right)$$
$$= \Phi\left(2.58 - \frac{(3470 - 3500)\sqrt{12}}{60}\right) - \Phi\left(-2.58 - \frac{(3470 - 3500)\sqrt{12}}{60}\right)$$
$= \Phi(4.312)- \Phi(-0.848) = 1 – 0.1982 = 0.8018$

d) $z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$3450 - 2.58\left(\frac{60}{\sqrt{12}}\right) \le \mu \le 3450 + 2.58\left(\frac{60}{\sqrt{12}}\right)$$

$3405.313 \le \mu \le 3494.687$

With 99% confidence, we believe the true mean tensile strength is between 3405.313 psi and 3494.687 psi. We can test the hypotheses that the true mean tensile strength is not equal to 3500 by noting that the value is not within the confidence interval. Hence we reject the null hypothesis.

9-41 a) 1) The parameter of interest is the true mean speed, μ.

2) $H_0 : \mu = 100$

3) $H_1 : \mu < 100$

4) $\alpha = 0.05$

5)

6) Reject H_0 if $z_0 < -z_\alpha$ where $-z_{0.05} = -1.65$

7) $\bar{x} = 102.2$, $\sigma = 4$

$$z_0 = \frac{102.2 - 100}{4/\sqrt{8}} = 1.56$$

8) Since 1.56> −1.65, do not reject the null hypothesis and conclude the there is insufficient evidence to conclude that the true speed strength is less than 100 at α = 0.05.

b) $z_0 = 1.56$, then p-value= $\Phi(z_0) \cong 0.94$

c) $\beta = 1 - \Phi\left(-z_{0.05} - \frac{(95-100)\sqrt{8}}{4}\right)$ = 1-Φ(-1.65 - −3.54) = 1-Φ(1.89) = 0.02938

Power = 1-β = 1-0.02938 = 0.97062

d) n =1 n ≅ 5

e) $\mu \le \bar{x} + z_{0.05}\left(\frac{\sigma}{\sqrt{n}}\right)$

$$\mu \le 102.2 + 1.65\left(\frac{4}{\sqrt{8}}\right)$$

$\mu \le 104.53$

Because 100 is included in the CI then we don’t have enough confidence to reject the null hypothesis.

5. 9-29 a) $H_0 : \mu = 10, H_1 : \mu > 10$

b) $H_0 : \mu = 7, H_1 : \mu \neq 7$

c) $H_0 : \mu = 5, H_1 : \mu < 5$

9-30 a) α=0.01, then a= $z_{\alpha/2}$ =2.57 and b=- $z_{\alpha/2}$ =-2.57

b) α=0.05, then a= $z_{\alpha/2}$ =1.96 and b=- $z_{\alpha/2}$ =-1.96

c) α=0.1, then a= $z_{\alpha/2}$ =1.65 and b=- $z_{\alpha/2}$ =-1.65

9-31 a) α=0.01, then a= $z_\alpha \cong$ 2.33

b) α=0.05, then a= $z_\alpha \cong$ 1.64

c) α=0.1, then a= $z_\alpha \cong$ 1.29

9-33 a) p-value=2(1- $\Phi(|Z_0|)$)=2(1- $\Phi(2.05)$) $\cong$ 0.04

b) p-value=2(1- $\Phi(|Z_0|)$)=2(1- $\Phi(1.84)$) $\cong$ 0.066

c) p-value=2(1- $\Phi(|Z_0|)$)=2(1- $\Phi(0.4)$) $\cong$ 0.69

9-34 a) p-value=1- $\Phi(Z_0)$ =1- $\Phi(2.05)$ $\cong$ 0.02

b) p-value=1- $\Phi(Z_0)$ =1- $\Phi(-1.84)$ $\cong$ 0.97

c) p-value=1- $\Phi(Z_0)$ =1- $\Phi(0.4)$ $\cong$ 0.34

9-35 a) p-value= $\Phi(Z_0) = \Phi(2.05)$ $\cong$ 0.98

b) p-value= $\Phi(Z_0) = \Phi(-1.84)$ $\cong$ 0.03

c) p-value= $\Phi(Z_0) = \Phi(0.4)$ $\cong$ 0.65

9-36 a.) 1) The parameter of interest is the true mean water temperature, μ.
2) $H_0 : \mu = 100$
3) $H_1 : \mu > 100$
4) $\alpha = 0.05$
5)
6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$
7) $\bar{x} = 98$, $\sigma = 2$

$$z_0 = \frac{98 - 100}{2/\sqrt{9}} = -3.0$$

8) Since -3.0 < 1.65 do not reject H_0 and conclude the water temperature is not significantly different

greater than 100 at $\alpha = 0.05$.

b) P-value = $1 - \Phi(-3.0) = 1 - 0.00135 = 0.99865$

c) $\beta = \Phi\left(z_{0.05} + \frac{100 - 104}{2/\sqrt{9}} \right)$
$= \Phi(1.65 + -6)$
$= \Phi(-4.35)$
$\cong 0$

6. n=100 $Z_0 = 1.2$, $Z_{\alpha/2} = 1.94$ accept H0;

n=400 $Z_0 = 2.4$ $Z_{\alpha/2} = 1.94$ reject H0