1. Page 657 of Text: 1, 3.

2. Consider a geometric Brownian motion, $S(t) = S_0e^{X(t)}$, where $X(t) = 3B(t) - t$. Since the drift is $< 0$, we know that $X(t) \to -\infty$, as $t \to \infty$, wp1. From here, show that in fact $S(t) \to 0$ wp1, while $E(S(t)) \to \infty$.

3. For a geometric BM, $S(t) = 50e^{2B(t)+3t}$, what is the probability that $S(t)$ goes up to 51 before going down to 49?

4. A stock price starting with price $S_0 = 2$ at time $t = 0$ moves according to a geometric BM with drift 3 and variance term $\sigma^2 = 4$; $S_1(t) = 2e^{2B_1(t)+3t}$. Independently, another stock starting at price $S_0 = 1$ at time $t = 0$ moves according to geometric BM with drift 2 and variance term $\sigma^2 = 1$; $S_2(t) = e^{B_2(t)+2t}$. What is the probability that at some time in the future, the two stocks have the same price?

5. For a stock with price following geometric BM, $S(t) = S_0e^{X(t)}$, where $X(t) = \sigma B(t) + \mu t$: Consider a derivative that gives a payoff of $C_T = S^2(T)$ at time $T$. Price this derivative, that is, compute

$$C_0 = e^{-rT}E^*(S^2(T)).$$