1. Consider the rat in the open maze (state space $S = \{0,1,2,3,4\}$). Given that the rat starts in cell 1, what is the expected total number of visits to cell 2 before the rat escapes?

2. Consider the Gambler’s ruin problem with $N = 4$, except that instead of only one fixed $p \in (0,1)$ (the “win per gamble” probability) there is a different $p$ for each state $i = 1, 2, 3$. If (for any $n$) $X_n = 1$, then $p = 0.2$, if $X_n = 2$, then $p = 0.5$, and if $X_n = 3$, then $p = 0.8$. Compute the probability $P_1$ = the probability the gambler reaches 4 before 0 given that $X_0 = 1$.

3. Families arrive to an island for vacation according to a Poisson process at rate $\lambda = 4$ per day. Each family, independently, is of random size, $J$, where $P(J = k) = (1-p)^{k-1}p$, $k \geq 1$, with $p = 0.2$. Each family, independently, remains on the island for an amount of time $V$ that has a continuous uniform distribution over the interval $(3,9)$ days. Initially, there are no families on the island.

   (a) Compute the variance of the number of families on the island at time $t = 5$ days.
   
   (b) Let
   
   $A = \{\text{There are 2 families of size 2 at time } t = 10\}$,
   
   $B = \{\text{There are 3 families of size 3 at time } t = 12\}$,
   
   $C = \{\text{There are 4 families of size 4 at time } t = 15\}$.
   
   Compute $P(A \cap B \cap C)$.

4. Consider an M/M/1 queue with “impatient” customers: Each customer, while waiting in line, will independently, leave (and never come back) after an amount of time that has an exponential distribution with rate $\gamma$ if they have not entered service yet. If they enter service, they stay and get served as usual. Let $X(t)$ denote the number of customers in the system at time $t$.

   (a) Argue that $\{X(t)\}$ is a birth and death process.
   
   (b) Set up the birth and death balance equations for the stationary probabilities $\{P_n : n \geq 0\}$.
   
   (c) Explain intuitively why this chain is always positive recurrent, for any values $\lambda > 0$, $\mu > 0$, $\gamma > 0$.
   
   (d) In terms of the $\{P_n : n \geq 0\}$, and basic system parameters ($\lambda$, $\mu$, $\gamma$) what is the rate at which people depart?