IEOR 4106, HMWK 5, Professor Sigman

1. Trucks arrive to the GW bridge according to a Poisson process at rate $\lambda_T = 100$ per hour. Independently, cars arrive to the GW bridge according to a Poisson process at rate $\lambda_C = 300$ per hour.
   (a) What is the probability that exactly 75 Trucks arrive during the hour from 1-2PM?
   (b) What is the probability that exactly 75 Trucks arrive during the hour from 1-2PM given that 102 Trucks already arrived during the earlier hour of 10-11AM?
   (c) What is the probability that both 75 Trucks arrive during the hour from 1-2PM and 320 Cars arrive during the hour from 10-11AM?
   (d) You go to the GW Bridge at time 12 noon to observe. What is the probability that you observe a Truck arrive before a Car?
   (e) Let *Vehicles* denote Trucks and Cars together (superposition). What is the probability that exactly 1 Vehicle arrives during a given minute of time (1/60 of an hour)?

2. Phone calls come in to your phone according to a Poisson process $\psi = \{t_n : n \geq 1\}$ at rate $\lambda = 5$ per hour. The counting process is given by $\{N(t) : t \geq 0\}$.
   (a) Conditional on $N(1) = 7$, what is the probability that exactly 2 of these 7 arrived in the first 15 minutes of the hour (e.g., during the first 1/4 of the hour)?
   (b) Explain why the above answer would remain the same if $N(1)$ is replace by $N(9) - N(8)$.
   (c) Suppose that the Poisson process $\psi = \{t_n : n \geq 1\}$ is actually the independent superposition of two Poisson processes: $\psi_1 = \{t_n(1) : n \geq 1\}$, a Poisson process at rate $\lambda_1 = 3$, with counting process $\{N_1(t) : t \geq 0\}$ (DOMESTIC CALLS), and $\psi_2 = \{t_n(2) : n \geq 1\}$, a Poisson process at rate $\lambda_2 = 2$, with counting process $\{N_2(t) : t \geq 0\}$ (FOREIGN CALLS).
      i. What is the probability that the first 4 calls you get are all Domestic?
      ii. Let K = the number of calls until you get the first Foreign call. Find $E(K)$.
      iii. Conditional on $N(1) = 7$, what is the probability that exactly 2 of these 7 are Foreign and 5 are Domestic?

3. According to a Poisson process at rate $\lambda = 20$ per day, a company buys units (100 share blocks) of stock A and holds on to each unit, independently of other units, for H days, where H has an exponential distribution with $E(H) = 60$ (days).

   Assume that initially (time $t = 0$) no units of stock A are held.
   (a) Compute the expected number of units held at times $t = 20$, $t = 50$ and $t = 90$ days.
   (b) Repeat (a), (b) when H has a uniform distribution on $(0, 120)$.
   (c) Repeat (a), (b) when H has a uniform distribution on $(40, 80)$.
   (d) As $t \to \infty$, what is the limiting probability distribution for the number of units held? Does it depend on which of the 3 distributions above are used for holding times?