The Role of International Institutions when Military Capacity is Endogenous: An equivalence result*

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Abstract

Most theories of negotiating and fighting treat the costs and benefits of war as exogenous. We take a step back and treat capacity as the result of strategic investment decisions that states make in anticipation of their interactions in the international system. This paper investigates the extent to which institutions, and in particular expectations about equilibrium play to these institutions, influence the incentives for arming by individual nations. Focusing on environments in which peaceful institutions cannot be constructed, we provide a characterization of the set of lotteries over arming and fighting that are attainable in an equilibrium to some institution/game. Our main substantive result is that changes in the institution(or in equilibrium selection) are offset by changes in the arming strategies of players. In the class of environment considered the probability of war conditional on military capacity is unaffected by the choice of the institution and must be the same in every equilibrium to every institution. This equivalence result has a straightforward but surprising interpretation; international institutions matter only because they effect the investment decisions in the first place.

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1 Introduction

On any list of tragic and large-scale ways resources and wealth are destroyed, war fighting must be highly ranked. Yet unlike earthquakes or hurricanes, the initiation and scale of armed conflict is a consequence of choices by interested actors. Given these facts, scholars and practitioners are left puzzled by the choices that lead to this destructive behavior. To better understand this puzzle, much of the theoretical literature has conceived of the outbreak of war as a form of bargaining failure. In many circumstances states disagree about how a scarce resource should be distributed or which of a set of policy alternatives should be adopted. States then are forced to bargain over the resource or policy choices and fight if a settlement cannot be reached. In most accounts fighting is thought to involve the destruction of resources and thus represents an inefficient outcome (or lottery) that is Pareto Dominated by some feasible settlement. It is well known that in such situations, expectations about the value of this outside option structure the incentives in bargaining, and in a very large class of problems, the payoffs from bargaining depend on the value of the outside option in a predictable manner (Banks 1990, Muthoo 1999, Powell 1999). In problems involving no uncertainty we tend to think that equilibrium play to a large class of bargaining games will result in efficient settlements. If, however, their is asymmetric information about the value of the outside option then war is possible (Fearon 1995), and generally, we should not expect equilibrium behavior to always reach efficient settlements in any institution, as has been shown in general negotiation problems and the crisis bargaining context in particular, (Myerson and Satterthwaite 1983, Fey and Ramsay 2008).

A common feature in nearly all of the work of this type is the assumption that the value of war is exogenous, albeit privately known. This assumption represents a convenient starting point, but this modeling approach leaves open several interesting questions. States clearly make choices that influence their millitary capactity and this capacity influences the distribution of possible utility streams that occur in war. A clear, albeit stylized, representation of this investment decision invloves the choice of how to allocate scarce resorces across distinct public goods of say defense and social services. A few papers have begun to investigate the incentives faced by nations that select their investments in millitary capacity (Meirowitz and Sartori 200x, Jackson and Morrelli 200x, POWELL?). In this paper we focus on an aspect of these arming decisions that has not been appreciated in the literature. Equilibrium investment decisions are likely to depend on expectations about how relevant nations will behave in the international institution when a conflict emerges. In particular, investment decisions may be influenced by expectations about downstream behavior which is influenced by the choice of the international institution. As such, not only does the choice of an institution influence the bargaining behavior of nations, but it may also influence the investment decisions of nations. Since our objective is an understanding of the relationship between instituion choice and the equilibrium investing and negotiating behavior that the institutional choice induces, we avoid focusing on a narrow set of game forms or making strong equilibrium selections.[1]

Our results show how international features, country specific features, and equilibrium behavior necessarily combine in any equilibrium to a broad class ofproblems in which nations invest and then bargain with bargaining failure resulting in conflict.. Our central results show that even in the absence of knowing the details of a particular institution, the requirement that behavior in the bargaining context forms an equilibrium and the requirement that investment decisions are mutual best responses when nations anticipate equilibrium play in subsequent periods alone provide enough structure to understand an important link between investment decisions and the probability of war. In particular, we show that independent of the details of an international institution or the equilibrium played in that institution, the conditional probability of war depends only on the primitive technologies–such as the marginal cost of arming and the functions relating arms levels to expected payoffs in war. In equilibrium the lottery over war conditional on investment levels is invariant to features of the institution's design. Put most strikingly, if one is interested in predicting the probability of war conditional on an investment level it is not neccesary to know anything about the international institution in which nations will negotiate and bargain. To be sure, changes in the institution can result in changes in equilibrium

[1] The literature and the authors of this paper, vascilate on whether it is best to use the term *institution* to describe the game form, the equilibrium to a game form or both. We have chosen to associate institution with a game form and through out we discuss institions (game forms) and equilibrium behavior in institutions.

behavior, the effect on arming strategies is so pronounced that it offsets the changes in bargaining/negotiating decisions. In otherwords, if the interaction of nation states is an equilibrium phenomena than the primary effect of international institutions is on the equilbrium arming decisions and not the bargaining behavior of nations in the institution.

The study of investment decisions prior to game-playing is limited to a small number of papers. Segal and Whinston (2002) study hold-up problems in which two parties each invest prior to the play of a contracting game. The investments influence the valuations of the disagreement option. In Segal and Whinston, however, the investment are not hidden information and so the connections to the current paper are limited. In a related literature Plott (1987) treats legal fees as an investment in the probability of winning a law suit, but these papers do not include the option of settling after the investments are made. Plott and Coughlan illustrate that the institutional choice (who pays legal fees) influences the investment decisions. Jackson and Morelli (2008) study investment and war fighting. Here, again the investment decisions are public and the relationship between institutiosn and investments cannot be studied as only one particular institution is analyzed. Meirowitz and Sartori (200x) consider models in which investment decisions are hidden actions and states bargain after making their investment decisions. They find a strong condition that is necessary and sufficient for the existence of equilibria in which disagreement/war is avoided. Moreover, the possibility of disagreement in equilibrium and the presence of investment strategies that involve randomizing are shown to be equivalent. That paper, however, is limited as it focuses only on unmediated bargaining games leaving open questions about the more general problem of designing institutions to influence arming, negotiating and warfighting. Moreover, Meirowitz and Sartori do not provide tight characterizations of equilibrium play, they instead focus on whether or not war can be avoided. As a consequence the paper does not tell us much about the relationship between institutional choice and equilibrium behavior.

Another literature that is relevant to this study involves the use of an area of game theory called mechanism design to establish "game free" results. These results can be thought of as characterizations of what is possible in any equilbrium to a large class of games. One of the most influential applications of this approach is

(Myerson and Satterthwaite 1983) in which it is shown that agreement cannot be guaranteed in problems of bilateral trade. In the study of negotiations and war fighting (Banks 1990) shows that in problems with one sided assymetric information, the equilibrium settlements and probability of fighting must be monotone in the unobserved capactify of the privately informed nation in any equilibrium to any bargaining game. More recently, (Fey and Ramsay 2008) consider problems in which both nations posses private information and investigate when it is possible to construct institions possessing equilibria in which the probability of war is 0. In these mechanism design papers a nation or both nations has private information but the realization of these types is exogenous. In this paper we take a step back and consider the mechanism design probabem when nations select their type (but these investments are privately observed).

The closest papers are Meirowitz and Sartori (2008) and Fey and Ramsay (2008, 2009). While not focused on investment decisions, Fey and Ramsay (2008) is also particularly connected to the current paper. They consider problems in which types are exogenous and there is private information. Fey and Ramsay focus on the mechanism design problem in a class of environments that includes the case of interdependent values of war, as studied here. The current paper focuses on the mechanism design problem when players first undertake endgoegenous investment in capacity (which is hidden information) as in Meirowitz and Sartori, and then play an arbitrary game (as in Fey and Ramsay). One conclusion that emerges from the analysis of each "half of the problem" (Meirowitz and Sartori on the investment side and Fey and Ramsy on the game free analysis side) is that there exist many reasonable circumstances where war with positive probability is unavoidable. This continues to be true in the situation where investment incentives and institutional design questions are take together. We, therefore, describe the limited circumstances that allow for peace with probability 1 and show that when peace with probability 1 is possible, institutional design is trivial. We then turn the bulk of our attention to the situations without these "nice" conditions and consider the hard problem. That is , much of our analysis focuses on characterizing the types of equilibria that can surface in games where always-peaceful equilibria do not exist. Our results show how international features, country specific features, and equilibrium behavior necessarily combine in any equilibrium to a broad class of bargaining problems.

The paper begins with a brief analysis of a motivating example where two states select armament levels prior to play of the ultimatum game. We then proceed to define the general institutional design problem. Our analysis begins by taking as given a fixed lottery over types and analyzes the induced problem of mechanism design with interdependent values. We then use the results from this problem to characterize equilibrium investment strategies and ultimately state and prove our main result.

2 An example

To motivate the analysis we consider the example of the ultimatum game, but extend the model to include an initial period in which the states select their armament level. The game allows the proposer to make an offer to the veto player and then allows veto player to decide whether to fight or accept the proposer's offer. For simplicity assume that each of the two players can have either a low armament level (0) or a high armament level (h). This is private information. Assume that the cost of selecting a high level of arms is c_i whereas not arming imposes no cost on nation i. We characterize a mixed investment strategy by $\alpha_p = \Pr(a_p = h)$ and $\alpha_v = \Pr(a_v = h)$. Assume that in the event of a war nation i attains the payoff $p(a_i, a_j)$ where this funciton is assumed to be symmetric, increasing in its first argument, and decreasing in its second argument. We begin by treating the investment decisions as exogenous. Suppose that states arm with probabilities α_p and α_v. Given an offer of s, a veto player that has armed has an expected utility from rejecting of $\alpha_p p(h,h) + (1-\alpha_p)p(h,0)$ and veto player that has not armed has an expected utility from rejecting of $\alpha_p p(0,h) + (1-\alpha_p)p(0,0)$ (net of arming costs).

Now consider an equilibrium of the following form: if the proposer has armed she makes an offer that causes an armed veto player to reject and an unarmed veto player to accept. The optimal such offer gives the veto player exactly $p(0,h)$. Given this, the expected utility to a proposer that arms is $[1 - p(0,h)](1-\alpha_v) + \alpha_v p(h,h)$. [2] If on the

[2]Note that this institution has a bit of commitment to prevent renegotiation built into it, as once the proposer sees such an offer rejected she learns that her oponent is armed and would be willing

other hand proposer does not arm then she makes an offer that both an armed and an unarmed veto player would accept. This offer must give the veto player, $p(h,0)$. The relevant conditions for an equilibrium of this form are fairly straightforward.

For a proposer to play both "arm" and "not arm" with positive probability she must be indifferent and

$$1 - p(h,0) = [1 - p(0,h)]\,(1 - \alpha_v) + \alpha_v p(h,h) - c_p.$$

Solving for the mixture, α_v, yields

$$\begin{aligned} 1 - p(h,0) + c_p &= [1 - p(0,h)] - [1 - p(0,h)]\,\alpha_v + \alpha_v p(h,h) \\ \frac{p(0,h) - p(h,0) + c_p}{p(h,h) + p(0,h) - 1} &= \alpha_v \end{aligned}$$

The denomnator is negative since $p(h,h) > p(0,h)$ and $2p(h,h) < 1$. In order for the numerator to be negative it must be the case that

$$c_p < p(h,0) - p(0,h.)$$

In order for $\alpha_v < 1$ we need

$$\begin{aligned} 1 - p(h,h) &> p(h,0) - c_p \\ c_p &> p(h,h) + p(h,0) - 1 \end{aligned}$$

which is true for any $c_p > 0$. So we have

$$\alpha_v = \frac{p(h,0) - p(0,h) - c_p}{1 - p(h,h) - p(0,h)}.$$

The veto player's indifference condition for arming is then

$$(1 - \alpha_p)(p(h,0)) + \alpha_p p(0,h) = (1 - \alpha_p)(p(h,0)) + \alpha_p p(h,h) - c_v.$$

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to make a different pareto efficient offer which the veto player would accept.

This condition simplifies to

$$\begin{aligned}\alpha_p p(0,h) &= \alpha_p p(h,h) - c \\ \alpha_p &= \frac{c_v}{p(h,h)-p(0,h)}\end{aligned}$$

Accordingly, the binding constraint for this type of equilibrium is that $c_v < \min\{p(h,0) - p(0,h), p(h,h) - p(0,h)\} = p(h,h) - p(0,h)$. It remains only to check that the offer strategy described is optimal for the proposer. A proposer that is unarmed could pretend to be strong and make a stonger offer. Such an offer would be rejected by veto player (subject to certain off-the-path belifs) and yield a payoff of $\alpha_v p(0,h) + (1-\alpha_v)p(0,0)$ to the proposer. In contrast the proposed strategy yields the payoff $1 - p(h,0)$. Substution yields the following expected utility for the deviation

$$\begin{aligned}&\alpha_v p(0,h) + (1-\alpha_v)p(0,0) \\ =\ &a_v(p(0,h) - p(0,0)) + p(0,0).\end{aligned}$$

Thus the relevant inequality that must be satisfied is

$$\begin{aligned}a_v(p(0,h) - p(0,0)) &\leq 1 - p(h,0) - p(0,0) \\ a_v &\geq \frac{1 - p(h,0) - p(0,0)}{p(0,h) - p(0,0)}.\end{aligned}$$

Since $1 - p(h,0) > p(0,0)$ the numerator is positive while the denominator is negative and thus and this condition is satisfied trivially. A proposer that is armed could make an offer that never gets accepted, or that always gets accepted. The former deviation yields her the expected utility $\alpha_v p(h,h) + (1-\alpha_v)p(h,0)$. Since $p(h,0) < [1 - p(0,h)]$ this quantity is less than $[1 - p(0,h)](1-\alpha_v) + \alpha_v p(h,h)$, the value to 1 of her equilibrium strategy and thus this deviation is unattractive. The latter deviation involves pooling with the unarmed type of proposer and results in a utility of $1 - p(h,0)$. The desired inequality is thus

$$\begin{aligned} 1 - p(h,0) &\leq [1 - p(0,h)]\,(1-\alpha_v) + \alpha_v p(h,h) \\ \frac{p(0,h) - p(h,0)}{p(0,h) + p(h,h) - 1} &\geq \alpha_v \\ \frac{p(h,0) - p(0,h)}{1 - p(0,h) - p(h,h)} &\geq \alpha_v \end{aligned}$$

The equilibrium value is, from above.

$$\alpha_v = \frac{p(h,0) - p(0,h) - c_p}{1 - p(h,h) - p(0,h)}$$

Thus the inequality to establish is

$$\frac{p(h,0) - p(0,h)}{1 - p(0,h) - p(h,h)} \geq \frac{p(h,0) - p(0,h) - c_p}{1 - p(h,h) - p(0,h)}.$$

This follows from the fact that $c_p \geq 0$. So if $c_p < p(h,0) - p(0,h.)$ then this seperating equilibrium exists. The first relevant point of this example is the fact that once a particular game form is selected the set of possible lotteries over strength that are consistent with equilibrium play to a larger game can be quite limited. In this example partial seperation of this form is only consistent with a particular profile of lotteries over the strenghts/types. Accordingly, the study of bargaining or mechanism design without connecting the investment levels to equilibrium play by agents that anticipate the subsequent game may miss important restrictions that we should expect data to satsify.

Now consider a game where the war winning function is:

$$\begin{aligned} p(h,h) = p(0,0) = 3/8 \\ p(0,h) = 1/8 \\ p(h,0) = 5/8. \end{aligned}$$

Also assume that country 1 has a cost of war ($c_1 = 1/6$), and country 2 can have a cost of $c_2 = 1/8$ or $c_2 = 3/16$. We start with the assumption that the lower cost country is the proposer, and has all the bargaining power. Then a simple calculation shows that the conditional probability of war, given that country 1 armed is 2/3. But, if we look at new game, where country 1 is the high cost player, the same kind of separating equilibrium exists. In fact, in that equilibrium a simple calculation shows that the probability of war conditional on country 1, now the high cost player, arming is again 2/3. That is, the "game form" or bargaining institution–which in one instance gave bargaining power to the low cost player and in the other gave it to the high cost player– did not effect the conditional probability of war.

In this example, one might then ask, does the changing role of the players effect the ex ante probability of war? Yes, another simple calculation shows that the probability of war is 1/2 when the low cost type is the proposer and 1/3 when the high cost type is the proposer. An important implication of this example–and the law of iterated expectations–is that if the institution did not effect the distribution of arming choices, it would not effect the probability of war given a set of choices. In what follows, we show that this tendency for the interaction between strategic incentives to arm and the bargaining procedures is quite general and that the important affect bargaining institutions have are through what might be called the "upstream" effects, on the incentives to arm. The rest of this paper then deals with the general case.

3 The model

Consider the interaction between two countries in anticipation of a crisis. Each country, $i \in \{1, 2\}$ must first select a level of investment in arms, $a_i \in \mathbb{R}_+$. To allow investments to be in mixed strategies we write $F_i(\cdot)$ to capture the distribution function of a_i. We assume that the cost of investment a_i is given by $c_i(a_i)$ where $c_i(\cdot)$ is a strictly increasing and differentiable function. By $c_i'(a_i)$ we denote the first derivitive of the cost at a_i and by $c_i^{-1}(\cdot)$ we denote the inverse of the cost function. The investment choices are assumed to be hidden information. The nations then negotiate over a resource or prize that is under dispute. We assume the prize is of size 1. If the states fail to reach an agreement they can fight. If they fight, investment levels enter into the contest payoff functions $p_1(a_1, a_2)$ and $p_2(a_2, a_1)$ which are increasing in their first argument and decreasing in their second argument. Throughout we assume that these functions are twice continuously differentiable. We assume that war is inefficient, $p_1(a) + p_2(a) < 1$ for all $a \in \mathbb{R}^2_+$. Payoffs for war are $p_1(a_1, a_2) - c_1(a_1)$ and $p_2(a_1, a_2) - c_2(a_2)$. If instead of fighting, the countries reach a settlement then they consume their share of the division.

We approach the problem without prescribing or assuming a particular model of negotiation. Instead of specifying a particualar negotiation game we consider a fairly large class of games or mechanisms. In particular we focus on mechanisms that result in transfers or war and assume only that the mechanism can commit to offers- but that nation don't need to accept offers they don't like. A mechanism is then a pair of message spaces M_1, M_2 and a triple of mappings, $q : M_1 \times M_2 \to [0, 1]$, $t_1 : M_1 \times M_2 \to \mathbb{R}^1_+$ and $t_2 : M_2 \times M_1 \to \mathbb{R}^1_+$. To give more intuition for these mechanisms it is helpful to think of a mechanism as a simple game in which the sequence of play is as follows: First (1) agents simultaneously select armament levels but these choices are hidden information, (2) then knowing only its own armament level, agents make reports $m_i \in M_i$ and (3) the mechanism then randomizes according to these three functions to assign payoffs. In particular it selects war with probability, $q(m_1, m_2)$. With probability $1 - q(m_1, m_2)$ the mechanism makes offers $t_1(m_1, m_2)$ to nation 1 and $t_2(m_2, m_1)$ to nation 2. (4) Finally, the states then decide whether to accept or reject the transfers. If either rejects then the outcome is fighting which results in payoffs $p_1(a_1, a_2)$ to nation 1 and $p_2(a_2, a_1)$ to nation 2. If both accpet the settlement then they get the payoffs $t_1(m_1, m_2)$ and $t_2(m_2, m_1)$ respectively. Note that the war payofffs hinge on the actual investments (a_1 and a_2) while the settlement payoffs depend on the reports or messages (m_1 and m_2).

A few observations are worth making. We do not require, at the onset, that transfers satisfy budget balance. In fact we will allow for transfers that sum to more than 1 and thus require external subsidy. The value of this approach is that it allows us to asses whether war can be prevented if a third party is willing to subsidize the settlements. We do, however require that transfers are non-negative. This assumption can be viewed as one of limited liability. We focus on institutions in which it is not possible to coerce a nation to give up more than just its possible share of the prize or territory at stake. One rationale for this assumption in the international relations context, is that we might think of institutions as bodies that states voluntarily join. If the institution could tax a state for expressing a claim on some territory then the state may be less willing to join the institution (or cede it the authority to adjudicate a claim). A second rationale is that if the institiont offered a negative settlment to state i then it would be the case that state i is better off fighting than accepting the settlement. Given our other assumption, that war is not a consensual act, it is unintersting to focus on institutions with negative transfers. The assumption that war is not consensual embodies the idea that the institution cannot prevent a war and that any of the two parties to a dispute can unilaterally initiate figthing. See Fey and Ramsay (2007) for an alternative interpretation of war and Fey and Ramsay (2008) for additional justification of this assumption.

4 Results

4.1 Preliminaries

Before proceeding to an analysis of incentive compatable and ex post individually rational direct mechanisms we provide a few preliminary results. Our first result illustrates that from a full implementation perspective, peace can not be attained. The assumption that war in non-consensual, i.e., that either player can initiate war implies that as long as the cannonical contest problem of simultaneously selection of a_i to maximize $p_i(a) - ca_i$ has a Nash equilibrium (in possibly mixed strategies) then there are pathological equilibria in which war occurs with probability one.

Theorem 1 *If the underlying contest functions $p_1(\cdot), p_2(\cdot)$ are such that the cannonical game in which each state i simultaneously selects a_i to maximize $p_i(a_i, a_{-i}) - ca_i$ has a Nash equilibrium then for every mechanism in the class described there exists an equilibrium in which war occurs with probability 1.*

To see why this is true, consider any mechanism in the class described and a strategy profile which selects a_i according to a Nash equilibrium in the cannonical contest problem and rejects any offer by the mechanism. No deviation by i will result in a peaceful outcome, and since the arming decisions are best responses in the cannonical contest, no deviation in the arming stage can improve i's payoff. It should be noted that refinements that require strategies to be the limit of best responses to completely mixed strategy equilibria do not automatically rule out this type of equilbrium. Once can carefully construct sequences of trembles in which the probability that player j trembleles to arm at level 0 with probability much higher than any other deviation from her conjectured strategy. If the offer is not too high, then rejection can be a best response in this game with trembles even if the offer exceeds the level $p_i(a_i^*, a_j^*)$.

A weaker criterion for what it means for a game to be peaceful is then justified.

Definition 2 *A mechanism is peaceful if it has an equilibrium in which war occurs with probability zero.*

Given this definition we have the following result.

Theorem 3 *In every equilibrium in which war occurs with probability zero, the arming choices must be* $a = (0, 0)$.

Proof. Suppose there is a peaceful mechanism and consider an equilibrium which war occurs with probability 0 but state i puts strictly positive probability on a set A not containing the point 0. Since war occurs with probability 0, state i's expected payoff conditional on selection from A is

$$\frac{\int_A \int \left[t_i(a_i', a_j) dF(a_j) - c_i a_i'\right] dF_i(a_i'')}{\int_A dF_i(a_i)}.$$

However, the deviation to $a_i'' = 0$ and a report of $m_i(a_i') = 0$ yields the strictly higher expected utility

$$\frac{\int_A \int \left[t_i(a_i', a_j) dF(a_j)\right] dF_i(a_i'')}{\int_A dF_i(a_i)}.$$

■

This result, captures a fundamental feature of this class of problems. If an equilibrium does not involve $a = (0, 0)$ with probability one than it must involve fighting with positive probability. This means that unless we have a non-arming equilibrium offers are not always accepted.

Our next result establishes necessary and sufficient conditions for peaceful equilibria. The argument involves determining when it is possible to construct mechanisms in which the designer offers each state a sufficient transfer that peace is better than the payoff $\max_{a_i} \{p_i(a_i, 0) - c_i(a_i)\}$

Theorem 4 *There exists a peaceful (and thus non arming mechanism i.e., $a_1, a_2 = 0$) without external subsidies iff*

$$1 \geq \max_{a_1} \{p_1(a_1, 0) - c_{i_i}(a_1)\} + \max_{a_2} \{p_2(a_2, 0) - c_2(a_2)\}. \tag{1}$$

Proof: In a peaceful mechanism theorem 1 requires that the peaceful equilibrium involve $a_i = 0$ and the mechanism must have $q(0, 0) = 0$. If $a_1 = 0$ is a best response to $a_2 = 0$ then it must be the case that $t_i(0, 0) \geq \max_{a_i} \{p_i(a_i, 0) - c_i(a_i)\}$ (to make deviation at arming undesirable) So for both players to be willing to select $a_1 = a_2 = 0$ the transfers must sum to at least

$$\max_{a_1} \{p_1(a_1, 0) - c_1(a_1)\} + \max_{a_2} \{p_2(a_2, 0) - c_2(a_2)\}$$

and thus this transfer is feasible without subsidy only if the inequality holds. Sufficiency follows from the observation that if each of the above transfers is offered a unilateral deviation from $(0, 0)$ is not profitable. If this condition is satisfied then a constant direct mechanism $t_i(m_i, m_j) = \max_{a_i} \{p_i(a_i, 0) - c_i(a_i)\}$ and $q(m_i, m_j) = 0$ works. With this mechanism, a deviation in reporting does not improve i's payoff unless war occurs, But expression (1) implies that neither player is better off unilaterally deviating and getting war. □

The logic of the previous theorem can be extended to answer the question, how large a subsidy does it take to construct an institution to prevent war. We have standardized the value of the resource under dispute to be 1, so if we let the quantity $r \geq 0$ represent a subsidy that some third party is willing to provide in order to avoid conflict then the minimal value of r needed to construct a peaceful and non arming mechanism is given by

$$r^{\min} = \max\{0, \max_{a_1}\{p_1(a_1,0) - c_1(a_1)\} + \max_{a_2}\{p_2(a_2,0) - c_2(a_2)\} - 1\}.$$

4.2 Properties of equilibria

We begin with a fairly standard analysis of incentive compatabile direct mechanisms, treating the distribution functions $F_i(\cdot)$ as fixed. A direct mechanism is represented by a function $t_i(m_i, m_j) : R_+^2 \to [0,1]$ that reports the transfer to i and a function $q(m_i, m_j) : R_+^2 \to [0,1]$ that determines the probability that fighting occurs.

Expected utility to i from investment a_i and report m_i can be written as

$$U_i(m_i, a_i) = \int [(1 - q(m_i, m_j))t_i(m_i, m_j) + q(m_i, m_j)p(a_i, m_j)]\, dF_j(m_j).$$

It is convenient to define $T_i(m_i) = \int [(1 - q(m_i, m_j))t_i(m_i, m_j)]\, dF_j(m_j)$ and $P_i(m_i; a_i) = \int q(m_i, m_j)p(a_i, m_j)dF_j(m_j)$. So

$$U_i(m_i, a_i) = T_i(m_i) + P_i(m_i; a_i)$$

We let $U_i(a_i) = U_i(a_i, a_i)$.[3] Incentive compatibility in a diect mechanism requires that for any a_i in i's support and any $a_i' \neq a_i$

[3]While the requirement of ex post individual rationality justifies the stronger notion of incentive compatibility (I.C.*), explored in Matthews and Postlewaite (1989), our study of necessary conditions focuses on the weaker and more convenient standard incentive compatibility conditions. In particular I.C.* requires that in addition to incentive compatibility it must be that for any $m_i \neq a_i$

$$\begin{aligned} &\int [(1 - q(a_i, m_j))\,[\max\{t_i(m_i, m_j), p(a_i, m_j)\}] + q(a_i, m_j)p(a_i, m_j)]\, dF(m_j) \\ \geq &\int [(1 - q(m_i, m_j))\,[\max\{t_i(m_i, m_j), p(a_i, m_j)\}] + q(m_i, m_j)p(a_i, m_j)]\, dF(m_j). \end{aligned}$$

While a full characterization requires that we work with I.C.*, the fact that I.C.* implies that I.C. is satisfied justifies our focus on the more tractable conditions in establishing results about properties that must be true in any equilibrium to any game.

$$
\begin{aligned}
U_i(a_i) &= T_i(a_i) + P_i(a_i; a_i) \geq T_i(a_i') + P_i(a_i'; a_i) \\
U_i(a_i') &= T_i(a_i') + P_i(a_i'; a_i') \geq T_i(a_i) + P_i(a_i; a_i')
\end{aligned}
$$

For fixed investment lotteries, standard arguments yield a revelation theorem, allowing us to focus on incentive compatable direcet mechanisms.

Theorem 5 *If there exists a mechanism with equilibrium arming decisions given by the mixtures F_1 and F_2 and the lottery $G(t_1, t_2, p_1, p_2)$ over transfers and war payoffs, then there is a direct mechanism posessing an equilibrium in which arming strategies are given by F_1 and F_2 and the staes report truthfully $m_i(a_i) = a_i$, which induces the same lottery over the outcomes.*

For fixed investment strategies the argument involves the standard composition strategy. Since investment decisions are privately observed and reports are unverifiable this first stage introduces no additional complications.

Our first characterization result is implied directly by incentive compatibility, and goes along way toward helping understand the incentives to arm. Rearranging the above incentive compatibility conditions in the standard way we have,

$$
\begin{gathered}
T_i(a_i) + P_i(a_i; a_i) - [T_i(a_i) + P_i(a_i; a_i')] \geq U_i(a_i) - U_i(a_i') \\
\geq T_i(a_i') + P_i(a_i'; a_i) - [T_i(a_i') + P_i(a_i'; a_i')]
\end{gathered}
$$

$$
P_i(a_i; a_i) - P_i(a_i; a_i') \geq U_i(a_i) - U_i(a_i') \geq P_i(a_i'; a_i) - P_i(a_i'; a_i')
$$

$$
\begin{aligned}
\int q(a_i, a_j)\, [p_i(a_i, a_j) - p(a_i', a_j)]\, dF_j(a_j) &\geq U_i(a_i) - U_i(a_i') \\
&\geq \int q(a_i', a_j)\, [p_i(a_i, a_j) - p(a_i', a_j)]\, dF_j(a_j) \qquad (2)
\end{aligned}
$$

Multiplying all terms by $\frac{1}{|a_i - a_i'|}$ and taking limits for a sequence $\{a_{i_n} - a_{i_n}'\}$, with $a_i > a_i'$, converging to zero allows us to conclude that

$$\lim_{a_i - a_i' \to 0} \int \frac{[p_i(a_i, a_j) - p(a_i', a_j)]}{|a_i - a_i'|} q(a_i, a_j) dF_j(a_j) \geq \lim_{a_i - a_i' \to 0} \frac{U_i(a_i) - U_i(a_i')}{|a_i - a_i'|}$$
$$\geq \lim_{a_i - a_i' \to 0} \int \frac{[p_i(a_i, a_j) - p(a_i', a_j)]}{|a_i - a_i'|} q(a_i', a_j) dF_j(a_j)$$

Since $p(\cdot, \cdot)$ is differnetiable and $q(a_i, a_j)[p_i(a_i, a_j) - p(a_i', a_j)]$ is bounded by an integrable function for all a_i, we obtain

$$\int \left[\frac{\partial p_i(a_i, a_j)}{\partial a_i}\right] \lim_{a_i - a_i' \to 0} q(a_i, a_j) dF_j(a_j)$$
$$\geq \lim_{a_i - a_i' \to 0} \frac{U_i(a_i) - U_i(a_i')}{|a_i - a_i'|}$$
$$\geq \int \left[\frac{\partial p_i(a_i, a_j)}{\partial a_i}\right] \lim_{a_i - a_i' \to 0} q(a_i', a_j) dF_j(a_j)$$

While $q(\cdot, \cdot)$ may not be continuous, the fact that $\int \left[\frac{\partial p_i(a_i, a_j)}{\partial a_i}\right] \lim_{a_i - a_i' \to 0} q(a_i, a_j) dF_j(a_j)$ is monotone and the domain is bounded (since dominance arguments ensure that nation i would not be willing to invest in a level greater than $c_i^{-1}(1)$) implies this function has at most a countable number of discontinuities, Therefore, for almost every a_i, we obtain the identity

$$U_i'(a_i) = \int \left[\frac{\partial p_i(a_i, a_j)}{\partial a_i}\right] q(a_i, a_j) dF_j(a_j)$$

The expected value of the mechanism to a country with investment a_i (net of costs) is then

$$U_i(\widehat{a}_i) = U_i(0) + \int_0^{\widehat{a}} \int \left[\frac{\partial p_i(a_i, a_j)}{\partial a_i}\right] q(a_i, a_j) dF_j(a_j) da_i.$$

For further reference, we collect these findings in the following theorem.

Theorem 6 *If (t, q) is an incentive compatable direct mechanism then the following* *is true for* $i \in \{1,2\}$ *(i)* $a_i > a_i'$ *implies*

$$\int q(a_i,a_j)\left[p_i(a_i,a_j) - p(a_i',a_j)\right] dF_j(a_j) \geq \int q(a_i',a_j)\left[p_i(a_i,a_j) - p(a_i',a_j)\right] dF_j(a_j) \tag{3}$$

and (ii) for almost every a_i

$$U_i'(a_i) = \int \left[\frac{\partial p_i(a_i,a_j)}{\partial a_i}\right] q(a_i,a_j)dF_j(a_j).$$

and (iii) the value of the mechanism (net of costs) is given by the function

$$U_i(\widehat{a}_i) = U_i(0) + \int_0^{\widehat{a}} \int \left[\frac{\partial p_i(a_i,a_j)}{\partial a_i}\right] q(a_i,a_j)dF_j(a_j)da_i.$$

The first part of the result does not imply that in every incentive compatible mechanism the conditional probability of war, given a_i, is itself monotone in a_i.[4]

With these results from the analsys of the problem while treating the distribution over investment levels as fixed we can now focus on the study of what types of investment strategies are actually possible in an equilibrium. It is useful to observe that since the value of winning is bounded by 1, the most that i will invest in any equilibrium is $c_i^{-1}(1)$. For any mixed strategy, take any levels of arming chosen with positive probability, a_i and a_i', then equilibrium requires that $U_i(a_i) - U_i(a_i') = c_i(a_i) - c_i(a_i')$. So for any two points in i's support we have

$$\begin{aligned}\int q(a_i,a_j)\left[p_i(a_i,a_j) - p(a_i',a_j)\right] dF_j(a_j) &\geq c_i(a_i) - c_i(a_i') \\ &\geq \int q(a_i',a_j)\left[p_i(a_i,a_j) - p(a_i',a_j)\right] dF_j(a_j)\end{aligned}$$

Taking $a_i - a_i' > 0$, and using the assumption that $p(a_i,a_j)$is differntiable and strictly increasing in its first argument yields

[4] In a seperate paper (Meirowitz and Ramsay 2009), we show that any function $q(\cdot,\cdot)$ which can be robustly implemented in the sense of Bergemann and Morris (2008) is in fact monotone.

$$\int q(a_i,a_j)[p_i(a_i,a_j)-p_i(a_i',a_j)]dF_j(a_j) \geq c_i(a_i)-c_i(a_i')$$
$$\geq \int q(a_i',a_j)[p_i(a_i,a_j)-p_i(a_i',a_j)]F_j(a_j)$$

$$\int \left[\frac{p_i(a_i,a_j)-p(a_i',a_j)}{a_i-a_i'}\right] q(a_i,a_j)dF_j(a_j) \geq \frac{c_i(a_i)-c_i(a_i')}{a_i-a_i'} \geq \int \left[\frac{p_i(a_i,a_j)-p(a_i',a_j)}{a_i-a_i'}\right] q(a_i',a_j)dF_j(a$$

Noting that the function under the integral is bounded because the support is bounded and taking the limit as $\{a_i - a_i'\}$ goes to zero gives,

$$\int \frac{\partial p_i(a_i,a_j)}{\partial a_i} \lim_{\{a_i-a_i'\}\to 0} q(a_i,a_j)dF_j(a_j) \geq c_i'(a_i) \geq \int \frac{\partial p_i(a_i,a_j)}{\partial a_i} \lim_{\{a_i-a_i'\}\to 0} q(a_i',a_j)dF_j(a_j).$$

This last conclusion implies that if $\int q(a_i,a_j)F(a_j)$ is continuous at a_i then

$$c_i'(a_i) = \int \frac{\partial p_i(a_i,a_j)}{\partial a_i} q(a_i,a_j)dF_j(a_j)$$

We cannot conclude that $c_i'(a_i) = \int \frac{\partial p_i(a_i,a_j)}{\partial a_i} q(a_i,a_j)dF(a_j)$ at every value of a_i because the function $q(\cdot,\cdot)$ need not be continuous. Again, however since the domain is bounded (recall the dominance arguments that $c_i^{-1}(1)$ is the most i will invest) and if the integral equation is monotone in its first argument, the number of discontinuities is at most countable. But if a_i is in i's support it must be optimal. This requires that a small deviation cannot improve i's payoff. If $c_i'(a_i) < \int \frac{\partial p_i(a_i,a_j)}{\partial a_i} q(a_i,a_j)dF(a_j)$ then for a small enough value of $\varepsilon > 0$ a deviation to $a_i+\varepsilon$ will result in a larger increase $U_i(a_i+\varepsilon)-U_i(a_i)$ than the change in cost (since c_i is differentiable and thus continuous). Accordingly at any a_i in the support of an equilibrium strategy it must be the case that $c_i'(a_i) = \int \frac{\partial p_i(a_i,a_j)}{\partial a_i} q(a_i,a_j)dF_j(a_j)$ and so this expression also has to be upper semi continuous at any point in an equilibrium support. This implies the following result.

Theorem 7 *In any equilibrium to any game, if a_i is in the support of i's mixed* *strategy then*

$$c_i'(a_i) = \int \frac{\partial p_i(a_i, a_j)}{\partial a_i} q(a_i, a_j) dF_j(a_j)$$

This equilibrium conditions illustrates that incentive compatability (and thus the envelope theorem) and the indifference condition for arming require that the equilibrium investment strategies are closely related to $q(\cdot, \cdot)$ and the primitives, c and $p_i(\cdot, \cdot)$. An even starker representation is possible, however. Consider two distinct functions $q(\cdot, \cdot)$ and $q'(\cdot, \cdot)$ that are both attained in equilibrium with mixed strategies $F_j(\cdot)$ and $F_j'(\cdot)$ respectively. Further assume that a_i is in the support of player i equilibrium mixture in both cases. Holding fixed the cost and war payoff function, we attain the following consequence of the equilibrium condition

$$\int \frac{\partial p_i(a_i, a_j)}{\partial a_i} q'(a_i, a_j) dF_j'(a_j) = \int \frac{\partial p_i(a_i, a_j)}{\partial a_i} q(a_i, a_j) dF_j(a_j).$$

Linearity of the intergral yields

$$\int \frac{\partial p_i(a_i, a_j)}{\partial a_i} \left(q'(a_i, a_j) dF_j'(a_j) - q(a_i, a_j) dF_j(a_j) \right) = 0$$

Using the mean value theorem we know that there is some value a_j^* s.t.

$$\frac{\partial p_i(a_i, a_j^*)}{\partial a_i} \int \left(q'(a_i, a_j) dF_j'(a_j) - q(a_i, a_j) dF_j(a_j) \right) = 0$$

Since $p_i(a_i, a_j)$ is assumed to be strictly monotone in both of its arguments, $\frac{\partial p_i(a_i, a_j^*)}{\partial a_i}$ is strictly positive for all values of a_j accordingly we know that $\frac{\partial p_i(a_i, a_j^*)}{\partial a_i} > 0$. This and the previous equlity require that

$$\int \left(q'(a_i, a_j) dF_j'(a_j) - q(a_i, a_j) dF_j(a_j) \right) = 0$$

And thus we have

$$\int q'(a_i, a_j) dF_j'(a_j) = \int q(a_i, a_j) dF_j(a_j)$$

But each of these terms is just the conditional probability of war given a_i in the environments $q(\cdot, \cdot)$ and $q'(\cdot, \cdot)$ respectively. Accordingly we have just concluded that

Theorem 8 *Fix c and consider two institutions (incentive compatable mappings) $q(\cdot,\cdot)$ and $q'(\cdot,\cdot)$. If investment level a_i is in the support of an equilibrium investment stratgy for player i in an equilibrium under each of these institutions, then* $\Pr(war \mid a_i)$ *is the same in both institutions.*

We can, in fact go slightly farther and provide a sometimes useful characterization of the conditional probability of war. Recal that the equilibrium condition requires that

$$c_i'(a_i) = \int \frac{\partial p_i(a_i, a_j)}{\partial a_i} q(a_i, a_j) dF_j(a_j)$$

And so the mean value theorem, again, allows us to reach the following conclusion

Corollary 9 *There exists some value a_j^{**} such that*

$$\Pr(war \mid a_i) = \int q(a_i, a_j) dF_j(a_j) = \frac{c_i'(a_i)}{\frac{\partial p_i(a_i, a_j^{**})}{\partial a_i}}$$

at any investment level a_i in the support of an equilibrium investment strategy for some incentive compatable institution.

This result establishes that as a function of i's investment the probability of war in any equilibrium is linear in the cost and decreasing in the marginal effect of investment on i's war payoff. While comparisons across games, or even across equilibria to the same game are complicated because the equilbrium mixtures (and supports might) vary the result is quite powerful. In the spirit of a revenue equivalence theorem it says that even though investment strategies vary, the overal probability of war conditional on a_i is the same in every equilibrium to every game in which a_i is in i's support. To compare this with revenue equivalence. Note that revenue equivalence provides an equivalence on the function relating realizations of types into revenue given an assignment rule. But expected revenue in any equilibrium to any auction (when the theorem applies) requires integrating this function over the exogenous distribution. For our problem in which the lotteries over types are endogenous the result does not depend on the analogue to an allocation rule (for us it would be $q(\cdot,\cdot)$. Equilibrium investment decisions (lotteries) respond to q (at least I.C. functions) in a way to completely offset variation in the q function.

5 Conclusion

In this paper we confront two problems that face the theoretical study of conflict. First, we take serious the idea that players choose their strengths and, to a great degree, make their choices with strategic foresight. In particular, decision-makers worry about how their choices of arms levels will effect their bargaining position in times of crisis. Second, we also worry that there is no natural or stylized model of crises or conflict, and that our theoretical results might be too closely tied to any particular choice of model that has assumptions that are not well motivated. To make progress on this problem we pursue game free results, and consider how the interaction between the institutional form of a crisis game and incentives to arm effect various equilibrium outcomes.

Even with minimal structure, we are able to learn a number of things about crisis bargaining with endogenous strength. For example, we know that strategic incentives to fight are only absent in a world with no arms, and a world with no arms is a necessary condition for there to exist peaceful equilibria. We also know that the strategic arming incentives do not depend on the size of the settlement offers from the bargaining game, and for games with additively separable technologies of war, the probability of war given a level of arms does not depend on the underlying institution. This is because any inducements toward peace a bargaining institution might create, then create incentives for players to "game the system" in a way that the conditional probability of war is unchanged. The other implication here, is that we can effect the probability of war with institutions that effect decisions to arm. That is, our results do not imply that institutions are irrelevant. They suggest that if we ignore the effect institutions have on strategic investment in arms, we may be missing the big effect institutions have on conflict. So while strategic investment decisions offset institutional effects on the probability of war downstream when institutions are given, institutions can effect investment and incentives to fight wars.

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