Do the Advantages of Incumbency Advantage Incumbents?

Sanford C. Gordon
New York University

Dimitri Landa
New York University

Abstract

We call into question whether some key sources of incumbency advantage frequently cited in the empirical and theoretical literature are, in fact, beneficial to incumbents. We show that increases in direct officeholder benefits such as the "campaign discount" incumbents enjoy relative to challengers and the pro-incumbent bias of interest groups making endorsements, as well as in district partisan bias favoring the officeholder, all encourage pooling among incumbents of higher and lower quality in equilibrium. While this means an improvement in the electoral prospects of lower-quality incumbents, it is harmful to those of higher quality. Whether the net electoral consequence for high-quality incumbents is positive or negative depends on whether the pooling effect is direct or mediated through voters' choices. Our findings suggest, further, that fundamental tensions may exist between accounts of incumbency advantage focusing on the sources noted above and on selection through repeated elections and challenger scare-off. They also point to obstacles to efforts to disaggregate the sources of incumbency advantage empirically.

1 Introduction

The "incumbency advantage" typically refers to the electoral margin a candidate might enjoy on account of her status as an incumbent running for reelection. An extensive literature in American politics beginning in the 1960s and 1970s has documented the existence of such a margin – first in congressional elections (e.g. Erikson 1971; Mayhew 1974), and later elsewhere (e.g. Ansolabehere and Snyder 2002). Other scholars have endeavored to disaggregate the impact on the incumbency advantage of what might be called the *advantages of incumbency* – that is, analytically distinct features of electoral politics perceived to contribute to that margin. A default presumption of these studies (and often a logical implication of their research designs) is that while these advantages may differ in the magnitude of their individual effects, they all point in the same direction. All are understood to work to the benefit of incumbents as such, rather than, for example, to the benefit of some types of incumbents occupying the same office and to the detriment of others. Further, none are thought to operate in opposition to one another.

We evaluate this presumption in a sequence of three simple models of electoral competition with strategic challengers, incumbents, and voters. In each of the models, a prospective challenger and incumbent, who may differ in their qualifications, make decisions concerning whether to participate in a race. The outcome of the race is determined by voters, who are less informed about the qualifications of the candidates than the candidates themselves. Each model captures a key feature of the electoral environment that affects the relative electoral fortunes of incumbents and prospective challengers even holding constant voters' beliefs about specific qualifications of individual politicians. (For convenience, we refer to these as "environmental" sources of the incumbency advantage.) The first two models consider two different sources typically associated with *direct officeholder benefits*: what we call the *campaign discount*, which corresponds to mechanisms that, ceteris paribus, make it less costly for an incumbent to run for reelection than for a challenger to mount a campaign, and *pro-incumbent endorser bias*, which may emerge from the unique opportunities for officeholders to cultivate relationships with influential interest groups or elites in a district. The third model considers ideological heterogeneity and *district partisan bias* toward an incumbent among the voters.[1]

[1] We discuss empirical literature associated with these advantages in further detail below.

In each of these models, voters cannot observe candidate quality directly, and must draw inferences about the relative merits of candidates from features of the electoral environment and a particular race. We demonstrate that a key implication of an increase in the campaign discount, endorser bias, and district partisan bias is an equilibrium *pooling effect* among incumbents of differing ability: because voters cue off candidates' decisions to enter or stay in a race, less-qualified incumbents will have a stronger incentive to remain in a contest against a serious challenger. While this effect may benefit those low-quality incumbents, it may harm those of higher quality by diluting the value of the signal entailed in their decision to run for reelection in the face of a serious challenge. Thus, these sources of incumbency advantage may advantage or *disadvantage* the electoral fortunes of incumbents depending on their relative abilities.

While the presence of the pooling effect is a key factor in determining the electoral fortunes of incumbents, our analysis shows that it is not, in and of itself, a sufficient determinant. Whether highly qualified incumbents are, in expectation, harmed by this effect depends on whether it is mediated by the voters' choices. An increase in the campaign discount or in endorser bias directly influences the behavior of incumbents, even holding constant the voter's choice. By contrast, an increase in pro-incumbent partisan bias produces pooling by making an incumbent more attractive to voters even holding constant candidate choices. In equilibrium, this distinction implies that a larger campaign discount or endorser bias does, in fact, have opposite effects on the electoral fortunes of high- and low-quality incumbents, whereas a larger pro-incumbent partisan bias is all upside.

These results suggest that some traditionally-noted sources of incumbency advantage may not uniformly benefit all incumbents: while some help incumbents indiscriminately, others help certain categories of incumbents while actually harming others. Our analysis further suggests a tension between some of the "environmental" sources of incumbency advantage and sources that may be thought to emerge *endogenously* in the immediate electoral context, most notably, selection and challenger scare-off. As discussed in further detail below, these are understood to put progressively higher quality incumbents in office. However, those same incumbents are the ones made worse off by the existence of campaign discount and greater endorser bias. The presence of this tension implies a slowing of the selection effect. It also suggests a methodological challenge for efforts to isolate the sources of incumbency advantage empirically.

The remainder of the paper is organized as follows. We begin by providing a brief overview of the advantages of incumbency discussed in previous empirical and theoretical research. Subsequently, we turn to the formal analysis of our three models. Finally, we discuss some of the implications of our research for future work on electoral politics. To focus the discussion in the main body on comparative statics and electoral welfare effects associated with the sources of incumbency advantage, we defer formal characterizations of the equilibria to the Appendix.

2 Incumbent Advantages and Electoral Politics

Our models focus explicitly on properties of three environmental (in the above sense) sources of incumbent electoral welfare. The first two correspond to direct officeholder benefits: sources of electoral advantage available to incumbents by virtue of their access to the powers of office. First, the *campaign discount* reflects greater name recognition within a district (which may stem from constituency service – see, e.g., Cain, Ferejohn, and Fiorina 1987), the franking privilege (Mayhew 1974), greater media coverage (Arnold 2004; Prior 2006; but see Ansolabehere, Snowberg, and Snyder 2004), and the ability to amass a war chest of contributions (Goodliffe 2005). All of these factors make it cheaper for an incumbent to mount a serious reelection campaign than for a candidate to mount a serious challenge, ceteris paribus.[2]

Second, *pro-incumbent endorser bias* is relevant in part because uninformed voters may rely on endorsements from respected elites or interest groups when deciding how to cast their ballots (McKelvey and Ordeshook 1985; Lupia 1994; Wittman 2007).[3] Because incumbent officeholders have opportunities to cultivate relationships or offer promises to those elites, this may lead to some form of bias toward the incumbent (c.f. Grossman and Helpman 1999).

The campaign discount and pro-incumbent endorser bias contrast with a third key source of incumbency advantage that also operates independently from voters' beliefs about specific qualifications of individual politicians. *Pro-incumbent district partisan bias* reflects the ideological orientation of an incumbent's constituency. Absent redistricting, the ideological characteristics of a district tend to be relatively stable. Thus, an incumbent from a political party favored in a district

[2] As this formulation implies, the factors that contribute to the campaign discount are not necessarily insurmountable obstacles to the challenger. Rather, they may be surmountable, albeit at cost.

[3] For a discussion of growing pro-incumbent tendencies in newspaper endorsements, see Ansolabehere, Lessem, and Snyder (2006).

may enjoy an advantage over challengers in an electorate that put her in power in the first place (Alford and Brady 1989; Gelman and King 1990; Hirano and Snyder 2007).[4]

Each of these three factors exist in an environment in which voters may possess different *prior beliefs* about the quality or valence of an incumbent officeholder when compared to the typical challenger. The sources of this disparity may thus operate as sources of incumbency advantage as well. One such source is electoral selection effects (Erikson 1971; Zaller 1998; Ashworth and Bueno de Mesquita 2008; Gowrisankaran, Mitchell, and Moro 2008): if voters systematically elect candidates of higher quality, then over time, we should anticipate officeholders to be of progressively higher quality. Moreover, given these selection effects, it is reasonable for relatively uninformed voters to rely on incumbency as a cue regarding a candidate's superior qualifications (Mayhew 1974; Ansolabehere, Hirano, Snyder, and Ueda 2006). An institutional source of disparate beliefs in the context of legislative elections is seniority (McKelvey and Riezman 1992): if agenda-setting power increases with experience, then, ceteris paribus, voters will anticipate higher performance from an incumbent than from her replacement. Finally, an incumbent's constituency service may contribute to the disparity in voter evaluations of incumbents and challengers (Fiorina and Rivers 1989; Cain, Ferejohn, and Fiorina 1987; Fiorina 1977).

The conjunction of disparate prior voter beliefs about quality and the three environmental factors may contribute further to *behavior* at the time of an election that may affect an incumbent's electoral prospects. Challengers reluctant to enter a race against an incumbent perceived to be unbeatable may be *scared off* (Banks and Kiewiet 1989; Cox and Katz 1996; Gordon, Huber, and Landa 2007). Incumbents facing a serious challenge may *strategically retire* rather than experience a likely defeat (Jacobson and Kernell 1983; Kiewiet and Zeng 1993; Hall and van Houweling 1995; Ansolabehere and Snyder 2004). Finally, given the presence of pro-incumbent endorser bias, elites must decide whether or not to endorse an incumbent given the potential electoral consequences of doing so (Juravich and Shergold 1988; Asher, Heberlig, Ripley, and Snyder 2001; Hill 2003).

3 The Campaign Discount

Our first model considers the campaign discount incumbents enjoy relative to challengers.

[4]Indeed, much of the empirical literature on incumbency since Gelman and King (1990) has sought to control for the confounding influence of party.

3.1 Primitives and Equilibrium Concept

There are three players: an incumbent i, a (potential) challenger c, and a representative voter v.[5] An incumbent officeholder's type $t_i \in \mathbb{R}$ is a draw from a distribution with density $p_i(\cdot)$ and associated cumulative distribution function $P_i(\cdot)$. The potential challenger's type, $t_c \in \mathbb{R}$, is an independent draw from a distribution with density $p_c(\cdot)$ and associated CDF $P_c(\cdot)$. Having incumbents and potential challengers come from separate distributions captures, in reduced form, the notion that the voter's prior beliefs about candidates who contemplate entering a race need not be symmetric.[6] We do assume, however, that these densities share common support, that they are common knowledge to all players, and that they are unimodal.[7]

We assume that the incumbent and potential challenger, consistent with their status as political elites, are better informed than voters. We model this asymmetry by assuming they know the specific values of t_i and t_c, while the voter initially knows only their distribution. Challengers and incumbents value the quality of the officeholder, and additionally place intrinsic value $q > 0$ on holding office. Further, challengers face an opportunity cost from running equal to $k > 0$, with $k < q$, while incumbents face an opportunity cost equal to $k - d$, with $0 < d < k$. The parameter d captures the difference between opportunity costs between challengers and incumbents, and may be thought of as a measure of the incumbent's campaign discount discussed above. The values of q, k, and d are commonly known to all players.

The restriction that the challenger's opportunity cost k exceeds zero can be interpreted as implying that the challengers considered here are are not "sacrificial lambs." A challenger who is a sacrificial lamb may choose to run even in the face of a certain electoral loss, for example to build name recognition for future contests or to provide a service to the party. These benefits may be interpreted as positive consumption value outside of the current model. However, in the context of our model, we can interpret k as a *net* cost to the candidate, with $k > 0$ reflecting common knowledge that the prospective challenger in question is "serious," that is, not a sacrificial lamb. In this regard, the absence of a challenge in this setting is equivalent to entry by a sacrificial lamb believed to be inferior to the incumbent.

[5] In Section 5, we consider a model with ideologically heterogeneous voters in which the median voter is decisive.

[6] Independence of incumbent and potential challenger types *ex ante* does not imply independence of incumbent and actual challenger types in equilibrium.

[7] Unless noted otherwise, we do not make any assumptions on the relative rankings of $E[t_i]$ and $E[t_c]$.

Table 1: Payoffs to Players

Actions		Challenger does not run	Challenger runs, incumbent quits	Both challenger, incumbent run
Voter retains incumbent	u_c	t_i	–	$t_i - k$
	u_i	$t_i + q$	–	$t_i + q - (k - d)$
	u_v	t_i	–	t_i
Voter elects challenger	u_c	–	$t_c + q - k$	$t_c + q - k$
	u_i	–	t_c	$t_c - (k - d)$
	u_v	–	t_c	t_c

The sequence of events is as follows:

1. The prospective challenger chooses whether to enter/challenge ($C = 1$) or not ($C = 0$). If prospective challenger does not enter, the game ends with the incumbent remaining in office.

2. If potential challenger enters, the incumbent decides whether to stay in the race ($S = 1$) or quit ($S = 0$). If the incumbent quits, the game ends with the challenger's election.

3. If the incumbent stays, the voter chooses whether to reelect the incumbent ($R = 1$) or not ($R = 0$).

The payoffs to all players under all action profiles are summarized in Table 1.

Our solution concept is perfect Bayesian equilibrium (PBE), which requires that (a) each player's choices be sequentially rational given her beliefs at the time of choice and the other player's strategy; (b) beliefs about the other player's type be consistent with prior beliefs, equilibrium strategies, and Bayes' Rule on the path of play. Other than the equilibrium we describe in detail below, there are two uninteresting, pooling equilibria as well. In the first, challengers never run, and voters believe a challenger who does run is inferior and vote to retain the incumbent. In the second, incumbents always retire if challenged, and voters believe an incumbent who stays in a race is inferior to the challenger, and vote to elect the challenger. Because we are interested in the relationship between the institutional environment and the inferences voters can draw in competitive races, and how those inferences affect the well-being of incumbents, we focus on the equilibrium in which competitive races can occur.

3.2 Preliminaries

The voter. As a shorthand, we will refer in what follows to situations in which the prospective challenger enters the race and the incumbent stays ($C = 1, S = 1$) as *contested races.* Given a contested race, the voter will vote to retain the incumbent if and only if $E[t_i|C = 1, S = 1] \geq E[t_c|C = 1, S = 1]$. The voter's posterior beliefs about the incumbent and the challenger for arbitrary information set Z are:

$$\begin{aligned} p(t_i|Z) &= \frac{\Pr(Z|t_i)p_i(t_i)}{\int_{-\infty}^{\infty} \Pr(Z|t)p_i(t)dt} \\ p(t_c|Z) &= \frac{\Pr(Z|t_c)p_c(t_c)}{\int_{-\infty}^{\infty} \Pr(Z|t)p_c(t)dt}. \end{aligned} \tag{1}$$

The incumbent. Let ρ denote the probability that the voter will vote to retain the incumbent conditional on a contested race. If the incumbent quits when challenged, the challenger will be elected with certainty. By contrast, if the challenger stays in the race, the incumbent's payoff depends on the voter's decision whether to retain. The expected utilities associated with these choices are thus

$$\begin{aligned} E[u_i(S = 0|C = 1)] &= t_c \\ E[u_i(S = 1|C = 1)] &= -(k - d) + (1 - \rho)t_c + \rho(t_i + q). \end{aligned}$$

Comparing these two values, the incumbent will stay to contest a challenge if and only if

$$t_i > t_c - q + \frac{k - d}{\rho}. \tag{2}$$

The challenger. Let $\sigma(t_c, t_i)$ denote the probability that an incumbent will stay in the race if challenged, as a function of the two candidates' respective competences. By the same rationale as that that applied in the case of the incumbent, a prospective challenger will enter if and only if

$$t_c > t_i - q + \frac{k}{(1 - \rho)\sigma(t_c, t_i)}. \tag{3}$$

It is immediate from the best response correspondences that the entry decision of the prospec-

tive challenger and the decision of the incumbent whether or not to remain to contest a race are monotonic in type; that is, if a prospective challenger of type t_c' chooses to enter, than any challenger of type $t_c'' > t_c'$ will do so as well, and symmetrically for the incumbent.

3.3 Results

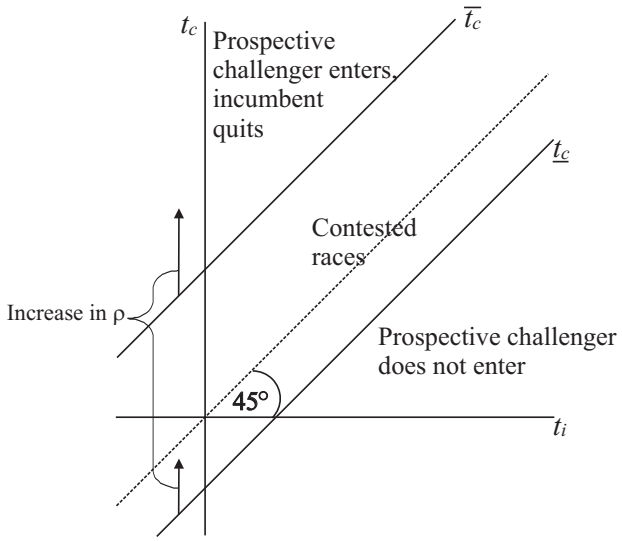
First, consider the inference a voter can draw from the mere fact that a race is contested by two serious candidates. First, observe that, *conditional on a contested race*, the voter must, in equilibrium, be indifferent between the candidates:

$$E[t_i|C = 1, S = 1] = E[t_c|C = 1, S = 1]. \tag{4}$$

We formalize the argument in Lemma 1 in the Appendix. Informally, if this were not the case, then the candidate the voter viewed as inferior would not run because of the opportunity cost of doing so. Condition (4) implies that if contested races are possible, there must be, for a given incumbent type t_i, challengers both inferior and superior to the incumbent who would be willing to enter. The voter's indifference is supported by the equilibrium choices of the voter as well as by the candidates. In particular, consider the effect of a change in the probability of retention, ρ. As ρ increases, a progressively narrower range of inferior prospective challengers will prefer entering the race, while an inferior incumbent will remain to contest a race against a broader range of superior challengers. This has the effect of decreasing the left side of equation (4) while increasing the right side. The equilibrium value of ρ, ρ^*, is the unique value that satisfies equation (4).

The equilibrium behavior of politicians emerges due to mutual deterrence. When the perquisites of office and the probability of victory are low, or when the opportunity cost of running is high, very incompetent prospective challengers will not find it in their interest to pursue office, and very incompetent incumbents will prefer leaving office to contesting a challenger who does enter. In order for races ever to be contested, the perquisites of holding office q must be relatively high ($q > 2k - d$, see Proposition 6 in the Appendix), otherwise, either the challenger will not run or, conditional on a challenge, the incumbent will always step down. The necessary and sufficient condition for the existence of prospective challenger and incumbent type-pairs that will lead to a contested race on the path of play is given by the conjunction of this condition and equation (4).

Figure 1: Candidate Behavior Given Candidate Competence in the Absence of an Endorser



Suppose the perquisites of holding office are sufficiently high that contested races sometimes emerge in equilibrium. Figure 1 displays candidate behavior in this case graphically as a function of the incumbent and prospective challenger's types. The quantity $\underline{t_c} < t_i$ represents the lowest quality prospective challenger willing to enter the race against an incumbent of type t_i, while the quantity $\overline{t_c} > t_i$ represents the highest quality challenger against whom an incumbent would remain in the race. Note that simple changes in the distributions of incumbent and challenger types can lead to each of the three cases depicted becoming more or less likely to occur on the path of play. The effect of a change in ρ is captured in the figure by an upward movement of the band of "contested races." Note that depending on the value of ρ, such a change can make this interval narrower or broader. Also note that, ceteris paribus, when the temptation of office is higher, the band of contested races will be broader.

Our first formal result describes how the incumbent's campaign discount d relates to candidate and voter behavior, and to the incumbent's *ex ante electoral prospects* – that is, her chances of retaining office averaging over the entire distribution of possible prospective challenger types.

Proposition 1 (Electoral Prospects and Campaign Discount) *In equilibrium, if contested races are possible ($q > 2k - d$), an increase in the incumbent's campaign discount d leads to:*

1. *a strict increase in the probability that a challenger enters the race, a strict increase in the probability that an incumbent stays in the race if challenged, and a strict decrease in the probability the voter retains the incumbent in a contested race; and*

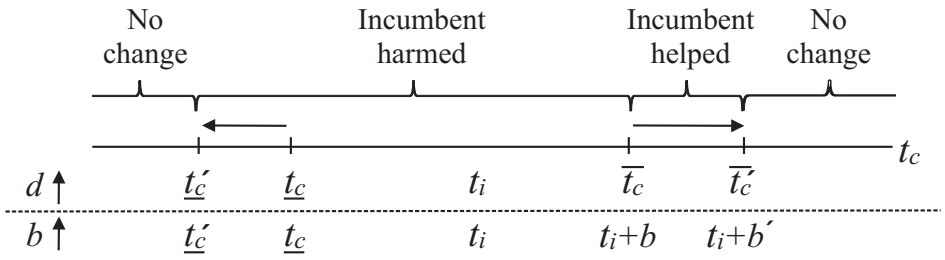
2. *a strict decrease in the electoral prospects of incumbents of sufficiently high type, and a strict increase in the electoral prospects of incumbents of sufficiently low type.*

The top of Figure 2 displays the intuition behind the result. The *direct* effect of an increase in the campaign discount d is to encourage incumbents to remain to contest a broader range of superior challengers than previously. The *indirect* effect comes via equilibrium responses of the voter and challenger. First, given a greater willingness of inferior incumbents to remain in the race, a heretofore indifferent voter will now strictly prefer to elect the challenger. A decrease in the retention probability ρ will restore the voter's indifference (and thus the equilibrium) by crowding out some (but not all) of those lower quality incumbents. Incumbents who would have faced contested races before and after the change in d will consequently win with lower probability. It will not be necessary to crowd out all of those lower quality incumbents to restore the voter's indifference, however, because, for a given incumbent type t_i, the decrease in ρ will also encourage some inferior challengers who were initially deterred to enter the race. Note that this works to the clear detriment of those incumbents, who would previously have been unopposed and won with certainty, but who might now face a contested race.

For those incumbents who would previously have stepped down and now remain in the race given the change in campaign discount d, electoral prospects are improved: they now have a positive probability of retaining office, whereas before that probability was zero. However, as the incumbent's type t_i increases, the likelihood of being in such a situation decreases, and will eventually be too small to offset the other negative consequences of the change in d. (Note that the value of the type of the incumbent for which the positive and the negative effects just balance each other is itself a decreasing function of d.) This points to what is, in effect, a disadvantage associated with the campaign discount, a feature of the electoral environment generally thought to advantage incumbents. Moreover, our result indicates that an increase in campaign discount actually *undermines* the "scare-off" of challengers – another purported source of incumbency advantage.[8]

[8]These results carry through if the voter learns candidate types with some positive probability.

Figure 2: Incumbent Electoral Fortunes Given Changes in Campaign Discount and Endorser Bias



The labels above the dotted line represent changes in critical values associated with an increase in d. The labels below the dotted line represent changes in critical values associated with an increase in b when endorsement is not redundant. For values of t_c on the range $(-\infty, \underline{t_c}')$, the incumbent is unchallenged before and after the change in d or b. On $(\underline{t_c}', \underline{t_c})$, challengers deterred from running prior to the change will now enter. On $(\underline{t_c}, \min\{\overline{t_c}, t_i+b\})$, candidate strategies are unchanged, but the probability the incumbent is retained decreases. On $(\min\{\overline{t_c}, t_i+b\}, \min\{\overline{t_c}', t_i+b'\})$, incumbents who would previously have dropped out now remain in the race. For challengers on $(\min\{\overline{t_c}', t_i+b'\}, \infty)$, incumbents drop out before and after the change.

4 Pro-incumbent Endorser Bias

4.1 Preliminaries

In this section, we introduce a second hypothesized source of incumbency advantage: the greater ease of obtaining the support of an influential interest group or other "endorser." We model this by introducing a fourth player, the endorser, e, who, like the challenger and incumbent (but unlike the voter) is fully informed about the types of the politicians. Consistent with our discussion of the sources of the incumbency advantage, the endorser is biased toward the incumbent. We model this bias by assuming that she would prefer the incumbent remain in office as long as $t_c < t_i + b$, where $b > 0$ is a commonly-known parameter capturing the extent of the endorser's bias toward the incumbent. Note that in the campaign discount model, there is no relationship between campaign discount and the incumbent's quality. By contrast, in the current model, endorser bias is directly related to quality. The sequence of play is identical to that described in the previous section, with the addition of the following step: conditional on a contested race, and prior to the voter's decision, the endorser announces her support either for the incumbent ($B = 1$) or the challenger ($B = 0$). As we demonstrate below, in equilibrium the endorser will pursue a cutpoint strategy, endorsing the incumbent if and only if $t_c > t_i + \beta$. To avoid proliferating notation, and without loss of generality,

below we use β as a shorthand for that strategy.[9]

The requirements of the solution concept (PBE) we employ are identical to those described above. We further refine the set of informative equilibria by requiring that the equilibrium be consistent with common knowledge that if the endorser endorses the incumbent, she prefers that the incumbent be elected, and likewise for an endorsement of the challenger. This refinement eliminates a strategically equivalent equilibrium in which the labels on endorsements are reversed.

First, consider the nature of an endorsement for the *challenger*. Such an endorsement is an unambiguous message to the voter of the challenger's superiority: "The relative competence of this challenger is so high that it overcomes my bias for the incumbent." Upon receiving such a message, the voter would simply vote to elect the challenger given a contested race. But then the incumbent would prefer not to remain in the race, owing to the opportunity cost of doing so. The net effect of this logic is encapsulated in the following remark:

Remark 1 *In equilibrium, there are no contested races in which the endorser endorses the challenger.*

Because the endorser is biased in favor of the incumbent, the endorsement for the incumbent is ambiguous – it is consistent with the incumbent being superior to the challenger, but also with the incumbent being inferior up to the difference equal to the size of the bias in the endorser's strategy. The effect is that sometimes, in equilibrium, the voter will ignore the endorsement for the incumbent as uninformative. Whether she chooses to do so will affect the nature of equilibrium play. To make the distinction between these cases precise, we first introduce the following definition:

Definition 1 *An endorsement for the incumbent is* ***redundant*** *if, on the path of play, it provides no additional information to the voter beyond that conveyed by the voter's prior beliefs about the candidates and the fact of a contested race.*

To understand the nature of redundancy, suppose that in equilibrium, the endorser endorses the incumbent if and only if the challenger's type is less than $t_i + \beta^*$, where β^* is the bias in the equilibrium strategy of the endorser If the pro-incumbent endorsement is redundant, then $t_i + \beta^*$

[9]Note that the endorser's strategy does not include gradations in the strength of endorsement. The possibility of such gradations does not alter the equilibrium in this model: conditional on wanting one candidate to win, the endorser has an incentive to offer the strongest recommendation for that candidate.

is outside the interval $(\underline{t_c}, \overline{t_c})$ as described in the equilibrium of the campaign discount model. Consequently, it will convey no additional information to the voter about the types of candidates. It bears pointing out that if $t_i + \beta^*$ fell outside this interval and the *challenger* were endorsed, this *would* convey additional information to the voter; however, via Remark 1, such endorsements do not occur on the path of play (and so the notion of redundancy makes sense only when defined with respect to endorsements for incumbents). If an endorsement is redundant, the set of (inferior) incumbents who would be deterred from running for reelection by the threat of not receiving an endorsement is a proper subset of those incumbents deterred by the threat that the voter may elect a challenger who runs.

4.2 Results

With the notion of redundancy in mind, we can describe the equilibrium (formally characterized in the Appendix) in the presence of an endorser who is biased in favor of the incumbent.

If endorsement is redundant, the equilibrium is precisely that described in the campaign discount game described in Section 3. Moreover, irrespective of whether endorsement is redundant, the challenger's best response correspondence is identical to that in inequality (3). By contrast, the incumbent's best response correspondence is determined jointly with (and so is a function of) whether the endorsement is redundant. Whether endorsements are redundant is determined endogenously by the location of $\overline{t_c}$, the highest quality challenger against whom an incumbent would remain in the race in the campaign discount game.

The interesting part of the current analysis therefore focuses on the situation in which endorsement is not redundant. First, we demonstrate in Proposition 7 in the Appendix that if endorsement is not redundant, contested races always occur with positive probability. Next, recall that in the equilibrium without endorsements, some inferior incumbents could, if challenged, be deterred from running for reelection by the threat that the voter might elect the challenger. The presence of endorsements adds an additional threat to an inferior incumbent's retention, because even the (relatively uninformed) voter can make a fully-informed judgment if the challenger is endorsed. An inferior incumbent who anticipates not receiving the endorsement will therefore choose not to run for reelection if challenged.

There is no incentive in this equilibrium for the endorser to deviate from a strategy of endorsing

the challenger if $t_c > t_i + b$ and otherwise endorsing the incumbent when endorsement is not redundant. If $t_c > t_i + b$, such that the endorser prefers that the challenger be elected, she can assure this outcome by endorsing that candidate. If $t_c < t_i + b$, such that the endorser prefers the incumbent, she maximizes the probability of an incumbent victory by endorsing the incumbent. Thus, the bias in the endorser's strategy is equivalent to the bias in her type on the path of play (cf. Wittman 2007).

Our next result concerns the relationship between the extent of endorser bias b and the behavior of challengers and voters, and on the incumbent's ex ante electoral prospects.

Proposition 2 (Electoral Prospects and Endorser Bias) *If endorsements are redundant, then a change in the extent of endorser bias b has no effect on equilibrium strategies, beliefs, or outcomes. If endorsements are not redundant, then:*

1. *an increase in the extent of endorser bias b leads to an increase in the probability that a challenger enters the race and in the probability that an incumbent stays in the race if challenged, and to a strict decrease in the probability the voter retains the incumbent in a contested race; and*

2. *a marginal increase in the extent of endorser bias strictly decreases the electoral prospects of incumbents of sufficiently high type and strictly increases the electoral prospects of incumbents of sufficiently low type.*

This proposition describes the second kind of disadvantage associated with the incumbency advantage. To see the intuition for these results, suppose, first, that a challenger has entered the race and endorsements are not redundant. The incumbent will drop out if the challenger's type t_c exceeds $t_i + b$. The larger the endorser bias b, the less likely a challenger will meet this criterion. Next, consider the voter's choice. An increase in endorser bias reduces the strength of the pro-incumbent signal to the uninformed voter given a contested race, so her posterior beliefs about the incumbent must therefore decline as well. As discussed above, the voter must be indifferent in equilibrium with respect to retention given a contested race. This indifference can only be restored by reducing the probability of retention, which has the effect of encouraging more inferior challengers to enter. Thus, similarly to the case of campaign discount, an increase in endorser bias has the effect of undermining the scare-off of challengers.

Note that if endorsement is redundant, then a change in the extent of the endorser's bias has no effect on the choices of any of the actors in the model. Consequently, such a change will have no effect on the electoral prospects of any incumbent. If endorsements are not redundant, the effect of a change in the extent of the endorser's bias would be both direct (holding the voter's choice fixed) and indirect (mediated by the change in ρ). The former comes in the form of more low-quality incumbents choosing to contest a challenge, and the latter in the form of an expansion of the set of (t_i, t_c) pairs producing contested races. The intuition for the effect of an increase in bias on these values and, consequently, on the electoral fortunes of high-quality incumbents, is captured in Figure 2, and is analogous to the effect of a change in d, as described above.

The second result of this section describes the kinds of incumbents that might or might not benefit from the existence of a biased endorser.

Proposition 3 (Electoral Prospects and the Presence of an Endorser) *The ex ante electoral prospects of incumbents of sufficiently high (low) type weakly, and sometimes strictly, benefit (suffer) from the existence of a pro-incumbent endorser.*

The underlying intuition of this proposition is that the absence of endorsements implies greater pooling of candidates. Low quality incumbents benefit in expectation from being able to pool with their higher quality counterparts, whereas the opposite is true for the highest quality incumbents. Proposition 3 shows that, although the higher types of incumbents prefer that an endorsement be less biased, this should not be understood to imply that they prefer that there be no endorsements at all. Operating in an environment with endorsements, even if biased ones, is better for such types than operating in environments in which there are no endorsements at all.

5 Partisan Bias in the Electorate

In this section we consider a third model of electoral politics, which incorporates partisan bias as the source of the incumbency advantage.

5.1 Preliminaries

In our previous two models, differences between candidates were formalized in terms of "competence." This evokes the existence of a valence dimension about which consensus among voters

regarding candidate quality might exist. In such an environment, models with a single voter such as those considered above are reasonable approximations. By contrast, the existence of ideological differences between candidates naturally corresponds to the existence of ideological differences within the electorate. Accordingly, a plausible model in which candidates differ with respect to ideology requires incorporating multiple voters who also differ in ideology.

Consider, then, a model identical to the one in Section 3 in all respects except the following. There is a set of N voters, N odd, with voter v's most preferred policy position in the underlying unidimensional policy space X denoted by $x_v \in \mathbb{R}$. Let $x_i \in \mathbb{R}$ and $x_c \in \mathbb{R}$ be the most preferred policies for the incumbent and the challenger, respectively. Voters' utilities are defined the same way as above but for the additional additive term, quadratic "ideological bias" $-(x_v - \hat{x}_{i,c})^2$, where $\hat{x}_{i,c} \in \mathbb{R}$ is the policy adopted by the candidate if elected: $\hat{x}_i$ if the candidate elected is the current incumbent and $\hat{x}_c$ if it is the current (potential) challenger. We assume that the position that a given candidate will implement if elected is the one he or she runs on. The utilities of the incumbent and the challenger are amended by adding a quadratic loss term $-(x_i - \hat{x}_{i,c})^2$ for the incumbent and $-(x_c - \hat{x}_{i,c})^2$ for the challenger.

In order to focus on exogenous partisan bias within the electorate, we will assume that platform positions, while divergent, are, without loss of generality, fixed at $\hat{x}_c = 0$ and $\hat{x}_i = 1$, respectively.[10] Fixing the platforms in this way allows us to avoid confounding the effects of partisan bias and ideological polarization between the parties, focusing our attention on the former. Further, this specification allows us to capture in reduced form the voters' bias that stems from the features of political parties rather than of the particular candidates. (This is the sort of bias that in the empirical literature on Congressional elections is associated with the "normal vote" for Republicans or Democrats – net of candidate-specific considerations.)

The preceding assumptions imply that the voters' preferences are *one-and-a-half dimensional* in the sense of Groseclose (2007, 323-24),[11] who demonstrates that under the conditions satisfied here, majority rule is transitive and selects the candidate most preferred by the median voter. Thus,

[10] See Groseclose 2001 and Aragones and Palfrey 2002 for models with a similar environment that generate platform divergence endogenously.

[11] That is, they satisfy the following set of conditions: (1) each candidate is describable by a vector (x, t) in $\mathbb{R}^2$; (2) for each voter v, there exists a parameter $x_v \in \mathbb{R}$ that is the voter's ideal point; (3) there exist functions $q(z)$ and $w(z)$ such that for all $v \in N$ and for any two candidates i and c, iR_vc if and only if $q(x_i - x_v) + w(t_i) \geq q(x_c - x_v) + w(t_c)$; where (4) $w(z)$ is finite for all z; (5) $q(z)$ is concave; and (6) $q(z)$ reaches a maximum at $z = 0$.

we can restrict our attention to the analysis of the median voter's choice problem. Let $x_m \in (0,1)$ be the most preferred policy of the median voter. Note that greater x_m corresponds to m's (and so the electoral district's) greater ideological bias toward the incumbent's party. Finally, we assume that each candidate's platform is closer to her ideal point than to that of her erstwhile competitor: that is, $|x_i - \hat{x}_i| < |x_i - \hat{x}_c|$ and with the symmetric condition holding for the challenger. (Given the normalization described above, this is equivalent to an assumption that $x_i < \frac{1}{2} < x_c$.) The sequence of play is identical to the one described in Section 3.

Let ρ_m be the probability the median voter (who is decisive here) retains the incumbent in a contested race. Note that unlike in the models above, this probability will be a function of the candidates' platforms $\hat{x}_i$ and $\hat{x}_c$, as well as the candidates' most-preferred policies x_i and x_c. Suppressing arguments of ρ_m and σ (the probability the incumbent stays in the race if challenged), the expected utilities to the prospective challenger of not entering and entering the race are given, respectively, by

$$\begin{aligned} E[u_c(C=0)] &= t_i - (x_c - \hat{x}_i)^2 \\ E[u_c(C=1)] &= -k + ((1-\sigma) + \sigma(1-\rho_m))(t_c + q - (x_c - \hat{x}_c)^2) + \sigma\rho_m(t_i - (x_c - \hat{x}_i)^2). \end{aligned}$$

Recalling the normalization that $\hat{x}_i = 1$ and $\hat{x}_c = 0$, the potential challenger will enter the race if and only if

$$t_c - t_i > -q + (2x_c - 1) + \frac{k}{(1-\rho_m)\sigma}. \tag{5}$$

By a similar logic, the incumbent will stay in the race if and only if

$$t_c - t_i < q + (2x_i - 1) - \frac{k-d}{\rho_m}. \tag{6}$$

The equilibrium strategies of the challenger and the incumbent in the game with ideological bias in the electorate (for a full characterization, see the Appendix) follow naturally from the best response correspondences above. For the same reasons as in the campaign discount and endorser games, the median voter in the electorate must, in equilibrium, be indifferent with respect to the incumbent and challenger conditional on a contested race. In the current game, however, this

indifference condition must take into account the ideological bias of the voter. In particular,

$$E[t_i|C=1,S=1]-(1-x_m)^2=E[t_c|C=1,S=1]-x_m^2. \tag{7}$$

As in the previous games, there exists a unique value of ρ_m, ρ_m^*, that satisfies the median voter's indifference condition. Note, however, that an ideological bias in favor of the incumbent's party ($x_m > \frac{1}{2}$) implies that the average competence of an incumbent in a contested race will be strictly less than that of the challenger (cf. Lemma 3 in Ashworth and Bueno de Mesquita 2008).

5.2 Results

Our first result in this section is the following remark:

Remark 2 (Robustness of Proposition 1) *The relationship of the campaign discount d to the probability a prospective challenger enters, the probability an incumbent stays in a race if challenged, the probability the voter retains the incumbent in a contested race, and the incumbent's electoral prospects are robust in the current strategic environment.*

This robustness result emerges immediately from the strategic logic of the equilibrium. In particular, the best response correspondences of the incumbent and prospective challenger are qualitatively similar to those in the campaign discount game above. From the perspective of the candidates, the introduction of ideology and platforms simply gives each of them an additional incentive to run beyond the perquisites q. From the perspective of the median voter, the introduction of ideological bias may alter the equilibrium probability that she retains the incumbent given a contested race; however, her indifference in that information set must be maintained as above in order for the race to be contested. Consequently, a change in the incumbent's campaign discount d has the same qualitative effects on the agents' choices as in the previous models.

A similar logic underlies the robustness of our results concerning endorser bias, although it requires redefining bias to reflect the introduction of ideology. In particular, suppose that in addition to non-ideological bias b (as above), the endorser has an ideal point x_e in the policy space.[12] Recall that $\hat{x}_c$ and $\hat{x}_i$ are normalized to 0 and 1, respectively. Then we may define her

[12] Cf. Grofman and Norrander 1990.

ideological bias for the incumbent as the difference in ideological loss associated with the victory by the challenger rather than the incumbent:

$$(x_e - \hat{x}_c)^2 - (x_e - \hat{x}_i)^2 = x_e^2 - (x_e - 1)^2 = 2x_e - 1.$$

The total pro-incumbent bias is therefore given by $2x_e - 1 + b$, and this quantity must be positive for the endorser to be meaningfully pro-incumbent. (Note that the non-ideological component of total bias, b, could be negative.) We have the following remark:

Remark 3 (Robustness of Proposition 2) *The relationship of endorser bias b to the probability a prospective challenger enters, the probability an incumbent stays in a race if challenged, the probability the voter retains the incumbent in a contested race, and the incumbent's electoral prospects are robust in the current strategic environment if $b > 1 - 2x_e$.*

Next, we consider the effects of district pro-incumbent partisan bias. To keep the exposition simple, we focus on the case in which the endorser's bias is redundant. Recall that x_m, the median voter's ideal policy, represents the partisan or ideological bias of the district. Our next result concerns the effects of this bias. Given the results from the previous two models, one might initially suspect that an increase in pro-incumbent district partisan bias will harm the electoral prospects of the incumbent. Indeed, in the present model, an increase in pro-incumbent bias will lead to greater pooling of lower quality with higher quality incumbents, as in the case of an increase in the campaign discount or endorser bias. However, the equilibrium consequences are quite different:

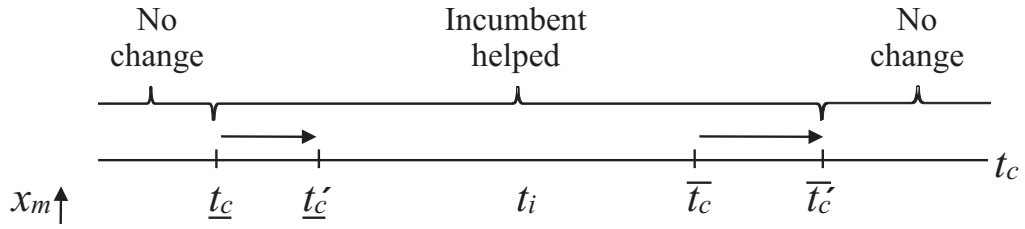
Proposition 4 (Electoral Prospects and Pro-Incumbent District Partisan Bias) *An increase in pro-incumbent district partisan bias x_m*

1. *leads to a strict decrease in the probability that a challenger enters the race, a strict increase in the probability that an incumbent stays in the race if challenged, and a strict increase in the probability the incumbent is reelected in a contested race; and*

2. *strictly increases the electoral prospects of the incumbent irrespective of the incumbent's type.*

The key reason for the difference in the effects of an increase in incumbency advantage between this and the previous models concerns how such an increase operates on the incentives of the players.

Figure 3: Incumbent Electoral Fortunes Given a Change in District Pro-Incumbent Partisan Bias



Note that t_i can be anywhere below $\overline{t_c}$. For values of t_c on the range $(-\infty, \underline{t_c})$, the incumbent is unchallenged before and after the change in partisan bias. On $(\underline{t_c}, \underline{t_c}')$, challengers willing to run prior to the change in bias are now deterred. On $(\underline{t_c}', \overline{t_c})$, candidate strategies are unchanged, but the probability the incumbent is retained increases. Given challengers on $(\overline{t_c}, \overline{t_c}')$, incumbents who would previously have dropped out now remain in the race. For challengers on $(\overline{t_c}', \infty)$, incumbents drop out before and after the change in bias.

An increase in district partisan bias makes the incumbent more attractive to the voter. A heretofore indifferent median voter will, then, prefer to transfer her support entirely to the incumbent. This will have the effect of encouraging entry by lower quality incumbents. In equilibrium, their influx is stemmed by a reduction in the probability the median voter retains the incumbent below 1 but higher than it was initially. The reason it is higher is that the equilibrium increase in ρ_m has the additional effect of crowding some (relatively lower quality) challengers who would have previously entered the race. Figure 3 displays this intuition graphically.

Note that all of the effect of the change in partisan bias on the candidates' strategies is *indirect*, that is, operating via the change in the median voter's strategy, ρ_m. By contrast, the immediate effect of the relevant change in the campaign discount and endorser bias on incumbents is *direct*, occurring even holding constant ρ_m. What hurts high quality incumbents in those cases is the response by the voter and challenger to those direct effects.

6 Effects on Selection on Quality

As noted in Section 2, the *selection account* of incumbency advantage refers to the feature of elections that, in expectation, leads to an improvement of officeholder quality in repeated elections (e.g. Zaller 1998; Ashworth and Bueno de Mesquita 2008). Indeed, positive selection on quality is also a property of equilibria in the models considered above. Given equilibrium play by the voter, the officeholder following an election will either be the incumbent (when there is no challenge or

when there is a contested race and the voter retains the incumbent) or a challenger who is, in expectation, as good as the incumbent (when there is a contested race and the voter elects the challenger) or better (when the challenger enters and the incumbent steps down).

In this section, we consider the interaction of selection and the three advantages of incumbency explicitly considered above. In particular, we analyze the effects of campaign discount d, endorser bias b, and district partisan bias x_m on the expected quality of an officeholder following an election.

Proposition 5 (Post-Election Expected Officeholder Quality)

1. *If the prior expected type of the incumbent, $E[t_i]$, is sufficiently high, then, regardless of whether the incumbent or challenger wins, the expected quality of an officeholder following an election is:*
   - *(a) decreasing in the campaign discount in the absence of an endorser or if endorsement is redundant; and*
   - *(b) decreasing in endorser bias if endorsements are not redundant.*
2. *The expected quality of an officeholder following an election is*
   - *(a) increasing in district partisan bias if the challenger wins, holding constant the voter's information set; and*
   - *(b) decreasing in district partisan bias if the incumbent wins, holding constant the voter's information set.*

How the difference between these quantities and the prior expected quality of an incumbent behaves can be thought of as a "rate of quality selection." The implication of Part 1 of the proposition is a kind of net "drag" on this rate. To understand the intuition, consider an increase in campaign discount d. By the logic described above, this increase will lead to an increase in the probability that a race is contested, because more inferior challengers will enter the race, and more inferior incumbents will choose to contest. By the same token, the probability that the voter receives a precise signal of superiority (either because a challenger is scared off or because an incumbent steps down) decreases. When $E[t_i]$ is sufficiently large, the fact of a contested race will cause the voter to update downward on the incumbent, and, given the voter's indifference in

a contested race, on the challenger as well. This effect will, in expectation, counteract the effect of the more precise signal conveyed by a single-candidate race. (By contrast, if $E[t_i]$ is low, the voter could update positively on the candidates given a contested race.) The same logic applies given an increase in endorser bias b. Because these effects are independent of whether the incumbent or the challenger wins, the average effect (across information sets) for sufficiently high $E[t_i]$ is always to slow down the speed of selection on type. Note that in an account of selection in repeated elections, the assumption that $E[t_i]$ is high (at least relative to $E[t_c]$) is quite natural.

Unlike the effects of increasing the campaign discount or endorser bias, the effect of an increase in partisan bias depends on whether the winner is the incumbent or the challenger, and on the conditions under which that individual won. As our analysis in the previous section shows, an increase in that bias leads to greater willingness by inferior incumbents to stay in a race and lesser willingness by inferior prospective challengers to enter. Consequently, when the incumbent wins, the effect of an increase in x_m is to decrease, and when the challenger wins, to increase the posterior mean on the officeholder. The fact that these effects have opposite signs, and that, in Part 2 of Proposition 5, it is necessary to condition on information sets (because a change in x_m changes their boundaries in the type-space), makes it difficult to ascertain the average effect of an increase in x_m on the speed of selection.[13] However, if selection is expected to decrease the probability of a challenge over time, part 2(b) of Proposition 5 becomes particularly relevant, its implication being the presence of a tension between the selection account and pro-incumbent partisan bias as well.

7 Discussion

The extensive empirical literature – some of which is cited above – reports significant variation in the magnitude of the effect of different hypothesized sources of incumbency advantage. Thus, what would seem to be a particularly needed contribution in the scholarship on the political economy of elections is a systematic theoretical assessment of the relationships and comparisons among these sources: whether and how they undermine or reinforce each other, and whether they confer advantages on the same or different categories of incumbents. This paper takes a step in the

[13] It is instructive to note that depending on the value of $E[t_i]$, it could be the case that the effect of an increase in x_m is to increase average candidate quality in contested races – something that is not possible given an increase in d or b. This can occur because for sufficiently high values of ρ (which can be induced by high values of $E[t_i]$), an increase in x_m will result in the dominance of the crowding out of inferior challengers over the encouragement of inferior incumbents to stay in a race.

direction of that kind of comparative analysis. Our key finding is that the presumption that all good things go together with respect to the sources of incumbency advantage is inaccurate. We show that this is true in two distinct ways. First, while some sources of incumbency advantage do indeed advantage incumbents as such, others help some types of incumbents while actually harming others. Second, we demonstrate that some sources of incumbency advantage may effectively undermine others. In this section, we first comment on our modeling choices in light or our findings, and then consider some interpretations and empirical implications of the results.

7.1 Further Considerations of Robustness

A common assumption in formal models of elections with uncertainty about candidates is that the challenger is randomly drawn from a prior distribution of types (see, e.g., Besley 2006, ch. 3). Our models depart from this by endogenizing the candidates' participation decisions. It is surely the case that our causal mechanisms, which rely on pooling by inferior incumbents, require retaining the assumption that the incumbent's choice to stay or quit is endogenous. It is worth considering, however, the value of endogenizing the challenger's choice as well.

The mechanisms we specify are robust to an environment in which challengers are not strategic. However, the effect of changes in the campaign discount, endorser bias, and partisan bias are not neutral if challenger entry is rational. Indeed, as we demonstrate in our analysis of the campaign discount and endorser bias models, an increase in either of those determinants encourages net entry by lower quality challengers in equilibrium. The effect on the inferences voters can draw from the fact of a contested race goes in the *opposite direction* from the effect on the incumbent in a model with exogenous challenges. A similar argument can be made concerning the result on the rate of selection in the presence of partisan bias. Maintaining the assumption of strategic challengers is important to ascertain the robustness of the result from the latter model.

A final consideration concerns our assumption that candidates care about policy in addition to the perquisites of office and the cost of running. In equilibrium, considerations of quality motivate candidate exit and entry decisions, and voters cue off of those decisions when voting. Suppose, though, that candidates did not care about policy. It is straightforward to demonstrate that there exist equilibria to the appropriately revised versions of our three models that are identical to those analyzed above, and so all of our above results are robust in those equilibria. However, the revised

models would support other equilibria in which contested races occur with positive probability as well. In those equilibria, all players are indifferent at the time of decision, and the relationship of model parameters and the probability that the prospective challenger enters, that the incumbent stays in the race, and that the voter retains the incumbent in a contested race are identical to those described above. Consider, for example, the campaign discount model. The direct effect of an increase in the campaign discount would be to make a heretofore indifferent incumbent strictly prefer to stay in the race. Equilibrium would be restored through a reduction in the probability of retention – i.e., in incumbent electoral prospects, and an increase in the probability of a contested race. However, in these equilibria, it need not be the case that candidate strategies are monotonic in type, and so making resolute claims about overall incumbent electoral prospects and rate of selection would be difficult. (That said, introducing a positive probability that the voter might learn the true candidate types would drive a wedge between the strategies of some inferior and superior candidates, as is the case in our models.)

7.2 Disadvantages of Incumbency and Incumbent Choices

If, as we have demonstrated, a greater campaign discount or endorser bias are bad for high-quality incumbents, and we expect from the (attenuated) quality-filtering effect that high-quality incumbents are more likely to be selected, it is instructive to consider why we continue to see those sources of incumbency advantage available to the incumbents. Why would incumbents not eliminate them?

At least two reasons stand out. First, a number of factors determining the ability of particular candidates to run a successful campaign, e.g., exogenous differences in the availability of resources, and the range of ways in which those resources translate into electoral advantages, lie outside of our model. In this sense, it is important to see our results on comparative electoral welfare as ceteris paribus. For example, high-quality incumbents may benefit from the campaign discount if they, unlike their expected challengers, do not have a personal fortune that could be spent on running. An initiative to eliminate these sources of incumbency advantage could thus run into resistance from both lower- and higher-quality incumbents.

Second, a number of the factors commonly associated with the campaign discount, such as the franking privilege and other influences on name recognition, interact with what we mean by "candidate quality." Eliminating these may entail undermining the ability of officeholders to engage

in activities that benefit their constituents, and so may be unpopular with the voters.

However, not all actions that officeholders could take in response to the incentives we identify have these perils. At an individual level, a high-quality incumbent may do well to lower her campaign discount if she could advertise that to the voters. Indeed, we see examples of such behavior. At the federal level, it is well-documented that incumbents have an easier time attracting contributions from political action committees (PACs), but in recent election cycles, a number of senators and House members have eschewed PAC contributions.[14] (Of course, how informative such a pledge will be to voters could itself vary depending on, e.g., the wealth of the legislator making it.[15]) However, if voters believe the campaign discount has a particular value, a candidate will, of course, want to take advantage of it.

At an institutional level, high-quality incumbents will want to decrease the campaign discount for the same reason, but also because in an environment of repeated elections, doing so will have an additional scare-off effect: in particular, it will further deter lower-quality challengers by reducing their expected stream of future payoffs associated with winning office. This additional effect suggests that, all else equal, the high-quality incumbents are better off with an institutional reform than with an individual action taken while keeping the institution fixed.

7.3 Some Empirical Implications

Our findings point to a number of implications for the empirical research on incumbency advantage. First, consider work examining the effects of endorsements on election outcomes. An obvious challenge to this research is omitted variables bias: ceteris paribus, newspapers and interest groups are more likely to endorse highly qualified candidates, so an endorsement may simply be proxying a difference in quality known to voters but unobservable to the analyst. Our findings concerning endorsers suggest that a problem of selection bias exists as well: if an endorser is sufficiently powerful to affect an election, then the candidate toward whom it is biased will drop out if she anticipates not receiving its endorsement. By contrast, the choices of endorsers to whom less attention is paid will not drive incumbents to retire, but the magnitude of their effects will be limited.

Second, much of the empirical literature on congressional elections has aimed at decomposing

[14] See, for example data from the Center for Responsive Politics, at www.opensecrets.org.

[15] Moreover, such pledges are also consistent with the interpretation that these politicians are attempting to send a self-verifying signal that they are not in the pocket of special interests – another way to signal their quality.

the sources of the incumbency advantage by controlling for sources other than the one of immediate interest. By focusing on "sophomore surge," Erikson (1971) and others have sought to control for time-invariant features of a candidate's quality. Likewise, the methodology of Gelman and King (1990) aims to control for latent party sympathies (the "normal vote") in a district. Cox and Katz (1996) seek to distinguish the direct effect on voters of incumbency from its indirect, "scare-off" effect on challengers. By confining their attention to repeat interactions between candidates, Levitt and Wolfram (1997) seek to "difference out" the effects of candidate quality and candidate scare-off. Hirano and Snyder (2007) apply this methodology to multi-member district elections, to remove the effects of quality, scare-off, and district partisan sympathies.

Most immediately, our analysis implies that some of the advantages of incumbency may actually be disadvantageous for some (high-quality) incumbents. It is not surprising, therefore, that a number of empirical studies have found that the effect of "direct officeholder benefits" on the reelection prospects of incumbents tend to be small. Moreover, the tensions discussed above between purported sources of the incumbency advantage imply that these sources are fundamentally *interactive*, and that further, those interactions are negative. Our analysis thus implies that a certain degree of caution is required when interpreting empirical results from analyses seeking to decompose the sources of incumbency advantage.

8 Conclusion

This paper has sought to determine whether some commonly-attributed sources of incumbency advantage do, in fact, benefit incumbents. Through a sequence of formal models, we have demonstrated that the answer to this question is not straightforward. In particular, large direct officeholder benefits – in the form of a discount incumbents receive on campaigning relative to challengers and the existence of pro-incumbent endorser bias – actually harm the electoral prospects of incumbents of sufficiently high quality. Further, these benefits undermine entry deterrence by challengers and the ability of voters to employ elections to select the most qualified candidates. Pro-incumbent district partisan bias benefits incumbents at all levels of quality and enhances challenger deterrence; its effects on selection, however, are more subtle. The complex relationships between different sources of incumbency advantage and the heterogeneity of their effects suggest that despite the existence of an extensive empirical literature on the subject, there still may be considerably more to say.

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Appendix

Characterizations of Equilibria

We begin with the following instrumental lemma:

Lemma 1 (Voter beliefs given a contested race) *If the potential challenger enters and the incumbent stays in the race, then in equilibrium, the uninformed voter must be indifferent between them, i.e.* $E[t_i|C=1,S=1] = E[t_c|C=1,S=1]$.

Proof. Suppose $E[t_i|C=1,S=1] < E[t_c|C=1,S=1]$. Then the voter would elect the challenger. But then the incumbent would not stay, so the race would be uncontested, a contradiction. A similar contradiction occurs given the supposition $E[t_i|C=1,S=1] > E[t_c|C=1,S=1]$. ■

Proposition 6 (Equilibrium in the Campaign Discount Game) *In equilibrium:*

1. *the prospective challenger enters iff* $t_c > t_i - q + \frac{k}{(1-\rho)\sigma(t_c,t_i)}$;
2. *if the prospective challenger enters, the incumbent stays in the race iff* $t_i > t_c - q + \frac{k-d}{\rho}$;
3. *if the race is contested, the voter retains the incumbent with probability* ρ^*, *which solves*

$$\int_{-\infty}^{\infty} p_c(t_c) \left[\frac{\int_{\underline{t_i}}^{\overline{t_i}} p_i(t_i) t_i dt_i}{P_i(\overline{t_i}) - P_i(\underline{t_i})} \right] dt_c = \int_{-\infty}^{\infty} p_i(t_i) \left[\frac{\int_{\underline{t_c}}^{\overline{t_c}} p_c(t_c) t_c dt_c}{P_c(\overline{t_c}) - P_c(\underline{t_c})} \right] dt_i, \tag{8}$$

$$\begin{aligned} \textit{where} \qquad \underline{t_i} &\equiv t_c - q + \frac{k-d}{\rho} \\ \overline{t_i} &\equiv t_c + q - \frac{k}{1-\rho} \\ \underline{t_c} &\equiv t_i - q + \frac{k}{1-\rho} \\ \overline{t_c} &\equiv t_i + q - \frac{k-d}{\rho}; \ \textit{and} \end{aligned} \tag{9}$$

4. *voter beliefs are derived via Bayes' Rule as per the system of equations in (1).*
5. *There exist pairs of incumbent and challenger types,* $\{t_i, t_c\}$, *that produce contested races if and only if* $q > 2k - d$.

Proof. 1. and 2. These are best response correspondences of the challenger and incumbent as described in inequalities (2) and (3).

3. Equation (8) expresses the voter's indifference condition (4) (required by Lemma 1) in terms of primitives and ρ. The quantities in equations (9) represent the types of lowest and highest quality incumbent (challenger) willing to run given the challenger's (incumbent's) type. These are derived from challenger and incumbent best response correspondences evaluated at equality; and noting, by monotonicity, that in equilibrium $\sigma^*(t_c, \overline{t_i}) = \sigma^*(\underline{t_c}, t_i) = 1$. It is straightforward to demonstrate that the left side of equation (8) is increasing, and the right side decreasing, in ρ; therefore, if a $\rho^* \in [0, 1]$ exists that satisfies equation (8), it is unique.

4. Equilibrium posterior beliefs are evaluated with $\Pr(Z|t_i)$ and $\Pr(Z|t_c)$ given by

$$\begin{aligned}
\Pr(C = 1, S = 1|t_i) &= P_c(\overline{t_c}) - P_c(\underline{t_c}) \\
\Pr(C = 1, S = 0|t_i) &= 1 - P_c(\overline{t_c}) \\
\Pr(C = 0|t_i) &= P_c(\underline{t_c}) \\
\Pr(C = 1, S = 1|t_c) &= P_i(\overline{t_i}) - P_i(\underline{t_i}) \\
\Pr(C = 1, S = 0|t_c) &= 1 - P_i(\overline{t_i}) \\
\Pr(C = 0|t_c) &= P_i(\underline{t_i}). \qquad \blacksquare
\end{aligned}$$

5. A necessary condition for the voter's indifference condition to be satisfied is $\underline{t_c} < t_i < \overline{t_c}$ (equivalently, $\underline{t_i} < t_c < \overline{t_i}$). Substituting and rearranging yields $\frac{k-d}{q} < \rho < \frac{q-k}{q}$. By assumption, $q < k < d$, so both $\frac{k-d}{q}$ and $\frac{q-k}{q}$ lie between 0 and 1. The condition for the interval $[\frac{k-d}{q}, \frac{q-k}{q}]$ to be nonempty is given in Part 5 of the Proposition. Note that this is also a sufficient condition, because $\rho = \frac{k-d}{q}$ implies $\overline{t_c} = t_i$, in which case $E[t_c|C = 1, S = 1] < E[t_i|C = 1, S = 1]$, and $\rho = \frac{q-k}{q}$ implies $\underline{t_c} = t_i$, in which case $E[t_c|C = 1, S = 1] > E[t_i|C = 1, S = 1]$. Therefore, given the condition in Part 5, there exists an interior value of ρ such that indifference is satisfied.

Proposition 7 (Equilibrium in the Endorser Bias Game) *In any equilibrium with an incumbent-biased endorser:*

1. *endorsement is redundant if and only if $t_i + \beta^* \geq \overline{t_c}$, where $\overline{t_c}$ is defined in part 3 of Proposition 6, and β^* is the bias in the equilibrium strategy of the endorser;*

2. *if endorsement is redundant, the endorser endorses the incumbent in a contested race, and the equilibrium characterization is otherwise identical to that described in Proposition 6;*

3. *if endorsement is not redundant,*

   (a) *the prospective challenger enters if and only if* $t_c > t_i - q + \frac{k}{(1-\rho)\sigma(t_c,t_i)}$*;*

   (b) *if the prospective challenger enters, the incumbent stays in the race if and only if* $t_c \leq t_i + \beta^*$*;*

   (c) *if the race is contested,*

      i. $\beta^* = b$*, that is, the endorser endorses the incumbent if and only if* $t_c \leq t_i + b$*; and*

      ii. *the voter elects the challenger with certainty if the endorser endorses the challenger (off the path of play), and otherwise retains the incumbent with probability* ρ^**, which solves*

$$\int_{-\infty}^{\infty} p_c(t_c) \left[\frac{\int_{t_c-b}^{\overline{t_i}} p_i(t_i) t_i dt_i}{P_i(\overline{t_i}) - P_i(t_c - b)} \right] dt_c = \int_{-\infty}^{\infty} p_i(t_i) \left[\frac{\int_{\underline{t_c}}^{t_i+b} p_c(t_c) t_c dt_c}{P_c(t_i + b) - P_c(\underline{t_c})} \right] dt_i, \quad (10)$$

      where $\underline{t_c}$ *and* $\overline{t_i}$ *are defined as in equations (9); and*

   (d) *voter beliefs on the path of play are derived via Bayes' Rule as per the system of equations in (1), and off the path of play (a contested race in which the incumbent does not receive an endorsement), they may be any posterior beliefs that support* $E[t_i | C = 1, S = 1, \textit{challenger endorsed}] < E[t_c | C = 1, S = 1, \textit{challenger endorsed}]$*.*

   (e) *There always exist pairs of incumbent and challenger types,* $\{t_i, t_c\}$*, that produce contested races.*

Proof. We begin with a proof of Part 3. Part 3(a) gives the best response correspondence for the challenger as given in inequality (3). To see that 3(c)i holds in any equilibrium in which the endorsement is non-redundant, suppose otherwise. Then the endorser sometimes endorses the incumbent if $t_c > t_i + b$ or the challenger when $t_c \leq t_i + b$. But, given non-redundancy, this would cause the voter to elect the endorser's less-preferred candidate with weakly higher probability than if the endorser were playing the strategy in 3(c)i. Thus, the supposition must be be false in any equilibrium in which the endorsement is non-redundant.

We next show that, given 3(c)i, any equilibrium must have the properties described in the remainder of Part 3. Given 3(c)i, an endorsement of the challenger would fully reveal the superiority of the challenger in a contested race, whom the voter would then elect. But then the incumbent would prefer not to stay in the race if challenged. By 3(c)i, the highest quality challenger against whom an incumbent would run has type $t_i + b$. By monotonicity of the incumbent's best response correspondence (2), she will also remain in the race against any challenger of lower type; this establishes 3(b). ρ^* is the unique value of ρ that establishes the voter's indifference given 3(a) and 3(b) as required by Lemma 1. Equation (10) in 3(c)ii expresses the voter's indifference as a function of ρ and primitives.

3(d). If endorsement is not redundant, equilibrium posterior beliefs on the path of play are evaluated with $\Pr(Z|t_i)$ and $\Pr(Z|t_c)$ given by

$$\begin{aligned}
\Pr(C=1,S=1|t_i) &= P_c(t_i+b) - P_c(\underline{t_c}) \\
\Pr(C=1,S=0|t_i) &= 1 - P_c(t_i+b) \\
\Pr(C=0|t_i) &= P_c(\underline{t_c}) \\
\Pr(C=1,S=1|t_c) &= P_i(\overline{t_i}) - P_i(t_c-b) \\
\Pr(C=1,S=0|t_c) &= 1 - P_i(\overline{t_i}) \\
\Pr(C=0|t_c) &= P_i(t_c-b),
\end{aligned}$$

where $\overline{t_i} = t_c + q - \frac{k}{1-\rho}$. If endorsement is redundant, $\Pr(Z|t_i)$ and $\Pr(Z|t_c)$ are the expressions given in the proof of Proposition 6 above.

3(e). A necessary condition for the voter's indifference and non-redundancy is $\underline{t_c} < t_i < t_i + b < \overline{t_c}$. Substituting, we obtain

$$-q + \frac{k}{1-\rho} < 0 < b < q - \frac{k-d}{\rho}. \tag{11}$$

Because $b > 0$ by assumption, (11) reduces to the conjunction of two inequalities: $q(1-\rho) > k$ and $(b-q)\rho < k-d$. Note that if endorsement is non-redundant, then $b - q < 0$. Then the first inequality simplifies to $\rho > \frac{k-d}{b-q}$, which, because the right side is negative, always holds. The second inequality simplifies to $\rho < \frac{q-k}{q} < 1$. If this holds at equality, then $\underline{t_c} = t_i$ and

$E[t_c|C=1,S=1] > E[t_i|C=1,S=1]$. As ρ approaches 0, then $\underline{t_c}$ approaches $t_i - q + k$. But then $\overline{t_c}$ would approach negative infinity, i.e. only very high incumbent types would run, and so endorsement would be redundant. Therefore, if endorsement is non-redundant, then there exists a $\rho^* \in [0, \frac{q-k}{q}]$ satisfying the voter's indifference, implying 3(e) must hold.

2. Immediate.

1. To establish that the condition in Part 1 is sufficient, note that for $\overline{t_c}$ as defined in Proposition 6, if $t_i + \beta > \overline{t_c}$, the threat of not receiving an endorsement would not deter any incumbents not already deterred given Proposition 6. Because that threat is not binding on any incumbents who would otherwise run, an endorsement for the incumbent is redundant. To see that the condition is necessary, note that if $t_i + \beta^* < \overline{t_c}$, there would exist challengers against whom an incumbent would run in the absence, but not in the presence of endorsements, as established above. But then endorsements would be informative, and thus not redundant. ■

Proposition 8 (Equilibrium in the Partisan Bias Game) *In equilibrium:*

1. *the prospective challenger enters iff* $t_c > t_i - q + (2x_c - 1) + \frac{k}{(1-\rho_m(\hat{x}_i,\hat{x}_c,x_i,x_c))\sigma(t_c,t_i)}$;

2. *if the prospective challenger enters, the incumbent stays in the race iff* $t_c < t_i + q + (2x_i - 1) - \frac{k-d}{\rho_m(\hat{x}_i,\hat{x}_c,x_i,x_c)}$;

3. *The incumbent is retained with probability* $\rho_m^*(\hat{x}_i, \hat{x}_c, x_i, x_c)$, *which solves*

$$\int_{-\infty}^{\infty} p_c(t_c) \left[\frac{\int_{\underline{t_i}}^{\overline{t_i}} p_i(t_i) t_i dt_i}{P_i(\overline{t_i}) - P_i(\underline{t_i})} \right] dt_c - (1 - x_m)^2 = \int_{-\infty}^{\infty} p_i(t_i) \left[\frac{\int_{\underline{t_c}}^{\overline{t_c}} p_c(t_c) t_c dt_c}{P_c(\overline{t_c}) - P_c(\underline{t_c})} \right] dt_i - x_m^2, \quad (12)$$

$$\begin{aligned} \textit{where} \qquad \underline{t_i} &\equiv t_c - q - (2x_i - 1) + \frac{k-d}{\rho_m(\hat{x}_i, \hat{x}_c, x_i, x_c)} \\ \overline{t_i} &\equiv t_c + q - (2x_c - 1) - \frac{k}{(1 - \rho_m(\hat{x}_i, \hat{x}_c, x_i, x_c))\sigma(t_c, t_i)} \\ \underline{t_c} &\equiv t_i - q + (2x_c - 1) + \frac{k}{(1 - \rho_m(\hat{x}_i, \hat{x}_c, x_i, x_c))\sigma(t_c, t_i)} \\ \overline{t_c} &\equiv t_i + q + (2x_i - 1) - \frac{k-d}{\rho_m(\hat{x}_i, \hat{x}_c, x_i, x_c)}; \ \textit{and} \end{aligned} \quad (13)$$

4. *voter beliefs are derived via Bayes' Rule as per the system of equations in (1).*

5. *There exist pairs of incumbent and challenger types, $\{t_i, t_c\}$, that produce contested races if and only if*

$$k < \frac{(2(q + (x_i - x_c))^2 + d}{8(q + (x_i - x_c))}; \tag{14}$$

$$\textit{and} \qquad q > (x_c - x_i) + \frac{k}{2(1 - \rho_m(\hat{x}_i, \hat{x}_c, x_i, x_c))} + \frac{k - d}{2\rho_m(\hat{x}_i, \hat{x}_c, x_i, x_c)}. \tag{15}$$

Proof. 1. and 2. These are best response correspondences given in inequalities (5) and (6).

3. By Groseclose's (2007) theorem, the median voter is decisive. Therefore, as argued in the text, she must be indifferent in a contested race. Equation (13) expresses the voter's indifference condition (7) in terms of primitives and ρ_m. The quantities in (13) represent lowest and highest quality incumbent (challenger) types willing to run given the challenger's (incumbent's) type.

4. Equilibrium posterior beliefs are evaluated with $\Pr(Z|t_i)$ and $\Pr(Z|t_c)$ identical to those given in Proposition 6, substituting the values of $\underline{t_c}$, $\overline{t_c}$, $\underline{t_i}$, and $\overline{t_i}$ obtained from Parts 3 of the current proposition.

5. For races to be contested, it must be the case that both of the inequalities (5) and (6) are satisfied. For there to exist pairs $\{t_i, t_c\}$ such that this is the case, the right side of (6) must be greater than the right side of (5). Solving for q, we obtain inequality (15).

Solving inequality (15) for ρ_m and simplifying, we get the equivalent inequality

$$\alpha\rho_m^2 - (\alpha - d)\rho_m + (k - d) < 0,$$

where $\alpha = 2(q + (x_i - x_c))$. For real values of ρ_m that solve this inequality to exist, the discriminant must be positive. The condition for this is given in (14).

To see that the conjunction of conditions (15) and (14) is both necessary and sufficient for the possibility of contested races, note that the densities $p_i(t_i)$ and $p_c(t_c)$ are defined over the entire real line. This insures, first, that if these inequalities are satisfied, there will exist pairs of incumbent and challenger types for which an election will be contested. ■

Proofs of Additional Results

Proof of Proposition 1

1. From definitions of equilibrium values of $\underline{t_c}$, $\overline{t_c}$, $\underline{t_i}$, and $\overline{t_i}$ given in (9), $\frac{\partial \overline{t_c}}{\partial d} = \rho^{-1} + (k -$

$d)\rho^{-2}\frac{\partial\rho}{\partial d}$, $\frac{\partial t_c}{\partial \overline{d}} = k(1-\rho)^{-2}\frac{\partial\rho}{\partial d}$, $\frac{\partial \overline{t_i}}{\partial d} = -k(1-\rho)^{-2}\frac{\partial\rho}{\partial d}$, and $\frac{\partial t_i}{\partial \overline{d}} = -\rho^{-1} - (k-d)\rho^{-2}\frac{\partial\rho}{\partial d}$. Suppose $\frac{\partial\rho}{\partial d} > 0$. Then, $\frac{\partial \overline{t_c}}{\partial d} > 0$, $\frac{\partial t_c}{\partial \overline{d}} > 0$, $\frac{\partial \overline{t_i}}{\partial d} < 0$, and $\frac{\partial t_i}{\partial \overline{d}} < 0$. But then by (8) an increase in d would lead to $E[t_c|C=1,S=1] > E[t_i|C=1,S=1]$, violating Lemma 1. Suppose $\frac{\partial\rho}{\partial d} = 0$. Then $\frac{\partial \overline{t_c}}{\partial d} > 0$, $\frac{\partial t_c}{\partial \overline{d}} = 0$, $\frac{\partial \overline{t_i}}{\partial d} = 0$, and $\frac{\partial t_i}{\partial \overline{d}} < 0$ also violating Lemma 1. Therefore $\frac{\partial\rho}{\partial d} < 0$. This implies $\frac{\partial t_c}{\partial \overline{d}} < 0$ and $\frac{\partial \overline{t_i}}{\partial d} < 0$. To retain voter indifference, it must also be the case that $\frac{\partial \overline{t_c}}{\partial d} > 0$ and $\frac{\partial t_i}{\partial \overline{d}} < 0$. The increase in $\overline{t_c}$ and decrease in $\underline{t_c}$ imply that the probability a challenger enters the race, $\int_{-\infty}^{\infty} p_i(t_i)(1-P_c(\underline{t_c}))dt_i$, and that an incumbent stays in the race if challenged, $\int_{-\infty}^{\infty} p_i(t_i)P_c(\overline{t_c})dt_i$, must each increase.

2. Let ρ', $\underline{t_c}'$, and $\overline{t_c}'$ be equilibrium values associated with d', and $\rho'' < \rho'$, $\underline{t_c}'' < \underline{t_c}'$, and $\overline{t_c}'' > \overline{t_c}'$ be equilibrium values associated with $d'' > d'$. For an incumbent of type t_i, ex ante electoral prospects given $d = d'$ are given by

$$P_c(\underline{t_c}') + (P_c(\overline{t_c}') - P_c(\underline{t_c}'))\rho'$$

and similarly for $d = d''$. The difference between ex ante electoral prospects given d' and d'' is

$$(P_c(\underline{t_c}') - P_c(\underline{t_c}''))(1-\rho'') + (P_c(\overline{t_c}') - P_c(\underline{t_c}'))(\rho' - \rho'') - (P_c(\overline{t_c}'') - P_c(\overline{t_c}'))\rho''.$$

Rearranging terms yields

$$(P_c(\underline{t_c}') - P_c(\underline{t_c}''))(1-\rho') + \left[(P_c(\overline{t_c}') - P_c(\underline{t_c}''))\rho' - (P_c(\overline{t_c}'') - P_c(\underline{t_c}''))\rho''\right]. \tag{16}$$

The first term in this expression is strictly positive. The term in square brackets is positive iff

$$\frac{(P_c(\overline{t_c}') - P_c(\underline{t_c}''))\rho'}{(P_c(\overline{t_c}'') - P_c(\underline{t_c}''))\rho''} > 1. \tag{17}$$

From definitions of $\underline{t_c}$ and $\overline{t_c}$ in (9), $\frac{\partial P_c(\underline{t_c})}{\partial t_i} = p_c(\underline{t_c})$ and $\frac{\partial P_c(\overline{t_c})}{\partial t_i} = p_c(\overline{t_c})$. By the mean value theorem, there exists an $\alpha \in [\underline{t_c}'', \overline{t_c}']$ and a $\beta \in [\underline{t_c}'', \overline{t_c}'']$, with $\alpha < \beta$, such that the left side of inequality (17) may be expressed as

$$\frac{(\overline{t_c}' - \underline{t_c}'')p_c(\alpha)\rho'}{(\overline{t_c}'' - \underline{t_c}'')p_c(\beta)\rho''}.$$

Substituting the values of $\overline{t_c}$ and $\underline{t_c}$ and rearranging yields

$$\left[\frac{2q\rho'(1-\rho'')-(k-d')(1-\rho'')-k\rho'}{2q\rho''(1-\rho'')-(k-d'')(1-\rho'')-k\rho''}\right]\frac{p_c(\alpha)}{p_c(\beta)}.$$

$p_c(\cdot)$ is unimodal by assumption. To the right of the mode, $p_c(\cdot)$ is strictly decreasing. Therefore, if t_i is sufficiently far to the right of the mode that $\underline{t_c}''$ also falls to the right of the mode, then $\frac{p_c(\alpha)}{p_c(\beta)} > 1$. The term in square brackets exceeds 1 iff

$$q - \frac{k}{1-\rho''} > -q + \frac{d'-d''}{\rho'-\rho''}.$$

The left side of this inequality is equal to $t_i - \underline{t_c}''$, which must be positive in equilibrium by Lemma 1. Because $d' < d''$ and $\rho'' < \rho'$, the right side is strictly negative. Therefore inequality (17) holds. This in turn implies that if t_i is sufficiently great, the expression in (16) is strictly positive, i.e. a shift from d' to $d'' > d'$ harms the incumbent's electoral prospects. A symmetric logic establishes the opposite for sufficiently low t_i. ■

Proof of Remark 1

Suppose $t_i > t_c - b$. An endorsement for the challenger is either uninformative, in which case the endorser is indifferent between endorsing the challenger and incumbent, or it is informative, in which case endorsing the incumbent increases the probability the incumbent is elected. Thus, holding fixed t_i, t_c, and b, any strategy for the endorser that includes endorsement of challengers with type $t_c < t_i + b$ is weakly dominated by one in which the endorses endorses the incumbent. Consequently, if the endorser endorses the challenger, it must be that $t_i \leq t_c - b$, and so the voter would strictly prefer electing the challenger. But then the race would not be contested because the incumbent would strictly prefer not to run. A similar argument establishes the conclusion for $t_i \leq t_c - b$. ■

Proof of Proposition 2

The proof is identical to that of Proposition 1. ■

Proof of Proposition 3

For incumbents with type $t_i > t_c - b$, the endorsement allows them to separate from incumbents of lower type, so the voter's posterior $E[t_i|C = 1, S = 1, \text{incumbent endorsed}]$ must be higher

than $E[t_i|C = 1, S = 1, \text{no endorser}]$. Because the equilibrium value of ρ is increasing in the voter's posterior mean on the incumbent, those types of incumbents must, in expectation, benefit electorally. (Incumbents who were not challenged are unaffected.) In contrast, incumbents with type $t_i < t_c - b$ choose $S = 0$ on the path of play because the endorser's equilibrium strategy identifies them as inferior. In the absence of the endorser, however, they can pool with higher types of incumbents. Incumbents whose types are below $\underline{t_i}$ (the lowest quality incumbent willing to stay in the race if challenged) are unaffected, but those whose types are between $\underline{t_i}$ and $t_c - b$ will now prefer $S = 1$. Consequently, their expected electoral welfare must be strictly higher. ■

Proof of Remark 2

Proof follows from the argument in the text. ■

Proof of Remark 3

Proof follows from the argument in the text. ■

Proof of Proposition 4

1. From definitions of equilibrium values of ρ, $\underline{t_i}$, $\overline{t_i}$, $\underline{t_c}$, $\overline{t_c}$ in (13), $\frac{\partial \overline{t_i}}{\partial x_m} = -k(1-\rho)^{-2}\frac{\partial \rho}{\partial x_m}$, $\frac{\partial \underline{t_i}}{\partial x_m} = -(k-d)\rho^{-2}\frac{\partial \rho}{\partial x_m}$, $\frac{\partial \overline{t_c}}{\partial x_m} = (k-d)\rho^{-2}\frac{\partial \rho}{\partial x_m}$, and $\frac{\partial \underline{t_c}}{\partial x_m} = k(1-\rho)^{-2}\frac{\partial \rho}{\partial x_m}$. The direct effect of an increase in x_m is to increase the left side of (12) and decrease the right. To restore voter indifference (and equilibrium) it is necessary for $E[t_i|C = 1, S = 1]$ to decrease and/or for $E[t_c|C = 1, S = 1]$ to increase. Suppose $\frac{\partial \rho}{\partial x_m} < 0$. Then $\frac{\partial \overline{t_i}}{\partial x_m} > 0$, $\frac{\partial \underline{t_i}}{\partial x_m} > 0$, $\frac{\partial \overline{t_c}}{\partial x_m} < 0$, and $\frac{\partial \underline{t_i}}{\partial x_m} < 0$. But then $E[t_i|C = 1, S = 1]$ would increase and $E[t_c|C = 1, S = 1]$ decrease, a contradiction. Suppose $\frac{\partial \rho}{\partial x_m} = 0$. Then $\frac{\partial \overline{t_i}}{\partial x_m} = \frac{\partial \underline{t_i}}{\partial x_m} = \frac{\partial \overline{t_c}}{\partial x_m} = \frac{\partial \underline{t_c}}{\partial x_m} = 0$. But then $E[t_i|C = 1, S = 1]$ and $E[t_c|C = 1, S = 1]$ would remain unchanged, also a contradiction. Therefore $\frac{\partial \rho}{\partial x_m} > 0$, which implies $\frac{\partial \overline{t_c}}{\partial x_m} > 0$ and $\frac{\partial \underline{t_c}}{\partial x_m} > 0$. This in turn implies that the probability a challenger enters a race, $\int_{-\infty}^{\infty} p_i(t_i)(1 - P_c(\underline{t_c}))dt_i$, is decreasing, while the probability an incumbent stays in the race if challenged, $\int_{-\infty}^{\infty} p_i(t_i)P_c(\overline{t_c})dt_i$, is increasing.

2. Let $x_m'' > x_m'$. Given Part 1, for any t_i, $t_c \in (-\infty, \underline{t_c}')$ or $t_c \in (\overline{t_c}'', \infty)$ implies no change in electoral prospects given a shift from x_m' to x_m''; $t_c \in (\underline{t_c}', \underline{t_c}'')$ implies retention with probability ρ' given x_m' and with certainty given x_m''; $t_c \in (\underline{t_c}'', \overline{t_c}')$ implies retention with probability ρ' given x_m' and $\rho'' > \rho'$ given x_m''; $t_c \in (\overline{t_c}', \overline{t_c}'')$ implies retention with probability zero given x_m' and retention with probability $\rho'' > 0$ given x_m''. The difference in electoral prospects given x_m'' and x_m' is $(P_c(\underline{t_c}'') - P_c(\underline{t_c}'))(1 - \rho') + (P_c(\overline{t_c}') - P_c(\underline{t_c}''))(\rho'' - \rho') + (P_c(\overline{t_c}'') - P_c(\overline{t_c}'))\rho'' > 0$.■

Proof of Proposition 5

1(a). The expected officeholder type following an election is given by

$$\int_{-\infty}^{\infty} p_i(t_i)\left(P_c(\underline{t_c})t_i + (P_c(\overline{t_c}) - P_c(\underline{t_c}))(\rho t_i + (1-\rho)E[t_c|t_c \in (\underline{t_c}, \overline{t_c}); t_i]) \right.$$
$$\left. +(1 - P(\overline{t_c}))E[t_c|t_c > \overline{t_c}; t_i]\right) dt_i,$$

which simplifies to

$$\int_{-\infty}^{\infty} p_i(t_i)\left((P_c(\underline{t_c}) + (P_c(\overline{t_c}) - P_c(\underline{t_c}))\rho)t_i \right.$$
$$\left. +(1-\rho)\int_{\underline{t_c}}^{\overline{t_c}} p_c(t_c)t_c dt_c + \int_{\overline{t_c}}^{\infty} p_c(t_c)t_c dt_c\right) dt_i.$$

Integrating by parts, this expression may be written as

$$\int_{-\infty}^{\infty} p_i(t_i)\left(\left(P_c(\underline{t_c}) + (P_c(\overline{t_c}) - P_c(\underline{t_c}))\rho\right)t_i \right. \tag{18}$$
$$\left. +(1-\rho)[\overline{t_c}P_c(\overline{t_c}) - \underline{t_c}P_c(\underline{t_c}) - G_c(\overline{t_c}) + G_c(\underline{t_c})] + G_c(\overline{t_c}) - \overline{t_c}P_c(\overline{t_c})\right) dt_i,$$

where $G_c(t) = \int_{-\infty}^{t} P_c(s)ds$. Differentiating with respect to d and collecting terms gives

$$\int_{-\infty}^{\infty} p_i(t_i)\left[(1-\rho)\underline{y}p_c(\underline{t_c})\frac{\partial \underline{t_c}}{\partial d} - \rho\overline{y}p_c(\overline{t_c})\frac{\partial \overline{t_c}}{\partial d}\right.$$
$$\left. + \left((G_c(\overline{t_c}) - \overline{y}P_c(\overline{t_c})) - (G_c(\underline{t_c}) - \underline{y}P_c(\underline{t_c}))\right)\frac{\partial \rho}{\partial d}\right] dt_i, \tag{19}$$

where $\overline{y} = \overline{t_c} - t_i$ and $\underline{y} = t_i - \underline{t_c}$. Recalling from above that $\frac{\partial \rho}{\partial d} < 0$, $\frac{\partial \underline{t_c}}{\partial d} < 0$, and $\frac{\partial \overline{t_c}}{\partial d} > 0$, the sum of the first two terms in square brackets integrated over $p_i(t_i)$ is strictly negative, while the third term is negative if and only if

$$\int_{-\infty}^{\infty} p_i(t_i)\left((G_c(\overline{t_c}) - \overline{y}P_c(\overline{t_c})) - (G_c(\underline{t_c}) - \underline{y}P_c(\underline{t_c}))\right) dt_i > 0.$$

It is straightforward to demonstrate that $G_c(t) - tP_c(t) = \int_t^{\infty} p_c(t_c)t_c dt_c$. Substituting yields

$$\int_{-\infty}^{\infty} p_i(t_i)\left(\int_{\overline{t_c}}^{\infty} p_c(t_c)t_c dt_c + t_i P_c(\overline{t_c})\right) dt_i > \int_{-\infty}^{\infty} p_i(t_i)\left(\int_{\underline{t_c}}^{\infty} p_c(t_c)t_c dt_c + t_i P_c(\underline{t_c})\right) dt_i,$$

and rearranging gives

$$\int_{-\infty}^{\infty} p_i(t_i) t_i dt_i > \int_{-\infty}^{\infty} p_i(t_i) \left(\frac{\int_{\underline{t_c}}^{\overline{t_c}} p_c(t_c) t_c dt_c}{P_c(\overline{t_c}) - P_c(\underline{t_c})} \right) dt_i. \tag{20}$$

Inequality (20) is equivalent to $E[t_i] > E[t_c|C = 1, S = 1]$. By Lemma 1, $E[t_c|C = 1, S = 1] = E[t_i|C = 1, S = 1]$. Therefore inequality (20) is equivalent to $E[t_i] > E[t_i|C = 1, S = 1]$. Therefore, as long as the prior mean is sufficiently high that a contested race causes the voter to update downward on the incumbent, the expression in (19) is strictly negative, implying that the average posterior mean officeholder type is decreasing in d.

1(b). The proof is identical to that of part 1(a), substituting b for d.

2(a). The expected challenger type conditional on a contested race may be written as

$$\int_{-\infty}^{\infty} p_i(t_i) \left[\frac{\int_{\underline{t_c}}^{\overline{t_c}} p_c(t_c) t_c dt_c}{P_c(\overline{t_c}) - P_c(\underline{t_c})} \right] dt_i.$$

Integrating by parts, this is equivalent to

$$\int_{-\infty}^{\infty} p_i(t_i) \left[\frac{\overline{t_c} P_c(\overline{t_c}) - \underline{t_c} P_c(\underline{t_c}) - G_c(\overline{t_c}) + G_c(\underline{t_c})}{P_c(\overline{t_c}) - P_c(\underline{t_c})} \right] dt_i, \tag{21}$$

where $G_c(t) = \int_{-\infty}^{t} P_c(s) ds$. Differentiating the term in square brackets with respect to x_m yields

$$\frac{\overline{t_c} p_c(\overline{t_c}) \frac{\partial \overline{t_c}}{\partial x_m} - \underline{t_c} p_c(\underline{t_c}) \frac{\partial t_c}{\partial x_m})}{P_c(\overline{t_c}) - P_c(\underline{t_c})} - \frac{(\overline{t_c} P_c(\overline{t_c}) - \underline{t_c} P_c(\underline{t_c}) - G_c(\overline{t_c}) + G_c(\underline{t_c}))(p_c(\overline{t_c}) \frac{\partial \overline{t_c}}{\partial x_m} - p_c(\underline{t_c}) \frac{\partial t_c}{\partial x_m})}{(P_c(\overline{t_c}) - P_c(\underline{t_c}))^2},$$

which is positive if and only if

$$\begin{aligned} & \frac{\partial \overline{t_c}}{\partial x_m} p_c(\overline{t_c})((G_c(\overline{t_c}) - G_c(\underline{t_c})) - (\overline{t_c} - \underline{t_c}) P_c(\underline{t_c})) \\ + \quad & \frac{\partial t_c}{\partial x_m} p_c(\underline{t_c})(-(G_c(\overline{t_c}) - G_c(\underline{t_c})) + (\overline{t_c} - \underline{t_c}) P_c(\underline{t_c})) \quad > 0. \end{aligned} \tag{22}$$

By the mean value theorem, there exists an $\alpha \in [\underline{t_c}, \overline{t_c}]$ such that $G_c(\overline{t_c}) - G_c(\underline{t_c}) = (\overline{t_c} - \underline{t_c}) P_c(\alpha)$.

Substituting into inequality (22) yields the condition

$$\frac{\partial \overline{t_c}}{\partial x_m} p_c(\overline{t_c})((\overline{t_c} - \underline{t_c})(P_c(\alpha) - P_c(\underline{t_c}))) + \frac{\partial \underline{t_c}}{\partial x_m} p_c(\underline{t_c})((\overline{t_c} - \underline{t_c})(P_c(\overline{t_c}) - P_c(\alpha))) > 0. \tag{23}$$

From Proposition 4, $\underline{t_c}$ and $\overline{t_c}$ are each increasing in x_m. Therefore, the left side of inequality (23) is strictly positive and the condition holds, i.e. the expected challenger type conditional on a contested race is increasing in x_m.

$p_c(t_c|C = 1, S = 0; t_i)$, the pdf of the challenger's type given entry and the incumbent's stepping down, conditional on t_i, is $p_c(t_c)$ truncated on $(\overline{t_c}, \infty)$. From Proposition 4, $\overline{t_c}$ is increasing in x_m. Therefore $E_c[t_c|C = 1, S = 0; t_i]$ is also increasing in x_m, which implies $E_i[E_c[t_c|C = 1, S = 0]]$, the expected challenger type given challenger entry and the incumbent stepping down, is increasing in x_m as well.

2(b). The logic of the proof of 2(b) is symmetric to that in 2(a). ■