Studies of a Latent Class Signal Detection Model for Constructed Response Scoring II: Incomplete and Hierarchical Designs

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Abstract

A basic consideration in large-scale assessments that use constructed response (CR) items, such as essays, is how to allocate the essays to the raters that score them. Designs that are used in practice are incomplete, in that each essay is scored by only a subset of the raters, and also unbalanced, in that the number of essays scored by each rater differs across the raters. In addition, all of the possible rater pairs may not be used. The present study examines the effects of these factors on parameter recovery and classification accuracy using simulations of a latent class model based on signal detection theory (SDT). Many tests also include more than one CR item, which introduces a nested or hierarchical structure into the design, in that raters are nested within essays (i.e., there are multiple raters per essay) and essays are nested within examinees (i.e., each examinee provides two or more essays). A hierarchical rater model (HRM) has previously been developed to recognize the nested structure. A version of the HRM that incorporates a latent class signal detection model in the first level, referred to as the HRM-SDT model, is presented. Parameter recovery in the HRM-SDT model is examined in simulations. The model is applied to data from several ETS tests.

Key words: Constructed responses, signal detection theory, balanced incomplete block, hierarchical rater model, latent class

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Many tests, such as the SAT® and GRE®, include constructed-response (CR) items, such as essays, in addition to multiple-choice (MC) items. The use of CR items necessitates the use of raters to score the items. A basic question is how to treat the rater scores. It is important to recognize, for example, that CR scores differ in a basic way from MC scores in that CR scores contain error, due to imperfect reliability of the raters and the possible presence of rater effects, whereas this is not the case for MC items, which can be objectively scored as right or wrong by either a person or a machine. The effects of rater reliability and rater effects in CR scoring are recognized in a latent class extension of signal detection theory (SDT; DeCarlo, 2002, 2005). In this approach, CR scores are viewed as being ordinal indicators of ordinal latent categories of essay quality. A prior report (DeCarlo, 2008a) examined parameter recovery and classification accuracy for latent class SDT models in fully crossed designs (where each essay is scored by every rater) and in incomplete designs (where each essay is scored by only some raters). The present report extends this research by examining some issues that arise when essays and raters are used in large-scale assessments.

First, in terms of the allocation of essays to raters, the designs used in large-scale assessments are necessarily incomplete, in that there are too many essays for a complete design to be used. Instead, each essay is typically scored by two raters out of a pool of perhaps several dozen raters. The effect of incompleteness was studied in prior research (DeCarlo, 2008a) by using a balanced incomplete block (BIB) design, which offers a useful baseline for comparison. In a BIB design, as applied to essay scoring, all possible rater pairs are used, each rater scores the same number of essays, and each pair of raters scores the same number of essays. Although a BIB design is statistically efficient, there are practical limitations to implementing it in large-scale assessments. For example, it is difficult to keep the number of essays scored by each rater equal given that, among other things, different numbers of examinees take the test each test day and the raters differ with respect to how long it takes to score a given set of essays. It also can be difficult to use all of the possible rater pairs, since there are a large number of possible pairs (e.g., 1,225 for 50 raters) and not all of the raters are available all of the time. So too, it is difficult to balance the number of essays scored by each pair of raters. As a result, in practice each rater typically scores a different number of essays and all of the rater pairs are not used, and so the design is not balanced with respect to the number of essays scored by each rater and with

respect to the number scored by each rater pair. The present study uses simulations to examine the effects of these factors on parameter recovery and classification for a latent class SDT model.

Second, many tests include more than one CR item, and so another issue that arises is how to model the multiple sets of CR scores for each examinee. It is important to recognize that, for multiple CR items, there are not only multiple raters nested within each CR item (i.e., each essay is scored by more than one rater), but also multiple CR items nested within examinees (e.g., each examinee writes more than one essay). The nested structure means that the sets of CR scores should not be treated as being independent. To recognize the nested structure, a hierarchical rater model (HRM) has been introduced (see Patz, Junker, Johnson, & Mariano, 2002). In the version of the model presented here, which will be referred to as the HRM-SDT model, a latent class SDT model is used in the first level and an item response theory (IRT) model is used in the second level (DeCarlo, 2008b). Specifically, in the first level of the model, the CR scores are used as ordinal indicators of the quality of an essay, as in the usual latent class SDT model. In the second level the essay qualities are used as ordinal indicators of a continuous underlying ability, as in the usual IRT model. The approach recognizes the dependence that arises from having each examinee provide two essays; it also provides information about the difficulty and discriminability of the CR items. The current study presents results for simulations where parameter recovery for the HRM-SDT was examined. Also presented are applications of the model to large-scale tests that include two essays or three problem-solving exercises.

This report is organized as follows: First, the latent class signal detection model is reviewed. Some issues that arise with incomplete and unbalanced designs are discussed, followed by a presentation of results for simulations of unbalanced incomplete designs. Next, the HRM-SDT model is introduced, followed by simulations that examine parameter recovery. The last section applies the model to several real-world data sets.

Latent Class Signal Detection Theory

Latent class signal detection theory has been discussed in a previous research report (DeCarlo, 2008a) and in several publications (DeCarlo, 2002, 2005), and so it is only briefly described here. Each essay is viewed as belonging to one of several latent categories of quality, with the latent categories defined by the scoring rubric. The task for each rater is to determine which category each essay belongs to. A basic idea of SDT is that a rater's judgment depends on

(a) his or her perception of the quality of the essay and (b) his or her use of response criteria, which reflect what the rater considers to be, for example, no mastery, little mastery, fair mastery, and so on; thus it is recognized in SDT that there may be individual differences in the way raters use the response categories. A rater is viewed as arriving at a judgment for each essay by using his or her perception of the essay together with response criteria.

SDT summarizes the data by providing two rater measures — a discrimination parameter *d*, which indicates how well the rater discriminates between the latent categories, and response criteria c_k, which reflect how the rater uses the response categories. In SDT, *d* has an interpretation as the distance between underlying probability distributions (e.g., of perception), whereas c_k are criteria located relative to the underlying distributions. Figure 1 shows an example with four latent categories (and so there are four perceptual distributions, one for each latent category) and responses of one to four (and so there are three response criteria).
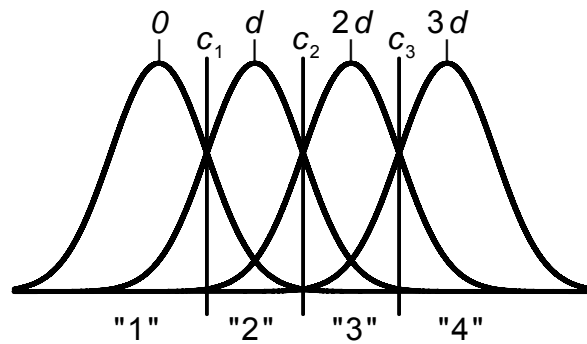
Figure 1. **A representation of signal detection theory.**

The Latent Class SDT Model

The above assumptions lead to a statistical model where the observed score of a rater serves as an ordinal indicator of the quality of an essay; the quality of an essay in turn is viewed as being a latent categorical variable on an ordinal scale. More specifically, the latent class SDT model for rater *j* is a model of the cumulative response probability given the latent category,

$$p(Y_j \leq k_j \mid X^{\#} = x^{\#}) = F(c_{jk} - d_j x^{\#}) \qquad (1)$$

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(cf. DeCarlo, 1998), where Y_j is the response variable for rater j, with values k_j that range from 1 to K_j, $X^{\#}$ is a latent categorical variable with values of $x^{\#} = 0$ to $K-1$ (i.e., it is assumed that there are K latent categories, defined by the scoring rubric), c_{jk} are $K-1$ response criteria for the jth rater and kth response category, with $c_{j0} = -\infty$, $c_{jK} = \infty$, and $c_{j1} < c_{j2} < ... < c_{j,K-1}$, d_j is a discrimination parameter for the jth rater, and F is a cumulative distribution function (CDF). Note that the use of values of 0, 1,..., $K-1$ for $x^{\#}$ in Equation 1 implements an equal distance restriction, in that it constrains the distances between the underlying distributions to be equal for adjacent distributions, and so the distances are multiples of each other (i.e., the first distance is d, the second is $2d$, the third is $3d$; see DeCarlo, 2002, 2005, 2008a). As noted earlier, the model can be viewed as a type of discrete factor model, and is also related to located latent class models and discrete IRT models (DeCarlo, 2002, 2005).

The latent class SDT model can be incorporated into a restricted latent class model by using differences between the cumulative response probabilities to get the probability for each response category. Thus, for K response categories,

$$p(Y_j = k_j \mid X^{\#} = x^{\#}) = F(c_{jk} - d_j x^{\#}) \qquad\qquad k_j = 1$$

$$p(Y_j = k_j \mid X^{\#} = x^{\#}) = F(c_{jk} - d_j x^{\#}) - F(c_{jk-1} - d_j x^{\#}) \qquad 1 < k_j < K_j. \qquad (2)$$

$$p(Y_j = k_j \mid X^{\#} = x^{\#}) = 1 - F(c_{jk-1} - d_j x^{\#}) \qquad\qquad k_j = K_j$$

The above probabilities are used in a restricted latent class model (see Clogg, 1995; Dayton, 1998), which is a model for the response patterns across raters,

$$p(Y_1 = k_1, Y_2 = k_2,..., Y_j = k_j) = \Sigma_{x\#}\, p(X^{\#} = x^{\#})\, \Pi_j\, p(Y_j = k_j \mid X^{\#} = x^{\#}), \qquad (3)$$

where the summation is over the values of the latent classes (i.e., $x^{\#}$), the product is over the J raters, and an assumption of local independence is made, that is,

$$p(Y_1 = k_1, Y_2 = k_2,..., Y_j = k_j \mid X^{\#}) = \Pi_j\, p(Y_j = k_j \mid X^{\#}).$$

The above represents a standard restricted latent class model, and so it can be fit with several software packages, such as LEM (Vermunt, 1997), Latent Gold (Vermunt & Magidson, 2007), or

Mplus (Muthén and Muthén, 2007; note that the latent class SDT model can be implemented in Mplus as a nonparametric confirmatory factory analysis model).

Incomplete Designs

Complete (fully crossed) designs, where each essay is scored by every rater, are not used in practice for large-scale assessments because there are typically a large number of essays to be scored and there are limitations to the number of essays each rater can score. This necessitates the use of incomplete designs; for example, each essay in large-scale assessments typically is scored by two raters out of a pool of raters. The essays can be allocated to the raters according to different rating designs (see Hombo, Donoghue, & Thayer, 2001). Prior research on the latent class SDT model has examined balanced incomplete block (BIB) designs (DeCarlo, 2008a), which are efficient (Fleiss, 1986) and serve as a useful baseline. In the current context, a BIB design uses all possible pairings of the raters, balances the number of essays scored by each rater, and balances the number of essays scored by each pair of raters. The present report extends prior research by examining, in addition to BIB, unbalanced designs, where raters score different numbers of essays and not all rater pairs are used; this is closer to the type of design that is used in real-world assessments, as noted below.

Estimation. The essays are allocated to the raters, and so the data in rating designs are missing by design. To use the terminology of Rubin (1976), data that are missing by design are *missing completely at random* (MCAR; note that the results are also valid with the weaker assumption of missing at random). The approach to fitting the model with missing data is to maximize the likelihood for the various subsets of raters that score each essay. First, consider the situation where three raters grade each and every essay in a fully crossed design. The log likelihood is

$$\log L = \Sigma_i \log \Sigma_{x\#} \, p(X^\# = x^\#) \, p(Y_{i1} = k_1 | \, X^\# = x^\#) \, p(Y_{i2} = k_2 | \, X^\# = x^\#) \, p(Y_{i3} = k_3 | \, X^\# = x^\#), \qquad (4)$$

where Y_{ij} is the score for examinee (essay) i for rater j. Next, consider an incomplete design where Essay 1 is scored by only Raters 1 and 2, Essay 2 is scored by only Raters 1 and 3, and so on. Then the log likelihood for the first case, with Raters 1 and 2, is

$$\log L_1 = \log \Sigma_{x\#} \, p(X^\# = x^\#) \, p(Y_{11} = k_1 | \, X^\# = x^\#) \, p(Y_{12} = k_2 | \, X^\# = x^\#),$$

and for the second case, with Raters 1 and 3, is

$$\log L_2 = \log \Sigma_{x\#}\, p(X^\# = x^\#)\, p(Y_{21} = k_1|\, X^\# = x^\#)\, p(Y_{23} = k_3|\, X^\# = x^\#),$$

and similarly for the remaining cases. Thus, in the presence of missing data, the log likelihood is based on the subsets of raters for which we have observations, and so all available information is used for each case. See the technical manual of Latent Gold (Vermunt & Magidson, 2005) for further details on estimation in latent class models with missing data using maximum likelihood (or posterior mode) estimation.

Connectedness. When using fewer than all of the possible pairs of raters, one has to take care that the raters are all *connected*, so that *d*, for example, is on a common scale and can be compared across all of the raters; any non-connected raters cannot be compared to the other raters. For example, if Raters 1 and 2 grade one set of essays and Raters 3 and 4 grade a second set of essays with no overlap of essays, then Raters 1 and 2 can be compared to each other, but they cannot be compared to Raters 3 or 4 (without making further assumptions). On the other hand, all of the raters can be compared if Rater 1 has some overlapping essays with Rater 3, for example (i.e., at least three rater pairs are needed for all of the raters to be connected when there are four raters). The issue is related to earlier discussions in the literature about comparing treatment effects in block designs, in terms of whether treatment contrasts are estimable (e.g., Eccleston & Hedayat, 1974; Weeks & Williams, 1964); note that in the current context, the essays (examinees) correspond to blocks and the raters correspond to treatments.

For the situation with *J* raters and two raters per block (essay), at least *J*−1 rater pairs must be used in order for all of the raters to be connected. For example, one can simply pair *J*−1 of the raters with the *J*th rater, that is, for 10 raters, one can use the nine pairs (1, 10), (2, 10), (3, 10), and so on, with the result that all of the raters will be fully connected. Simply put, each rater is paired with the same rater (e.g., Rater 10), and so all of the raters can be compared to each other. Another option is to use a spiral-like design, with the nine pairs being (1,2), (2,3), (3,4), and so on (which is simple to implement and helps with balancing). This design was used here for the unbalanced condition because it is simple to implement, is used in practice, and allows one to create unbalanced data with some control over how many essays each rater scores.

Lack of balance. The present report examines the effect of a lack of balance, that is, situations where the raters score different numbers of essays instead of an equal number. The

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type of design used is shown in Table 1, with the rows referring to groups of essays and the columns to raters. For example, the first row of Table 1 shows that Raters 2 and 5 score the same 20 essays. The last row of the table shows that the 10 raters score a total of 50, 60, 120, 140, 200, 230, 280, 310, 370, or 400 essays each. Note that the basic nine adjacent pairs needed for the raters to be fully connected are used, along with a 10th pair that was added so that each rater is paired with two other raters. Thus $(10/45) \times 100 = 22\%$ of the possible 45 rater pairs are used, which is close to the lower limit for connectedness of 20% (i.e., for 10 raters, a minimum of nine rater pairs are required, giving $(9/45) \times 100 = 20\%$). The total number of essays scored is 1,080, with each rater scoring an average of 216 essays (to match the BIB design). Note that for the BIB condition (and in the original fully crossed data), the population values of d were ordered, in increasing magnitude, from Rater 1 to 10 (and the incomplete data were created from the original fully crossed data, see below). Thus for the incomplete design the raters were randomly allotted to the 10 columns of the design, as shown in Table 1, so that the sample size was not systematically related to the value of d.

Table 1

An Unbalanced Incomplete Design (N = 1,080)

	Rater										
	2	5	6	4	3	9	10	1	8	7	Total
	20	20									
		40	40								
			80	80							
				60	60						
					140	140					
						90	90				
							220	220			
								150	150		
									250	250	
	30									30	
											1080
Total/Rater	50	60	120	140	200	230	310	370	400	280	

Linkage. As noted above, a design with two raters per essay can be fully connected with the use of only $J-1$ rater pairs out of a possible $[(J \times (J-1)]/2$ rater pairs. For example, if there are 40 raters, then the raters will be fully connected with the use of 39 rater pairs out of a possible 780 pairs (i.e., 5% of the total possible unique pairs). Another issue that arises has to do with the *degree of linkage* in the design. The degree of linkage refers (in part) to the percent of the possible rater pairs used in the design. A basic question of interest is whether there is any benefit to using more than the minimum number of rater pairs needed for connectedness. This will be examined in the present report in both balanced and unbalanced designs by increasing the number of rater pairs and examining the effects on estimation and classification.

An aspect of the degree of linkage in incomplete designs that has not, to my knowledge, been noted before is that it affects the relative sparseness of the data. For example, consider the situation with 40 raters using a 1 to 5 response. For a BIB design with two raters per essay, there are $5^2 \times (40 \times 39)/2 = 19{,}500$ possible response patterns, and so unless the sample size is very large, there likely will be many patterns with few or no observations, and so the data will be quite sparse. However, for a spiral design, using the minimum number of rater pairs for connectedness (i.e., 39), there are only $5^2 \times 39 = 975$ possible response patterns, and so the number of patterns with no observations will be smaller and the frequencies per response pattern will be larger. In addition, with the spiral design the number of essays scored by each rater pair will be larger than for the BIB design. Both of these factors might have effects on estimation that offset any loss due to having fewer rater pairs. The effects of linkage will be examined here using conditions where both the balance and linkage are varied.

Boundary problems and posterior mode estimation. It was noted in a previous research report that a problem arises when the latent class SDT model is applied to data from incomplete designs (DeCarlo, 2008a). The problem is that estimates of the parameters that are on the boundary are often found, such as estimates of zero (or one) for one or more of the latent class sizes, or estimates of d that are infinite or large with indeterminate standard errors. Problems of this sort are well known in latent class analysis (e.g., Clogg & Eliason, 1987; Maris, 1999), and several solutions have been proposed. One approach is (partly) Bayesian, in that the parameters are treated as random variables instead of as fixed values to be estimated. For example, the latent classes have a multinomial distribution with parameters, $p(X^{\#})$, that are the

latent class sizes; the parameters can be assumed to also have a probability distribution, such as the Dirichlet, which is a conjugate prior for the multinomial, in that the posterior is also Dirichlet.

This approach is used in *posterior mode estimation* (PME, also called *maximum a posteriori estimation*; see Galindo-Garre & Vermunt, 2006; Gelman, Carlin, Stern, & Rubin, 1995; Maris, 1999; Schafer, 1997; Vermunt & Magidson, 2005). In posterior mode estimation, the maximum of the log posterior function is found, where the log posterior function is the log likelihood function plus a log prior. Posterior mode estimation can be viewed as maximum likelihood estimation (MLE) with a penalty for solutions that are close to the boundary; it basically smooths the estimates away from the boundary. Galindo-Garre and Vermunt (2006) presented a simulation that suggested PME gave more reliable parameter estimates and standard errors than either MLE or parametric bootstrapping. Galindo-Garre, Vermunt, and Bergsma (2004) presented a simulation that suggested advantages of PME over a full Bayesian approach via Monte Carlo Markov chain (MCMC) methods; they also noted that PME is less computationally intense than MCMC. In the context of latent class SDT, simulations presented in DeCarlo (2008a) suggest that PME (using Bayes constants of 1, see below) ameliorated boundary problems and led to good recovery of the population parameters and standard errors (for a range of values found in real-world data).

With respect to priors, Galindo-Garre and Vermunt (2006) examined several noninformative priors (Jeffreys, normal, and Dirichlet) in a simulation and suggested versions of the Dirichlet prior that appeared to perform well (and are simple to implement); these priors are implemented in Latent Gold 4.5, which was used here. With respect to the latent class sizes, the prior smooths the estimates, slightly, towards equality, which helps prevent the occurrence of class sizes of zero or one. With respect to the rater responses, the prior makes the conditional response probabilities slightly more equal and so, in the present context, the prior smooths estimates of d towards zero and thereby prevents indeterminate values; the version of the Dirichlet prior used in Latent Gold for the response variables also preserves their marginal distributions (for details see Vermunt & Magidson, 2005; in the context of SDT this basically means that the smoothing pertains primarily to d, and not c_{jk}). The hyperparameters of the Dirichlet prior are specified in Latent Gold through the use of Bayes constants (which also have an interpretation in terms of adding observations to the data). The simulations presented below

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offer additional information about the performance of PME with Dirichlet priors in latent class SDT and in a hierarchical SDT rater model.

Simulations: Unbalanced, Incomplete Designs

This section presents simulations of BIB designs and unbalanced designs with both full and partial linkage. It should be noted that the design of the simulations was guided in part by the observations that, for several large-scale tests that were examined, anywhere from 200 to 10,000 essays were obtained on any given test day; the essays for a given test day were in turn scored by anywhere from 10 to 80 raters, with each rater scoring anywhere from 1 to 600 essays. Thus for the simulations, the basic design consisted of 10 raters scoring 1,080 essays (because the BIB is fully balanced for 1,080), with each rater scoring 216 essays (or an average of 216 in the unbalanced condition), so that the number of raters, number of essays, and essays scored per rater are all comparable to those found in practice.

For the BIB design with full linkage, all 45 possible rater pairs were used, each rater scored 216 essays, and each rater pair scored 24 essays. Also examined is an unbalanced design with full linkage; the number of essays scored per rater ranged from 60 to 400, and all 45 rater pairs were used. In BIB and unbalanced designs with near minimum linkage, close to the minimum number of rater pairs was used (10 out of 45, or 22%; note that 9 out of 45, or 20%, is the minimum for connectedness) to obtain information about estimation with a near-minimum number of rater pairs (while still maintaining balance in that each rater pair appears twice). BIB and unbalanced designs with moderate linkage, 20 out of 45 rater pairs, also were used to examine the effects of degree of linkage.

Methods

The simulated data were generated using SAS macros written by the author; earlier macros were modified as needed for the current studies. Data for 10 raters discriminating between six latent classes by giving 1-6 responses were simulated. The latent class sizes were chosen to approximate a normal distribution (.08, .17, .25, .25, .17, .08), which is consistent with results found in previous research. Prior research has found a mean value of d for several large-scale assessments in the vicinity of 3 to 4 with an approximate normal distribution. Thus for all conditions a range of values of d from 1 to 5 were used, which covers a range of detection from poor to excellent (for the logistic model) and is consistent with that found for real-world data.

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The distribution of d was approximately normal, with values for the 10 raters of 1, 2, 2, 3, 3, 3, 3, 4, 4, and 5. As in prior research (DeCarlo, 2008a), the response criteria were located at the intersection points of adjacent distributions, which has the convenient property that the relative locations of the criteria remain the same as d varies; it also appears to be a reasonable approximation to what is found with real-world raters (see DeCarlo, 2008a). Specifically, relative to d, the first through last criteria are located at ½d, 1½d, 2½d, and so on, because the intersection points for symmetrical distributions are midway between two adjacent distributions. So, for example, for six latent classes and a d of 2, the six distributions will be at 0, 2, 4, 6, 8, and 10, and the five response criteria will be at 1, 3, 5, 7, and 9, and similarly for other values of d.

Specifics of data generation are given in a prior report (DeCarlo, 2008a). Data were first generated for a fully crossed design. The BIB and unbalanced incomplete data were then created from the fully crossed data by inserting missing values according to the design. For example, for the BIB, blocks of all possible pairs were created and all other data points were set to missing. For the unbalanced simulation, missing values were inserted according to the design (e.g., Table 1). Each condition consisted of 100 replications. The specific designs used for both the balanced and unbalanced conditions, with BIB and spiral designs, are shown in Appendix A.

To see how the total sample size was determined for the BIB condition, note that the following conditions hold for a BIB design (see Fleiss, 1986):

$$gr = nk,$$

$$g \leq n,$$

$$\lambda (g - 1) = r (k - 1), \tag{5}$$

where g is the number of raters, r is the number of essays scored by each rater, n is the number of examinees (essays), k is the number of raters that score each essay (i.e., the block size), and λ is the number of essays scored by each pair of raters. For the design examined here, it follows from the first equation that $10 \times 216 = 1{,}080 \times 2$, which shows that when each essay is scored by 2 raters out of 10, a total of 1,080 essays are scored if each rater scores 216 essays. Further, substituting 10 for g, 216 for r, and 2 for k in the last relation of Equation 5 shows that each rater pair scores 24 essays (which is λ). Note that if one wanted to use 1,000 essays (n) with each rater scoring 200 essays (r), then it follows from Equation 5 that λ is not a whole number (it is 22.22);

this shows that Equation 5 imposes constraints on n and r (given a choice of the number of raters, g, and the number of raters per essay, k) if λ (the number of essays scored by each rater pair) is to be a whole number.

Several software packages can be used to fit the latent class SDT model, such as LEM (Vermunt, 1997), Latent Gold (Vermunt & Magidson, 2007), and Mplus (Muthén & Muthén, 2007); Latent Gold (Version 4.5) was used here. Latent Gold uses the EM algorithm followed by the Newton-Raphson procedure to obtain maximum likelihood estimates of the parameters. Only minor modifications of the algorithms for MLE are needed for PME; Schaefer (1997) noted, for example, that any algorithm used for MLE, such as the iterative proportional fitting algorithm commonly used for log-linear models, can easily be modified to find posterior modes when Dirichlet priors are used for the conditional response probabilities and latent class probabilities (p. 307; also see Gelman et al., 1995).

A SAS macro written by the author was used to generate 100 input files for the Latent Gold analysis and also a DOS batch file, which was used to call Latent Gold repeatedly to perform the analysis. Other SAS macros stripped out information from the Latent Gold output for each replication, and the results were combined in a file for the remaining analyses. The SAS macro that stripped out and summarized the data checked and corrected for label switching, as described in a previous report (DeCarlo, 2008a). Another problem is that the solution could represent a local maximum; to decrease the likelihood of this, the number of sets of starting values was increased from the default of 10 to 20. One also has to check that the solution converged before reaching the maximum number of iterations.

Results

  Rater parameters and latent class sizes. Appendix B presents, for the rater parameters and latent class sizes, the population parameters, the mean parameter estimates, the bias, the percent bias (the bias divided by the population value, times 100), and the mean squared error (MSE) for fits of the model to the 100 sets of simulated data.

  Table B1 shows results for the BIB condition, where all 45 possible rater pairs were used. The table shows that, for the rater discrimination parameter d, recovery is good, with a percent bias of less than 10% for all 10 raters; the MSE is also small, less than 0.6. The bias tends to be negative, which means that d tends to be underestimated. The table shows that the percent bias is

largest for the largest value of *d* (5, for Rater 10). With respect to *c*, the bias and MSE tend to be larger than those for *d*, but with percent bias still generally under 10%. The percent bias tends to be large for the first criterion for each rater; however, this depends in part on the arbitrary location of the zero point (which is why the percent bias is more meaningful for slope parameters like *d* and less so for location parameters like *c*). The percent bias is larger for criteria associated with the largest values of *d*, such as for Rater 10. With respect to estimation of the latent class sizes, the percent bias is less than 10% except for the end classes, which have large positive bias; in particular, the latent class sizes of 0.08 tend to be over-estimated as 0.10.

Table B2 shows results for the unbalanced design, where all 45 rater pairs were again used, with the raters scoring different numbers of essays. With respect to *d*, the bias is less than 10% in all cases, with three notable exceptions. First, the percent bias is large for the two raters who scored the fewest number of essays (Raters 2 and 5, who scored 50 and 60 essays, respectively, as shown in Table 1). Second, as in the BIB condition, the bias is large for the largest value of *d* (5, for Rater 10). Thus the results suggest that a lack of balance leads to larger bias. With respect to the response criteria *c*, the same patterns as found in the BIB condition appear, in that the percent bias is larger for Raters 2 and 5, who graded the smallest number of essays, and Rater 10, who has the largest value of *d*. With respect to the latent class sizes, there is virtually no difference from the balanced design, in that the middle class sizes are well recovered, with a percent bias of less than 10%, whereas the end classes are overestimated (again with 0.08 estimated as 0.10).

Table B3 presents results for 10 raters each scoring 216 essays in a balanced design with "near-minimum" linkage, in that only 10 out of 45 possible rater pairs were used (the design is shown in Appendix A). The percent bias is again generally less than 10%, and the bias tends to be negative. The percent bias for raters with the largest values of *d*, Raters 4 and 5, is large and negative, and so high values of discrimination tend to be underestimated, as found above. Compared to the BIB design with 45 rater pairs, the bias for *d* is larger but only slightly so, with just two raters (Raters 8 and 10, with the largest values of *d*) having a percent bias greater than 10%. The bias tends to be larger for the response criteria, with the largest bias for the raters with the largest values of *d*. With respect to the estimates of the latent class sizes, the results are nearly identical to those for the BIB design, with good estimation of the middle class sizes and positive bias for the end classes.

Table B4 shows results for 10 raters in an unbalanced design with near-minimum linkage (i.e., 10 out of 45 possible rater pairs). The effect of a lack of balance again appears to be an increase in the bias, with the largest effects generally for the raters with the smallest sample sizes or the largest values of d. With respect to estimation of the latent class sizes, the results are nearly identical to those found in the other conditions.

Tables B5 and B6 show results for a spiral-like design with moderate linkage (20 out of 45 possible rater pairs). With respect to the rater parameters, the results are comparable to those obtained for the near-minimum linkage condition, except that the bias and MSE are slightly smaller. Estimation of the latent class sizes is also comparable to that found in the other conditions. Thus it appears that increasing the number of rater pairs from 10 to 20 offers at most a small improvement in recovery of the rater parameters or the latent class sizes.

Standard errors. Appendix C presents results for the evaluation of the estimates of the standard errors of d and the standard errors of the latent class sizes. The standard error estimates are computed using standard asymptotic theory (i.e., using the inverse of the observed information matrix; for details see Vermunt & Magidson, 2005). The bias is obtained by computing the standard deviation of the parameter estimates across the 100 replications (SD in the tables, which serves as the population value), and subtracting it from the mean of the estimated standard errors (i.e., across the 100 replications, which is the Mean SE shown for each parameter in the table).

Table C1 shows that the bias in the estimates of the standard errors is generally less than 10%, except for large values of d (4 or 5), where the SEs tend to be overestimated. The bias for the SEs of the latent class sizes is less than 10%, except for the sixth latent class. Table C2 shows that a lack of balance leads to larger bias (greater than 20%) for the SEs of d for the two raters with the smallest samples sizes (50 and 60); the bias is positive, and so the SEs are overestimated. For the remaining raters, however, the bias is similar to that found in Table C1, with overestimation of the SEs for large values of d. The bias for the SEs of the latent class sizes in Table C2 is also comparable to that found in Table C1.

Tables C3 and C4 show results for the condition with only 10 rater pairs. Table C3 shows that the percent bias for the SEs of the d parameters is generally large and positive, and so the SE's tend to be overestimated. The bias for the latent class sizes is smaller and similar to that found in Tables C1 and C2. Table C4 shows similar results, with about the same magnitude of

bias. Thus, whereas the tables in Appendix B show that using a near-minimum number of rater pairs has little deleterious effect on estimation of the rater parameters, Tables C1 and C3 show that there is a fairly large effect on the estimates of the standard errors of the rater parameters, which are generally overestimated. The estimates of the standard errors for the latent class sizes, on the other hand, do not appear to be heavily affected by using fewer rater pairs or by having a lack of balance, as shown in Tables C1 through C4.

Table C5 shows that, for the condition with 20 rater pairs, the bias of the SEs is generally small (i.e., under 10%) for the rater parameters and latent class sizes, except for large values of d. In contrast, Table C6 shows that the bias for the SEs of the rater parameters is large in the unbalanced condition. As in Tables C1 to C4, Tables C5 and C6 suggest that the SEs of the latent class sizes are adequately estimated.

Overall, the results shown in Appendices B and C suggest that the estimation of d and its standard error are good for values of d in the range of 1 to 5; estimation of c tends to be poorer, depending in part on the value of d. Tables B1 to B6 show that using an unbalanced design increases bias for the rater parameters in both complete and incomplete linkage conditions. However, estimates of the latent class sizes and their standard errors do not appear to be heavily affected by a lack of balance. With respect to linkage, decreasing the linkage by using 10 or 20 rater pairs in lieu of 45 led to perhaps a small increase in the bias, but the effect was relatively small. The effect of decreasing linkage appears to be primarily on the estimates of the standard errors of the rater parameters, which are overestimated; the latent class sizes and their standard errors, on the other hand, are reasonably well estimated in all conditions.

Classification. Table 2 shows results for classification accuracy (proportion correctly classified) for the six conditions discussed above. Proportion correct (PC) is the estimated proportion of cases that are correctly classified and is obtained from the posterior probabilities (for a fit of the model see, e.g., Clogg, 1995); note that PC is available for both simulated and real-world data (i.e., upon fitting the model, the PC can be estimated). In contrast, this is not the case for PC_{obt}, which is only available in a simulation; specifically, PC_{obt} is the obtained (not estimated) proportion of cases that were actually correctly classified in the simulation (i.e., PC_{obt} is computed by comparing the classifications obtained from the posterior probabilities to the true latent classes; of course the true latent class for each case is only known in a simulation). Similarly, PC_{av} is the proportion of cases that were correctly classified in the simulation by using the obtained average

score (rounded both up and down; the rounding that gave the largest value of PC_{av} is the one that is reported). Table 2 also shows two measures of association between the classifications and the true latent classes, namely the Pearson correlation r and tau-b, again for both the model-based classifications (with subscript *obt*) and the average scores (with subscript *av*).

Table 2

Estimated and Obtained Proportion Correct and Correlations With True Latent Classes, Balanced and Unbalanced Designs, Varying Linkage

Pairs	PC	PC_{obt}	PC_{av}	r_{obt}	r_{av}	$\tau_{b\text{-}obt}$	$\tau_{b\text{-}av}$
45 balanced	0.747	0.730	0.661	0.927	0.910	0.874	0.846
45 unbalanced	0.770	0.765	0.646	0.939	0.903	0.892	0.835
20 balanced	0.740	0.719	0.668	0.922	0.911	0.868	0.848
20 unbalanced	0.755	0.735	0.658	0.925	0.902	0.873	0.835
10 balanced	0.744	0.733	0.654	0.931	0.910	0.880	0.846
10 unbalanced	0.763	0.767	0.646	0.940	0.904	0.894	0.836

Note. PC = estimated proportion correct; PC_{obt} = obtained (in the simulation) proportion correct; PC_{av} = obtained (in the simulation) proportion correct using the average score; r_{obt} and $\tau_{b\text{-}obt}$ are the obtained Pearson correlation and tau-b; r_{av} and $\tau_{b\text{-}av}$ are the obtained correlation and tau-b for the average scores.

Several results are apparent in Table 2. First, the estimated proportion correctly classified, PC, tends to overestimate the proportion actually correctly classified in the simulation (i.e., PC_{obt}), although the overestimation is very small, generally around .02 or less. Overestimation of PC_{obt} by PC has been noted earlier (DeCarlo, 2005, 2008a); note that a comparison of the results shown in Table 2 with those shown in Table 6 of DeCarlo (2005) shows that the overestimation is larger for smaller sample sizes (about 5% for a sample size of 300 and over 20% for a sample size of 100, compared to the 2% found here for a sample size of 1,080).

Second, Table 2 shows that the proportion correctly classified using the average score, $PC_{av,}$ is in every case 5% to 10% lower than the proportion correctly classified using the model (i.e., PC_{obt}). This shows that there is clearly a benefit to using the model-based classifications over the average scores, as found in other studies (DeCarlo, 2008a), assuming of course that the SDT model is appropriate. Table 2 also shows that the correlations of the average scores with the

true latent classes, r_{av} and $\tau_{b\text{-}av}$, also tend to be smaller than the correlations for the model-based classifications, r_{obt} and $\tau_{b\text{-}obt}$.

With respect to a lack of balance, Table 2 shows that PC_{obt} is actually slightly larger in the unbalanced conditions than in the balanced conditions (perhaps because some of the raters score more essays), and so a lack of balance appears to have no detrimental effect on classification accuracy. With respect to the classifications obtained by using the average rater scores, a lack of balance reduces PC_{av} by only about 1%.

With respect to linkage, an interesting result shown in Table 2 is that the number of rater pairs used (i.e., 45, 20, or 10) appears to have little influence on classification accuracy. For example, correct classification (PC_{obt}) for the spiral design conditions with only 10 rater pairs is as high (.733 and .767 for balanced and unbalanced) as in the BIB conditions with 45 rater pairs (.730 and .765).

Discussion

These simulations examine parameter recovery and classification in balanced and unbalanced designs with varying linkage. The results show that estimation is affected by a lack of balance and by having less than full linkage; however, the rater discrimination parameter *d* is generally well recovered, as are the latent class sizes. The largest bias is associated with the largest values of *d*, which tend to be underestimated. Note that underestimation of large values of *d* means that the raters actually are performing better than indicated by the estimate (which is not a problem, but it would be a problem if the opposite occurred — if small values of *d* were overestimated, then raters who perform poorly would appear to be better than they really were). The results also suggest that a lack of balance has a larger effect on parameter recovery than the number of rater pairs used (i.e., the degree of linkage); the effect of linkage on parameter estimation was fairly small (in terms of bias). Thus the results show that using a design with less than full linkage appears to result in little loss with respect to parameter recovery, with the main effect appearing to be larger standard errors.

The results for classification accuracy are of particular interest. First, the results in Table 2 show that there is nearly a 10% increase in classification accuracy obtained by using model-based classifications in lieu of average scores. Of course the increase in PC depends on the parameters and design (and on the validity of the SDT model). However, other studies that have

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used different parameters and designs also have found increases in classification accuracy in the range of 5% to 10% when model-based classifications were used in lieu of average scores (DeCarlo, 2008a). These results point to the potential advantage of using a model-based approach over simply averaging the raters' scores; the next step is to conduct validity studies with real-world data.

Second, Table 2 shows that a lack of balance has only a small effect on classification accuracy, which was reduced by less than 2%. An interesting result shown in Table 2 is that the degree of linkage appears to have virtually no effect on classification accuracy. Thus the degree of linkage appears to have little effect on either parameter estimation (in terms of bias for d) or on classification accuracy. This might occur because, as noted above, designs with less than full linkage are less sparse, in the sense that there are fewer possible response patterns; the fact that the number of essays for each rater pair is also larger in the spiral design than in a BIB design might also affect estimation and classification.

In sum, the current results suggest that a design with the near-minimum number of rater pairs can be used with little loss in estimation or classification; the main loss appears to be an increase in the bias of the standard errors. The finding of little or no detrimental effect on classification of using a design with near-minimum linkage has important implications, in that it suggests one can use a simpler design, such as that used above, in lieu of a BIB design. A practical consequence is that, because there are considerably fewer rater pairs in the spiral design as compared to the BIB, allocation of the essays to raters is much easier to manage. For example, for a situation with 32 raters, as in the real-world data analysis presented below, there are 496 rater pairs in a BIB design, but only 32 rater pairs in the spiral-like design used here. An advantage of having fewer rater pairs to manage is that it is easier to keep the design from getting highly unbalanced, whereas this is more difficult to do when there are hundreds of rater pairs to manage. As shown here, a lack of balance appears to have a more deleterious effect on estimation than using fewer rater pairs. Thus, whereas some large-scale assessments attempt to use nearly all (or all) of the possible rater pairs, the results presented here suggest that this may not be necessary.

The next section examines another practical problem that arises in large-scale assessments, which is that some assessments include more than one CR item. This raises

questions about how to analyze multiple CR items within the framework of latent class signal detection theory.

A Hierarchical Rater Model

Many tests, such as the SAT, include only one CR item, such as an essay. However, some tests include more than one CR item; for example, the GRE has two CR items (essays on issue and argument tasks), whereas the Praxis™ Middle School Mathematics test (MSMAT) and Reading Across the Curriculum: Elementary (RACE) test (which is also a part of the Praxis series) each include three CR items, which are problem-solving exercises. Although one can analyze each essay separately, a more comprehensive model considers the multiple essays simultaneously. In particular, as discussed above, it is important to recognize the nested structure of the data, in that raters are nested within the essays and, when there is more than one essay, essays are nested within examinees. This structure is explicitly recognized by a hierarchical rater model (HRM), which was introduced by Patz (1996) and is discussed in Patz et al. (2002). Here it is shown that the latent class SDT model can easily be used for the first level of the HRM (DeCarlo, 2008b), which offers some advantages over the SDT-like model used by Patz et al. The model also can easily be fit using MLE or PME, whereas Patz et al. used a (more computationally intense) Markov Chain Monte Carlo (MCMC) approach.

An HRM-SDT Model

Figure 2 shows a representation of the HRM-SDT model with two essays per examinee. The first level of the model, which is simply a latent class SDT model, relates the observed scores of the raters (Y_j), which are ordinal, to the latent class variables ($X^\#_l$), which also are ordinal. As discussed above, the observed responses Y_j arise from the rater's perception Ψ and their use of response criteria c_{jk}, in that the distance of c_{jk} from the conditional mean of Ψ determines the response probability; curved arrows are used in the figure to indicate the nonlinear nature of this relation. The arrows from $X^\#$ to Ψ indicate that the mean of Ψ is shifted by d_j as the latent category increases by one. In the second level of the model, the ordinal latent variables $X^\#$ serve as indicators of the examinee's ability (θ) via an item response theory (IRT) model (the generalized partial credit model is used here). The arrows from θ to Ψ are curved to indicate that the latent class variables $X^\#$ have a nonlinear relation to an examinees' ability, via

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an IRT model, with parameters *a* and *b,* for the slope (discrimination) and category steps, respectively.
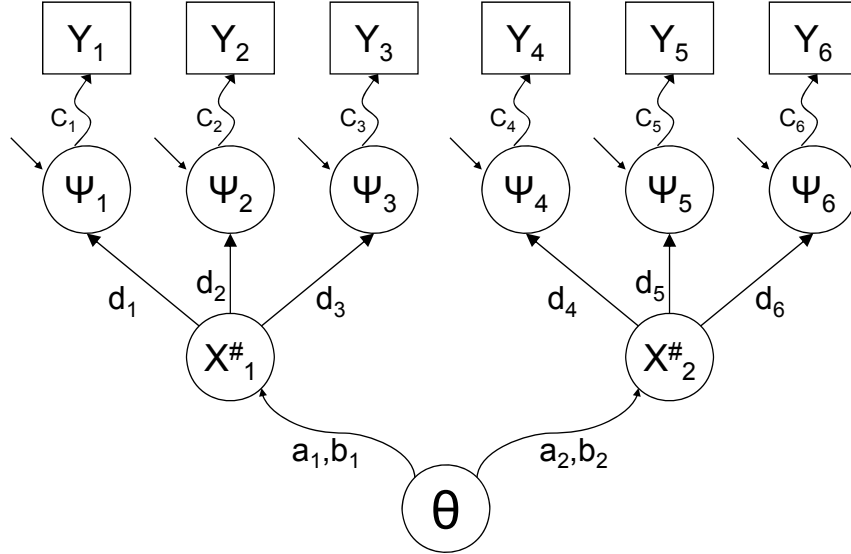
Figure 2. **A representation of the HRM-SDT model.**

With matrices **Y** for the response patterns and $\mathbf{X}^{\#}$ for the latent class variables, and writing $p(\mathbf{y})$ for $p(\mathbf{Y} = \mathbf{y})$ and $p(\mathbf{x}^{\#})$ for $p(\mathbf{X}^{\#} = \mathbf{x}^{\#})$, the model can be written as

$$p(\mathbf{y}) = \Sigma_{x\#} \int_{\theta} p(\mathbf{x}^{\#}|\theta)\, p(\mathbf{y}|\mathbf{x}^{\#},\theta)\, p(\theta)\, d\theta, \tag{6}$$

where $p(\mathbf{y}|\mathbf{x}^{\#},\theta)$ is the rater component of the model (the first level) and $p(\mathbf{x}^{\#}|\theta)$ is the model for the CR items (the second level). As before, an assumption of independence given the latent class variables $\mathbf{X}^{\#}$ is made,

$$p(\mathbf{y}|\mathbf{x}^{\#},\theta) = \Pi_{jl}\, p(y_{jl}|\mathbf{x}^{\#},\theta), \tag{7}$$

where *j* indicates the rater and *l* indicates the essay. An assumption of independence of the *l* latent classes given θ is also made,

$$p(\mathbf{x}^{\#}|\theta) = \Pi_{l}\, p(x_{l}^{\#}|\theta). \tag{8}$$

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As before, the first level of the model consists of a latent class SDT model,

$$p(Y_{jl} \leq y_j \mid x_l^{\#}) = F(c_{jkl} - d_{jl} x_l^{\#}).$$

The above is incorporated into the right side of Equation 7 by differencing the cumulative probabilities, as shown above. The second level of the model treats the latent classes as ordinal indicators of ability using an IRT model. For example, using adjacent categories logits (see Agresti, 2002) gives the generalized partial credit (GPC) model (Muraki, 1992),

$$\log [p(X_l^{\#} = x_l^{\#} \mid \theta)/p(X_l^{\#} = x_l^{\#}+1 \mid \theta)] = b_{lx\#} - a_l \theta, \tag{9}$$

where $X_l^{\#}$ are ordinal latent categories for the lth CR item, $x_l^{\#}$ are discrete values that range from zero to one minus the number of latent classes, θ is a continuous latent variable (i.e., ability), and $b_{lx\#}$ and a_l are item step and discrimination parameters, respectively, for item l; note that the model is parameterized in a manner similar to that for the SDT model, in that lower categories are modeled (whereas in the usual version of the GPC model, higher categories are modeled). Starting with Equation 9, one can also write the model in terms of probabilities, as done by Muraki (1992) and others.

There are many possible versions of the HRM-SDT model that can be examined in simulations. Examined here is a basic version of the model with two or three constructed response items and three indicators per CR item (fully crossed). This version of the model arises in practice when the data are pooled across raters (as is commonly done), thereby giving a fully crossed design (that ignores rater effects). Also note that, although each essay in large-scale assessments is typically scored by two raters, a third rater (an adjudicator) also is used for many tests, giving a total of three raters per essay; DeCarlo and Kim (2008) showed that including the third score is feasible and provides useful information. The real-world data examined below are also of this form. The simulations provide information about parameter recovery for the different components of the HRM-SDT model as well as information about the effect of using more CR items (three instead of two).

Methods

The hierarchical rater model previously has been fit using a fully Bayesian approach and the MCMC algorithm (e.g., Patz et al., 2002). The model as presented here, however, can easily

be fit using maximum likelihood estimation (or posterior mode estimation), employing any one of several widely available software packages. For example, the models were fit here using the syntax version of Latent Gold 4.5 (Vermunt & Magidson, 2007). The model can also be fit with the freely available software Lem (Vermunt, 1997), though only with MLE and not PME.

Data

For the simulation, data for the hierarchical rater model, parameterized as shown above (using the GPC for the IRT portion), were generated using SAS macros written by the author. Each dataset consisted of 3,000 observations; a total of 100 datasets were generated for each of two conditions: an HRM-SDT with two CR items and an HRM-SDT with three CR items. The population values of the parameters are shown in Appendix D; the values used were chosen because they are typical of those found in other studies. SAS macros were used to strip out and summarize the results from the Latent Gold output; further details about the simulations can be found in a prior research report (DeCarlo, 2008a). As before, the SAS macro that computed the final results checked and corrected for label switching. Note that the problem of label switching is more complex for the HRM-SDT model because it can occur both for the latent categorical variable $X^{\#}$ and for the latent continuous variable θ, and so there are four possibilities that must be considered. In particular, when label switching occurs for $X^{\#}$, the sign of d is reversed, c has to be corrected by adding $K - 1$ times d to the obtained estimates of c, and the order of the latent class sizes is reversed; this is the same as above for the simple latent class SDT model. In addition, for HRM-SDT, label switching can occur for θ and not for $X^{\#}$, in which case the sign of a will be reversed, but the order for b will be correct. If label switching occurs for both θ and for $X^{\#}$, then the sign of a is correct, but the order of b is reversed. The SAS macro that combined the results for the 100 replications checked for these different possibilities and made the appropriate corrections to the parameter estimates.

Some preliminary runs showed that problems with boundary problems occasionally occurred, and so PME was used, with Bayes constants of unity for the rater responses and the latent classes (as also used in the studies above). Note that the computer time for the situation with three CR items was found to be much longer than that for two CR items. The time to complete one of the (100) replications is determined by the time it takes to complete each iteration (which is slower for Newton-Raphson steps than EM steps), the number of iterations

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needed for convergence, the number of sets of starting values that are used (10 were used here), and the number of nodes used for the continuous variable theta (10 nodes were used here). For the situation with two CR items, each replication took about 10 minutes to complete, however, for the situation with three CR items, each replication took about 2 hours to complete (and so the simulation took several weeks of continuous computer time).

Results: Two CR Items

The first section of Table D1 shows the parameter estimates for the signal detection part of the model, along with the bias, percent bias, and the mean squared error. With respect to the signal detection parameters, recovery of the discrimination parameter d is excellent, with a bias of 1.2% or less; the MSE is also small. The bias is also small for the response criteria, with a percent bias of under 5%. Overall, recovery of the signal detection parameters is excellent.

The second section of Table D1 shows results for the IRT part of the model, that is, a GPC model that relates the six ordinal latent classes for each CR item to theta. The table shows that, for both CR items, the bias for estimates of the discrimination parameters a_1 and a_2 is fairly large, with a percent bias of 22.7% and 15.7%, respectively. The bias for the category step estimates b_{lm} is also generally large (greater than 10%). Computing the Monte Carlo standard error for the a parameters (as the standard deviation across replications divided by the square root of the number of replications) gives 95% CIs (confidence intervals) of (1.18, 1.27) and (1.24, 1.28), neither of which contain the population values of 1.0 and 1.5, respectively. This shows that the 100 replications were enough to detect significant bias. In contrast, a 95% CI for the estimate of the first d gives (1.99, 2.01), which contains the population value of 2.0 and simply reflects that the bias is very small; the results are similar for the other d parameters. Thus there is negligible bias for estimates of the rater parameters but significant bias for the CR item parameter estimates.

It is interesting to note that the population value of a for the first essay (1.0) is overestimated (as 1.2), whereas the population value of 1.5 for the second essay is underestimated (as 1.3). This suggests that the estimates of a are shrunk towards a mean value, which would happen if the a parameters were random, as in multilevel models with random slope parameters (Raudenbush & Bryk, 2002). However, the a parameters are fixed, not random, in the HRM-SDT model, and so the reason for the shrinkage (if it is in fact occurring) is not

known. It is not due to the use of posterior mode estimation, because that smooths the *a* parameters towards zero, and not towards the mean (which can be verified easily with Latent Gold by using large values for the Bayes constants). An examination of the correlation matrix of the parameter estimates showed that the correlation of the estimates of the *a* parameters for the first and second item was large and negative (around -0.95), which would account for the over- and under-estimation noted above for *a*; note that the correlation was considerably smaller (-0.50 or less) for the situation with three CR items (examined next).

Overall, the simulation suggests that parameters for the SDT part of the HRM model are well recovered, particularly the discrimination parameter *d*; recovery of the parameters for the IRT part of the model appears to be poorer. This likely occurs because there are only two indicators for the IRT model (i.e., two essays); this is examined in the next section, where an additional CR item is added. Also note that a prior simulation of the HRM-SDT with only two raters for each CR item (and two CR items; DeCarlo, 2008b) found large bias for both the SDT and IRT parts of the model, which suggests that the use of only two indicators, at either level, results in poor recovery.

Results: Three CR Items

Table D2 shows results for fits of the HRM-SDT model for the simulation with three CR items, again with three raters per item. The top section of the table shows that the signal detection parameters are again well estimated, with a bias of generally less than 5%. The next section of the table shows that, in contrast to the results found for two CR items, the parameters of the second-level IRT model are well estimated, with bias under 5%. Thus Table D2 shows that the addition of an additional indicator at level two of the model, namely a third CR item, markedly improves estimation of the level two parameters, that is, the parameters of the IRT model for the CR items.

Overall, the results suggest that, with at least two CR items, the rater parameters are well recovered when there are three scores per essay, and so one can usefully evaluate rater performance and classification accuracy. On the other hand, the CR item parameters are well recovered primarily when there are at least three CR items per examinee. This means that if one wishes to compare different CR items in terms of item characteristics, then one should try to have at least three CR items. Another possibility (that potentially eliminates the need for more

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CR items) is to include multiple-choice items in Level 2 as indicators of theta, which is being examined in current research (DeCarlo & Kim, 2009; Kim, 2009).

ETS Data

This section applies the HRM-SDT model to the writing section of two ETS datasets. The first example is the writing portion of a large-scale language test that includes two writing tasks for each examinee, whereas the second example involves a test with three problem-solving exercises.

Language Test: Data

The data come from 42,608 examinees. Each essay was scored by two raters, with some essays scored by an additional rater, an adjudicator, when the first two scores differed by more than one. For the first writing task, 3.9% of the essays had a third (adjudicated) score, whereas 2.6% of the essays for the second task had a third score. Data for the third scores can be viewed as being missing at random (Rubin, 1976), in that the probability that a value is missing is determined by an observed variable – the difference between the two observed scores (i.e., the value is missing if it is less than 2). The analyses presented here include the third scores; DeCarlo and Kim (2008) showed that estimation is good for adjudicated scores, as along as a sufficient number are available, as is the case here. Thus there are three scores per essay (one with a large percentage of missing values), giving a total of six scores for the two essays.

The scoring rubric consisted of categories from 0 to 5. Note that a score of 1 to 5 indicates a judgment of the quality of an essay, whereas a score of 0 indicates that there was no essay to be judged (i.e., a blank) or that it was written in the wrong language, and so on, and so one can argue for not including essays with scores of zero (e.g., a score of zero for a blank is not a judgment of an essay's quality), which was done here; note that, for the sample used here, only 124 essays out of 42,732 essays received scores of zero.

An HRM-SDT model, as described above, was fit. The SDT component of the model, which is the first level, used the 1-to-5 essay scores from the three raters as ordinal indicators of five latent classes of essay quality for each essay. The IRT component (a GPC model), which is the second level of the model, used the 1-to-5 latent classes for the two essays as two ordinal indicators of examinee ability. In the analysis presented here, the data are treated as coming from a fully crossed design (i.e., the data are pooled across raters), in order to obtain information about

the HRM-SDT model as applied to pooled data. The model was fit using Latent Gold 4.5; as in the simulations above, posterior mode estimation with Bayes constants of unity for the response variable and latent categories was used.

Results

With respect to the rater parameters (in this case for the pooled raters), Table 3 shows that discrimination is in the range of 3.0 to 3.5, which is good discrimination (for the logistic model; for example, for $d = 3.5$, the odds ratio is 33 to 1; also see the previous research report). For the first writing task the estimate of d for the adjudicated score is slightly smaller than that obtained for the other two scores, whereas for the second writing task the estimates of d are all about the same. The criteria estimates are also similar across the three scores; however, those for the third score are to the left of (i.e., smaller than) those for the other two scores for the first writing task; this indicates that, for the second writing task, the adjudicated scores tended to be slightly more liberal. It is also interesting to note that in all cases the response criteria estimates in Table 3 are close to the intersection point locations. For example, for the first score an estimate of d of 3.4 means that, if the criteria were located at the intersection points, then they would be at 1.7, 5.1, 8.5, and 11.8, and the estimates shown in Table 3 (1.7, 5.4, 9.2, and 12.7) are quite close to these values. Thus it appears that, at least for pooled data, the response criteria tend to be located close to the intersection points of the underlying logistic distributions, at least for the language test examined here (cf. DeCarlo, 2008a).

With respect to the CR item parameters, discrimination is high (4.19 for the first writing task and 2.94 for the second). The item step parameters are fairly similar across the two writing tasks. The last section of the table shows estimates of the latent class sizes. The largest latent class size is for Category 4, followed by Category 3. It is interesting to note that for both writing tasks the latent class sizes are slightly negatively skewed, which is in contrast to the approximately normal distribution of latent class sizes found for other tests (see DeCarlo, 2008a); this might be a characteristic of language tests, but more research on this is needed.

Mathematics Test: Data

The second example is for a large-scale mathematics test used in certification. The test includes three CR items, which are problem-solving exercises, each of which is scored by two raters. The raters score the exercises on a 4-point scale, and so a latent class model with four

latent classes was used for the first level of the model (the SDT part); the generalized partial credit model was again used for the second level of the model (the IRT part).

Table 3

Results for HRM-SDT Model, Two Essays, Language Test

Rater parameters

	First writing task					
	First score		Second score		Third score	
Parameter	Estimate	SE	Estimate	SE	Estimate	SE
d	3.42	0.04	3.46	0.04	2.94	0.16
c_1	1.68	0.05	1.75	0.05	1.13	0.26
c_2	5.38	0.08	5.41	0.08	4.25	0.31
c_3	9.22	0.12	9.32	0.12	7.73	0.43
c_4	12.70	0.14	12.83	0.14	10.80	0.52
	Second writing task					
d	3.28	0.04	3.33	0.04	3.32	0.23
c_1	0.27	0.08	0.29	0.08	0.18	0.54
c_2	4.54	0.10	4.59	0.10	4.28	0.45
c_3	8.94	0.12	9.06	0.13	8.85	0.66
c_4	12.47	0.15	12.64	0.15	12.45	0.78

CR item parameters (generalized partial credit model)

	First writing task		Second writing task	
Parameter	Estimate	SE	Estimate	SE
a	4.19	0.43	2.95	0.16
b_1	−5.17	0.54	−6.24	0.31
b_2	−2.84	0.26	−3.66	0.18
b_3	0.37	0.06	−0.40	0.05
b_4	4.21	0.41	3.03	0.16

Latent class sizes

	First writing task		Second writing task	
Parameter	Estimate	SE	Estimate	SE
p_1	.12	< .01	.03	< .01
p_2	.15	< .01	.11	< .01
p_3	.27	< .01	.32	< .01
p_4	.29	< .01	.37	< .01
p_5	.17	< .01	.18	< .01

Results

Table 4 shows results for a fit of the HRM-SDT model. A notable difference, compared to the results found above, is that the estimates of the discrimination parameters *d* are quite large, in the range of 8-13 across the three problems. This indicates excellent discrimination; it suggests that the mathematics problems can be classified into the scoring categories more accurately than the writing samples examined above (and for other writing tests, where rater parameter estimates in the range of 2 to 5 have been found). This has not been noted before; comparisons of discrimination across other tests might be informative in future research.

The middle section of Table 4 shows that the CR item discrimination parameter estimates (*a*) are all around unity. The bottom section of Table 4 shows estimates of the latent class sizes. In this case the class sizes are skewed to the left. For example, for the first problem, Class 3 is the largest (.44), followed by Class 4 (.37). For the second and third problems Classes 3 and 2, respectively, have the largest class sizes. Thus, most examinees receive scores of 3 or 4 for the first task, and scores of 2 or 3 for the second and third tasks. The skew might arise because the exam was a test of minimal competency; this merits closer attention in future research. It is interesting to note that the results in Table 4 suggest that the first problem was more often "passed" than the second or third problems, which could reflect a systematic difference in the type of problem-solving exercises that were used for the second and third problems, or an order effect; again, this merits closer attention in future research.

It is also interesting to note that the standard errors of the rater parameters are larger in Table 4 than in Table 3, which likely reflects that there were only two raters per essay in Table 4, whereas there were three raters per essay in Table 3. Similarly, the standard errors of the CR item parameters are larger in Table 3 than in Table 4, which reflects that there were only two CR items in Table 3, but three CR items in Table 4. Thus if one's interest is in obtaining accurate estimates of either the rater parameters or the CR item parameters, then the results suggest that a minimum of three raters or three items should be used (another option is to include multiple choice items in Level 2; DeCarlo & Kim, 2009; Kim, 2009).

Table 4

Results for HRM-SDT, Three Problem-Solving Exercises, Mathematics Test

Rater parameters

	First problem			
	First score		Second score	
Parameter	Estimate	SE	Estimate	SE
d	8.42	0.71	11.27	1.55
c_1	3.28	0.48	4.88	1.06
c_2	15.05	1.42	20.90	3.10
c_3	24.26	2.13	32.92	4.65
	Second problem			
d	12.46	1.74	10.74	1.06
c_1	5.11	1.20	3.93	0.69
c_2	13.77	1.74	12.21	1.06
c_3	26.54	3.49	23.19	2.13
	Third problem			
d	10.28	0.72	12.64	1.50
c_1	4.53	0.53	4.88	0.82
c_2	11.64	0.72	14.12	1.51
c_3	22.07	1.44	27.08	3.01

CR item parameters

	First problem		Second problem		Third problem	
Parameter	Estimate	SE	Estimate	SE	Estimate	SE
a	1.16	0.09	0.99	0.07	1.12	0.08
b_1	−3.01	0.18	−3.33	0.13	−2.51	0.11
b_2	−1.63	0.08	−0.47	0.05	−0.15	0.04
b_3	0.47	0.06	1.63	0.08	2.22	0.11

Latent class sizes

	First problem		Second problem		Third problem	
Parameter	Estimate	SE	Estimate	SE	Estimate	SE
p_1	.03	< .01	.03	< .01	.08	< .01
p_2	.15	< .01	.35	< .01	.44	< .01
p_3	.44	< .01	.47	< .01	.38	< .01
p_4	.37	< .01	.16	< .01	.10	< .01

In sum, the HRM-SDT model provides information about the performance of the raters, about characteristics of the tasks, and about the constructs. For example, the scoring of the mathematics test appears to differ from other tests, such as the language test analyzed above, in that rater discrimination is quite high, which to my knowledge has not been noted before. The high discrimination likely reflects the different nature of the tasks used in the mathematics test, in that the CR items were problem-solving tasks, rather than essays from the writing section of a test such as the GRE or SAT (or the language test examined here). Note that the finding of high values of d for the mathematics test also has implications with respect to classification accuracy, in that it should be higher for higher values of d. For example, the estimates of the proportion correctly classified, PC, for the mathematics test are .96, .97, and .97, for the first through third problems, respectively (with estimates of λ of .93, .95, and .95; see DeCarlo, 2008a, or Dayton, 1998, for a discussion of lambda). In contrast, for the language test PC is .83 and .83 (with λ of .76 and .73) for the two essays, which is still good but is lower than that found for the mathematics test (in part because of the lower rater discrimination). Thus, given the high values of discrimination, classification accuracy for the mathematics test is quite high, around 97%; note that one could use this result to argue for simply using the average rater scores for the mathematics test, because classification accuracy will likely still be quite high, given the high discrimination.

Conclusions

The latent class SDT model offers a useful approach to the analysis of constructed response data. The model can easily be incorporated into a more elaborate framework, such as in models with higher-order structures, like the HRM-SDT model. The approach provides information not only about the performance of the raters and the accuracy of the classifications, but also about characteristics of the CR items and aspects of the underlying constructs. Both the basic SDT and higher-order model, and variations, can also easily be fit using widely available software for latent class analysis or structural equation modeling, which should help to encourage applied researchers to use the models.

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Appendix A

Balanced and Unbalanced Designs Used in the Simulations

Table A1

Balanced Design, 45 Rater Pairs, 24 per Pair

| | Rater | | | | | | | | | | |
	1	2	3	4	5	6	7	8	9	10	Total
	24	24									
	24		24								
	24			24							
	24				24						
	24					24					
	24						24				
	24							24			
	24								24		
	24									24	
		24	24								
		24		24							
		24			24						
		24				24					
		24					24				
		24						24			
		24							24		
		24								24	
			24	24							
			24		24						
			24			24					
			24				24				
			24					24			
			24						24		
			24							24	
				24	24						
				24		24					
				24			24				
				24				24			
				24					24		
				24						24	
					24	24					
					24		24				
					24			24			
					24				24		
					24					24	
						24	24				
						24		24			
						24			24		
						24				24	
							24	24			
							24		24		
							24			24	
								24	24		
								24		24	
									24	24	
											1080
Total/Rater	216	216	216	216	216	216	216	216	216	216	

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Table A2

Unbalanced Design, 45 Rater Pairs

				Rater						
1	2	3	4	5	6	7	8	9	10	Total
5	5									
15		15								
20			20							
5				5						
20					20					
5						5				
100							100			
15								15		
185									185	
	5	5								
	5		5							
	5			5						
	5				5					
	5					5				
	10						10			
	5							5		
	5								5	
		20	20							
		5		5						
		20			20					
		25				25				
		40					40			
		40						40		
		30							30	
			5	5						
			10		10					
			30			30				
			15				15			
			20					20		
			15						15	
				5	5					
				5		5				
				20			20			
				5				5		
				5					5	
					10	10				
					30		30			
					10			10		
					10				10	
						105	105			
						90		90		
						5			5	
							35	35		
							45		45	
								10	10	
										1080
Total/Rater 370	50	200	140	60	120	280	400	230	310	

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Table A3

Balanced Design, 10 Rater Pairs, 108 per Pair

	2	5	6	4	3	9	10	1	8	7	Total
	108	108									
		108	108								
			108	108							
				108	108						
					108	108					
						108	108				
							108	108			
								108	108		
									108	108	
	108									108	
											1080
Total/Rater	216	216	216	216	216	216	216	216	216	216	

Table A4

Unbalanced Design, 10 Rater Pairs

	2	5	6	4	3	9	10	1	8	7	Total
	20	20									
		40	40								
			80	80							
				60	60						
					140	140					
						90	90				
							220	220			
								150	150		
									250	250	
	30									30	
											1080
Total/Rater	50	60	120	140	200	230	310	370	400	280	

Table A5

Balanced Design, 20 Rater Pairs, 54 per Pair

				Rater							
	1	2	3	4	5	6	7	8	9	10	Total
	54	54									
		54	54								
			54	54							
				54	54						
					54	54					
						54	54				
							54	54			
								54	54		
									54	54	
	54									54	
	54		54								
		54		54							
			54		54						
				54		54					
					54		54				
						54		54			
							54		54		
								54		54	
	54								54		
		54								54	
											1080
Total/Rater	216	216	216	216	216	216	216	216	216	216	

Table A6

Unbalanced Design, 20 Rater Pairs

	Rater										
	1	2	3	4	5	6	7	8	9	10	Total
	35	35									
		5	5								
			50	50							
				15	15						
					5	5					
						40	40				
							185	185			
								35	35		
									35	35	
	95									95	95
	120		120								
		5		5							
			25		25						
				70		70					
					15		15				
						5		5			
							40		40		
								175		175	
	120								120		
		5								5	
											1080
Total/Rater	370	50	200	140	60	120	280	400	230	310	

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Appendix B

Parameter Estimates, Bias, Percent Bias, and MSE

Table B1

BIB Design, Normal d, N = 1,080, 45 Rater Pairs, 24 per Pair

Size	Parameter	Value	Estimate	Bias	%Bias	MSE
			Rater parameters			
216	d_1	1.0	0.954	−0.0462	−4.620	0.023
216	d_2	2.0	1.928	−0.0719	−3.595	0.061
216	d_3	2.0	1.928	−0.0722	−3.610	0.081
216	d_4	3.0	2.971	−0.0290	−0.967	0.252
216	d_5	3.0	2.908	−0.0916	−3.053	0.222
216	d_6	3.0	2.941	−0.0591	−1.970	0.194
216	d_7	3.0	2.969	−0.0310	−1.030	0.167
216	d_8	4.0	3.844	−0.1560	−3.900	0.370
216	d_9	4.0	3.828	−0.1719	−4.298	0.402
216	d_{10}	5.0	4.553	−0.4475	−8.950	0.575
	c_{11}	0.5	0.362	−0.1384	−27.680	0.143
	c_{12}	1.5	1.372	−0.1277	−8.513	0.166
	c_{13}	2.5	2.391	−0.1094	−4.376	0.193
	c_{14}	3.5	3.398	−0.1024	−2.926	0.239
	c_{15}	4.5	4.399	−0.1010	−2.244	0.326
	c_{21}	1.0	0.728	−0.2723	−27.230	0.217
	c_{22}	3.0	2.788	−0.2124	−7.080	0.332
	c_{23}	5.0	4.785	−0.2149	−4.298	0.429
	c_{24}	7.0	6.808	−0.1917	−2.739	0.671
	c_{25}	9.0	8.918	−0.0825	−0.917	0.982
	c_{31}	1.0	0.763	−0.2370	−23.700	0.268
	c_{32}	3.0	2.817	−0.1827	−6.090	0.373
	c_{33}	5.0	4.822	−0.1778	−3.556	0.591
	c_{34}	7.0	6.850	−0.1502	−2.146	0.854

Size	Parameter	Value	Estimate	Bias	%Bias	MSE
	c_{35}	9.0	8.894	−0.1065	−1.183	1.326
	c_{41}	1.5	1.149	−0.3506	−23.373	0.562
	c_{42}	4.5	4.284	−0.2160	−4.800	0.992
	c_{43}	7.5	7.426	−0.0742	−0.989	1.809
	c_{44}	10.5	10.505	0.0046	0.044	3.185
	c_{45}	13.5	13.777	0.2768	2.050	5.423
	c_{51}	1.5	1.033	−0.4671	−31.140	0.521
	c_{52}	4.5	4.143	−0.3569	−7.931	0.846
	c_{53}	7.5	7.255	−0.2449	−3.265	1.519
	c_{54}	10.5	10.293	−0.2073	−1.974	2.429
	c_{55}	13.5	13.420	−0.0797	−0.590	4.213
	c_{61}	1.5	1.103	−0.3970	−26.467	0.499
	c_{62}	4.5	4.304	−0.1962	−4.360	0.641
	c_{63}	7.5	7.330	−0.1703	−2.271	1.238
	c_{64}	10.5	10.417	−0.0831	−0.791	2.187
	c_{65}	13.5	13.539	0.0390	0.289	3.686
	c_{71}	1.5	1.112	−0.3878	−25.853	0.450
	c_{72}	4.5	4.308	−0.1923	−4.273	0.672
	c_{73}	7.5	7.446	−0.0538	−0.717	1.475
	c_{74}	10.5	10.526	0.0261	0.249	2.128
	c_{75}	13.5	13.695	0.1954	1.447	3.155
	c_{81}	2.0	1.285	−0.7152	−35.760	0.993
	c_{82}	6.0	5.469	−0.5309	−8.848	1.629
	c_{83}	10.0	9.658	−0.3423	−3.423	2.888
	c_{84}	14.0	13.643	−0.3568	−2.549	4.633
	c_{85}	18.0	17.778	−0.2225	−1.236	6.724
	c_{91}	2.0	1.307	−0.6927	−34.635	0.976
	c_{92}	6.0	5.486	−0.5143	−8.572	1.529
	c_{93}	10.0	9.556	−0.4442	−4.442	2.959

Size	Parameter	Value	Estimate	Bias	%Bias	MSE
	c_{94}	14.0	13.591	−0.4089	−2.921	4.748
	c_{95}	18.0	17.748	−0.2518	−1.399	7.103
	c_{101}	2.5	1.493	−1.0072	−40.288	1.747
	c_{102}	7.5	6.438	−1.0620	−14.160	2.583
	c_{103}	12.5	11.493	−1.0069	−8.055	4.414
	c_{104}	17.5	16.226	−1.2741	−7.281	6.645
	c_{105}	22.5	21.266	−1.2341	−5.485	9.175

			Latent class sizes			
	Class 1	0.080	0.101	0.0210	26.250	
	Class 2	0.170	0.160	−0.0100	−5.882	
	Class 3	0.250	0.239	−0.0110	−4.400	
	Class 4	0.250	0.244	−0.0060	−2.400	
	Class 5	0.170	0.157	−0.0130	−7.647	
	Class 6	0.080	0.099	0.0190	23.750	

Note. Size is the number of essays scored by each rater.

Table B2

Unbalanced Design, Normal d, N = 1,080, 45 Rater Pairs

Size	Parameter	Value	Estimate	Bias	%Bias	MSE
Rater parameters						
370	d_1	1.0	0.970	−0.0302	−3.020	0.013
50	d_2	2.0	1.771	−0.2292	−11.460	0.194
200	d_3	2.0	1.967	−0.0334	−1.670	0.081
140	d_4	3.0	2.903	−0.0974	−3.247	0.266
60	d_5	3.0	2.542	−0.4576	−15.253	0.449
120	d_6	3.0	2.847	−0.1534	−5.113	0.363
280	d_7	3.0	2.914	−0.0865	−2.883	0.134
400	d_8	4.0	4.208	0.2084	5.210	0.371
230	d_9	4.0	3.828	−0.1719	−4.298	0.391
310	d_{10}	5.0	4.137	−0.8635	−17.270	1.103
	c_{11}	0.5	0.385	−0.1153	−23.060	0.064
	c_{12}	1.5	1.407	−0.0928	−6.187	0.080
	c_{13}	2.5	2.424	−0.0760	−3.040	0.091
	c_{14}	3.5	3.448	−0.0525	−1.500	0.107
	c_{15}	4.5	4.484	−0.0157	−0.349	0.141
	c_{21}	1.0	0.510	−0.4904	−49.040	0.792
	c_{22}	3.0	2.564	−0.4358	−14.527	0.787
	c_{23}	5.0	4.469	−0.5313	−10.626	1.342
	c_{24}	7.0	6.431	−0.5690	−8.129	1.827
	c_{25}	9.0	8.467	−0.5328	−5.920	2.885
	c_{31}	1.0	0.748	−0.2516	−25.160	0.241
	c_{32}	3.0	2.844	−0.1557	−5.190	0.352
	c_{33}	5.0	4.933	−0.0667	−1.334	0.597
	c_{34}	7.0	6.985	−0.0150	−0.214	0.912
	c_{35}	9.0	9.105	0.1045	1.161	1.410
	c_{41}	1.5	1.084	−0.4162	−27.747	0.573

Parameter	Value	Estimate	Bias	%Bias	MSE
c_{42}	4.5	4.187	−0.3126	−6.947	0.849
c_{43}	7.5	7.277	−0.2234	−2.979	1.918
c_{44}	10.5	10.360	−0.1396	−1.330	3.110
c_{45}	13.5	13.341	−0.1587	−1.176	4.412
c_{51}	1.5	0.809	−0.6908	−46.053	1.133
c_{52}	4.5	3.587	−0.9129	−20.287	1.785
c_{53}	7.5	6.464	−1.0357	−13.809	2.839
c_{54}	10.5	9.250	−1.2501	−11.906	4.443
c_{55}	13.5	11.863	−1.6367	−12.124	7.426
c_{61}	1.5	0.945	−0.5554	−37.027	0.776
c_{62}	4.5	4.097	−0.4034	−8.964	1.380
c_{63}	7.5	7.162	−0.3383	−4.511	2.433
c_{64}	10.5	10.131	−0.3693	−3.517	4.131
c_{65}	13.5	13.304	−0.1959	−1.451	6.671
Parameter	Value	Estimate	Bias	%Bias	MSE
c_{71}	1.5	1.027	−0.4730	−31.533	0.469
c_{72}	4.5	4.184	−0.3164	−7.031	0.647
c_{73}	7.5	7.293	−0.2072	−2.763	0.878
c_{74}	10.5	10.402	−0.0981	−0.934	1.491
c_{75}	13.5	13.461	−0.0386	−0.286	2.237
c_{81}	2.0	1.407	−0.5927	−29.635	0.766
c_{82}	6.0	5.995	−0.0051	−0.085	0.970
c_{83}	10.0	10.572	0.5717	5.717	2.702
c_{84}	14.0	15.047	1.0466	7.476	5.501
c_{85}	18.0	19.582	1.5821	8.789	9.741
c_{91}	2.0	1.342	−0.6577	−32.885	0.888
c_{92}	6.0	5.515	−0.4850	−8.083	1.362
c_{93}	10.0	9.620	−0.3805	−3.805	2.637
c_{94}	14.0	13.782	−0.2181	−1.558	4.827
c_{95}	18.0	17.828	−0.1723	−0.957	7.126
c_{101}	2.5	1.240	−1.2604	−50.416	2.021

c_{102}	7.5	5.803	−1.6968	−22.624	3.913
c_{103}	12.5	10.344	−2.1562	−17.250	6.891
c_{104}	17.5	14.833	−2.6666	−15.238	11.576
c_{105}	22.5	19.315	−3.1847	−14.154	18.167
Latent class sizes					
Class 1	0.080	0.102	0.0220	27.500	
Class 2	0.170	0.162	−0.0080	−4.706	
Class 3	0.250	0.235	−0.0150	−6.000	
Class 4	0.250	0.234	−0.0160	−6.400	
Class 5	0.170	0.165	−0.0050	−2.941	
Class 6	0.080	0.101	0.0210	26.250	

Table B3

Balanced Design, Normal d, N = 1,080, 10 Rater Pairs, 108 per Pair

Size	Parameter	Value	Estimate	Bias	%Bias	MSE
			Rater parameters			
216	d_1	1.0	0.993	−0.0072	−0.720	0.019
216	d_2	2.0	1.932	−0.0685	−3.425	0.077
216	d_3	2.0	1.882	−0.1177	−5.885	0.058
216	d_4	3.0	2.786	−0.2140	−7.133	0.220
216	d_5	3.0	2.758	−0.2422	−8.073	0.215
216	d_6	3.0	2.991	−0.0090	−0.300	0.167
216	d_7	3.0	3.095	0.0954	3.180	0.193
216	d_8	4.0	3.195	−0.8052	−20.130	0.851
216	d_9	4.0	4.203	0.2034	5.085	0.249
216	d_{10}	5.0	4.210	−0.7904	−15.808	0.890
	c_{11}	0.5	0.405	−0.0952	−19.040	0.089
	c_{12}	1.5	1.453	−0.0466	−3.107	0.101
	c_{13}	2.5	2.460	−0.0405	−1.620	0.139
	c_{14}	3.5	3.488	−0.0125	−0.357	0.217
	c_{15}	4.5	4.528	0.0280	0.622	0.231
	c_{21}	1.0	0.783	−0.2170	−21.700	0.219
	c_{22}	3.0	2.845	−0.1554	−5.180	0.385
	c_{23}	5.0	4.858	−0.1419	−2.838	0.595
	c_{24}	7.0	6.937	−0.0633	−0.904	0.873
	c_{25}	9.0	8.960	−0.0405	−0.450	1.227
	c_{31}	1.0	0.692	−0.3085	−30.850	0.217
	c_{32}	3.0	2.723	−0.2774	−9.247	0.274
	c_{33}	5.0	4.750	−0.2504	−5.008	0.385
	c_{34}	7.0	6.749	−0.2508	−3.583	0.546

Size	Parameter	Value	Estimate	Bias	%Bias	MSE
	c_{35}	9.0	8.775	−0.2251	−2.501	0.808
	c_{41}	1.5	0.978	−0.5223	−34.820	0.656
	c_{42}	4.5	3.970	−0.5300	−11.778	0.934
	c_{43}	7.5	6.969	−0.5310	−7.080	1.639
	c_{44}	10.5	9.962	−0.5378	−5.122	2.377
	c_{45}	13.5	12.947	−0.5526	−4.093	3.487
	c_{51}	1.5	1.001	−0.4992	−33.280	0.602
	c_{52}	4.5	3.904	−0.5965	−13.256	1.048
	c_{53}	7.5	6.970	−0.5303	−7.071	1.405
	c_{54}	10.5	9.926	−0.5736	−5.463	2.239
	c_{55}	13.5	12.917	−0.5833	−4.321	3.421
	c_{61}	1.5	1.145	−0.3548	−23.653	0.522
	c_{62}	4.5	4.328	−0.1720	−3.822	0.670
	c_{63}	7.5	7.529	0.0289	0.385	1.184
	c_{64}	10.5	10.637	0.1365	1.300	2.009
	c_{65}	13.5	13.920	0.4197	3.109	3.626
	c_{71}	1.5	1.190	−0.3106	−20.707	0.484
	c_{72}	4.5	4.530	0.0302	0.671	0.752
	c_{73}	7.5	7.808	0.3075	4.100	1.419
	c_{74}	10.5	11.053	0.5528	5.265	2.566
	c_{75}	13.5	14.366	0.8656	6.412	4.403
	c_{81}	2.0	0.874	−1.1260	−56.300	1.627
	c_{82}	6.0	4.497	−1.5035	−25.058	3.112
	c_{83}	10.0	8.018	−1.9816	−19.816	5.520
	c_{84}	14.0	11.539	−2.4606	−17.576	8.459
	c_{85}	18.0	15.059	−2.9406	−16.337	12.715
	c_{91}	2.0	1.513	−0.4867	−24.335	0.790
	c_{92}	6.0	6.099	0.0987	1.645	0.788

Size	Parameter	Value	Estimate	Bias	%Bias	MSE
	c_{93}	10.0	10.515	0.5153	5.153	1.628
	c_{94}	14.0	14.915	0.9151	6.536	3.232
	c_{95}	18.0	19.590	1.5904	8.836	6.603
	c_{101}	2.5	1.451	−1.0486	−41.944	1.537
	c_{102}	7.5	5.992	−1.5076	−20.101	3.252
	c_{103}	12.5	10.590	−1.9099	−15.279	5.647
	c_{104}	17.5	15.019	−2.4808	−14.176	9.425
	c_{105}	22.5	19.753	−2.7471	−12.209	13.152

Latent class sizes

Class 1	0.080	0.103	0.0230	28.750	
Class 2	0.170	0.159	−0.0110	−6.471	
Class 3	0.250	0.234	−0.0160	−6.400	
Class 4	0.250	0.241	−0.0090	−3.600	
Class 5	0.170	0.159	−0.0110	−6.471	
Class 6	0.080	0.105	0.0250	31.250	

Table B4

Unbalanced Design, Normal d, N = 1,080, 10 Rater Pairs

	Parameter	Value	Estimate	Bias	%Bias	MSE
			Rater parameters			
370	d_1	1.0	0.986	−0.0139	−1.390	0.009
50	d_2	2.0	1.883	−0.1175	−8.780	0.194
200	d_3	2.0	1.960	−0.0404	−2.515	0.070
140	d_4	3.0	2.696	−0.3042	−13.607	0.275
60	d_5	3.0	2.435	−0.5646	−17.430	0.484
120	d_6	3.0	2.996	−0.0043	−13.790	0.201
280	d_7	3.0	3.410	0.4100	2.167	0.350
400	d_8	4.0	3.249	−0.7510	−19.070	0.693
230	d_9	4.0	4.259	0.2592	11.953	0.291
310	d_{10}	5.0	4.115	−0.8850	−33.388	1.034
	c_{11}	0.5	0.447	−0.0533	−14.660	0.060
	c_{12}	1.5	1.460	−0.0398	−1.313	0.056
	c_{13}	2.5	2.501	0.0010	1.552	0.069
	c_{14}	3.5	3.517	0.0167	2.417	0.094
	c_{15}	4.5	4.533	0.0331	2.660	0.113
	c_{21}	1.0	0.700	−0.3005	−41.090	0.674
	c_{22}	3.0	2.721	−0.2791	−14.237	0.866
	c_{23}	5.0	4.724	−0.2757	−8.862	1.393
	c_{24}	7.0	6.788	−0.2121	−6.306	2.238
	c_{25}	9.0	8.904	−0.0959	−4.751	3.150
	c_{31}	1.0	0.792	−0.2082	−23.450	0.327
	c_{32}	3.0	2.830	−0.1696	−1.973	0.383
	c_{33}	5.0	4.917	−0.0833	−2.956	0.573
	c_{34}	7.0	6.993	−0.0075	−4.414	0.922
	c_{35}	9.0	9.064	0.0639	5.504	1.326
	c_{41}	1.5	0.902	−0.5982	−51.147	0.800

Parameter	Value	Estimate	Bias	%Bias	MSE
c_{42}	4.5	3.828	−0.6719	−18.473	1.271
c_{43}	7.5	6.749	−0.7513	−14.465	1.885
c_{44}	10.5	9.646	−0.8541	−10.626	2.978
c_{45}	13.5	12.571	−0.9291	−8.887	4.293
c_{51}	1.5	0.788	−0.7119	−60.727	1.198
c_{52}	4.5	3.466	−1.0336	−24.851	2.022
c_{53}	7.5	6.089	−1.4109	−17.681	3.307
c_{54}	10.5	8.849	−1.6511	−15.172	4.928
c_{55}	13.5	11.557	−1.9428	−13.393	7.349
c_{61}	1.5	1.125	−0.3752	−50.807	0.608
c_{62}	4.5	4.256	−0.2439	−19.693	0.993
c_{63}	7.5	7.514	0.0135	15.033	1.683
c_{64}	10.5	10.716	0.2161	11.291	2.471
c_{65}	13.5	13.955	0.4548	9.986	4.016
c_{71}	1.5	1.468	−0.0318	−36.427	0.426
c_{72}	4.5	5.006	0.5064	3.853	1.039
c_{73}	7.5	8.464	0.9640	1.711	2.085
c_{74}	10.5	12.052	1.5516	4.038	4.337
c_{75}	13.5	15.723	2.2230	6.173	8.508
c_{81}	2.0	1.110	−0.8902	−58.085	1.063
c_{82}	6.0	4.664	−1.3365	−26.143	2.298
c_{83}	10.0	8.134	−1.8665	−18.622	4.402
c_{84}	14.0	11.621	−2.3786	−16.774	7.406
c_{85}	18.0	15.160	−2.8398	−15.198	10.747
c_{91}	2.0	1.606	−0.3945	−50.550	0.771
c_{92}	6.0	6.219	0.2185	18.187	1.068
c_{93}	10.0	10.683	0.6832	11.929	2.007
c_{94}	14.0	15.098	1.0981	9.118	3.965
c_{95}	18.0	19.658	1.6576	7.248	6.944

Parameter	Value	Estimate	Bias	%Bias	MSE
c_{101}	2.5	1.387	−1.1130	−65.588	1.681
c_{102}	7.5	5.943	−1.5566	−38.940	3.385
c_{103}	12.5	10.387	−2.1126	−33.582	6.402
c_{104}	17.5	14.805	−2.6952	−30.991	10.584
c_{105}	22.5	19.262	−3.2382	−29.630	15.763

Latent class sizes

Class 1	0.080	0.100	0.0200	25.000	
Class 2	0.170	0.161	−0.0090	−5.294	
Class 3	0.250	0.238	−0.0120	−4.800	
Class 4	0.250	0.239	−0.0110	−4.400	
Class 5	0.170	0.159	−0.0110	−6.471	
Class 6	0.080	0.102	0.0220	27.500	

Table B5

Balanced Design, Normal d, N = 1,080, 20 Rater Pairs, 54 per Pair

Size	Parameter	Value	Estimate	Bias	%Bias	MSE
			Rater parameters			
216	d_1	1.0	0.969	−0.0312	−3.120	0.019
216	d_2	2.0	1.995	−0.0054	−0.270	0.093
216	d_3	2.0	1.943	−0.0572	−2.860	0.100
216	d_4	3.0	2.728	−0.2720	−9.067	0.254
216	d_5	3.0	2.823	−0.1768	−5.893	0.289
216	d_6	3.0	2.954	−0.0459	−1.530	0.211
216	d_7	3.0	2.914	−0.0858	−2.860	0.160
216	d_8	4.0	4.043	0.0427	1.068	0.313
216	d_9	4.0	3.996	−0.0044	−0.110	0.352
216	d_{10}	5.0	4.511	−0.4889	−9.778	0.625
	c_{11}	0.5	0.383	−0.1168	−23.360	0.119
	c_{12}	1.5	1.427	−0.0728	−4.853	0.101
	c_{13}	2.5	2.459	−0.0411	−1.644	0.121
	c_{14}	3.5	3.466	−0.0345	−0.986	0.163
	c_{15}	4.5	4.463	−0.0375	−0.833	0.215
	c_{21}	1.0	0.776	−0.2245	−22.450	0.278
	c_{22}	3.0	2.870	−0.1299	−4.330	0.432
	c_{23}	5.0	4.966	−0.0344	−0.688	0.589
	c_{24}	7.0	7.062	0.0624	0.891	1.011
	c_{25}	9.0	9.211	0.2110	2.344	1.621
	c_{31}	1.0	0.762	−0.2378	−23.780	0.267
	c_{32}	3.0	2.794	−0.2060	−6.867	0.452
	c_{33}	5.0	4.849	−0.1506	−3.012	0.649
	c_{34}	7.0	6.912	−0.0880	−1.257	1.100

Size	Parameter	Value	Estimate	Bias	%Bias	MSE
	c_{35}	9.0	8.910	−0.0896	−0.996	1.673
	c_{41}	1.5	0.939	−0.5607	−37.380	0.645
	c_{42}	4.5	3.895	−0.6048	−13.440	0.975
	c_{43}	7.5	6.818	−0.6816	−9.088	1.695
	c_{44}	10.5	9.720	−0.7801	−7.430	2.697
	c_{45}	13.5	12.652	−0.8477	−6.279	4.485
	c_{51}	1.5	1.035	−0.4647	−30.980	0.567
	c_{52}	4.5	4.057	−0.4434	−9.853	0.965
	c_{53}	7.5	7.048	−0.4517	−6.023	1.964
	c_{54}	10.5	10.030	−0.4704	−4.480	3.356
	c_{55}	13.5	13.045	−0.4554	−3.373	4.957
	c_{61}	1.5	1.124	−0.3759	−25.060	0.400
	c_{62}	4.5	4.208	−0.2920	−6.489	0.687
	c_{63}	7.5	7.373	−0.1266	−1.688	1.599
	c_{64}	10.5	10.433	−0.0671	−0.639	2.621
	c_{65}	13.5	13.615	0.1148	0.850	4.137
	c_{71}	1.5	1.085	−0.4153	−27.687	0.506
	c_{72}	4.5	4.155	−0.3451	−7.669	0.630
	c_{73}	7.5	7.303	−0.1966	−2.621	1.072
	c_{74}	10.5	10.350	−0.1497	−1.426	1.731
	c_{75}	13.5	13.537	0.0370	0.274	2.739
	c_{81}	2.0	1.418	−0.5818	−29.090	0.683
	c_{82}	6.0	5.728	−0.2718	−4.530	0.984
	c_{83}	10.0	10.103	0.1030	1.030	1.916
	c_{84}	14.0	14.423	0.4230	3.021	4.064
	c_{85}	18.0	18.816	0.8163	4.535	7.243
	c_{91}	2.0	1.370	−0.6303	−31.513	1.030
	c_{92}	6.0	5.707	−0.2928	−4.880	1.579

Size	Parameter	Value	Estimate	Bias	%Bias	MSE
	c_{93}	10.0	10.050	0.0499	0.499	2.716
	c_{94}	14.0	14.180	0.1798	1.284	4.435
	c_{95}	18.0	18.556	0.5560	3.089	6.848
	c_{101}	2.5	1.395	−1.1047	−44.188	1.957
	c_{102}	7.5	6.401	−1.0988	−14.650	2.676
	c_{103}	12.5	11.337	−1.1629	−9.303	4.070
	c_{104}	17.5	16.158	−1.3417	−7.667	6.416
	c_{105}	22.5	20.990	−1.5104	−6.713	9.830

Latent class sizes

		Value	Estimate	Bias	%Bias	
Class 1		0.080	0.101	0.0210	26.250	
Class 2		0.170	0.161	−0.0090	−5.294	
Class 3		0.250	0.237	−0.0130	−5.200	
Class 4		0.250	0.242	−0.0080	−3.200	
Class 5		0.170	0.159	−0.0110	−6.471	
Class 6		0.080	0.100	0.0200	25.000	

Table B6

Unbalanced Design, Normal d, N = 1,080, 20 Rater Pairs

Size	Parameter	Value	Estimate	Bias	%Bias	MSE
			Rater parameters			
370	d_1	1.0	0.973	−0.0273	−2.730	0.011
50	d_2	2.0	1.582	−0.4178	−20.890	0.369
200	d_3	2.0	1.875	−0.1250	−6.250	0.145
140	d_4	3.0	2.935	−0.0651	−2.170	0.252
60	d_5	3.0	2.546	−0.4541	−15.137	0.464
120	d_6	3.0	2.816	−0.1844	−6.147	0.282
280	d_7	3.0	2.869	−0.1312	−4.373	0.147
400	d_8	4.0	4.282	0.2822	7.055	0.344
230	d_9	4.0	3.754	−0.2459	−6.148	0.423
310	d_{10}	5.0	4.640	−0.3596	−7.192	0.446
	c_{11}	0.5	0.423	−0.0774	−15.480	0.072
	c_{12}	1.5	1.441	−0.0586	−3.907	0.072
	c_{13}	2.5	2.446	−0.0545	−2.180	0.086
	c_{14}	3.5	3.455	−0.0449	−1.283	0.120
	c_{15}	4.5	4.450	−0.0498	−1.107	0.146
	c_{21}	1.0	0.345	−0.6551	−65.510	1.133
	c_{22}	3.0	2.152	−0.8478	−28.260	1.611
	c_{23}	5.0	3.994	−1.0056	−20.112	2.506
	c_{24}	7.0	5.777	−1.2232	−17.474	3.937
	c_{25}	9.0	7.533	−1.4671	−16.301	5.552
	c_{31}	1.0	0.747	−0.2532	−25.320	0.348
	c_{32}	3.0	2.705	−0.2948	−9.827	0.583
	c_{33}	5.0	4.715	−0.2849	−5.698	0.999
	c_{34}	7.0	6.700	−0.3001	−4.287	1.624

Size	Parameter	Value	Estimate	Bias	%Bias	MSE
	c_{35}	9.0	8.726	−0.2741	−3.046	2.412
	c_{41}	1.5	1.036	−0.4639	−30.927	0.675
	c_{42}	4.5	4.083	−0.4166	−9.258	1.072
	c_{43}	7.5	7.300	−0.1997	−2.663	1.911
	c_{44}	10.5	10.501	0.0011	0.010	2.994
	c_{45}	13.5	13.634	0.1340	0.993	5.074
	c_{51}	1.5	0.821	−0.6795	−45.300	1.236
	c_{52}	4.5	3.601	−0.8994	−19.987	1.987
	c_{53}	7.5	6.419	−1.0809	−14.412	3.037
	c_{54}	10.5	9.148	−1.3523	−12.879	4.943
	c_{55}	13.5	12.045	−1.4547	−10.776	7.354
	c_{61}	1.5	0.985	−0.5154	−34.360	0.762
	c_{62}	4.5	3.987	−0.5132	−11.404	1.243
	c_{63}	7.5	7.065	−0.4349	−5.799	1.964
	c_{64}	10.5	10.138	−0.3618	−3.446	3.316
	c_{65}	13.5	13.014	−0.4856	−3.597	4.985
	c_{71}	1.5	1.073	−0.4275	−28.500	0.411
	c_{72}	4.5	4.126	−0.3741	−8.313	0.581
	c_{73}	7.5	7.201	−0.2994	−3.992	0.882
	c_{74}	10.5	10.209	−0.2911	−2.772	1.513
	c_{75}	13.5	13.254	−0.2457	−1.820	2.390
	c_{81}	2.0	1.624	−0.3764	−18.820	0.513
	c_{82}	6.0	6.216	0.2163	3.605	0.960
	c_{83}	10.0	10.714	0.7136	7.136	2.414
	c_{84}	14.0	15.253	1.2526	8.947	5.108
	c_{85}	18.0	19.874	1.8735	10.408	9.231
	c_{91}	2.0	1.372	−0.6277	−31.384	0.800
	c_{92}	6.0	5.509	−0.4910	−8.184	1.497

Size	Parameter	Value	Estimate	Bias	%Bias	MSE
	c_{93}	10.0	9.479	−0.5215	−5.215	2.769
	c_{94}	14.0	13.461	−0.5391	−3.851	4.650
	c_{95}	18.0	17.447	−0.5534	−3.074	7.740
	c_{101}	2.5	1.607	−0.8930	−35.722	1.363
	c_{102}	7.5	6.727	−0.7730	−10.307	1.834
	c_{103}	12.5	11.564	−0.9365	−7.492	3.177
	c_{104}	17.5	16.724	−0.7757	−4.432	5.381
	c_{105}	22.5	21.717	−0.7833	−3.481	7.844

Latent class sizes

Class 1	0.080	0.098	0.0180	22.500	
Class 2	0.170	0.160	−0.0100	−5.882	
Class 3	0.250	0.241	−0.0090	−3.600	
Class 4	0.250	0.234	−0.0160	−6.400	
Class 5	0.170	0.167	−0.0030	−1.765	
Class 6	0.080	0.100	0.0200	25.000	

Appendix C

Evaluation of the Estimated Standard Errors for *d* and the Latent Class Sizes

Table C1

BIB Design, Normal d, N = 1,080, 45 Rater Pairs, 24 per Pair

Size	Parameter	SD	Mean SE	Bias	%Bias
216	d_1	0.145	0.136	−0.009	−6.071
216	d_2	0.238	0.258	0.020	8.476
216	d_3	0.276	0.256	−0.020	−7.340
216	d_4	0.504	0.469	−0.035	−6.933
216	d_5	0.464	0.453	−0.011	−2.453
216	d_6	0.439	0.460	0.021	4.748
216	d_7	0.410	0.464	0.053	13.018
216	d_8	0.591	0.684	0.094	15.834
216	d_9	0.613	0.678	0.065	10.580
216	d_{10}	0.615	0.856	0.241	39.150
	Class Size 1	0.020	0.020	0.000	1.574
	Class Size 2	0.026	0.026	0.000	0.990
	Class Size 3	0.027	0.030	0.003	9.971
	Class Size 4	0.029	0.030	0.001	2.634
	Class Size 5	0.024	0.026	0.002	6.952
	Class Size 6	0.017	0.020	0.003	20.627

Table C2

Unbalanced Design, Normal d, N = 1,080, 45 Rater Pairs, 24 per Pair

Size	Parameter	SD	Mean SE	Bias	%Bias
370	d_1	0.110	0.101	−0.009	−8.514
50	d_2	0.377	0.455	0.078	20.562
200	d_3	0.284	0.258	−0.026	−9.251
140	d_4	0.509	0.544	0.035	6.834
60	d_5	0.492	0.619	0.127	25.864
120	d_6	0.585	0.558	−0.027	−4.681
280	d_7	0.358	0.379	0.021	5.951
400	d_8	0.575	0.694	0.119	20.699
230	d_9	0.604	0.671	0.067	11.060
310	d_{10}	0.601	0.816	0.215	35.730
	Class Size 1	0.019	0.021	0.002	12.903
	Class Size 2	0.027	0.028	0.001	2.941
	Class Size 3	0.030	0.032	0.002	8.108
	Class Size 4	0.030	0.031	0.001	1.974
	Class Size 5	0.025	0.027	0.002	7.143
	Class Size 6	0.019	0.020	0.001	5.263

Table C3
BIB Design, Normal d, N = 1,080, 10 Rater Pairs, 108 per Pair

Size	Parameter	SD	Mean SE	Bias	%Bias
216	d_1	0.137	0.134	−0.003	−2.047
216	d_2	0.270	0.301	0.031	11.481
216	d_3	0.211	0.250	0.039	18.483
216	d_4	0.420	0.569	0.149	35.573
216	d_5	0.398	0.569	0.171	42.965
216	d_6	0.411	0.631	0.220	53.678
216	d_7	0.431	0.681	0.250	58.035
216	d_8	0.453	0.747	0.294	64.863
216	d_9	0.458	0.855	0.396	86.491
216	d_{10}	0.518	0.871	0.353	68.213
	Class Size 1	0.019	0.021	0.002	9.948
	Class Size 2	0.030	0.027	−0.003	−11.184
	Class Size 3	0.031	0.031	0.000	0.977
	Class Size 4	0.033	0.031	−0.002	−6.344
	Class Size 5	0.028	0.028	0.001	1.818
	Class Size 6	0.017	0.022	0.005	27.168

Table C4

Unbalanced Design, Normal d, N = 1,080, 10 Rater Pairs

Size	Parameter	SD	Mean SE	Bias	%Bias	
370	d_1	0.097	0.106	0.009	9.731	
50	d_2	0.427	0.489	0.062	14.627	
200	d_3	0.262	0.268	0.006	2.290	
140	d_4	0.429	0.605	0.176	40.894	
60	d_5	0.409	0.654	0.245	59.980	
120	d_6	0.450	0.731	0.281	62.300	
280	d_7	0.429	0.763	0.334	77.951	
400	d_8	0.361	0.695	0.334	92.463	
230	d_9	0.476	0.902	0.426	89.613	
310	d_{10}	0.504	0.859	0.355	70.484	
	Class Size 1	0.018	0.021	0.003	19.318	
	Class Size 2	0.029	0.028	−0.001	−2.778	
	Class Size 3	0.033	0.031	−0.002	−5.775	
	Class Size 4	0.033	0.031	−0.002	−6.907	
	Class Size 5	0.027	0.028	0.001	2.190	
	Class Size 6	0.017	0.020	0.003	14.943	

Table C5

BIB Design, Normal d, N = 1,080, 20 Rater Pairs, 54 per Pair

Size	Parameter	SD	Mean SE	Bias	%Bias	
216	d_1	0.133	0.140	0.007	5.042	
216	d_2	0.307	0.311	0.004	1.270	
216	d_3	0.313	0.327	0.014	4.473	
216	d_4	0.426	0.481	0.055	12.913	
216	d_5	0.510	0.486	−0.024	−4.689	
216	d_6	0.459	0.462	0.003	0.570	
216	d_7	0.392	0.423	0.031	7.779	
216	d_8	0.561	0.658	0.097	17.278	
216	d_9	0.596	0.700	0.104	17.390	
216	d_{10}	0.624	0.852	0.228	36.447	
	Class Size 1	0.017	0.020	0.003	14.416	
	Class Size 2	0.028	0.026	−0.002	−6.609	
	Class Size 3	0.031	0.030	−0.001	−3.007	
	Class Size 4	0.028	0.030	0.002	7.720	
	Class Size 5	0.024	0.025	0.001	3.263	
	Class Size 6	0.018	0.019	0.001	4.110	

Table C6

Unbalanced Design, Normal d, N = 1,080, 20 Rater Pairs

Size	Parameter	SD	Mean SE	Bias	%Bias
370	d_1	0.103	0.111	0.008	7.725
50	d_2	0.444	0.564	0.120	27.122
200	d_3	0.361	0.396	0.035	9.547
140	d_4	0.500	0.694	0.194	38.828
60	d_5	0.510	0.683	0.173	33.843
120	d_6	0.500	0.582	0.082	16.286
280	d_7	0.362	0.369	0.007	2.032
400	d_8	0.516	0.674	0.158	30.576
230	d_9	0.605	0.681	0.076	12.489
310	d_{10}	0.566	0.818	0.252	44.623
	Class Size 1	0.017	0.018	0.001	3.926
	Class Size 2	0.027	0.025	−0.002	−6.472
	Class Size 3	0.029	0.029	0.000	0.694
	Class Size 4	0.035	0.029	−0.006	−16.378
	Class Size 5	0.024	0.025	0.001	2.375
	Class Size 6	0.017	0.018	0.001	2.975

Appendix D

Evaluation of Parameter Estimates: HRM-SDT Model

Table D1

Two CR Items, Three Scores per CR Item, Fully Crossed, N = 3,000

Parameter	Value	Estimate	Bias	%Bias	MSE
Signal detection parameters					
d_1	2	2.003	0.003	0.150	0.004
d_2	3	3.037	0.037	1.233	0.011
d_3	4	4.015	0.015	0.375	0.028
d_4	2	1.998	−0.002	−0.100	0.003
d_5	3	3.002	0.002	0.067	0.010
d_6	4	3.983	−0.017	−0.425	0.023
c_{11}	1	1.022	0.022	2.200	0.019
c_{12}	3	3.016	0.016	0.533	0.020
c_{13}	5	5.013	0.013	0.260	0.030
c_{14}	7	7.009	0.009	0.129	0.043
c_{15}	9	9.008	0.008	0.089	0.063
c_{21}	1.5	1.562	0.062	4.133	0.046
c_{22}	4.5	4.573	0.073	1.622	0.057
c_{23}	7.5	7.596	0.096	1.280	0.085
c_{24}	10.5	10.630	0.127	1.210	0.140
c_{25}	13.5	13.650	0.154	1.141	0.200
c_{31}	2	2.016	0.016	0.800	0.076
c_{32}	6	6.048	0.048	0.800	0.114
c_{33}	10	10.036	0.036	0.360	0.193
c_{34}	14	14.052	0.052	0.371	0.364
c_{35}	18	18.056	0.056	0.311	0.518
c_{41}	1	0.978	−0.022	−2.200	0.017

Parameter	Value	Estimate	Bias	%Bias	MSE
c_{42}	3	2.975	−0.025	−0.833	0.020
c_{43}	5	4.984	−0.016	−0.320	0.025
c_{44}	7	6.990	−0.010	−0.143	0.033
c_{45}	9	9.004	0.004	0.044	0.040
c_{51}	1.5	1.477	−0.023	−1.533	0.037
c_{52}	4.5	4.483	−0.017	−0.378	0.044
c_{53}	7.5	7.482	−0.018	−0.240	0.067
c_{54}	10.5	10.52	0.017	0.162	0.109
c_{55}	13.5	13.53	0.034	0.252	0.162
c_{61}	2	1.921	−0.079	−3.950	0.101
c_{62}	6	5.936	−0.064	−1.067	0.107
c_{63}	10	9.940	−0.060	−0.600	0.161
c_{64}	14	13.954	−0.046	−0.329	0.227
c_{65}	18	17.974	−0.026	−0.144	0.370
Parameter	Value	Estimate	Bias	%Bias	MSE

CR Item parameters

Parameter	Value	Estimate	Bias	%Bias	MSE
a_1	1	1.227	0.227	22.700	0.105
a_2	1.5	1.265	−0.235	−15.667	0.091
b_{11}	−2	−2.344	−0.344	−17.200	0.290
b_{12}	−1	−1.177	−0.177	−17.700	0.071
b_{13}	0	−0.045	−0.045	—	0.011
b_{14}	0.5	0.604	0.104	20.800	0.029
b_{15}	1	1.232	0.232	23.200	0.133
b_{21}	−3	−2.599	−0.401	−13.367	0.283
b_{22}	−1.5	−1.326	−0.174	−11.600	0.059
b_{23}	0	0.006	0.006	—	0.007
b_{24}	1.5	1.315	−0.185	−12.333	0.067
b_{25}	3	2.616	−0.384	−12.800	0.243

Note. %Bias is not defined when the population parameter is zero.

Table D2

Three CR Items, Three Scores per CR Item, Fully Crossed, N = 3,000

Parameter	Value	Estimate	Bias	%Bias	MSE
		Rater parameters			
d_1	2	2.008	0.008	0.400	0.007
d_2	3	3.044	0.044	1.467	0.016
d_3	4	4.02	0.020	0.500	0.029
d_4	2	2.001	0.001	0.050	0.004
d_5	3	3.006	0.006	0.200	0.011
d_6	4	3.987	−0.013	−0.325	0.023
d_7	2	2.001	0.001	0.050	0.003
d_8	3	3.012	0.012	0.400	0.006
d_9	4	4.007	0.007	0.175	0.023
c_{11}	1	1.031	0.031	3.100	0.123
c_{12}	3	3.024	0.024	0.800	0.142
c_{13}	5	5.022	0.022	0.440	0.149
c_{14}	7	7.018	0.018	0.257	0.163
c_{15}	9	9.016	0.016	0.178	0.175
c_{21}	1.5	1.575	0.075	5.000	0.269
c_{22}	4.5	4.584	0.084	1.867	0.299
c_{23}	7.5	7.607	0.107	1.427	0.333
c_{24}	10.5	10.639	0.139	1.324	0.375
c_{25}	13.5	13.662	0.162	1.200	0.427
c_{31}	2	2.025	0.024	1.199	0.391
c_{32}	6	6.050	0.050	0.833	0.423
c_{33}	10	10.037	0.037	0.370	0.444
c_{34}	14	14.052	0.052	0.371	0.570
c_{35}	18	18.049	0.049	0.272	0.695

Parameter	Value	Estimate	Bias	%Bias	MSE
c_{41}	1	0.967	−0.033	−3.300	0.034
c_{42}	3	2.963	−0.037	−1.233	0.041
c_{43}	5	4.971	−0.029	−0.580	0.048
c_{44}	7	6.978	−0.022	−0.314	0.057
c_{45}	9	8.992	−0.008	−0.089	0.061
c_{51}	1.5	1.459	−0.041	−2.733	0.069
c_{52}	4.5	4.463	−0.037	−0.822	0.087
c_{53}	7.5	7.460	−0.040	−0.533	0.111
c_{54}	10.5	10.493	−0.007	−0.067	0.151
c_{55}	13.5	13.510	0.010	0.074	0.204
c_{61}	2	1.896	−0.104	−5.200	0.161
c_{62}	6	5.912	−0.088	−1.467	0.189
c_{63}	10	9.912	−0.088	−0.880	0.258
c_{64}	14	13.922	−0.078	−0.557	0.327
c_{65}	18	17.941	−0.059	−0.328	0.458
c_{71}	1	1.009	0.009	0.900	0.015
Parameter	Value	Estimate	Bias	%Bias	MSE
c_{72}	3	2.995	−0.005	−0.167	0.016
c_{73}	5	5.002	0.002	0.040	0.023
c_{74}	7	6.997	−0.003	−0.430	0.032
c_{75}	9	9.020	0.020	0.222	0.045
c_{81}	1.5	1.503	0.003	0.200	0.026
c_{82}	4.5	4.527	0.027	0.600	0.035
c_{83}	7.5	7.527	0.027	0.360	0.052
c_{84}	10.5	10.550	0.050	0.476	0.082
c_{85}	13.5	13.580	0.080	0.593	0.127
c_{91}	2	2.014	0.014	0.700	0.067
c_{92}	6	6.018	0.018	0.300	0.118
c_{93}	10	10.014	0.014	0.140	0.191
c_{94}	14	14.042	0.042	0.300	0.278

Parameter	Value	Estimate	Bias	%Bias	MSE
c_{95}	18	18.047	0.047	0.261	0.454

Parameter	Value	Estimate	Bias	%Bias	MSE
		CR Item parameters			
a_1	1	1.016	0.016	1.600	0.014
a_2	1.5	1.572	0.072	4.800	0.836
a_3	0.5	0.503	0.003	0.600	0.001
b_{11}	−2	−2.058	−0.058	−2.900	0.291
b_{12}	−1	−1.022	−0.022	−2.200	0.053
b_{13}	0	−0.016	−0.016	—	0.022
b_{14}	0.5	0.510	0.010	2.080	0.022
b_{15}	1	1.054	0.054	5.400	0.326
b_{21}	−3	−3.059	−0.059	−1.967	2.080
b_{22}	−1.5	−1.525	−0.025	−1.667	0.290
b_{23}	0	0.003	0.003	—	0.075
b_{24}	1.5	1.550	0.050	3.333	0.340
b_{25}	3	3.135	0.135	4.500	2.214
b_{31}	−1	−0.999	0.001	0.100	0.016
b_{32}	−0.5	−0.504	−0.004	−0.800	0.008
b_{33}	0	0.001	0.001	—	0.006
b_{34}	0.5	0.502	0.002	0.400	0.007
b_{35}	1	0.996	−0.004	−0.400	0.017

Note. %Bias is not defined when the population parameter is zero.