Appendix

Derivation of Multinomial Logit Model from Extreme Value SDT Choice Model

As shown in the text, the basic (without signal) SDT choice model with an extreme value Type I distribution for the CDF and PDF in Equation 5 is,

$$p\left(Y_{ij} = m \middle| \boldsymbol{b}_{jm}\right) = \int_{-\infty}^{\infty} \left(\prod_{\substack{k=1 \\ k \neq m}}^{M} e^{-e^{-(b_{ijm} - b_{ijk} + \varepsilon_{ijm})}} \right) e^{-\varepsilon_{ijm}} e^{-e^{-\varepsilon_{ijm}}} \, d\varepsilon_{ijm}.$$

The form of the model leads to some algebraic simplifications that yield a simple integral. Note that the last term, $e^{-e^{-\varepsilon_{ijm}}}$, can be re-written as $e^{-e^{-(b_{ijm} - b_{ijk} + \varepsilon_{ijm})}}$ for $k = m$, and so this term can be brought into the product term, which eliminates the $k \neq m$ restriction,

$$p\left(Y_{ij} = m \middle| \boldsymbol{b}_{jm}\right) = \int_{-\infty}^{\infty} \left(\prod_{k=1}^{M} e^{-e^{-(b_{ijm} - b_{ijk} + \varepsilon_{ijm})}} \right) e^{-\varepsilon_{ijm}} \, d\varepsilon_{ijm}.$$

$$= \int_{-\infty}^{\infty} e^{-\sum_{k=1}^{M} e^{-(b_{ijm} - b_{ijk} + \varepsilon_{ijm})}} \, e^{-\epsilon_{ijm}} \, d\varepsilon_{ijm}.$$

$$= \int_{-\infty}^{\infty} e^{-\sum_{k=1}^{M} e^{-\varepsilon_{ijm}} e^{-(b_{ijm} - b_{ijk})}} \, e^{-\varepsilon_{ijm}} \, d\varepsilon_{ijm}.$$

The term $e^{-e_{ijm}}$ in the exponent of the first term is a constant over the summation of k, and so it can be taken outside of the summation,

$$p(Y_{ij} = m | \boldsymbol{b}_{jm}) = \int_{-\infty}^{\infty} e^{-e^{-\varepsilon_{ijm}} \sum_{k=1}^{M} e^{-(b_{ijm} - b_{ijk})}} \, e^{-\varepsilon_{ijm}} d\varepsilon_{ijm}.$$

Let $t = e^{-\varepsilon_{ijm}}$. This changes the limits of the integral, in that, when $\varepsilon_{ijm} = -\infty$, $t = \infty$ and when $\varepsilon_{ijm} = \infty$, $t = 0$. Also note that $dt/d\varepsilon_{ijm} = -e^{-\varepsilon_{ijm}}$ and so $dt = -e^{-\varepsilon_{ijm}} d\varepsilon_{ijm}$. Substituting in the above gives,

$$p(Y_{ij} = m | \boldsymbol{b}_{jm}) = \int_{\infty}^{0} e^{-t \sum_{k=1}^{M} e^{-(b_{ijm} - b_{ijk})}} \, (-dt)$$

$$= \int_{0}^{\infty} e^{-t \sum_{k=1}^{M} e^{-(b_{ijm} - b_{ijk})}} \, dt.$$

The above is a standard form, e^{kt}, with known integral, which in this case is,

$$\int_0^{\infty} e^{-t\sum_{k=1}^{M} e^{-(b_{ijm}-\eta_{ijk})}}\, dt = \left.\frac{e^{-t\sum_{k=1}^{M} e^{-(b_{ijm}-b_{ijk})}}}{-\sum_{k=1}^{M} e^{-(b_{ijm}-b_{ijk})}}\right]_0^{\infty}$$

$$= 0 - \frac{1}{-\sum_{k=1}^{M} e^{-(b_{ijm}-b_{ijk})}}$$

$$= \frac{1}{\sum_{k=1}^{M} e^{-(b_{ijm}-b_{ijk})}}$$

$$= \frac{e^{b_{ijm}}}{\sum_{k=1}^{M} e^{b_{ijk}}},$$

which is the multinomial logit version of the basic SDT model given in the body of the article as Equation 7.

Model for Two Alternatives

The true-false signal detection model presented earlier (DeCarlo, 2020) is a special case of the SDT choice model presented here. For the choice model, when $M = 2$, the conditional probability of choosing the first alternative, from Equation 10, is

$$p(Y_{ij} = 1|b_{j1}, d_j, X_{j1}, \lambda_i) = \lambda_i \frac{e^{b_{j1}+d_jX_{j1}}}{e^{b_{j1}+d_jX_{j1}}+e^{d_jX_{j2}}} + (1-\lambda_i)\frac{e^{b_{j1}}}{1+e^{b_{j1}}} \qquad \text{A1}$$

with bias for the last alternative as zero, $b_{j2} = 0$. The true-false SDT model introduced in DeCarlo (2020; Equation 4), is

$$p\left(Y_{ij} = 1\middle|b_{j1}, d_j, Z_j, \lambda_i\right) = \lambda_i \frac{e^{-b_{j1}+d_jZ_j}}{1+e^{-b_{j1}+d_jZ_j}} + (1-\lambda_i)\frac{e^{-b_{j1}}}{1+e^{-b_{j1}}}$$

The subscript '$j1$' is used in line with the current notation but is not needed when there are only two alternatives. Changing the signal coding from $Z_j = (-1, 1)$ to dummy coding, X_{jm}, the above can be re-written as,

$$p(Y_{ij} = 1|b_{j1}, d_j, X_{j1}, \lambda_i) = \lambda_i \frac{e^{-b_{j1}+d_jX_{j1}}}{e^{-b_{j1}+d_jX_{j1}}+e^{d_jX_{j2}}} + (1-\lambda_i)\frac{e^{-b_{j1}}}{1+e^{-b_{j1}}},$$

where $X_{j2} = (1 - X_{j1})$. The above is identical to Equation A1 with the sign of the bias parameter b reversed. The sign change occurs because of the arbitrary direction of the coding of the bias – in DeCarlo (2020) a positive value of b indicated a bias towards a response of 'false', whereas here a positive value of b indicates a bias towards a response of 'true' (with 1=true and 2=false); the latter approach is more useful for $M > 2$ in that a positive value then always indicates a bias towards that particular alternative. Thus, the true-false SDT model introduced earlier is simply a special case of the current model with $M = 2$.

A Latent Gold Program to fit the Mixture SDT Model

```
//LG5.1//
version = 5.1
infile 'C:\Users\decar\Desktop\Choice\SAT12\SAT12_resp.sav'

model
options
   maxthreads=all;
   algorithm
      tolerance=1e-008 emtolerance=0.01 emiterations=450 nriterations=150 ;
   startvalues
      seed=0 sets=16 tolerance=1e-005 iterations=50;
   bayes
      categorical=1 variances=1 latent=1 poisson=1;
   montecarlo
      seed=0 sets=0 replicates=500 tolerance=1e-008;
   quadrature  nodes=11;
   missing excludeall;
   output
      parameters=last standarderrors profile probmeans=posterior bvr standarderrors
estimatedvalues=regression predictionstatistics setprofile setprobmeans;
//remove slashes on next line if you want an output file//
//outfile 'SAT12_out.sav' classification;//
choice = 1;
variables
  caseid case;
  groupid person;
  choicesetid item;
  dependent ch choice;
  independent itemno nominal;
  attribute _Constants_,signal;
```

```
latent
   del ordinal 2 score=(0 1), theta group continuous;
equations
 (1)theta;
  del <- 1 + (+)theta;
  ch <- _Constants_ |itemno + (+)signal del|itemno;
end model
```

Table A1

Parameter Estimates for the Nominal Response Model, SAT12 data

Item	b_{j1}		b_{j2}		b_{j3}		b_{j4}		b_{j5}	
1	*1.15	(0.26)	0.98	(0.26)	1.21	(0.25)	1.07	(0.25)	–4.41	(0.98)
2	0.83	(0.15)	–2.24	(0.43)	–0.59	(0.23)	*1.86	(0.15)	0.14	(0.18)
3	–0.04	(0.09)	0.07	(0.09)	0.37	(0.08)	–0.64	(0.12)	*0.25	(0.09)
4	–0.09	(0.09)	*0.69	(0.07)	–0.20	(0.09)	–0.06	(0.09)	–0.34	(0.10)
5	–0.42	(0.16)	0.24	(0.12)	*1.70	(0.09)	–0.36	(0.15)	–1.16	(0.22)
6	*–0.11	(0.13)	1.56	(0.08)	–0.13	(0.12)	–1.06	(0.17)	–0.27	(0.13)
7	–1.53	(0.50)	*3.22	(0.25)	–2.24	(0.71)	1.72	(0.25)	–1.17	(0.46)
8	*–0.07	(0.09)	0.11	(0.08)	–0.01	(0.09)	0.31	(0.08)	–0.34	(0.10)
9	0.81	(0.26)	–1.11	(0.41)	*3.42	(0.24)	–0.49	(0.37)	–2.62	(0.79)
10	*1.16	(0.12)	0.58	(0.12)	0.19	(0.14)	–2.30	(0.38)	0.37	(0.13)
11	2.57	7.(08)	*10.70	(6.95)	5.52	(6.97)	2.96	(7.07)	–21.70	(27.6)
12	–0.82	(0.13)	–0.69	(0.13)	0.31	(0.09)	*0.95	(0.07)	0.25	(0.09)
13	–0.09	(0.14)	*1.82	(0.09)	–0.48	(0.15)	–0.17	(0.15)	–1.09	(0.20)
14	*2.35	(0.13)	–1.04	(0.26)	0.17	(0.18)	–1.89	(0.39)	0.41	(0.16)
15	–1.43	(0.40)	–0.11	(0.25)	0.22	(0.22)	–1.59	(0.41)	*2.90	(0.17)
16	–0.99	(0.16)	–0.57	(0.14)	*0.93	(0.08)	0.36	(0.09)	0.27	(0.09)
17	–2.96	(1.18)	–0.57	(0.59)	–0.77	(0.70)	*4.68	(0.39)	–0.38	(0.60)
18	0.91	(0.11)	–1.80	(0.30)	0.21	(0.13)	*0.69	(0.13)	–0.01	(0.14)
19	*2.52	(0.24)	–0.07	(0.30)	2.12	(0.24)	–1.29	(0.41)	–3.28	(0.80)
20	–0.45	(0.67)	–2.4	(1.41)	1.66	(0.50)	*4.49	(0.48)	–3.30	(1.21)
21	0.25	(0.24)	–1.56	(0.47)	*3.36	(0.17)	–0.89	(0.33)	–1.15	(0.39)
22	0.03	(0.40)	–1.21	(0.57)	*4.12	(0.28)	–1.73	(0.65)	–1.21	(0.58)
23	0.55	(0.08)	–0.03	(0.10)	–0.28	(0.11)	*0.54	(0.08)	–0.78	(0.14)
24	*2.45	(0.14)	0.82	(0.16)	–0.82	(0.27)	–1.71	(0.39)	–0.73	(0.26)
25	0.44	(0.09)	0.07	(0.10)	*0.78	(0.08)	–1.13	(0.17)	–0.16	(0.11)
26	–2.61	(0.55)	1.27	(0.20)	–2.08	(0.45)	1.49	(0.20)	*1.93	(0.21)
27	*3.66	(0.25)	0.29	(0.32)	–1.92	(0.65)	–1.25	(0.55)	–0.78	(0.40)
28	–0.05	(0.20)	–2.57	(0.55)	*1.92	(0.16)	1.47	(0.16)	–0.77	(0.24)
29	*0.65	(0.08)	0.60	(0.08)	0.25	(0.09)	–0.60	(0.12)	–0.89	(0.14)
30	–0.11	(0.09)	–0.44	(0.11)	–0.49	(0.11)	0.09	(0.09)	*0.94	(0.07)
31	0.28	(0.39)	–0.39	(0.52)	–2.66	(0.91)	*3.88	(0.39)	–1.11	(0.52)
32	–0.23	(0.10)	0.01	(0.10)	0.99	(0.07)	–0.78	(0.13)	*0.01	(0.10)
Item	a_{j1}		a_{j2}		a_{j3}		a_{j4}		a_{j5}	
1	*1.21	(0.19)	0.73	(0.19)	0.25	(0.18)	0.27	(0.18)	–2.46	(0.64)

2	0.12	(0.17)	–1.00	(0.40)	–0.43	(0.24)	*1.54	(0.18)	–0.23	(0.19)
3	–0.10	(0.12)	–0.05	(0.11)	–0.30	(0.10)	–0.42	(0.15)	* 0.87	(0.11)
4	–0.11	(0.11)	*0.44	(0.08)	–0.10	(0.11)	–0.15	(0.11)	–0.07	(0.12)
5	–0.36	(0.18)	0.17	(0.14)	*0.94	(0.12)	–0.26	(0.17)	–0.49	(0.24)
6	*0.84	(0.13)	–0.24	(0.09)	–0.08	(0.14)	0.20	(0.19)	–0.73	(0.16)
7	–1.04	(0.41)	*1.31	(0.24)	–0.45	(0.64)	0.45	(0.24)	–0.27	(0.44)
8	*0.56	(0.10)	–0.05	(0.10)	–0.59	(0.11)	–0.11	(0.09)	0.19	(0.11)
9	0.70	(0.25)	0.32	(0.43)	*0.76	(0.21)	–0.64	(0.33)	–1.14	(0.60)
10	*1.04	(0.13)	0.10	(0.13)	–0.24	(0.15)	–1.24	(0.34)	0.33	(0.14)
11	1.10	(3.16)	*4.01	(3.03)	2.91	(3.06)	1.38	(3.16)	–9.40	(11.90)
12	–0.17	(0.15)	–0.18	(0.15)	0.22	(0.10)	*0.20	(0.08)	–0.06	(0.10)
13	–0.18	(0.16)	*0.85	(0.12)	–0.04	(0.19)	–0.47	(0.17)	–0.16	(0.24)
14	*0.99	(0.15)	0.10	(0.30)	–0.28	(0.19)	–0.85	(0.36)	0.05	(0.18)
15	–0.95	(0.34)	–0.11	(0.25)	0.55	(0.24)	–0.78	(0.36)	*1.29	(0.18)
16	–0.53	(0.19)	–0.50	(0.16)	*0.73	(0.10)	0.09	(0.11)	0.21	(0.11)
17	–1.88	(0.74)	1.22	(0.63)	–0.38	(0.58)	*1.30	(0.32)	–0.27	(0.50)
18	–0.01	(0.13)	–0.96	(0.32)	–0.24	(0.16)	*1.61	(0.17)	–0.40	(0.17)
19	*1.45	(0.21)	0.06	(0.27)	0.71	(0.20)	–0.73	(0.35)	–1.49	(0.56)
20	0.08	(0.62)	0.06	(1.37)	–0.03	(0.43)	*1.69	(0.43)	–1.81	(0.79)
21	–0.26	(0.27)	–0.30	(0.51)	*0.52	(0.19)	0.20	(0.37)	–0.15	(0.44)
22	–0.09	(0.38)	0.71	(0.64)	*1.33	(0.27)	–1.23	(0.49)	–0.72	(0.48)
23	–0.04	(0.09)	–0.40	(0.12)	0.36	(0.12)	*0.57	(0.09)	–0.49	(0.16)
24	*1.14	(0.16)	0.14	(0.18)	–0.34	(0.27)	–0.63	(0.37)	–0.32	(0.27)
25	0.12	(0.10)	–0.02	(0.12)	*0.72	(0.10)	–0.62	(0.20)	–0.20	(0.13)
26	–1.46	(0.44)	0.33	(0.19)	–1.34	(0.37)	0.55	(0.20)	*1.92	(0.21)
27	*1.42	(0.26)	–0.86	(0.30)	–0.99	(0.56)	–0.87	(0.49)	1.30	(0.45)
28	–0.21	(0.22)	–0.83	(0.53)	*1.03	(0.18)	0.04	(0.17)	–0.02	(0.28)
29	*0.65	(0.10)	–0.25	(0.10)	–0.16	(0.11)	0.01	(0.15)	–0.25	(0.17)
30	0.05	(0.11)	–0.14	(0.13)	–0.22	(0.13)	–0.03	(0.10)	*0.34	(0.08)
31	–0.25	(0.36)_	0.55	(0.57)	–1.29	(0.75)	*2.33	(0.40)	–1.34	(0.45)
32	0.03	(0.12)	–0.51	(0.12)	0.40	(0.08)	–0.20	(0.16)	*0.29	(0.11)

Table notes: standard errors are in parentheses; posterior mode estimation with Bayes constants of 1 was used; * indicates the correct alternative.

Simulations

Presented here are the results for two simulations with values of the logistic-normal parameter similar to those found for the SAT12 analysis, that is $\mu = -0.3$, $\sigma = 4.0$, which gives a bimodal distribution with a higher left mode (see Figure 2). The population bias and discrimination parameters were similar to those found for real–data, with bias parameters ranging from −2 to 2 and discrimination ranging from 1 to 4. Sample sizes of 600 and 1000 were used with 20 items.

The results show good parameter recovery, with percent bias generally under 10% and coverage rates close to the nominal 95% value. Table A2 (top) shows that μ of the logistic normal was well recovered, however σ tends to be slightly underestimated. The lower part of the table shows that the discrimination parameter *d* was generally well recovered with percent bias around 10% or lower. This also holds for most of the bias parameters, b_{jm} with some exceptions; note that in cases where the percent bias was larger (e.g., Items 4 and 12), the bias estimates still have the correct order. The right side of Table A2 shows that, for the larger sample size of 1000, percent bias was generally small, with excellent coverage rates. The parameters were again generally well recovered.

Table A2

Parameter Recovery for Simulated Data, 4 Response Categories

	N = 600					*N = 1000*				
Par.	Pop.	Mean	Bias	%Bias	%Cov	Pop.	Mean	Bias	%Bias	%Cov.
μ	–0.3	–0.30	–0.00	0.40	0.82	–0.3	–0.27	–0.03	8.96	0.84
σ	4	3.52	–0.48	12.03	0.95	4	3.61	–0.39	9.68	0.91
b_{11}	2	1.98	–0.02	1.03	0.95	2	1.99	–0.01	0.41	0.95
b_{12}	−1	−0.97	0.03	2.89	0.96	−1	−1.03	−0.03	2.88	0.97
b_{13}	−0.5	−0.59	−0.09	17.52	0.95	−0.5	−0.52	−0.02	3.72	0.97
b_{21}	−1.5	−1.53	−0.03	1.91	1	−1.5	−1.51	−0.01	0.64	0.93
b_{22}	−2	−2.15	−0.15	7.68	0.98	−2	−2.13	−0.13	6.37	0.99
b_{23}	−1	−1	0	0.06	0.95	−1	−1	0	0.08	0.95
b_{31}	0.5	0.53	0.03	5.37	0.95	0.5	0.54	0.04	8.35	0.91
b_{32}	1	1.02	0.02	2.24	0.96	1	1.03	0.03	3.28	0.93
b_{33}	2	1.99	−0.01	0.72	0.92	2	1.98	−0.02	0.86	0.95
b_{41}	2	2.29	0.29	14.41	0.98	2	2.25	0.25	12.74	0.98
b_{42}	1	1.28	0.28	28.21	0.99	1	1.26	0.26	26.41	0.98
b_{43}	0.5	0.78	0.28	55.97	0.98	0.5	0.76	0.26	52.87	0.99
b_{51}	1	0.91	−0.09	8.58	0.96	1	0.88	−0.12	12.14	0.95
b_{52}	0.5	0.52	0.02	3.19	0.96	0.5	0.51	0.01	1.28	0.97
b_{53}	2	3.51	1.51	75.29	0.96	2	3.49	1.49	74.73	0.95
b_{61}	0.5	0.51	0.01	1.09	0.96	0.5	0.52	0.02	4.4	0.96
b_{62}	1	0.97	−0.03	3.19	0.98	1	0.96	−0.04	3.57	0.97
b_{63}	1	1	0	0.07	0.97	1	1.02	0.02	1.87	0.91
b_{71}	0.2	0.2	0	2.29	0.93	0.2	0.2	0	0.78	0.97
b_{72}	1	0.99	−0.01	1.17	0.94	1	1.01	0.01	1.06	0.97

b_{73}	1.5	1.45	−0.05	3.38	0.98	1.5	1.46	−0.04	2.5	0.94
b_{81}	2	2.08	0.08	4.16	0.99	2	2.02	0.02	0.97	0.94
b_{82}	1	1.08	0.08	7.59	0.99	1	1.01	0.01	0.76	0.93
b_{83}	0.5	0.56	0.06	12.19	0.98	0.5	0.5	0	0.87	0.95
b_{91}	0.5	0.36	−0.14	28.83	0.96	0.5	0.37	−0.13	26.23	0.92
b_{92}	1	0.99	−0.01	0.66	0.91	1	1.02	0.02	1.66	0.93
b_{93}	2	1.99	−0.01	0.5	0.93	2	2.01	0.01	0.33	0.96
b_{101}	0.75	0.75	0	0.52	0.96	0.75	0.76	0.01	1.57	0.91
b_{102}	1	0.98	−0.02	1.87	0.98	1	0.99	−0.01	0.9	0.95
b_{103}	0.5	0.49	−0.01	2.86	0.98	0.5	0.51	0.01	1.54	0.94
b_{111}	1	1.03	0.03	2.61	0.97	1	1	0	0.07	0.96
b_{112}	−1	−0.99	0.01	1.11	0.98	−1	−0.98	0.02	1.53	0.92
b_{113}	0.5	0.44	−0.06	11.26	0.97	0.5	0.43	−0.07	13.91	0.96
b_{121}	0.3	0.44	0.14	48.24	1	0.3	0.45	0.15	50.55	0.97
b_{122}	0.9	1.04	0.14	15.09	1	0.9	1.05	0.15	16.75	0.97
b_{123}	1.2	1.35	0.15	12.56	0.99	1.2	1.35	0.15	12.59	0.99
b_{131}	−0.7	−0.86	−0.16	22.69	1	−0.7	−0.87	−0.17	24.96	0.99
b_{132}	0.2	0.21	0.01	5.76	0.97	0.2	0.19	−0.01	3.78	0.93
b_{133}	0.5	0.52	0.02	4.26	0.96	0.5	0.5	0	0.9	0.92
b_{141}	1	1	0	0.11	0.97	1	1.01	0.01	0.5	0.93
b_{142}	1.25	1.2	−0.05	3.77	0.93	1.25	1.19	−0.06	5.1	0.96
b_{143}	1	1.01	0.01	1	0.97	1	1	0	0.01	0.95
b_{151}	−0.4	−0.42	−0.02	4.09	0.93	−0.4	−0.4	0	0.7	0.97
b_{152}	0.2	0.18	−0.02	7.68	0.92	0.2	0.2	0	0.19	0.98
b_{153}	−0.2	−0.22	−0.02	11.26	0.96	−0.2	−0.21	−0.01	4.63	0.97
b_{161}	−0.8	−0.79	0.01	1.43	0.96	−0.8	−0.75	0.05	6.61	0.92
b_{162}	−1	−0.97	0.03	3.41	0.96	−1	−0.94	0.06	5.89	0.93
b_{163}	−0.5	−0.47	0.03	6.41	0.98	−0.5	−0.44	0.06	11.5	0.93
b_{171}	1	0.93	−0.07	6.75	0.94	1	0.95	−0.05	5.1	0.93
b_{172}	1.5	1.5	0	0.01	0.92	1.5	1.5	0	0.17	0.91
b_{173}	0.5	0.53	0.03	5.21	0.94	0.5	0.51	0.01	2.4	0.97
b_{181}	0.8	0.8	0	0.31	0.96	0.8	0.8	0	0.14	0.93
b_{182}	−1	−1.46	−0.46	46.29	0.95	−1	−1.52	−0.52	52.36	0.92
b_{183}	0.5	0.5	0	0.53	0.95	0.5	0.49	−0.01	2.13	0.94
b_{191}	−0.5	−0.48	0.02	3.4	0.93	−0.5	−0.53	−0.03	6.69	0.93
b_{192}	0.25	0.23	−0.02	6.13	0.94	0.25	0.25	0	0.32	0.97
b_{193}	2	2	0	0	0.95	2	1.99	−0.01	0.52	0.97
b_{201}	0.6	0.74	0.14	22.89	0.99	0.6	0.69	0.09	14.94	0.97
b_{202}	1.2	1.32	0.12	10.26	0.98	1.2	1.29	0.09	7.87	0.99
b_{203}	0.9	1.04	0.14	15.21	0.98	0.9	0.99	0.09	10.09	0.97
d_1	1	1.09	0.09	8.75	0.97	1	1.02	0.02	2.16	0.94
d_2	2	2.17	0.17	8.59	0.98	2	2.13	0.13	6.27	0.98
d_3	3	3.38	0.38	12.82	0.99	3	3.21	0.21	7.06	0.96
d_4	3.5	3.83	0.33	9.31	0.98	3.5	3.76	0.26	7.45	0.99
d_5	4	4.3	0.3	7.44	0.99	4	4.2	0.2	5.06	0.99

d_6	3	3.12	0.12	4.03	0.97	3	3.11	0.11	3.52	0.97
d_7	2	2.05	0.05	2.63	0.99	2	2.07	0.07	3.58	0.95
d_8	1	1.06	0.06	6.39	0.99	1	1.02	0.02	1.95	0.95
d_9	2	2.16	0.16	8.01	0.99	2	2.15	0.15	7.43	0.92
d_{10}	1	1.04	0.04	3.67	0.97	1	1.03	0.03	2.7	0.93
d_{11}	3.5	3.72	0.22	6.21	1	3.5	3.53	0.03	0.92	0.98
d_{12}	3	3.18	0.18	5.87	0.99	3	3.15	0.15	4.84	0.96
d_{13}	3	3.19	0.19	6.2	1	3	3.15	0.15	5.12	0.99
d_{14}	4	4.41	0.41	10.34	0.95	4	4.17	0.17	4.37	0.99
d_{15}	1	0.99	−0.01	1.35	0.95	1	1	0	0.1	0.97
d_{16}	2	2.04	0.04	1.92	0.97	2	2.09	0.09	4.61	0.99
d_{17}	2	2.11	0.11	5.37	0.97	2	2.06	0.06	3.19	0.97
d_{18}	4	4.49	0.49	12.31	0.98	4	4.53	0.53	13.16	0.99
d_{19}	2.5	2.76	0.26	10.32	0.96	2.5	2.48	−0.02	0.82	0.95
d_{20}	2	2.14	0.14	7.12	0.97	2	2.09	0.09	4.66	0.97