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An Application of Signal Detection Theory With Finite Mixture Distributions to Source Discrimination

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A mixture extension of signal detection theory is applied to source discrimination. The basic idea of the approach is that only a portion of the sources (say A or B) of items to be discriminated is encoded or attended to during the study period. As a result, in addition to 2 underlying probability distributions associated with the 2 sources, there is a 3rd distribution that represents items for which sources were not attended to. Thus, over trials, the observed response results from a mixture of an attended (A or B) distribution and a nonattended distribution. The situation differs in an interesting way from detection in that, for detection, there is mixing only on signal trials and not on noise trials, whereas for discrimination, there is mixing on both A and B trials. Predictions of the mixture model are examined for data from several recent studies and in a new experiment.

In a detection study, an event is either presented or not presented on each trial, and the observer's task is to decide whether or not an event was presented. For example, in a recognition-memory study, words from a previously studied list are presented during a test, along with new words, and the observer's task is to decide whether each word is old or new. In a discrimination study, however, one of two types of events, say A or B, is presented on each trial, and the observer's task is to decide whether A or B was presented. In terms of memory research, the difference is that, for detection, a new or an old item is presented on each trial, whereas for discrimination, only old items are presented. The present article shows that this difference is rather interesting when viewed from the perspective of a mixture extension of signal detection theory (SDT; DeCarlo, 2000, 2002b). In particular, the mixture approach assumes that mixtures arise in SDT because of the influence of an additional process. For example, with respect to detection, it is assumed that there is mixing for old items because some items are not attended to during the study period; it has previously been shown that this mixing leads to receiver operating characteristic (ROC) curves on inverse-normal coordinates (i.e., *z*-ROC curves) with nonunit slopes (DeCarlo, 2000, 2002b). With respect to discrimination, if the same process that leads to mixing for detection (e.g., attention) also operates in discrimination, then there will be mixing on both A and B trials, because some of the A and B items will not have been attended to during the study period. In this article, it is shown that mixture SDT in this case predicts curved *z*-ROC curves; this prediction is examined below for data from several recent source discrimination experiments and one new experiment. The mixture approach as previously applied to detection is first reviewed briefly, and then the approach is applied to discrimination.

Sample programs are available at http://www.columbia.edu/~ld208.

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Mixture SDT for Detection

Consider the basic signal detection experiment in which, on each trial, either a signal or noise is presented. According to SDT, the effect of a presentation of a signal or noise can be represented by probability distributions on an underlying psychological continuum. The distance between the two distributions, denoted here as *d*, reflects an observer's ability to detect a signal. In the mixture extension of SDT, the effect of a presentation of a signal is represented by two (or more) probability distributions rather than one. The two distributions correspond to different representations of the signal and can be motivated in more than one way, depending on the particular research context. A simple and general interpretation is that the distributions arise because over trials, observers attend to the signal at different levels. As a result, there are latent classes of items, such as attended or nonattended items, on signal trials. Thus, the observed response on each signal trial is based on one or the other of two distributions, depending on whether the signal was attended to or not; it is not known which signals were attended to or not, and so the two distributions are mixed over trials. Note that, in the application to detection, there is mixing only on signal trials.[1]

It has previously been shown that the mixture extension of SDT provides an alternative to another extension of SDT, the unequal variance SDT model (Green & Swets, 1966). To formalize the ideas of SDT and introduce notation, first consider the equal variance normal SDT model, which can be written as

$$p(Y \leq k | X = x) = \Phi\left(\frac{c_k - dx}{\sigma}\right), \quad (1)$$

where Y is the response variable (e.g., a confidence-rating response) that takes on values $k = 1 - K$, where K is the number of response categories; X is a dummy-coded variable ($x = 0, 1$) that indicates signal or noise (A or B, for the application to discrimination); $p(Y \leq k | X = x)$ is the cumulative probability of a response

[1] However, a situation in which there might be mixing on noise trials was noted in DeCarlo (2002b).

of k or less, conditional on X; Φ is the cumulative distribution function for the normal distribution; c_k is the kth response criterion (where the criteria are strictly ordered); d is the detection parameter; and σ is the standard deviation.

The unequal variance extension of SDT allows the variances of the underlying distributions to differ across signal and noise,

$$p(Y \leq k|X = x) = \Phi\left(\frac{c_k - d_N x}{\sigma_S^x}\right), \quad (2)$$

which shows that the model generalizes the equal variance normal model by introducing the parameter $\sigma_S > 0$, which is the standard deviation of the signal distribution; note that the model can be generalized further by using distributions other than the normal (via generalized linear models; see DeCarlo, 1998, 2000). If the standard deviation of the noise distribution is set to unity (i.e., $\sigma_N = 1$), then $1/\sigma_S$ gives the ratio of the standard deviations of the new and signal distributions, which corresponds to the slope of the z-ROC curve. The parameter d_N is the detection parameter scaled with respect to the standard deviation of the noise distribution. The unequal variance normal SDT model and the mixture normal SDT model are compared for data from detection and discrimination studies in DeCarlo (2000, 2002b); see DeCarlo (2003b) for some additional examples of the unequal variance SDT model and comparisons with other models.

Mixture SDT for Discrimination

Source monitoring has been studied intensively and applied to a variety of research areas (for a review, see Johnson, Hashtroudi, & Lindsay, 1993). In the typical procedure, items are presented by two sources, say A or B (e.g., words are read by a man or a woman), and in a subsequent test, observers are required to decide if an item is new, old and presented by Source A, or old and presented by Source B. Thus, the task involves both detection (a decision as to whether an item is new or old) and discrimination (a decision as to the source of an old item). Recently, due in part to the large number of studies that have examined source monitoring, there has been an increased interest in source discrimination. To examine source discrimination in and of itself, recent studies have used a variant of the procedure that is typically used in source monitoring studies. In particular, observers were shown items from two sources, as in the usual source monitoring study, but the subsequent test differed in that it was solely a test of source discrimination; that is, the previously studied A and B words were presented (without new words), and the observers' task was to decide whether the words were presented by Source A or by Source B (e.g., Hilford, Glanzer, Kim, & DeCarlo, 2002; Yonelinas, 1999). Thus, the procedure was one of pure discrimination, without detection, because all of the words presented during the test were old. In addition, confidence-rating responses were used, because the studies were concerned with the form of the ROC curve for discrimination. For example, observers indicated their confidence about the source of an item on a 1 to 6 scale, with a response of 1 indicating *sure A*, 2 indicating *slightly sure A*, and so on up to 6 indicating *sure B.*

The discrimination procedure just described is rather interesting when viewed from the perspective of the mixture extension of SDT. What makes it interesting is that, during the test, only old items are presented, namely the A and B items that were previously presented during the study phase. So the test phase presents two types of signals, rather than signal and noise (i.e., old and new items). It follows from the idea used to motivate mixture SDT for detection that each of the two signal distributions might consist of a mixture of distributions; that is, in addition to the A and B distributions, corresponding to items that were attended to during the study period, there are A′ and B′ distributions, corresponding to A and B items that received different levels of attention or processing during the study period. Further, if the A′ and B′ distributions correspond to nonattended items, then it is reasonable to assume that they have the same location.

Figure 1 illustrates the theory. In the context of research on source memory, the A and B distributions might represent perceptions of features associated with the two sources; the distributions are referred to here simply as *discrimination* distributions. The distances of the A and B distributions from the nonattended distribution (which is used as the zero point) are given by the means Ψ_A and Ψ_B, which can be viewed as measures of source memory strength; the measures are denoted here simply as d_A and d_B, respectively (in line with SDT notation). Of primary interest is the distance between the A and B distributions, $d_{AB} = d_B - d_A$, as shown in Figure 1, which is a measure of an observer's ability to discriminate between the two sources.

A General Mixture SDT Model for Discrimination

A general mixture SDT model for source discrimination follows directly from the ideas illustrated in Figure 1 and can be written, for normal distributions, as

$$p(Y \leq k|X = x) = \begin{cases} \lambda_A \Phi(c_k - d_A) + (1 - \lambda_A)\Phi(c_k) & x = 0 \\ \lambda_B \Phi(c_k - d_B) + (1 - \lambda_B)\Phi(c_k) & x = 1 \end{cases}, \quad (3)$$

where the variances are set to unity (without loss of generality). The remaining terms are as defined above, except for the addition of the mixing parameters λ_A and λ_B, which can be interpreted as indicating the proportion of A and B items for which features were attended to during the study period. A comparison of Equation 3,

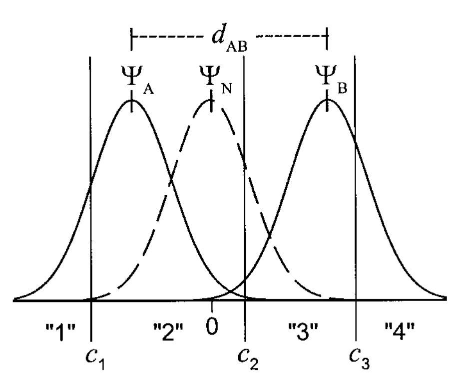

Figure 1. Signal detection theory with finite mixture distributions as applied to source discrimination for attended (solid lines) and nonattended (dashed line) items. The distances (d) of the A and B distributions from the nonattended distribution (Ψ_N) are given by Ψ_A and Ψ_B, respectively. Responses of 1, 2, 3, and 4 are delineated by the criteria c_1, c_2, and c_3.

which is a mixture signal detection model for discrimination, with Equation 2 of DeCarlo (2002b), which is a mixture signal detection model for detection, shows that the difference is that for discrimination, there is mixing on both A and B trials (i.e., for $x = 0, 1$), whereas for detection, there is mixing only on signal trials (i.e., only for $x = 1$).

Parameter Restrictions

Given Equation 3, there are several parameter restrictions that are of substantive interest. For example, the restriction $\lambda_A = \lambda_B$ means that the level of attention is equal across the A and B items, whereas $-d_A = d_B$ means that the A and B distributions are the same distance (but in opposite directions) from the nonattended distribution (and so the absolute values of both $-d_A$ and d_B are equal to ½ d_{AB}). The implication of the latter restriction is that the two source distributions are strengthened equally when attended to during the study period (i.e., shifted by the same amount but in opposite directions), which represents a symmetrical strengthening effect.

A simple mixture model for discrimination follows from an application of both of the restrictions noted above—that is, $\lambda_A = \lambda_B$ and $-d_A = d_B = ½\, d_{AB}$. The restricted version of Equation 3 can then be written as

$$p(Y \le k|X = x) = \lambda\Phi(c'_k - ½ d_{AB}x) + (1 - \lambda)\Phi(c'_k), \quad (4)$$

where x is coded as -1 for presentation of one event, say A, and as 1 for presentation of the other event, say B, and c'_k are the criteria located with respect to the intersection point of the A and B distributions (see the choice theory parameterization discussed in Appendix A of DeCarlo, 1998). With this (effect) coding, Equation 4 shows that the coefficient of X gives one half of the discrimination parameter d_{AB} (and ½ $d_{AB} = -d_A = d_B$).

The restricted model of Equation 4 generalizes the equal variance normal SDT model of Equation 1 with the addition of a single parameter, λ. The model offers a useful starting point for the study of source discrimination; it was introduced in DeCarlo (2000) and was also considered by Hilford et al. (2002). The more general model of Equation 3, introduced here, is useful in situations in which a need to relax the parameter restrictions of Equation 4 may be motivated. For example, if the sources are treated in the same way, referred to here as a *symmetric* treatment of the sources, then Equation 4 should be adequate (though of course it is still possible that participants may not attend to the sources equally or remember them equally well). However, in situations in which the sources are not treated symmetrically, as discussed below, the more general model of Equation 3 may be needed. Thus, in the examples that follow, particular attention is paid to exactly how the sources were presented.

The model can be fit using software for latent class analysis. In particular, the Appendix provides notes on the use of the software LEM (Vermunt, 1997), which is freely available on the Internet (the Web site is given in the Appendix); sample LEM programs are also available at my Web site (http://www.columbia.edu/~ld208). With respect to identifiability, Prakasa Rao (1992) has shown that the family of finite mixtures of univariate normal distributions is identifiable, and so the full model of Equation 3 is identified if a rating response with at least five response categories is used (so that the number of parameters is less than or equal to the number of observations, which is a necessary condition for identification). If a binary response is used, then multiple sessions are needed, which is the usual case for constructing ROC curves from binary data (and it must be assumed that the detection and attention parameters are constant across the multiple sessions).

Inverse Normal ROC Curves

The ROC curves that follow from the theory can be derived by noting that a z-ROC curve is simply a plot of the inverse normal transformation of $p(Y > k|X = 0)$ on the x-axis against the inverse normal transformation of $p(Y > k|X = 1)$ on the y-axis (e.g., see DeCarlo, 1998). For the discrimination mixture model of Equation 3, it follows that the z-ROC curve is a plot of

$$\Phi^{-1}\{1 - [\lambda_A\Phi(c_k - d_A) + (1 - \lambda_A)\Phi(c_k)]\}$$

on the x-axis (i.e., the inverse normal transformation of 1 − Equation 3 for $x = 0$) against

$$\Phi^{-1}\{1 - [\lambda_B\Phi(c_k - d_B) + (1 - \lambda_B)\Phi(c_k)]\}$$

on the y-axis (i.e., the inverse normal transformation of 1 − Equation 3 for $x = 1$). The above equations are simply the equations for the theoretical mixture z-ROC curves. Note that the fitted z-ROC curves presented below were obtained by substituting maximum-likelihood estimates of d_A, d_B, λ_A, and λ_B in the above equations and plotting the curves generated by varying c_k.

Figure 2 presents examples of z-ROC curves that follow from the mixture SDT model as applied to discrimination. The upper left panel shows a curve for the restricted model (Equation 4) with equal attention and memory strength (i.e., detection parameters) across the sources, using values suggested by the results obtained below. The figure shows that the z-ROC curve is symmetrical and the effect of the mixing is to pull the curve down toward the diagonal, which results in a nonlinear curve that is curved upward. The upper right panel shows an example in which the attention parameters λ_A and λ_B are not equal. The effect is to move the point of greatest dip away from the center (to the right in this case), and so the curve is not symmetrical. The two lower panels show examples in which attention is equal across the sources but memory for the sources differs (these panels show a large difference, with detection being 1 for one source and 3 for the other). The effect on the z-ROC curves is apparent, in that the curves have slopes other than unity, are curved, and are asymmetrical.

The general model of Equation 3 offers a rich means for examining data from research in which a source discrimination procedure was used. The next section examines data from several recent experiments involving source discrimination, with attention paid to how the sources were presented. In particular, in a symmetric procedure, words presented from the two sources (e.g., in a male or a female voice) are intermingled during the study period (i.e., they differ only with respect to voice, with all other aspects of the word presentations kept the same across the two sources). In this case, it seems reasonable to assume that memory and attention are equal across the sources, and Equation 4 is used to examine the data. In contrast, in an asymmetric procedure, the two sources are not treated symmetrically. For example, in one version of an asymmetric procedure, all of the words from one source are

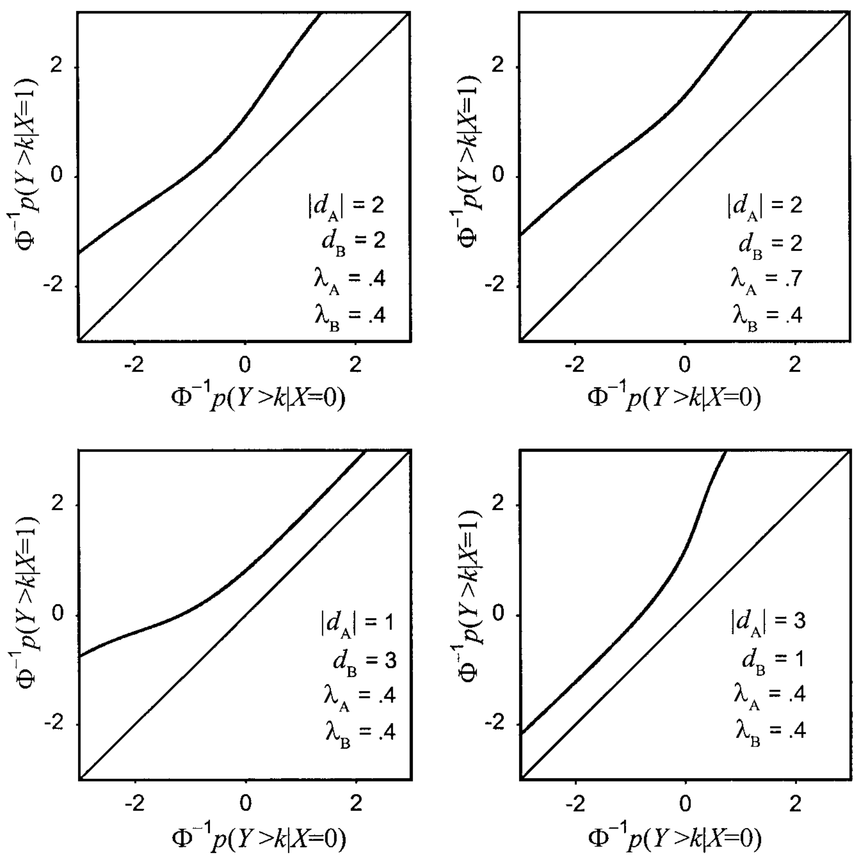

Figure 2. Examples of mixture receiver operating characteristic curves for discrimination on inverse normal coordinates.

presented first, followed by words from the other source. In this case, one would expect that memory for the more recent source might be better, in which case Equation 3 with $|d_A| \neq d_B$ would be appropriate. This prediction is examined below in several experiments.

Applications to Recent Research

Source Discrimination With Rating Responses: Sources Treated Symmetrically

In this section, experiments that used a source discrimination procedure in which the sources were treated symmetrically are examined—specifically, Experiment 1 of Yonelinas (1999) and Experiments 2 and 3 of Hilford et al. (2002).[2] In all three experiments, participants saw two lists of words, denoted here simply as A and B. During a subsequent test, they were shown words from both lists and were asked to rate their confidence as to which list each word was from (from 1 = *sure A* to 6 = *sure B*). Thus, the task consisted solely of source discrimination. For Experiment 1 of Yonelinas, the two sources were the position of the word on the screen during the study period (left or right); for Experiment 2 of Hilford et al., the two sources were whether the word had been read by a man or a woman; and for Experiment 3 of Hilford et al., the two sources were whether the word had appeared on the top or the bottom of the screen during the study period.

Table 1 presents goodness-of-fit statistics and information criteria for maximum-likelihood fits of the unequal variance normal SDT model and the mixture normal SDT model of Equation 4. The likelihood ratio (*LR*) statistic provides information about the absolute fit of the model, whereas Akaike's information criterion (AIC) and the Bayesian information criterion (BIC) are used to compare the different models, with smaller values indicating a preferred model (see Agresti, 1990; Burnham & Anderson, 2002; DeCarlo, 2002a, 2002b). The information criteria are used here, for example, to compare the unequal variance model and the mixture models (which are not nested, in that one model cannot be obtained by restricting parameters of the other model).

Table 1 shows that, for all three experiments, the unequal variance model provides a poor description of the data, in that all of the *LR* goodness-of-fit statistics are large and significant. As shown by the *z*-ROC plots presented in Figure 3, this lack of fit reflects the fact that the curves are nonlinear, which is inconsistent

[2] Experiment 1 of Hilford et al. (2002) gave similar results, but it used a different procedure in that a source discrimination response was made contingent on a detection response of "old."

Table 1
Goodness-of-Fit Statistics and Information Criteria for Experiment 1 of Yonelinas (1999) and Experiments 2 and 3 of Hilford et al. (2002): Sources Treated Symmetrically

Model	*LR*	*df*	*p*	AIC	BIC
Yonelinas, Experiment 1 (N = 2,400)					
Unequal variance SDT	37.87	3	< .01	7,368	7,409
Mixture SDT	4.58	3	.21	7,335	7,375
Hilford et al., Experiment 2 (N = 7,850)					
Unequal variance SDT	125.34	3	< .01	26,378	26,426
Mixture SDT	6.82	3	.08	26,259	26,308
Hilford et al., Experiment 3 (N = 7,198)					
Unequal variance SDT	23.22	3	< .01	23,889	23,937
Mixture SDT	3.24	3	.36	23,869	23,917

Note. *LR* = likelihood ratio goodness-of-fit test; AIC = Akaike's information criterion; BIC = Bayesian information criterion; SDT = signal detection theory; Mixture SDT = Equation 4 in the text.

with unequal variance SDT; AIC and BIC are also considerably larger (i.e., >10; see Burnham & Anderson, 2002) for the unequal variance model than they are for the mixture model. In contrast, Table 1 shows that the fit of the simple mixture SDT model of Equation 4 is adequate for all three experiments, in that the *LR* statistics are not significant.

Figure 3 shows the fitted mixture *z*-ROC curves and maximum-likelihood estimates of the detection and mixture parameters for all three experiments for a fit of Equation 4. The figure shows that the fitted mixture *z*-ROC curves provide good descriptions of the data in all cases, which is as expected in light of the goodness-of-fit statistics shown in Table 1. Thus, the mixture SDT model accurately describes the curvature that is evident in the data. The reason why the unequal variance SDT model fits poorly is also apparent—the *z*-ROC curves are clearly nonlinear. The *z*-ROC curves are remarkably similar across the three experiments, in that they all have slopes near unity and are symmetrically curved upward.

With respect to the parameter estimates and standard errors, Figure 3 shows that, for all the experiments, the estimates of d_{AB} are in the range of 3.5 to 4.9, which indicates good discrimination, and the estimates of λ are in the range of .3 to .4, which can be interpreted as showing that the source of about 30%–40% of both the A and B items was attended to during the study period. Note that, for the experiments of Hilford et al. (2002), discrimination was higher when the source was a male versus a female voice (the estimate of d_{AB} = 4.8) than it was when the source was whether the word appeared on the top or the bottom of the screen (the estimate of d_{AB} = 3.5), yet attention was not higher for the male–female voices (the estimates of λ are .32 for male–female voices and .37 for top–bottom word position). This finding provides some evidence that the discrimination parameter can vary independently of the mixing parameter λ; some additional evidence on this is noted below.

In summary, the mixture SDT approach to discrimination predicts that, if the source is not attended to on a proportion of the study trials, then *z*-ROC plots of the data will be curved, as shown by the upper left panel of Figure 2. This result was found for the experiments of Yonelinas (1999) and Hilford et al. (2002). In all three experiments, the sources were treated symmetrically, in that the sources were intermixed during the study trials. The results, in terms of *z*-ROC plots, were curves that were symmetrically curved upward, with slopes close to unity. It is important to recognize that these two results have a specific interpretation in terms of the mixture SDT model of Equation 4, which is that during the study

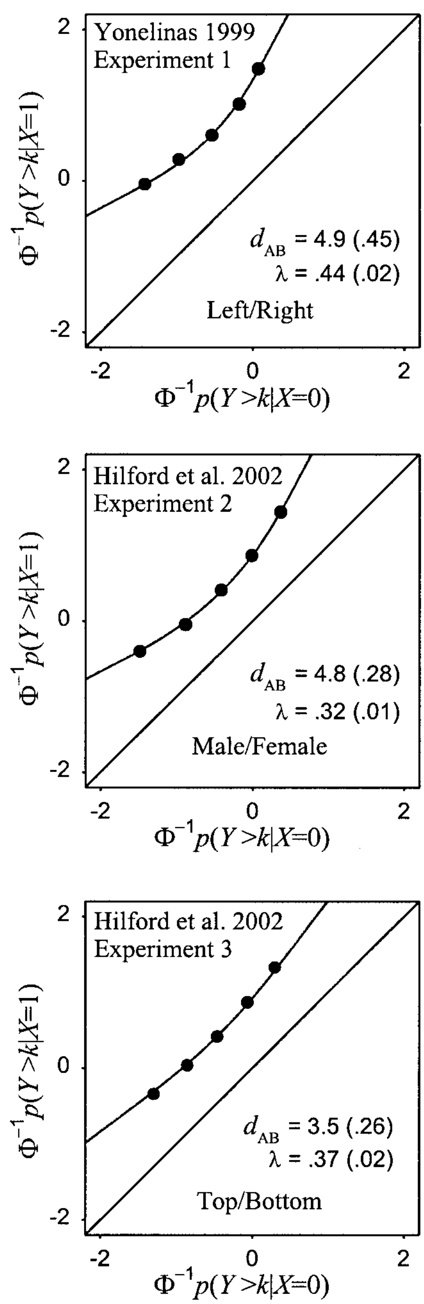

Figure 3. Data and fitted mixture receiver operating characteristic curves on inverse normal coordinates for Experiment 1 of Yonelinas (1999) and Experiments 2 and 3 of Hilford et al. (2002). Standard errors appear in parentheses.

Table 2
Goodness-of-Fit Statistics and Information Criteria for Experiments 2–4 of Yonelinas (1999): Sources Not Treated Symmetrically

Model	*LR*	*df*	*p*	AIC	BIC
Experiment 4 (N = 3,840)					
Unequal variance SDT	70.12	3	< .01	12,429	12,437
Mixture SDT	147.39	3	< .01	12,506	12,550
Equation 3 with $-d_A \neq d_B$	2.69	2	.26	12,363	12,414
Experiment 3 (N = 4,344)					
Unequal variance SDT	15.18	3	< .01	14,629	14,673
Mixture SDT	8.14	3	.04	14,621	14,666
Equation 3 with $-d_A \neq d_B$	5.46	2	.07	14,621	14,672
Experiment 2 (N = 3,840)					
Unequal variance SDT	63.07	3	< .01	12,914	12,958
Mixture SDT	6.13	3	.11	12,857	12,901
Equation 3 with $-d_A \neq d_B$	1.71	2	.43	12,855	12,905

Note. *LR* = likelihood ratio goodness-of-fit test; AIC = Akaike's information criterion; BIC = Bayesian information criterion; SDT = signal detection theory; Mixture SDT = Equation 4 in the text.

period, the sources were attended to equally, and memory for both sources was strengthened equally.

Source Discrimination With Rating Responses: Sources Not Treated Symmetrically

This section examines situations in which the sources were not treated symmetrically. For example, in Experiment 4 of Yonelinas (1999), the sources were whether words were (a) on a list read aloud immediately before the test or (b) on a list read aloud 5 days earlier. In this case, one would expect that memory for the source of words that were presented 5 days earlier would be poorer than memory for the source of words that were presented immediately before the test, and so the mixture model with $|d_A| \neq d_B$ should be appropriate. This model can be implemented by using Equation 3; it seems reasonable to assume that manipulating the delay between study and test should not affect the proportion of items attended to during each study period (because the delay occurs after the first list is presented), and so the assumption $\lambda_A = \lambda_B$ is maintained.

Also examined here are two other experiments of Yonelinas (1999) in which the sources were not treated symmetrically; the procedure also differed somewhat from that used in the other experiments.[3] In Experiment 2 of Yonelinas, first one list of words was read aloud in a male voice, and then a second list of words was read aloud in a female voice. The procedure in Experiment 3 of Yonelinas was similar, except that the first list (in a male voice) was presented twice, whereas the second list (in a female voice) was presented only once. Because of the asymmetric nature of the procedures, it would be expected that, for both experiments, there might be differences in memory strength (as reflected by d_A and d_B) across the two sources.

Table 2 shows goodness-of-fit statistics and information criteria for fits of the unequal variance normal SDT model, the restricted mixture model of Equation 4, and the mixture model of Equation 3 with $|d_A| \neq d_B$ but $\lambda_A = \lambda_B$. The table shows that, for all three experiments, the *LR* statistics are large and significant for the unequal variance model. As the z-ROC curves presented in Figure 4 show, this lack of fit occurs because the curves are again nonlinear. With respect to the restricted mixture SDT model of Equation 4, the table shows that the *LR* statistic is quite large for Experiment 4 and is also significant for Experiment 3. As shown by the z-ROC plots presented in Figure 4, this occurs because the plots are not symmetrical. Table 2 also shows that the fit of Equation 3 with $|d_A| \neq d_B$ is acceptable, in terms of the *LR* statistic, for all three experiments. With respect to the information criteria, for Experiment 4 of Yonelinas (1999), both AIC and BIC are clearly smallest for Equation 3 with $|d_A| \neq d_B$; for Experiments 2 and 3, AIC suggests that Equations 3 and 4 are about equal, whereas BIC favors Equation 4. As shown by the z-ROC plots and parameter estimates for Experiments 2 and 3, the asymmetry, if any, is small, which is why the information criteria suggest that Equation 4 is adequate for these experiments.

Figure 4 shows, for Experiments 2, 3, and 4 of Yonelinas (1999), plots of the data, fitted mixture z-ROC curves, and maximum-likelihood estimates of the parameters and their standard errors. With respect to Experiment 4, which used a 5-day delay between the lists, the top panel of Figure 4 shows that the z-ROC curve is clearly neither linear nor symmetrical, which is why relaxing the restriction $|d_A| = d_B$ provides a considerable improvement in fit over Equation 4. The estimates of d_A and d_B are −1.0 and 2.9, respectively (and so the estimate of d_{AB} is 3.9), which shows that memory for items presented 5 days earlier ($|d_A|$

[3] The procedure differed from the other experiments in that new items were presented during the test phase, and both discrimination and recognition judgments were made on each trial; only the discrimination judgments are examined here. A more general model that simultaneously considers both types of judgments can also be formulated (see DeCarlo, 2003a).

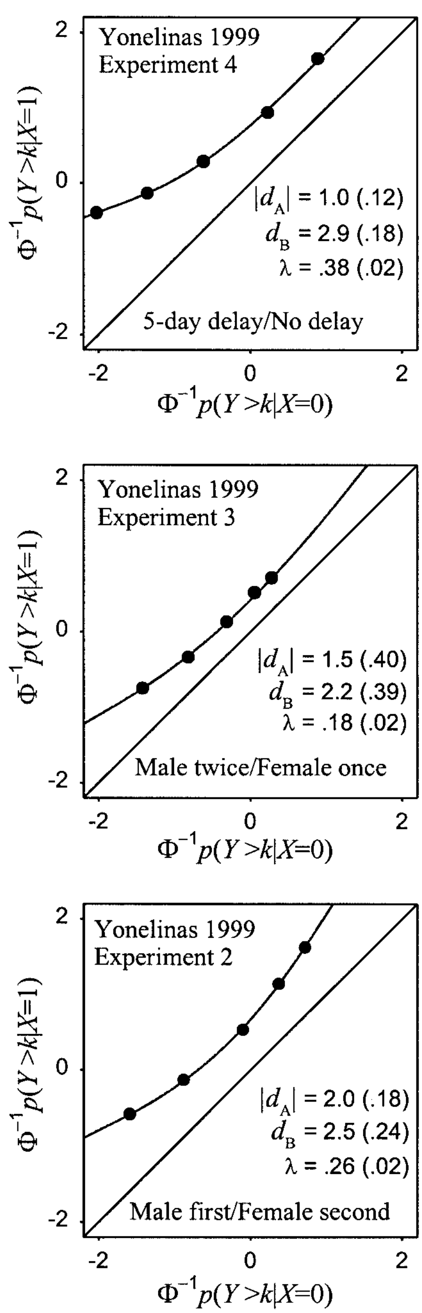

Figure 4. Data and fitted mixture receiver operating characteristic curves on inverse normal coordinates for Experiments 2, 3, and 4 of Yonelinas (1999).

= 1.0) is considerably poorer than memory for items presented immediately before the test (d_B = 2.9). This finding provides important evidence that the parameters of Equation 3 behave as expected in response to an experimental manipulation. The interpretation in terms of mixture SDT is simple yet compelling: The A item distribution might have originally been shifted as much as the B item distribution during the study period (i.e., about 2.9 standard deviations to the left), but the distribution drifted back toward the nonattended distribution during the 5-day intervening period (to 1 standard deviation), possibly due to decay in memory. From this perspective, the estimates of d_A and d_B quantify effects of decay in memory.

The middle and lower panels of Figure 4 show results for Experiments 2 and 3 of Yonelinas (1999). The *z*-ROC curve for Experiment 3 suggests possible, though small, asymmetry. The *z*-ROC curve for Experiment 2, in contrast, is nearly symmetrical, which is why Equation 4 was found to be adequate in terms of fit. Note, however, that for both experiments the point estimates of *d* differ slightly across the two sources. For Experiment 3, the middle panel of Figure 4 shows that the estimate of d_B is 2.2 for words on the more recent list, whereas the estimate of $|d_A|$ is 1.5 for words on the list presented earlier, in spite of the fact that the first list was presented twice (the standard errors are relatively large, however). For Experiment 2, the lower panel of Figure 4 shows that the estimate of d_B is 2.5 for words on the more recently presented list, and the estimate of $|d_A|$ is 2.0 for words on the list presented earlier (the difference is small, however, relative to the standard errors, and it is not statistically significant). Thus, in terms of Equation 3, the results for both Experiments 2 and 3 suggest that memory might have been slightly better for more recently presented words, which is as expected (the results suggest that a larger sample size might be needed to detect the small effect of simply presenting one list before the other). Note that although differences in the parameter estimates were small for Experiments 2 and 3, they were nevertheless in the correct direction, in that *d* was consistently larger for the most recently presented list. Finding a smaller effect in Experiments 2 and 3 than in Experiment 4 is not surprising, because Experiment 4 used a 5-day delay, which should have had a larger effect on memory than simply presenting one list before the other in the same session, which represents a much smaller delay.

In summary, for the three experiments of Yonelinas (1999) with asymmetric source presentations, plots of the data on inverse normal coordinates gave *z*-ROC curves that were curved upward, as was also found for the three experiments discussed earlier and shown in Figure 3. Additionally, there was some evidence of asymmetry in the *z*-ROC curves. Asymmetry showed up most clearly for an experiment in which a strong experimental manipulation—namely a 5-day delay—was used, whereas presenting one list before the other in the same session appears to have had at most only a small effect on memory strength, as shown by the *z*-ROC plots and the parameter estimates of Equation 3.

Summary: Source Discrimination

Together, the six experiments discussed above show that, for source discrimination data, (a) *z*-ROC curves were curved upward, as predicted by mixture SDT; (b) *z*-ROC curves for experiments in which the sources were treated symmetrically were symmetric; and (c) *z*-ROC curves for experiments in which the sources were not treated symmetrically showed deviations from symmetry (at least in two of three cases). The mixture SDT model helps to organize and summarize these results in terms of effects on memory, as reflected by the estimates of d_A and d_B, and in terms of effects on attention, as reflected by the mixture parameter λ.

The above experiments varied a factor that one would expect to affect memory strength, namely the length of time between presenting a list and testing memory. The approach in the current experiment, reported below, differed in that a factor that might affect attention was manipulated. Specifically, it seemed reasonable to assume that presenting an item for a shorter period of time might decrease the probability that it would be attended to. An analysis of data of Ratcliff, McKoon, and Tindall (1994; presented in DeCarlo, 2002b) provided some evidence that this would be the case. The current experiment examined the effects of varying the presentation time in a source discrimination study, with the expectation that the mixing parameter would be affected if indeed it reflects attention. In addition, both symmetric and asymmetric conditions were used. With respect to the asymmetric condition, unequal presentation times were used across the sources to see if this manipulation would affect the *z*-ROC curves; these conditions were motivated by the results found for the experiments of Yonelinas (1999). In terms of the mixture SDT model, the predictions were that (a) *z*-ROC curves should be curvilinear; (b) increasing the presentation time should affect the mixing parameter λ; and (c) asymmetry in the treatment of the sources should be reflected by an asymmetry in the parameters of Equation 3, and in particular the model with $\lambda_A \neq \lambda_B$ should be appropriate if presentation time affects the proportion of items that are attended to.

Method

Participants

The participants were 48 graduate students enrolled in courses in cognitive psychology or measurement. Four conditions were examined, with 12 participants randomly assigned (using sampling without replacement) to each condition. Each student participated in one session, which was 30 min or less in duration.

Materials

A list of 120 words was selected from the list provided by Coltheart (1981), with the criteria that the words were 5 or 6 letters in length and had a word-frequency count between 40 and 60 per million (Kučera & Francis, 1967).

Design and Procedure

The experiments were run on personal computers. The experiment was controlled using E-prime (Version 1.0, Beta 5.0; Schneider, 2000). The participants were first given a short practice session, during which 10 words were presented on either the left or the right side of the computer screen (i.e., random sampling without replacement, with 5 words presented on the left and 5 on the right). For the test, participants rated their confidence that a word had been presented on the left or right side of the screen by using a 1 to 6 response, with 1 = *sure left*, 2 = *fairly sure left*, 3 = *slightly sure left*, 4 = *slightly sure right*, 5 = *fairly sure right*, and 6 = *sure right*.

For the experiment, 120 words were presented in two blocks of 60 (i.e., a block of 60 words followed by a test, then another block of 60 words followed by a test). During the study period, a fixation cross appeared on each trial for 500 ms on either the left or the right side of the screen, indicating where the word was to appear. The study word then appeared for either 1 s or 3 s, depending on the condition (described below). The screen was then cleared, and the next fixation cross and word were presented. For the test, each word was presented in the center of the screen, and the response scale (i.e., numbers with labels as given above) appeared below the word. The participant then entered his or her response by using the numbered keys located at the top of the computer keyboard. The screen was then cleared and the next trial began. Participants were told that they should try to use each response category at least once.

There were four conditions, with different participants in each condition. The conditions differed with respect to how long the words appeared on the left or right side of the screen. In two symmetric conditions, the words appeared either for 1 s on the left or right side of the screen (half on each side) or for 3 s on either side. These conditions allowed me to see whether curved ROC curves would again be found for a source discrimination procedure; they also provided information as to whether increasing the presentation time would lead to an increase in attention, as measured by the mixture parameter. In two asymmetric conditions, the presentation times were different across the two sources: The times were 3 s left, 1 s right for one condition and 1 s left, 3 s right for the other condition; the effects of asymmetry of presentation time on the ROC plots and mixture parameter estimates are examined below.

Results

Symmetric Conditions

The top part of Table 3 shows *LR* statistics and information criteria for fits of the unequal variance model and the mixture model of Equation 4 to the data from the two symmetric conditions, with 1-s and 3-s presentation times for each source. The *LR* statistics indicate rejection of the unequal variance SDT model for both conditions. However, the fit of the mixture model of Equation 4 is adequate, in that the *LR* statistics are not significant; both information criteria are also considerably smaller for the mixture model.

Figure 5 shows the data and fitted mixture *z*-ROC curves (for Equation 4). The figure shows why the unequal variance SDT model fits poorly, in that the curves are clearly curved upward, as was also found for the experiments of Hilford et al. (2002) and Yonelinas (1999). Thus, the present results join a growing body of evidence showing that the unequal variance SDT model does not describe data from source discrimination studies, in that *z*-ROC curves are not linear but, rather, are curved upward. The mixture SDT model, in contrast, accurately describes the curvature and provides a simple explanation for it (i.e., it results from mixing).

The parameter estimates and standard errors for a fit of Equation 3 are also shown in Figure 5. The estimates of the discrimination parameter d_{AB} are large (5.2 and 4.2) for both presentation times; with the standard errors taken into account (and noting that a 95% confidence interval is formed by multiplying the standard errors by 1.96 and adding and subtracting), the results indicate that discrimination did not differ significantly across the two conditions (in that 95% confidence intervals overlap). The estimates of λ, however, differed across the conditions; the estimate is .27 for the 1-s left, 1-s right condition and .41 for the 3-s left, 3-s right condition (and 95% confidence intervals do not overlap). If λ indeed reflects attention, then the results suggest that an increase in presentation time for both sources from 1 s to 3 s led to an increase in the percentage of items for which source was attended to, from 27% to 41%, with no apparent effect on discrimination strength (i.e., d_{AB}). Thus, the results add to earlier evidence (see DeCarlo, 2002b) that manipulating presentation time affects the mixing parameter, which supports the view that the mixing parameter provides a measure of attention. In addition, the results also suggest that λ and d_{AB} can vary independently, in that the estimate of λ is clearly larger for the 3-s presentation times than for the 1-s

Table 3
Goodness-of-Fit Statistics and Information Criteria for the Four Conditions of the Current Experiment (N = 1,440)

Model	1-s left, 1-s right					3-s left, 3-s right				
	LR	*df*	*p*	AIC	BIC	*LR*	*df*	*p*	AIC	BIC
Symmetric conditions										
Unequal variance SDT	34.11	3	< .01	4,924	4,961	17.01	3	< .01	4,734	4,771
Mixture SDT	0.87	3	.83	4,752	4,789	2.02	3	.57	4,581	4,618
	3-s left, 1-s right					1-s left, 3-s right				
	LR	*df*	*p*	AIC	BIC	*LR*	*df*	*p*	AIC	BIC
Asymmetric conditions										
Unequal variance SDT	11.36	3	.01	4,919	4,956	5.90	3	.12	4,525	4,562
Mixture SDT	13.06	3	< .01	4,921	4,958	0.59	3	.90	4,520	4,557
Equation 3 with $\lambda_A \neq \lambda_B$	4.60	2	.10	4,776	4,818	0.58	2	.75	4,522	4,564

Note. *LR* = likelihood ratio goodness-of-fit test; AIC = Akaike's information criterion; BIC = Bayesian information criterion; SDT = signal detection theory; Mixture SDT = Equation 4 in the text.

presentation times, but the estimate of d_{AB} is not larger for the 3-s presentation times.

Asymmetric Conditions

The lower part of Table 3 shows that for the 3-s left, 1-s right condition, the *LR* statistic is significant for both the unequal variance SDT model and the mixture model of Equation 4. As shown by the plots presented in Figure 6, this significance occurs because the *z*-ROC curve is nonlinear and asymmetric. The table also shows that relaxing the restriction $\lambda_A = \lambda_B$ for this condition results in a model with acceptable fit in terms of the *LR* statistic; both of the information criteria also are considerably smaller than those for the unequal variance model or Equation 4. Thus, an effect of asymmetric presentation times is clearly apparent in this condition. For the 1-s left, 3-s right condition, however, Table 3 shows that fits of the unequal variance SDT model, Equation 4, and Equation 3 with $\lambda_A \neq \lambda_B$ are all acceptable. There is little difference with respect to fit for Equation 4 and Equation 3 because, as shown by the *z*-ROC plot presented in Figure 6, the asymmetry, if any, is small (as is the amount of curvature, which is why the unequal variance model also is not rejected). In this case, both information criteria are smallest for Equation 4; the criteria favor the simpler model of Equation 4 because Equation 3 provides little improvement in fit, the unequal variance model provides no improvement, and the criteria include a penalty term for additional parameters (with BIC using a heavier penalty).

Figure 6 shows plots of the data and fitted mixture *z*-ROC curves. Asymmetry in the *z*-ROC plot is apparent only for the 3-s left, 1-s right condition, shown in the left panel; note that the data suggest a curve with a slope greater than unity (for a fit of the unequal variance SDT model, the estimates of the slope differed significantly from unity only for the 3-s left, 1-s right condition). For the 1-s left, 3-s right condition, shown in the right panel of

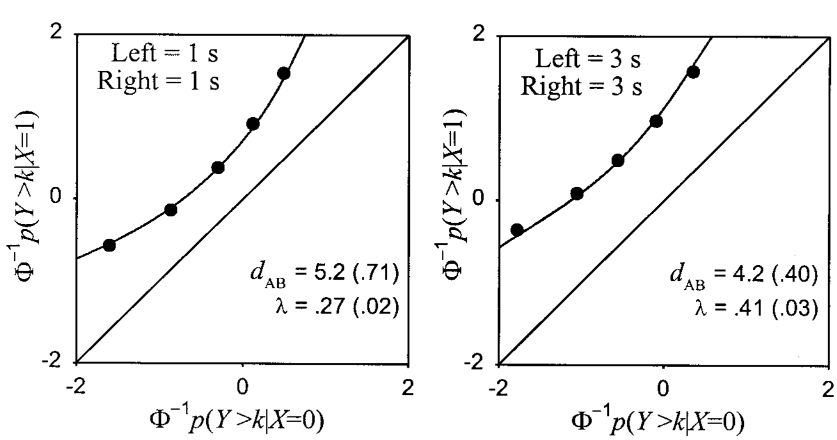

Figure 5. Data and fitted mixture receiver operating characteristic curves on inverse normal coordinates for the two symmetric conditions of the current experiment.

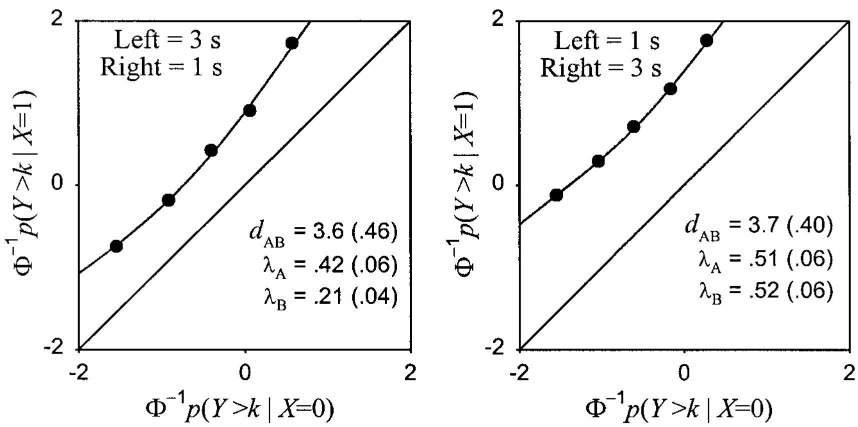

Figure 6. Data and fitted mixture receiver operating characteristic curves on inverse normal coordinates for the two asymmetric conditions of the current experiment.

Figure 6, the *z*-ROC plot appears to be slightly curved but nearly symmetrical.

Figure 6 also shows that the estimates of the discrimination parameter (3.6 and 3.7) are close in magnitude across the two asymmetric conditions. The left panel of Figure 6 shows that the estimates of λ_A and λ_B for the 3-s left, 1-s right condition are .42 for 3-s left words and .21 for 1-s right words. This difference can be interpreted as showing that, with a presentation time of 3 s for left words, the source of 42% of the words was attended to, whereas with a presentation time of 1 s for right words, the source of only 21% of the words was attended to. Thus, an asymmetric effect of presentation time was found; the results are also consistent with those found above for the symmetric conditions, in that the estimate of λ was larger for a longer presentation time. The right panel of Figure 6 shows that the estimates of λ_A and λ_B are close in value for 1-s left and 3-s right presentation times, and so an asymmetric effect of presentation time was not found in this case. Note that both estimates of λ are rather large (around 51%) as compared with those obtained in the other conditions; the large estimates of λ suggest that observers in this condition might simply have had higher levels of attention overall, which could reflect a sampling fluctuation or possibly a position effect of some sort. Additional studies are needed.

In summary, the results for all four conditions of the current experiment show that curved *z*-ROC curves are again found for source discrimination experiments. This is important evidence in favor of the mixture SDT model, which provides a simple account of why curvature is found: because of mixing for both A and B items. In addition, the results suggest that increasing the presentation time from 1 s to 3 s in symmetric conditions led to an increase in the percentage of words for which the source was attended to. This relationship was shown by the larger estimate of λ obtained in the 3-s left, 3-s right condition (41%) as compared with that in the 1-s left, 1-s right condition (27%). Further, the results for the 3-s left, 1-s right asymmetric condition suggest that the source of twice as many 3-s words (42%) received attention than did the source of 1-s words (21%). Thus, this condition provides evidence, in addition to that found for the symmetric conditions, that a manipulation of presentation time affects λ. The results for the 1-s left, 3-s right condition, in contrast, did not show differences in the estimates of λ (51%) across the sources, and so the simple model of Equation 4, with equal attention and equal strengthening across the sources, was adequate. The *z*-ROC plot in this case is still slightly curved, which is consistent with the curvature found in the other conditions; however, the curve does not show any apparent deviations from symmetry, and whether this is an anomalous result remains to be determined.

The results discussed above were for the data pooled across participants. Equation 4 was also fit to the individual data of the 48 participants. The main limitation associated with analysis of individual data is the small sample size for each participant (120 trials), which can lead to estimation problems. For example, the LEM program (Vermunt, 1997) did not converge on a solution for 3 of the 48 participants, and in some other cases, the ratio of largest to smallest eigenvalues was very large (e.g., $>$1,000), which indicates weak identification, meaning basically that the standard errors of the parameter estimates were large or could not be estimated. For examples of these problems in the context of SDT with latent classes, see DeCarlo (2002a). With respect to the estimates of λ, the problem of weak identification occurred for only 2 participants (in that the estimates of the standard errors of λ were greater than 1.0, whereas they were generally less than 0.2 in all other cases). Thus, 5 cases were dropped for the individual analysis (3 because of failure of convergence and another 2 because of weak identification). For the remaining 43 participants, the mean estimates of λ computed from the individual estimates were .30 for the 1-s left, 1-s right condition and .45 for the 3-s left, 3-s right condition (for 11 and 10 participants, respectively), which is consistent with the estimates obtained for the pooled data (as shown in Figure 5): .27 and .41, respectively. So, for both individual and pooled data analysis, the estimate of λ was clearly larger for the 3-s condition. For the 3-s left, 1-s right condition, the mean estimates were $\lambda_A = .26$ and $\lambda_B = .51$ (with 11 participants in each condition, for Equation 3 with $d_A = d_B$), which are similar to the estimates of .21 and .42 obtained for the pooled data. For the 1-s left, 3-s right condition, the mean estimate of λ (for Equation 4, with 11 participants) was .49 for the individual analysis, which is close to the estimate of .51 obtained for the pooled analysis. Thus, with respect to λ, the results for the individual analysis lead to the same conclusions as do those for the pooled analysis, as was also found by Yonelinas (1999), Hilford et al. (2002), and DeCarlo (2002b).

Discussion

The extension of SDT through finite mixtures of the underlying distributions yields a general class of signal detection models. The mixtures are viewed in mixture SDT as arising from the action of a second process, which gives rise to latent classes of items, with the mixing parameter λ indicating the proportion of each type of item, such as attended or nonattended items. For detection, it is assumed in mixture SDT that there is mixing only on signal (old-item) trials, and the result is that *z*-ROC curves have slopes less than unity. For source discrimination, it is assumed in mixture SDT that there is mixing on both A- and B-item trials, with the result that *z*-ROC curves tend to be curved upward. Thus, the mixture approach to SDT provides a unified account of detection and discrimination by showing that the same process (e.g., attention) can lead to the rather different results that are found across the two situations—that is, nearly linear curves with nonunit slopes for detection and nonlinear curves with upward curvature for discrimination. This account represents an important unification of two seemingly disparate results.

Equation 4 offers a simple model that is clearly useful for the analysis of source discrimination data; in a recent comparison of this model with other models of source memory, Hilford et al. (2002) arrived at a similar conclusion. The more general model of Equation 3 introduced here offers two basic ways to relax assumptions of Equation 4: It allows for unequal source memory strength across the two sources—conceptualized in mixture SDT as unequal distances from a nonattended distribution (i.e., d_A and d_B)—and it allows for unequal levels of attention across the sources (i.e., λ_A and λ_B). This article examines situations in which Equation 4 should suffice (sources treated symmetrically) and situations in which the more general model of Equation 3 might be needed (sources treated asymmetrically).

For experiments that used a procedure in which the sources were treated symmetrically, the simple source discrimination mixture model of Equation 4 was apparently sufficient, as reflected by the finding of symmetric (and curved) *z*-ROC curves for the experiments of Hilford et al. (2002), Yonelinas (1999), and the current study. For experiments in which the sources were treated asymmetrically, *z*-ROC curves were curved upward in all cases, which is consistent with the results found for the other experiments. Deviations of the curves from symmetry ranged from absent to large. For example, Experiment 4 of Yonelinas, with a 5-day delay between the study lists, provided clear evidence of a differential effect of delay on memory for source and, most important, this effect was reflected by the parameters of the general model of Equation 3. This is evidence in favor of the mixture model, in that it shows that the parameters behave as expected in response to a strong experimental manipulation. The delay between lists was much smaller for the other two experiments of Yonelinas, in that one list was simply presented after the other in the same session, and a differential effect on the parameters of Equation 3 was not as apparent; the point estimates of d_A and d_B suggested, at most, a small effect. Overall, the results suggest that source memory strength, as measured by d_A and d_B, can be affected by using a delay between presentation of a source and test.

The experiment presented here differs from earlier ones in that it represents an attempt to manipulate another parameter of the model, namely the mixture parameter λ. In certain ways, it is easier to manipulate discrimination strength in source discrimination experiments, because one can use a long delay between the lists—as done by Yonelinas (1999), for example—which appears to have a large effect on parameter estimates and *z*-ROC curves. The situation is more difficult with respect to attention, in that it is harder to gain control over attention. As shown here and in earlier studies, manipulating the presentation time appears to have an effect, though the effect is clearly not as large as that seen with a 5-day delay. One could try to increase the size of the effect by using longer presentation times, but longer times might not necessarily mean greater attention—participants could become bored and actually end up paying less attention; note that the relatively small estimate of λ (.18) found in Experiment 3 of Yonelinas suggests a case in which this may have happened, in that the results for the mixture model indicate that attention was low (18%) in a condition in which a list was presented twice. In any case, for the current experiment, the estimate of λ was larger in a symmetric condition in which both sources were presented for 3 s than it was in a condition in which the sources were presented for 1 s, which is consistent with results found for presentation time in detection studies (DeCarlo, 2002b). The results for the asymmetric conditions were mixed, in that different estimates of λ_A and λ_B were found in one condition but not in the other. Additional research that attempts to manipulate attention in source discrimination experiments is needed.

It is assumed, in this study, that the source of each word is either attended to or not. The present results show that this assumption appears to be adequate, in that the mixture model clearly described the data. Here I note that one can, however, extend the model to include a greater number of discrete levels of attention, as long as the number of parameters is less than or equal to the number of observations (so that the model is identified). It is also possible to include additional latent classes by relaxing the assumption that nonattended A and B words have the same location. The simple two-level model (i.e., with attended and nonattended sources) fit quite well for all of the studies analyzed here, however, so there is little motivation for adding additional parameters (which does little or nothing in terms of improving fit, and the information criteria include a penalty for the introduction of additional parameters). In my view, conceptualizing the source of the items as being either attended to or not, as done here, is both theoretically plausible and empirically defensible.

In conclusion, the mixture extension of SDT unifies results obtained across both detection and discrimination studies. The mixture approach offers a new lens through which to view and interpret source discrimination data; it offers a framework that, ideally, will help researchers to develop and refine further experimental tests of the model, to compare the model with other models (which need to be formally developed and tested, as done here), and to delineate the advantages and limitations of the mixture approach as compared with other approaches.

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Appendix

Notes on Fitting the Mixture SDT Model

The mixture SDT model can be fit using software packages that offer latent class analysis. For example, maximum-likelihood estimates of the model's parameters were obtained here using the software LEM (Vermunt, 1997), which is freely available on the Internet (http://www.uvt.nl/faculteiten/fsw/organisatie/departementen/mto/software2.html). The general model shown by Equation 3 can be implemented in LEM by rewriting the model with the use of a latent dummy-coded variable W (i.e., which indicates attention). Specifically, the general model of Equation 3 can be written as

$$p(Y \leq k|X = x) = \sum_w p(W = w|X = x)p(Y \leq k|W, XW),$$

where $p(W|X)$ gives the (conditional) mixing parameters (λ_A and λ_B in Equation 3), and the second term can be written as a cumulative probit model,

$$p(Y \leq k|W = w, XW = xw) = \Phi(c_k - d_B w - d_{AB} xw).$$

Note that this parameterization gives direct estimates of the discrimination parameter d_{AB} and the location of one distribution (B) with respect to the location of the nonattended distribution (given by the parameter d_B). The model is specified in LEM by the terms $p(W|X)$ and $p(Y|XW)$, with a cumulative probit model specified for $p(Y|XW)$. To fit the model with the restriction $\lambda_A = \lambda_B$, the term $p(W|X)$ is replaced by $p(W)$; to fit the model with the restriction of $|d_A| = |d_B|$, only the term XW is included in the second equation, with the term W dropped and the coding for X changed from dummy coding to effect coding (-1, 1). An LEM program for the data of Experiment 1 of Yonelinas (1999) is available at http://www.columbia.edu/~ld208.

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