Does Wage Volatility Matter in Labor Markets? Theory and Evidence on Labor Mobility*

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Abstract

We present theory and evidence on the effects of wage volatility on labor mobility. Our model of job turnover explicitly incorporates variance of within-job wages by assuming that wages evolve as random walk processes. With the additional assumption that job changes entail "switching" costs, the key theoretical result is that the optimal threshold of turnover – the minimum wage difference between outside and inside jobs necessary for a job change – is positively related to wage volatility. Data from the National Longitudinal Surveys of Youth show that workers who hold more volatile jobs get bigger wage gains when they quit and move to a new job, and that they quit less frequently if their jobs are characterized by high within-job wage growth rates. These findings are consistent with the implications of our theoretical model.

Key words: Labor mobility, Within-job wage volatility, Random walk, Multi-armed bandit problem

JEL Classification: J30, J60

1 Introduction

The shift in focus from labor mobility as a mere means of allocative efficiency in the labor market to labor mobility as an outcome of individual investment decisions has provided important insights into the structure of inter-firm mobility. Since the prospect of higher wages is a major impetus for voluntary job changes, the analysis of inter-firm labor mobility and its associated (mobility) wage gains is a central topic in modern labor economics. However, it is somewhat surprising that the role of wage volatility has been largely unexplored in the voluminous literature on labor mobility and wages, especially since within-job wage evolution is often modeled as a stochastic process. In this paper we attempt to fill this gap by presenting a theoretical and empirical analysis of wage volatility and its effects on labor mobility. First, we present a theory of job turnover that explicitly models the role of wage volatility in predicting labor mobility. Second, using data from the National Longitudinal Surveys of Youth, we analyze the effects of wage volatility on quit rates and mobility wage gains. Our empirical findings are consistent with the implications of the theory.

The modern literature on earnings dynamics views the evolution of labor productivity and wages as stochastic processes. For example, in their classic 1982 article "A Theory of Wages," Harris and Holmstrom assume that a worker's productivity on a job is subject to random shocks. Since firms are risk neutral and workers are risk averse, they derive an equilibrium wage contract that provides partial insurance against productivity shocks to the worker. Although this wage contract (which is non-decreasing) is stochastic, it is clearly designed to attenuate the presumed volatility in worker productivity. In another theoretical article on wages and turnover, Mortensen (1988) explicitly models wage evolution as a stochastic process. Topel and Ward (1992) present evidence that within-job wages do in fact evolve as a random walk process with drift. Although the stochastic nature of productivity and wage evolution is central in these studies, the literature has yet to address volatility effects in labor markets. This omission is particularly striking given the enormous theoretical and empirical significance of volatility effects in financial markets.

Our model of job turnover is based on a two-job framework where an employed worker in every period has the option of either staying with the current (inside) job or switching to an outside (future) job.[1] The important feature of the model is that the within-job wage evolves as a random walk process. This random walk assumption is of course the source of variance of within-job wages – i.e. of wage volatility. With the additional assumption that job changes entail irrecoverable switching costs, the key theoretical result is that the optimal threshold of turnover – i.e. the minimum wage difference between the outside job and current job necessary for a worker to switch jobs – is positively related to wage volatility.[2] The intuition follows from the fact that volatility increases the value of delaying the decision to switch jobs. At the optimum threshold the worker is indifferent between switching jobs now versus later. However, if volatility increases the worker can avoid the increased downside risk by waiting. And this delay is achieved by setting a higher optimal threshold of turnover. Although a few earlier studies (Kiefer and Neumann 1989; Macdonald 1988; Sargent 1987) refer to the possible link between "labor market volatility" and the relative valuation of outside versus inside jobs, no previous theoretical study has derived a link between volatility and labor mobility. To our knowledge this is the first model of inter-firm labor mobility to explicitly derive a relationship between within-job wage volatility and the optimal threshold of turnover.[3]

The theoretical result that volatility is positively related to the optimal threshold of turnover of course cannot be directly tested because we do not observe this threshold of turnover. However, by considering the threshold effects on quit rates and mobility wage gains, we can discern some testable implications of volatility on these observed mobility

[1]We consider embedding this two-job framework within a more realistic search framework where the outside job is interpreted as arising periodically from a stationary distribution of outside jobs. In Section 2.4 we discuss how our model is related to job search theory.

[2]The model here is an adaptation of modern investment theory – which is in turn a generalization of financial option pricing theory – into the labor mobility framework. The options pricing approach to investment is based on the insight that "irreversibility and the possibility of delay are very important characteristics of most investments in reality" (p. 6 in Dixit and Pindyck 1994). These two features are well-known in the labor mobility literature: first, job changes entail irrecoverable moving costs; and second, workers typically have the option of delaying a switch from one job to another. What is absent in the labor literature, unlike in the investment literature, is an analysis of the ramifications of wage evolution as a random walk process on other related outcomes. As a consequence when the random walk assumption is explicitly introduced into the mobility framework, the standard result from modern investment theory, which says that higher volatility increases the value of delaying costly investments because the increased downside risk can be avoided by waiting, also holds in the context of labor mobility.

[3]Search and mobility costs are of course key features of a variety of models that underpin empirical analyses of the joint determination of wages and mobility (e.g. Black and Loewenstein 1991; Barron et al 1993; Kuhn 1993). Given this extensive literature on mobility costs we do not focus on the ramifications of switching costs *per se* on mobility outcomes.

outcomes. Moreover, our model identifies within-job wage growth as a factor that influences the effect of volatility on quit rates that we can explicitly test.

The effects of volatility on quit rates and mobility wage gains are somewhat complicated because volatility not only affects the threshold of turnover (as predicted by theory) but also the distribution of the wage difference between the outside and inside jobs. For example, if volatility only increased the threshold, then volatility will be negatively correlated with quit rates because a higher threshold simply reduces the likelihood of a job switch. However, this negative effect is counteracted by the fact that any positive wage difference between the outside and inside jobs increases if wage volatility increases; thus making job switches more likely for any given threshold. Hence the net effect of volatility on quit rates will depend on which of these two effects dominate. In the model section we explicitly characterize this condition and identify an implication to test the theory. Since the model shows that volatility is more likely to have a negative effect on turnover if the within-job wage growth rate (the drift parameter) is larger, we can explicitly test this implication by including an interaction term between volatility and mean within-job wage growth in a quit regression.[4]

The effect of volatility on mobility wage gains is far less ambiguous. For instance, if volatility does not lead to a higher quit rate – which is indeed possible if the positive effect of volatility on the optimal threshold is sufficiently large – then volatility will necessarily increase mobility wage gains. Of course, this positive relationship can hold even if the quit rate increases, although not necessarily. Hence a positive coefficient on volatility in a mobility wage gain equation and a negative coefficient on the interaction between volatility and mean wage growth in a quit equation are empirical results that would support the theory.

The major challenge of testing these implications is the construction of an appropriate empirical counterpart of within-job wage volatility. The difficulties are both conceptual and practical. The first question is the employment context over which volatility is likely to significantly vary. In econometric parlance, is wage volatility a job-, person-, industry- or occupation-specific effect? If we follow the theoretical model then the most disaggregate measure of volatility would be based on job-specific wage data. Panel data sources notwith-

[4]In the next section we present a simpler version of our model to more easily illustrate these theoretical predictions and the conditions under which an increase in volatility leads to a reduction in quit rates.

standing, the practical difficulty of obtaining such a job-specific measure is the loss of a huge fraction of the job sample due to under-selection of short duration jobs.[5]

An alternative measure – adopted in this paper – is to compute a person-specific wage volatility measure based on within-job wage changes over the entire observed individual life cycle of employment. The obvious advantage of a person-specific volatility measure is that we avoid sample selection problems that plague the construction of a job-specific measure. The disadvantage is that if a significant component of the variation in wage volatility is job-specific and workers randomly move across jobs characterized by different volatilities, then our person-specific measure could obscure this variation in volatility. As a result, any lack of evidence of volatility effects using person-specific measures, would not necessarily rule out the existence of job-specific volatility effects. On the other hand, evidence of person-specific volatility effects would not also necessarily suggest that volatility is a person effect. For example, if workers move among similar volatility jobs or industries or occupations then our person-specific measure would simply proxy volatility variations across these different employment dimensions. Evidence of person-specific volatility effects, however, would provide support for our theory and highlight the role of wage volatility in predicting labor market outcomes even though we might not be able to pinpoint the exact source of volatility variation. A decomposition analysis of wage volatility is a topic for a future research agenda.

In the empirical section we compute person-specific within-job wage growth and within-job wage volatility measures, and estimate the effects of these measures on the likelihood of quit rates and on mobility wage gains due to voluntary job changes. To preview our findings: workers who hold more volatile jobs get bigger wage gains when they quit and move to a new job, and they quit less frequently especially if their jobs are characterized by high within-job wage growth rates. These findings are highly significant and robust to various model specifications, sample selection corrections, and the inclusion of an extensive array of control variables related to individual, job and other relevant characteristics.

[5] Note that most micro survey data provide only annual wage data and job durations are relatively short especially among workers in the early part of their careers. Since a minimum of four within-job wage observations are required to compute a sample variance of (de-trended) job-specific wages, many jobs will have missing volatility information. Of course, even in principle, time series of job-specific wage data will be limited because (nominal) wage adjustments, unlike say stock price data, occur infrequently for a variety institutional and other reasons.

Our empirical analysis of volatility effects is a significant departure from the existing literature because no previous study has analyzed the effects of wage volatility on outcomes related to labor mobility. The empirical literature on turnover and mobility wage gains is largely motivated by testing implications of specific human capital and job search theories. For example, Antel (1986) argues that the comparison of wages between movers and stayers underestimates mobility wage gains (due to search) because stayers experience higher within-job wage growth. Topel (1991) makes the converse case that the comparison of wages of stayers and movers underestimates the true returns to specific skills because movers move to more lucrative jobs. Mincer (1993) analyses differential wage gains conditional on quits and layoffs across older and younger workers. He finds wage gains are typically positive, higher for quits than for layoffs, and higher for younger than older movers. Bartel and Borjas (1981) find that workers who quit due to finding a better job experience significant wage gains compared to their counterparts who move for other reasons.

One final issue of interpretation is whether our empirical findings might be compatible with risk aversion on the part of workers. For example, the fact that volatility reduces quit rates and increases mobility wage gains could be consistent with risk aversion if volatility pertains to wages of future jobs. Clearly, risk averse workers would be less likely to switch to more volatile jobs, and if they did switch to such jobs, they would need a compensating wage differential to offset the risk. A direct test of this hypothesis would correlate wage volatilities of past versus future jobs with quit rates and mobility wage gains and evaluate whether the effects of future job volatilities on these mobility outcomes are stronger. However, as mentioned earlier, data limitations prohibit the computation of job-specific volatility measures. We do however present an indirect test of sorts by exploiting the exact timing of the incidence of quits and observed mobility wage gains. For example, our person-specific measure will largely reflect wage volatility of future/past jobs if the incidence of quits and mobility wage gains occur early/late in a worker's career.[6] We can empirically address whether our estimated effects of volatility are stronger in the early part of a worker's career since our

[6] This interpretation of risk aversion of course hinges on the assumption that volatility is job specific. Under this presumption the proposed test is somewhat imprecise because our person-specific measure of volatility represents an aggregate over several jobs, even though we can determine at every point in time whether our person-specific volatility measure is based on (largely) past or future wage observations.

measure would then reflect wage volatility of future jobs. The evidence does not support the risk aversion hypothesis.

The remainder of the paper is arranged as follows. The next section presents the theoretical analysis. In the first part we present a simpler version of our theory to clarify the intuition behind the main theoretical result – i.e. the positive relation between volatility and the optimal threshold of turnover – and to highlight the effects of volatility on mobility wage gains and quit rates and to derive the various implications that we can test. Next we present the general model of turnover, and derive the optimal switching rule and various comparative static results. We conclude the theory section with a detailed discussion of the relation of our model to job search theory. Section 3 presents the empirical analysis. First, we describe the NLSY data and the construction of our volatility measure. Second, we detail the estimation framework. Third, we present and discuss our empirical findings. Section 4 concludes with a brief summary.

2 Theoretical Analysis

2.1 A Motivating Example

In this section we present a simple example of our model of turnover to motivate the key theoretical result of the paper and to illustrate the links between wage volatility and labor mobility. Here we consider a simple model in which wages are permanently shocked only once at the beginning of the second period. As a consequence the worker simply decides whether to switch jobs in the current period or to postpone this decision to the next period. Despite this restriction, our example helps to clarify the intuition for why volatility increases the optimal threshold of turnover and to illustrate why wage volatility leads to higher mobility wage gains and lower turnover especially if the within-job wage growth rate is high. In the next subsection we present the full model where within-job wages evolve as a random walk process.

Denote w^i for $i = a, b$ as wage of job i in period 1. Assume that job b is the currently-held job. Suppose that for all $i = a, b$, w^i can either increase by h with probability p, or decrease by h with probability $1 - p$ at the beginning of the second period. Hence the only decision for the worker is whether to switch now or to delay this decision to the second period.

The present value of expected net income V_{switch} if the worker switches jobs in period 1 is given by:

$$V_{switch} = w^a - C + \frac{\beta}{1-\beta}\left(w^a + ph - (1-p)h\right), \tag{1}$$

where C is the switching cost, and β is the discount factor. If the worker decides to delay the switching decision to the second period then the present value of net income V_{wait} is given by:

$$\begin{aligned} V_{wait} &= w^b + \frac{\beta}{1-\beta}\left(\begin{array}{c} p^2 \max\left(w^a + h, w^b + h\right) + p\left(1-p\right)\max\left(w^a - h, w^b + h\right) \\ +p\left(1-p\right)\max\left(w^a + h, w^b - h\right) + \left(1-p\right)^2 \max\left(w^a - h, w^b - h\right) \end{array}\right) \\ &= w^b + \frac{\beta}{1-\beta}\left(\left(w^a - w^b - C\right)\left(1 - p\left(1-p\right)\right) + w^b - h\left(1 + 2p^2 - 4p\right)\right) \end{aligned} \tag{2}$$

Equating (1) and (2) at the threshold wage differential $s = w^a - w^b$, and rearranging terms, we get

$$s = \gamma_1 C + \gamma_2 \tag{3}$$

where $\gamma_1 = \frac{1-\beta(2-p(1-p))}{1-\beta(1-p(1-p))}$ and $\gamma_2 = \frac{2\beta p(1-p)h}{1-\beta(1-p(1-p))}$. Note that mean within-job wage growth, $wjwg$, is given by:

$$wjwg \equiv ph - (1-p)h = h\left(2p-1\right). \tag{4}$$

Assume that $p > 1/2$ to make $wjwg$ positive. Now consider the case where $wjwg$ is held constant. Clearly s increases as switching costs C increase. The more interesting comparative static result is the effect of wage volatility on s. Note that from (4), p can be expressed as follows:

$$p = \frac{h+k}{2h}, \tag{5}$$

where $k = wjwg$. Substituting (5) into equation (3), and taking the derivative with respect to h, we get

$$\frac{\partial s}{\partial h} > 0,$$

which follows from the fact that both γ_1 and γ_2 are increasing in h. Therefore the change in the optimal threshold of turnover due to a change in volatility (h) is positive. Clearly an increase in wage volatility leads to a higher optimal threshold of turnover.

Since we do not observe s we cannot of course directly test this theoretical result of the model. However, in the context of this simple example we can inquire about the likely ramifications of an increase in wage volatility on the quit rate and mobility wage gains. Note, the quit rate is determined by the frequency with which a given wage difference between the outside job and current job reaches s. Starting from a given s, this frequency is a positive function of h and a negative function of s. So whether an increase in h implies a higher or lower quit rate depends precisely on how volatility (h) affects the optimal threshold s. If s increases substantially due to an increase in h – i.e. where the derivative ds/dh is sufficiently large – then the likelihood that volatility will have a negative effect on the quit rate will also increase. So the question is under what conditions will this derivative of s with respect to h will be relatively large. Note that $\frac{d^2}{dhdk}\gamma_1 > 0$ and $\frac{d^2}{dhdk}\gamma_2 > 0$. So, from equation (3), the cross derivative with respect to h and k – mean within-job wage growth – turns out to be positive:

$$\frac{\partial^2 s}{\partial h \partial k} > 0.$$

This condition implies that the positive volatility effect on the threshold of turnover is increasing with the within-job wage growth rate.[7] Hence it is more likely that volatility will have a negative effect on the quit rate if within-job wage growth is higher. This model implication can be tested by including an interaction between volatility and within-job wage growth in a quit regression. The model predicts that the estimated coefficient on this interaction term should be negative.

The effect of an increase in h on mobility wage gains is much less ambiguous. If an increase in volatility does not lead to an increase in the quit rate then mobility wage gains will necessarily increase with volatility. Note that every observation s that implies a job change will have a correspondingly higher wage difference because of the volatility increase. But even if the quit rate increases with volatility, mobility wage gains are still likely to increase with volatility. If the quit rate increases then some positive wage differences that were smaller than the threshold prior to the volatility increase would have to increase sufficiently to cross the new threshold (even though this new threshold is now higher than it was before

[7] See the discussion following Theorem 2 below for a more formal explanation of this result.

the volatility increase). Such observations could in principle have a negative effect on the expected value of mobility wage gains. However, for volatility to have a net negative effect on mobility wage gains, this possible negative effect must dominate the two positive effects – via the increases in the threshold and in the positive wage differences – that result from the increase in volatility.

Before we take these predictions to our empirical analysis we present the full model where wages evolve as a random walk process in every period.

2.2 Model of Inter-Firm Mobility

In this section we present our full model where inter-firm mobility is based on irrecoverable switching costs and the stochastic evolution of within-job wages. First, assume that a worker is currently employed at say job b. The wage at this "inside" job evolves over time as a random walk process with drift. Suppose at the beginning of each time period the worker has the option to switch to an outside job a. If the worker switches to job a then the wage in job a also evolves as a random walk process with drift.[8] For analytical simplicity, assume that all jobs evolve as symmetric random walk processes with drift. We begin with the definition of wage dynamics:

Definition 1 (Wage Dynamics) *For job $i \in \{a, b\}$, the incremental process $\{x_t\}$ is an i.i.d. sequence with*

$$\Pr(x_t = h) = p, \ \Pr(x_t = -h) = 1 - p,$$

where $p \in [0, 1]$ is the drift parameter and $0 < h < \infty$ is the scale parameter. Then the wage of job i at time t is simply $w_t^i = \sum_{n=0}^{t} x_n$.

At the beginning of every time period t the worker decides whether to switch to the outside job or to remain in the current job till the next period. If the worker decides to switch jobs then she must incur a fixed and irrecoverable cost C.[9] In addition, assume that the worker is risk neutral – i.e. utility is a linear function of wages – and infinitely

[8] The modeling details below assume that both jobs a and b evolve simultaneously as independent and identical random walk processes. The main conclusions of the model are robust to the alternative assumption that either the inside or the outside job is frozen (Jun 2004). As it turns out the only relevant statistic for labor mobility in this framework is the wage difference between the outside and inside jobs as long as jobs are symmetric.

[9] Although we do not explicitly model the source of switching costs, the obvious reasons are direct monetary and psychological costs of moving, and possible costs associated with relocation of family.

lived. Hence the worker's problem is to maximize the expected discounted stream of lifetime net income, where net income at any given time is the wage minus the switching cost C, conditional on a job switch.

Since all jobs evolve as symmetric random walk processes the worker's decision to switch jobs depend only on the wage difference between the outside job option and the current job. Therefore, given switching cost C, the worker only needs to evaluate the wage difference between the outside job option and the current job to decide whether to switch or not. This implies that there is a *reservation* value such that whenever the potential mobility wage gain exceeds this value the worker will switch jobs. This reservation value will depend on switching costs C, and more importantly, on the properties of the random walk process of wages – i.e. within-job wage volatility. The point, however, is that the switching rule depends on neither the level of wages nor the identity of jobs.

The Bellman representation of the decision problem faced by a worker can be stated as follows:

$$v^b\left(w^a, w^b\right) = \max\left[w^a - C + \beta V^a,\ w^b + \beta V^b\right], \tag{6}$$

where $v^i\left(w^a, w^b\right)$ denote the value function for a worker who is currently employed at job i, given w^a and w^b. The continuation value of job i to worker is:

$$\begin{aligned} V^i = {} & p^2 v^i\left(w^a + h, w^b + h\right) + (1-p)^2 v^i\left(w^a - h, w^b - h\right) + \\ & p(1-p)\, v^i\left(w^a + h, w^b - h\right) + p(1-p)\, v^i\left(w^a - h, w^b + h\right). \end{aligned}$$

The solution to this problem is to find the optimal switching rule – the minimum wage difference between the outside job and current job for a worker to switch jobs – given the fixed cost C of switching jobs.

2.3 Optimal Threshold and Some Comparative Statics

We now derive the minimum wage difference between the outside job option and the current job that is necessary for a worker to switch jobs. Let s denote this threshold for job turnover. Theorem 1 below details the solution to the optimal job switching rule.

Theorem 1 (Optimal Job Switching Rule) *The worker will switch jobs if the wage difference between the outside and inside jobs is greater than or equal to the optimal threshold of turnover s, which is given by:*

(1.1) $s = C(1-\beta) + 2\beta p(1-p)$ *if* $s < h$,
(1.2) $s = C(1-\beta) + 2h\rho\frac{1-\rho^{s/h}}{(1-\rho)\left(1+\rho^{s/h+1}\right)}$ *if* $s \geq h$ *and* s/h *is an integer,*
(1.3) $s\left(1-\frac{\rho^l(1+\rho)}{1+\rho^{2l+1}}\right) = C(1-\beta)\left(1+\frac{\rho^l(1+\rho)}{1+\rho^{2l+1}}\right) + \frac{2h}{1+\rho^{2l+1}}\left(\rho\frac{1-\rho^{2l}}{(1-\rho)} - (\rho+1)k\rho^l\right)$,
if $s \geq h$, s/h *is not an integer, and where* $s/h < l < s/h+1$, *where*
$\rho = \frac{2p(1-p)\beta}{1-2\beta p^2-\beta+2\beta p+\sqrt{(1-4\beta p^2-\beta+4\beta p)(1-\beta)}}$.

We present the details of the derivation in the Appendix. Here we outline a sketch of our proof strategy. First, consider a worker who always switches jobs whenever the current wage gain is at least as large as some minimum wage gain. Second, we compute the sum of present discounted expected net incomes from this threshold rule of mobility. This enables us to calculate the sequences of switching times, and the sum of present discounted net income for each completed job spell. Third, for any arbitrary threshold wage differential we can compute: (1) the expected wages from switching and then following some threshold rule, and (2) the expected wages from staying and then also following the same threshold rule. Finally, the optimal threshold wage differential, s, is defined and solved for where (1) and (2) are equal.

Theorem 2 summarizes the main comparative static results.

Theorem 2 (Comparative Statics) *Fix* $wjwg = h(2p-1)$ *and* $p > 1/2$. *Then the minimum mobility wage gain* s*: (1) increases in* C*, and (2) increases in* h *and this increase in* s *with respect to* h *is larger when* $wjwg$ *is high.*

Using the Implicit Function Theorem, it is easy to show that the minimum mobility wage gain s increases in C, increases in h, and decreases in p for $p \in (1/2, 1)$ (See Jun 2001 for details). Now, fix $wjwg = ph - (1-p)h = h(2p-1)$. Even if we fix within-job wage growth the minimum mobility wage gain still increases in switching costs C. Assume that mean within-job wage growth is positive, $p > 1/2$. Note, for within-job wage growth to remain constant, p and h must trade-off each other. For example, an increase in h must be accompanied by a decrease in p. Therefore if h increases, then the minimum mobility wage gain s increases more when $wjwg$ is fixed because an increase in h is accompanied by a decrease in p, which further increases the minimum mobility wage gain. This simply says that an increase in h induces a higher minimum mobility wage gain when $wjwg$ is fixed. Moreover since a high p is equivalent to a high within-job wage growth rate, the increase in the minimum mobility wage gains due to an increase in volatility h, is larger when within-job wage growth rate is at a higher level. The corollary of this result is that the negative effect of volatility on turnover is more likely when within-job wage growth is high.

Some of the specifics of this model can be extended in several ways. First, it can be generalized to allow the evolution of outside and inside wages to be correlated. The implications are discerned as follows: if the correlation is positive then the threshold wage differential decreases, and if the correlation is negative then the threshold wage differential increases. These results raise an empirical challenge because we do not observe the simultaneous evolution of wages in the current and outside jobs. Second, we can model optimal switching in an environment where either the inside or outside wage is frozen. The same results hold since only the wage difference between the outside and inside job matters for switching decisions.

In the next subsection we briefly discuss some related theory and especially how our model might be related to search theory.

2.4 Job Search and Other Related Theory

The obvious shortcoming of our model is the unrealism of the two-job framework since workers typically do not switch between the same two jobs over the life cycle of employment. Of course, the outside wage option can arise from a distribution of outside job offers. But the question is whether our simple two-job model and its theoretical implications can be interpreted within a proper job search framework.

Although job search theory has various renditions, a non-degenerate distribution of outside job offers and imperfect information about the quality and location of job matches are some of the salient characteristics of search models. In Jovanovic's (1979) well-known mismatch theory of turnover workers search from a non-degenerate distribution of outside job offers. Although in equilibrium this distribution of outside offers is supported by the assumption of heterogeneity of match quality, *ex ante* all outside jobs are identical because jobs are treated as experience goods. Hence all jobs offer the same initial wage. Worker-firm separations result because of the arrival of signals about match quality over the course of an employment relationship. Since priors about match quality are presumed to be identically non-informative across all jobs and Bayesian learning implies that within-job wage volatility decreases with job tenure, the mismatch theory is not well suited to study the effects of wage volatility on labor mobility.

Both the mismatch theory and our model of turnover are also related to a larger theoretical literature on multi-armed bandit problems. The well-known Gittins index rule, from the classical formulation of the multi-armed bandit framework (Gittins 1979), shows that the optimal strategy is to pick the arm with the highest Gittins index, where the computation of the index of an arm is based only on the properties of that arm. Banks and Sundaram (1994) argued that in the presence of switching costs any index rule will not be optimal. The theoretical contribution of our model is that we derive an optimal rule in the presence of switching costs. However, we do so by restricting our model to the two-armed bandit framework. Bergemann and Välimäki (2001) have shown that if the distribution of arms is stationary then a modified index rule is optimal even in the presence of switching costs. Note that Jovanovic's mismatch theory of turnover is structurally identical to this multi-armed bandit framework since the stochastic element of each arm arises from the learning process about underlying productivity of the arm. Hence, this framework, like Jovanovic's, is not ideally suited to study the effects of volatility on switching behavior. By contrast, our model – because it allows volatility to play an exogenous role in predicting labor mobility – is better suited to illustrate the links between wage volatility and the optimal threshold of turnover.

To return to the question posed earlier, can our two-job model of turnover be interpreted within a job search framework? For example, suppose that the outside wage option arises from a stationary distribution and that neither the worker nor the firm knows the match quality at the time of job start. Hence *ex ante* all jobs must offer the same initial wage. However, once a job commences, the evolution of within-job wages are determined by two distinct stochastic processes – i.e. by a random walk component (which is exogenously given) and a match quality component (which arises from Bayesian updating). With the further assumption that job changes entail an irrecoverable switching cost, this framework clearly maintains all the elements of the mismatch theory and our model of job turnover. The key prediction of the mismatch model is that worker-firm separations are less likely to occur as the employment relationship ages. Hence holding job tenure constant, wage volatility (due to the random walk assumption) is still likely to be positively related to the optimal threshold of turnover – the key theoretical result of our model. The formalization of this conjecture is a part of an ongoing research project.

3 Empirical Analysis

3.1 NLSY Data and Wage Volatility

We use the National Longitudinal Surveys of Youth (NLSY) data from 1979 to 1994 for our empirical analyses. The NLSY tracks a panel of 12,686 young women and men, first interviewed in 1979. The availability of work histories of early careers, including detailed information on job duration and separation, labor market experience, wage rates, and other individual and job characteristics, make this data ideal for our analyses of volatility effects on labor mobility outcomes. We only track CPS (Current Population Survey) designated jobs, although the NLSY tracks additional jobs. The CPS job is typically the main or most recent job, and more information is available about CPS jobs. Wage rates are deflated by the consumer price index from the Report of the President, where the base year is 1987.

One of our dependent variables is mobility wage gains. It is defined as the percentage difference between the first wage in the future job and the last wage in the current job, conditional on a voluntary job change. We ensure that the two wage observations occur in contiguous years. The other dependent variable is the quit rate, where a quit is defined as a voluntary job change between two contiguous survey years. Since these surveys are conducted each year our quit models estimate an annual turnover rate.

The more contentious issue is the construction of our wage volatility measure. Since we assume that wages evolve as a random walk with drift, our empirical analyses focus on volatility of within-job wage changes. For each individual in our sample we estimate a simple OLS within-job wage difference equation based on wage observations over the entire life cycle. Obviously, we do not include wage changes due to job changes in this sample. In these individual regressions, job tenure and experience are included as right hand side variables to account for the concavity of the wage-experience and wage-tenure profiles. We then take the sample variance of the error term from each of these individual regressions as our sample measure of person-specific wage volatility. More explicitly, we can write the estimation equation for measuring volatility of worker i ($volatility_i$) as follows:

$$wjwc_{it}\left(=w_{it}^c-w_{it-1}^c\right)=\alpha_1+\alpha_2 experience_{it}+\alpha_3 tenure_{it}+\epsilon_{it},$$

where w_{it}^c is the log wage of current job of worker i at time t. Then $volatility_i$ is simply the variance of the error term of above regression:

$$volatility_i=\sum\left(wjwc_{it}-\widehat{wjwc}_{it}\right)^2/\left(n_i-3\right),$$

where n_i is the number of $wjwc$ observations for individual i.

This wage volatility measure is of course the key predictor in our subsequent analyses of quit rates and mobility wage gains. In our empirical analyses we also include the mean of within-job wage growth among our independent variables.[10] Also note that a key prediction of the model is that it is more likely for volatility to have a negative effect on turnover if within-job wage growth is higher.

We impose various sample restrictions on our data by excluding: (1) any observation where the real wage (in 1987 dollars) is less than \$1.50 or greater than \$200.00, (2) if mobility wage changes fall by more than 50% or exceed 300%, and (3) if within-job wage changes fall by 50% or exceed by 300%. The primary purpose of these restrictions is to exclude obvious outliers and data errors. Although these criteria are arbitrary, we check for robustness of our results by experimenting with various alternative cutoffs with no qualitative changes and only minor quantitative changes in our results.

3.2 Estimation Framework

In this section we outline the estimation framework to test volatility effects on mobility wage gains and quit rates. We first derive our mobility wage gain equation as a reduced form version of a standard human capital earnings function. Hence, consider wages in the last period of the current job and the first period of the future job as

$$w_{it}^c=Z_{it}\beta+\mu_i^1+\mu_i^2 t+\varepsilon_{it} \text{ and } w_{it+1}^f=Z_{it+1}\beta+\mu_i^1+\mu_i^2 t+\varepsilon_{it+1},$$

[10] Here we depart from Topel and Ward (1992). Their evidence shows that "heterogeneity among jobs in predictable wage growth is not an important feature of the data" (p. 459). As a consequence, their evidence is consistent with the same drift parameter across all jobs. Since our framework is based on person-specific effects, we proceed on the assumption that individuals are likely to differ in terms of this drift parameter. Hence we include mean wage growth over the life cycle as a control variable in both our mobility wage gains and quit equations.

respectively.[11] Note that w is the log wage and Z is a vector of worker characteristics such as age, education, race, gender, and other determinants of wages subscripted by time t; superscripts c and f on w designate wages in the "current" and "future" job, respectively. We also allow for individual fixed effects as well as individual time trends, denoted by μ_i^1 and $\mu_i^2 t$, respectively. Since our focus is on mobility wage gains, we take the difference in wages across the future job and current job. This yields an equation that is one step closer to our estimating equation:

$$\Delta w_{it} \equiv w_{it+1}^f - w_{it}^c = \Delta Z_{it}\beta + \mu_i^2 + \Delta\varepsilon_{it}. \tag{7}$$

An individual's mobility wage gain is now a function of the changes in explanatory variables such as changes in marital status. The individual fixed effects in the error structure of course drop out with the exception of the time trend terms which generate individual fixed-effects in the mobility wage gain specification. We assume, however, that mobility wage gains continue to be a function of time invariant factors such as education, gender, and race. Our framework up to this point closely parallels Goldberg and Tracy (2001).

Our theoretical analysis of course implies that mobility wage gains will also be a function of wage volatility. As a consequence we augment equation (7) to generate our estimating equation:

$$\Delta w_{it} = X_{it}\beta + \beta_1 volatility_i + \beta_2 growth_i + \mu_i^2 + \Delta\varepsilon_{it},$$

where $volatility_i$ is wage volatility and $growth_i$ is mean of within-job wage growth rate for worker i. The vector X includes not only worker characteristics such as age, education, gender and race, but also changes in time dependent covariates.

Since we observe mobility wage gains only if a worker voluntarily changes jobs, our subsample of mobility wage gain observations is not a random sample of individuals with a current job and a potential outside job option. As a result we need to explicitly introduce a quit model in order to correct for potential sample selection bias in our estimation. Of course this first stage quit model is of direct interest to us since our theoretical considerations suggest that volatility is a likely determinant of the quit rate also. In particular, the model

[11] We assume the same β for both equations only for notational convenience. In our empirical specifications we include current and lagged time dependent variables separately and thus estimate a different β for the current and future wage equations.

implies that the volatility effect on turnover is more likely to be negative if within-job wage growth is higher.

We implement various Heckman-type sample selection corrections. Our selection equation is based on whether or not an individual quits her current job voluntarily. The basic framework can be outlined by first considering a latent quit equation, and the corresponding quit outcome.

$$q_{it}^{*} = z_{it}\gamma_1 + volatility_i\gamma_2 + growth_i\gamma_3 + volatility_i * growth_i\gamma_4 + \eta_{it},$$
$$q_{it} = 1 \text{ if } q_{it}^{*} > 0, \text{ and } q_{it} = 0, \text{ otherwise.}$$

The q_{it} variable indicates whether worker i quits her current job at time t. Note that our model predicts that $\gamma_4 < 0$. Of course sample selection arises because mobility wage gains are observed only if a worker quits her current job and moves to a new job. Hence:

$$\Delta w_{it} = X_{it}\beta_1 + volatility_i\beta_2 + growth_i\beta_3 + \Delta\varepsilon_{it} \text{ if } q_{it} = 1.$$

This model can be estimated either via maximum likelihood (Greene 1981) or by using Heckman's "two-step" procedure (Heckman 1976, 1979). In our empirical results section we present estimates from both techniques.

Since in the presence of individual fixed effects this sample correction does not yield consistent estimates, Woodbridge (1995) suggests an alternative procedure that yields consistent estimates with fixed effects. The procedure exploits the panel nature of our data. The basic idea is to generate Inverse Mills Ratios for each year and then to include not only this ratio in our mobility wage gain regression (the standard two-step Heckman procedure), but also interactions of this ratio with time dummies.[12] Hence our final estimating equation is:

$$\Delta w_{it} = X_{it}\beta_1 + volatility_i\beta_2 + growth_i\beta_3 + \beta_4\lambda_{it} + T\lambda_{it}\beta_5 + \Delta\varepsilon_{it},$$

where λ is the Inverse Mills Ratio and T is a vector of time dummies. This procedure provides consistent estimates in the presence of fixed effects.[13]

[12] See Chapter 17 of Woodbridge (2002) for details.

[13] See Woodbridge (1995) for details.

3.3 Empirical Results

3.3.1 Summary Statistics

Table 1 lists the labels and descriptions of the key variables. Tables 2 through 4 present various summary statistics. Table 2 pertains to person-specific variables. Tables 3 and 4 are based on samples specific to the mobility wage gains and quit models, respectively. Table 2 is restricted to a sample of 6,099 individuals that have information on wage volatility. To compute a volatility measure, an individual needs to have at least four valid within-job wage observations. The average individual has about 7 such within-job wage observations, where the range goes from the minimum of 4 to a maximum of 15 observations. In our sample the average number of mobility wage gain observations per individual is about 1.7. The key variable is the sample variance based on individual within-job wage regressions – our measure of within-job wage volatility. The mean volatility is .075 and the standard deviation is .16. This substantially high standard deviation bodes well for our efforts to study the effects of volatility variations across individuals on their labor mobility outcomes. The within-job wage growth is .08. Our sample is composed of slightly more than 50% male and about 60% white. The average completed years of schooling in the sample is a little over 13 years.

The sample in Table 3 is based on observations from our mobility wage gain equations. Since each individual can have multiple wage gain observations, individuals are repeated in this sample. We include summary statistics for experience, tenure, on-the-job training, and whether the job change entailed a change in the 2-digit industry classification. Table 4 repeats the same for the sample of observations on which our quit regressions are based. Clearly this sample includes not only observations with valid mobility wage gains – i.e. when a person quits – but also all the within-job observations when individuals do not quit their jobs. Hence this sample size is considerably larger.

3.3.2 Volatility Effects on Mobility Wage Gains

Table 5 presents coefficients estimates from various mobility wage gains models. The first and second columns show estimates from OLS and random effects specifications, respectively. Here we do not explicitly take into account the first stage quit equation. The remaining three columns are estimates from models that explicitly take into account the first stage selection equation. Column 3 is based on the maximum likelihood specification that jointly estimates both the mobility wage gain equation and the quit equation. (We show the coefficients of the quit equation in column 4 of Table 6.) Column 4 includes the Inverse Mills Ratio from the first stage selection equation that is based on Heckman's two-step correction – i.e. the quit equation. Column 5 includes in addition the interaction of this Inverse Mills Ratio with time dummies to account for individual fixed effects, as discussed in the earlier section. All the standard errors (except column 2) are corrected for multiple individual observations and all are weighted regressions by the number of observations used to compute the volatility measure. A host of control variables are included in all the models. Variables with a t subscript pertain to the time period coinciding with the first period on the future job.

The key finding is a positive and highly significant coefficient on volatility. This positive effect of volatility on mobility wage gains holds across all the different model specifications, including those that correct for sample selection. Simple calculations show that an increase of one standard deviation of our volatility measure has about a 10% increase in the mobility wage gain rate. This positive correlation between volatility and mobility wage gains is of course consistent with our model of job turnover. Also note that within-job wage growth is negatively correlated with mobility wage gains.

The variable Career is the ratio of the number of prior within-job wage change observations at a given point in time to the total number of within-job wage change observations over the entire observed life cycle of an individual.[14] The negative coefficient on Career simply says that mobility wage gains decrease as individuals progress along their careers. Among other noteworthy findings are: men experience high mobility wage gains, entering a union covered job leads to high gains, and conversely, leaving an union covered job leads to lower gains.

3.3.3 Volatility Effects on Quit Rates

Table 6 presents coefficients from four quit models: a linear probability model, a probit model, a random effects probit model, and the first stage of the maximum likelihood joint estimation of mobility wage gains equation, respectively. (The last column here corresponds

[14] Note that our volatility measure of within-job wage changes is based on the latter number of observations.

to the third column of Table 5.) We include a similar set of independent variables in these models as in the mobility wage gain models except that we do not use information about future jobs to predict the likelihood of a quit probability. In addition we also include the current wage as a control for job value. The key difference however is that in these quit models we include an interaction between volatility and within-job wage growth. Across all these different model specifications we find that the estimated effect of this interaction term is negative as predicted by our model. The coefficient on volatility is positive (and significant) and the coefficient on within-job wage growth is negative (and not significant). But the key model prediction is of course that volatility is more likely to have a negative effect on turnover as within-job wage growth increases, which finds support in the negative coefficient estimate on the interaction term. Previous studies have shown that past wage growth on a job reduces the likelihood of turnover (Topel and Ward, 1992). Our negative but insignificant effect of within-job wage growth on turnover should not be interpreted as past wage growth on the job since our measure of within-job wage growth is a person- and not a job-specific measure.

3.3.4 What about Risk Aversion?

If our measure of volatility pertained to wages of future jobs then these joint findings – namely, that volatility reduces quit rates and increases mobility wage gains – would be consistent with risk aversion on the part of workers. However, since our measure of volatility is person-specific and based on the entire life cycle of wage observations these findings cannot be interpreted as evidence of risk aversion. Although we are unable to directly test this risk aversion hypothesis because job-specific volatility measures are infeasible, we attempt to provide a discriminating test of sorts by exploiting the exact timing of when a quit occurs in the course of an individual's life cycle.

Note that a low Career value (close to zero) indicates not only an early point in an individual's observed career, but more precisely the fact that our person-specific volatility measure is based more on wage observations of future jobs. Conversely, a high value (close to one) would indicate that our volatility measure is based more on wage observations of past jobs. With this variable we can address whether the negative effect of volatility on quits and the positive effect of volatility on mobility wage gains are stronger when our volatility measure is based more on future than on past job wage observations by including Career and its interaction with volatility as independent variables in our mobility wage gains and quit regression models. Such evidence would provide some support for the risk aversion hypothesis.

In Tables 5 and 6 we also include coefficient estimates of Career interacted with volatility. In the wage gain models (Table 5) the sign of the interaction term is positive across all model specifications, although the estimates are insignificant. A Negative coefficient would indicate that past wage volatility has a negative impact on mobility wage gains and would lend support to the risk aversion explanation. However given that these estimates are positive (although insignificant) implies that we find no evidence of risk aversion. We also include this interaction term in all of our quit models (Table 6). As we argued earlier, risk aversion would predict a positive sign on the interaction term in the quit model. However, all the coefficients of the interaction term are negative (and significant). Hence if anything this set of results contradict the risk aversion explanation. In conclusion, we fail to find any supporting evidence for the risk aversion hypothesis.

4 Conclusion

In this paper we present a theory about the role of wage volatility in predicting labor mobility. The model is structurally equivalent to the options pricing approach to investments where switching costs associated with moving from one job to another are explicitly incorporated and within-job wages are assumed to evolve as a random walk process. The key theoretical result is that an increase in wage volatility implies a higher optimal threshold of turnover. As a consequence volatility can reduce quit rates and increase mobility wage gains conditional on workers voluntarily switching jobs. We present supporting evidence from the National Longitudinal Surveys of Youth. The data show, under various specifications, that volatility of within-job wage changes is negatively correlated with quits especially if within-job wage growth is high, and positively correlated with wage gains, controlling for a variety of individual and other characteristics.

5 Appendix: Proof of Theorem 1

Proof. of Theorem 1.

We begin the proof of Theorem 1 by considering the difference:

$$v^a\left(w^a+h, w^b+h\right)-v^b\left(w^a+h, w^b+h\right).$$

Let $w^a - w^b$ equal the threshold s. Note that it is optimal to stay in job a at $v^a\left(w^a+h, w^b+h\right)$. The worker is indifferent between switching now and delaying at $v^b\left(w^a+h, w^b+h\right)$ since mobility wage gain is equal to the threshold. Hence $v^b\left(w^a+h, w^b+h\right)$ has the same value as $v^a\left(w^a+h, w^b+h\right)-C$ if a switch is made to job a now. Therefore it follows:

$$v^a\left(w^a+h, w^b+h\right)-v^b\left(w^a+h, w^b+h\right)=C. \tag{8}$$

The same argument leads to:

$$v^a\left(w^a-h, w^b-h\right)-v^b\left(w^a-h, w^b-h\right)=C. \tag{9}$$

Now consider the difference $v^a\left(w^a+h, w^b-h\right)-v^b\left(w^a+h, w^b-h\right)$. By the threshold definition, it is strictly optimal to choose job a in both $v^a\left(w^a+h, w^b-h\right)$ and $v^b\left(w^a+h, w^b-h\right)$. Hence $v^b\left(w^a+h, w^b-h\right)$ and $v^a\left(w^a+h, w^b-h\right)-C$ has the same value if a switch is made to the job a. This implies:

$$v^a\left(w^a+h, w^b-h\right)-v^b\left(w^a+h, w^b-h\right)=C. \tag{10}$$

Hence only the following difference remains to be derived:

$$v^a\left(w^a-h, w^b+h\right)-v^b\left(w^a-h, w^b+h\right).$$

We first derive this difference if the threshold s is less than the jump h, which is the case of Theorem 1.1. Notice that the wage differential in $\left(w^a-h, w^b+h\right)$ is less than $-s$. Hence a worker will switch to job b in $v^a\left(w^a-h, w^b+h\right)$ and a worker with $v^b\left(w^a-h, w^b+h\right)$ will remain in job b. As a result, the value of $v^a(s-h, h)+C$ is identical to the value $v^b(s-h, h)$ if a switch is made to job b now. This implies $v^a(s-h, h)-v^b(s-h, h)=-C$. Substituting this and $(8)-(10)$ into (6) and rearranging terms, we prove Theorem 1.1.

Theorem 1.2 and 1.3 use identical arguments, and we only present the proof of Theorem 1.2. With the initial wage differential $\Delta_0 (= w^a - w^b)$ at time zero, suppose that a worker is currently in job b and remains there until the wage differential becomes $\Delta_\tau = \Delta_0 + 2h$ for the first time, where

$$\tau \in \{T > 0 | \Delta_j < \Delta_0 + 2h \text{ for all } 0 < j < T \text{ and } \Delta_T = \Delta_0 + 2h\}. \tag{11}$$

We assume that $\Delta_0 < s$ and thus it is optimal to stay in job b until the time τ.

We derive the sum of present discounted value of wages between time 0 and τ. This sum can be separated into two components: a "constant term" G and a fluctuation term. The constant term denotes the sum of present discounted value of starting wage w^b, and the fluctuation term is the remainder. By definition, the fluctuation term does not depend on the starting wage. We focus on the constant term. For each $\tau \geq 1$, there are multiple sample paths that lead to the target wage difference $\Delta_0 + 2h$ at random time τ. Let N_j denote the set of such sample paths where the first hitting time to the target wage difference $\Delta_0 + 2h$ is a random time j, and n_j is the cardinality of the set. Fixing j, the constant term on a particular sample path is given by $w^b \sum_{i=1}^{j} \beta^i$ (there are n_j of such sums). Denote ϕ_j as the probability associated with the random time j. Then the constant term with starting wage w^b is:[15]

$$G\left(w^b\right) = w^b + \sum_{j=1}^{\infty} \phi_j n_j w^b \sum_{i=1}^{j} \beta^i = w^b \left[1 + \sum_{j=1}^{\infty} \phi_j n_j \sum_{i=1}^{j} \beta^i\right]$$

where $\phi_j \equiv \Pr[j > 0 | \Delta_l < \Delta_0 + 2h \text{ for all } 0 < l < j \text{ and } \Delta_j = \Delta_0 + 2h]$.

We now proceed to derive the value function at the end of the "travel" to the target difference $\Delta_0 + 2h$ from Δ_0. Consider the value function $v^b\left(w^a, w^b\right)$. For each random time j defined above, this value function at the end of the travel will be $\beta^j v^b\left(\Delta_0 + 2h\right)$.[16] This occurs on n_j different paths, each with the probability ϕ_j of occurring. Since the stochastic process of the wage differential Δ_t does not keep track of wage at each job, the wage level of each job can be anything at a random time j. However, all the wage pairs are equivalent as long as they have the identical difference since the optimal switching rule only depends on

[15] Note that the first period wage w^b is earned regardless of sample paths.

[16] For this notation only, we put the wage differential in the value function.

the wage differential (as we showed earlier). Therefore we can show:[17]

$$v^b\left(w^a+h,w^b+h\right)-v^b\left(w^a,w^b\right)=G\left(w^b+h\right)-G\left(w^b\right)+\frac{h}{1-\beta}\sum_{j=1}^{\infty}\phi_j n_j \beta^j, \tag{12}$$

which reduces to

$$1+\sum_{j=1}^{\infty}\phi_j n_j \sum_{i=1}^{j}\beta^i=\frac{1-\sum_{j=1}^{\infty}\phi_j n_j \beta^j}{1-\beta}. \tag{13}$$

Note that $\sum_{j=1}^{\infty}\phi_j n_j \beta^j$ is simply the moment generating function of the "β-discounted" first hitting time probability that the wage differential is reduced by $2h$. Denote ρ as this m.g.f. A standard exercise[18] shows that ρ can be expressed as in Theorem 1. Rewrite (13) such that:

$$1+\sum_{j=1}^{\infty}\phi_j n_j \sum_{i=1}^{j}\beta^i=\frac{1-\rho}{1-\beta}. \tag{14}$$

The equality in (14) is critical in deriving the optimal switching policy. In Lemma 1 below, we derive $v^a(s-h,h)-v^b(s-h,h)$.

Lemma 1. Let $k=s/h$.

$$v^a(s-h,h)-v^b(s-h,h)=-2h\frac{(1-\rho)\left(1-\rho^k\right)}{(1-\beta)\left(1+\rho^{k+1}\right)}+C.$$

We prove the lemma from the two following claims.

Claim 1.

$$v^a(s,0)-v^a(s-h,h)=h\frac{1-\rho^{2k+2}-2\rho^k+2\rho^{2k+1}}{(1-\beta)\left(1-\rho^{2k+2}\right)}.$$

Proof of Claim 1: The proof is done by constructing optimal paths. For $v^a(s,0)$, we construct the following optimal rule. First define switching rule π_{11} as:

$$\Gamma_t=A \text{ for } 0\le t<\tau_1, \text{ where } \left\{\tau_1>0|\Delta_t>-s\ \forall t<\tau_1, \Delta_{\tau_1}=-s\right\}.$$

[17] The argument is as follows. For the value functions $v^b\left(w^a,w^b\right)$ and $v^b\left(w^a+h,w^b+h\right)$, note that for every sample path arriving at the wage differential Δ_0+2h from the pair $\left(w^a,w^b\right)$, there exists an identical sample path from the pair $\left(w^a+h,w^b+h\right)$ except that it is always h higher until the first hitting time τ, since the wage differential between the wage pairs remains identical on every "matched" path, and the optimal rule only depends on wage differential. Hence the sum of present discounted value of the difference between $v^b\left(w^a,w^b\right)$ and $v^b\left(w^a+h,w^b+h\right)$ over all sample paths until the time τ can be expressed as the difference between two constant terms: $G\left(w^b\right)$ and $G\left(w^b+h\right)$. The existence of the identical sample paths makes computation of the fluctuation term redundant since it will be canceled out in the difference between the two value functions.

[18] Interested readers can find a derivation of ρ in Cox and Miller (1965).

and next define π_{12} as:

$$\Gamma_t = B \text{ for } \tau_1 \leq t < \tau_1' \text{ where } \left\{\tau_1' > \tau_1 | \Delta_t < s + 2h \ \forall \tau_1 \leq t < \tau_1', \Delta_{\tau_1'} = s + 2h\right\},$$
$$\Gamma_t = A \text{ for } \tau_1' \leq t < \tau_1'' \text{ where } \left\{\tau_1'' > \tau_1' | \Delta_t > -s \ \forall \tau_1' \leq t < \tau_1'', \Delta_{\tau_1''} = -s\right\}.$$

We first apply π_{11} and then apply π_{12} at random time τ_1. Clearly, the constructed policy π_{11} and the subsequent application of π_{12} satisfy optimality.

For $v^a(s-h,h)$, we start by defining switching rule π_{21} as follows:

$$\Gamma_t = A \text{ for } 0 \leq t \leq \tau_2 \text{ where } \{\tau_2 > 0 | \Delta_t > -s - 2h \ \forall t < \tau_2, \Delta_{\tau_2} = -s - 2h\}.$$

Next define π_{22} as follows:

$$\begin{aligned}
&\Gamma_t = B \text{ for } \tau_2 \leq t < \tau_2', \\
&\text{where } \left\{\tau_2' > \tau_2 | \Delta_t < s \ \forall \tau_2 \leq t < \tau_2', \Delta_{\tau_2'} = s\right\}; \\
&\Gamma_t = A \text{ for } \tau_2' \leq t < \tau_2'', \\
&\text{where } \left\{\tau_2'' > \tau_2' | \Delta_t > -s - 2h \ \forall \tau_2' \leq t < \tau_2'', \Delta_{\tau_2''} = -s - 2h\right\}.
\end{aligned}$$

We first apply π_{21} and then apply π_{12} at random time τ_2. As in the previous case, it is easy to check that the policy π_{21} and the subsequent application of π_{22} are optimal.

At the end of applying policy π_{12} and π_{22}, the wage differential is $-s$ and $-s-2h$ for $v^a(s,0)$ and $v^a(s-h,h)$, respectively, which is identical to the wage differential at time $t = \tau_1$ and $t = \tau_2$ for each respective value function. This implies that starting with π_{11} and repeating π_{12} over and over again is an optimal rule. Denote this optimal rule as π_1^*. The optimal rule π_2^* can be constructed in a similar fashion. Let $\{\tau_1^*\}$ be the sequence of switching times under π_1^* and $\{\tau_2^*\}$ be the sequence of switching time under π_2^*.

Note that each $\{\tau_i^*\}$ can be decomposed into the sequence of independent hitting times when the wage differential widens or shrinks by $2h$ for the first time from the previous wage differential. Notice further that random times in $\{\tau_1^*\}$ and $\{\tau_2^*\}$ can be matched by matching paths under the optimal rule. This implies that $\tau_1 = \tau_2$, $\tau_1' = \tau_2'$, and $\tau_1'' = \tau_2''$. This means that switching costs will disappear in the difference $v^a(s,0) - v^a(s-h,h)$.

We are now ready to derive the difference $v^a(s,0) - v^a(s-h,h)$. Note that the optimal path before the first job switch to job b under π_1^* (and also π_2^*) can be decomposed into $k\,(= s/h)$ independent "steps" where the wage differential shrinks by $2h$ in each step. (12) –

(14) imply that the difference $v^a(s,0) - v^a(s-h,h)$ between any two adjacent steps until the first job switch is exactly $h(1-\rho)/(1-\beta)$. This is because, for each sample path from $v^a(s,0)$ under the optimal rule π_1^*, there exists an identical sample path from $v^a(s-h,0)$ under the optimal rule π_2^* except that it is always h below. After k steps of $2h$ decreases in each step, the wage differential will be $-s$ and $-s-2h$ for $v^a(s,0)$ and $v^a(s-h,h)$, respectively. At this point in time, it is optimal to switch to job b for both value functions. Similar arguments as above show that the difference between the two value functions from the time of the first switch and up until the second switch is: $-h(1-\rho)/(1-\beta)$. Since $\{\tau_i^*\}$ is constructed as a repetition of optimal rules applied until the second (random) switch defined in π_{12} and π_{22}, the difference between the two value functions will also alternate accordingly as the optimal rules.[19] Using this argument, we get:

$$\begin{aligned} & v^a(s,0) - v^a(s-h,h) \\ &= h\frac{1-\rho^k}{1-\beta} - \rho^k h \frac{1-\rho^{k+1}}{1-\beta}\left[\sum_{j=0}^{\infty}\rho^{2j(k+1)} - \sum_{j=0}^{\infty}\rho^{(2j+1)(k+1)}\right]. \end{aligned}$$

Rearrangement completes the proof. $\|$

Claim 2. With $k = s/h$

$$v^b(s,0) - v^b(s-h,h) = h\frac{2\rho - \left(1+\rho^{k+1}\right)}{(1-\beta)\left(1+\rho^{k+1}\right)}.$$

Proof of Claim 2 is based on the same argument as the proof of Claim 1, and it is omitted here. Lemma 1 is then a straightforward implication of Claim 1 and Claim 2. $\|$

Proof of Theorem 1: Since worker is indifferent between switching now and delaying at the threshold s, plugging the result from Lemma 1 into (6) and using (8) – (10), we get:

$$s - C + \beta\left(p^2 + (1-p)^2 + p(1-p)\right)C - \beta p(1-p)\frac{2h(1-\rho)\left(1-\rho^k\right)}{(1-\beta)\left(1+\rho^{k+1}\right)} + C = 0.$$

Rearrangement completes the proof. $\|$

[19] In other words, whenever job a is chosen, for each sample path under π_1^*, there exists an identical sample path of π_2^* except that it is h below, and whenever job b is chosen, for each sample path of π_1^*, there exists an identical sample path under π_2^* except that it is h above.

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Table 1. Variable Descriptions

Variable	Description	# Obs	Mean
mwg	Mobility wage gain	19,220	.1658
volatility	Sample variance from wage change OLS regressions	55,709	.0722
growth	Mean within-job wage changes over entire career	69,707	.0849
gender	If male	160,502	.5181
nonwhite	If non-white	160,502	.4019
grad	Highest grade completed	159,716	12.6670
wage	Hourly pay rate deflated (in 1987 dollars)	113,659	7.6538
afqt	Armed Forces Qualifying Test Scores	151,344	40.9813
tenure	Job tenure, years	103,568	2.6983
union	If wages set by collective bargaining agreement	95,044	.1803
quit	If quit job since last interview	91,808	.2437
training	Required years of occupational training	115,689	1.0693
married	If married and living with spouse	137,340	.4134
change_ind	If changed industry at job change	92,104	.3224
career	Ratio of current to total completed experience	55,353	.5328

Table 2. Summary Stastistics for Person-Specific Variables
(# Observations = 6,099)

Variable	Mean	Std.	Minimum	Maximum
n_mwg	1.6980	1.4960	0	9
volatility	.0754	.1630	0	2.4303
growth	.0797	.1032	−.1926	1.3407
n_vol	7.3777	2.7734	4	15
nonwhite	.4030	.4905	0	1
gender	.5302	.4991	0	1
afqt	45.0340	28.4611	1	99
grad	13.1623	2.3801	0	18

Note: *n_mwg* is the number of mobility wage gain observations for a worker, and *n_vol* is the number of residual observations from which volatility is computed.

Table 3. Summary Statistics: Moblity Wage Gain Variables
(# Observations = 10,355)

Variable	Obs	Mean	Std.	Minimum	Maximum
mwg	$10,355$	.1682	.4455	$-.4997$	2.9506
volatility	$10,355$	.0763	.1654	0	2.3028
growth	$10,355$	.0829	.1082	$-.1926$	1.3407
grad	$10,355$	13.2673	2.3516	0	18
$union_{t-1}$	$9,949$	.1302	.3366	0	1
$tenure_{t-1}$	$10,277$	1.9861	2.2244	.0192	19.6731
$training_{t-1}$	$10,295$	1.0415	.7350	.1667	5
change_ ind	$9,584$	.5581	.4966	0	1

Table 4. Summary Statistics: Quit Variables
(# Observations = 55,705)

Variable	Obs	Mean	Std.	Minimum	Maximum
quit	$55,705$	.1859	.3890	0	1
volatility	$55,705$	.0721	.1516	0	2.4303
growth	$55,705$	.0768	.0957	$-.1926$	1.3407
grad	$55,705$	13.1525	2.3185	0	18
$union_{t-1}$	$53,457$	.2094	.4069	0	1
$tenure_{t-1}$	$55,332$	3.2957	3.0873	.0192	19.6731
$training_{t-1}$	$55,490$	1.1127	.7508	.1667	5
change_ ind	$49,538$	.1956	.3966	0	1

Table 5. Mobility Wage Gain Regressions
(# Observations = 10,268)

Dependent Variable: mwg_t	OLS	OLS (RE)	Sample Selection MLE	Sample Selection Two-Step	Sample Selection Modified Two-Step
Intercept	.0287	.0281	.1255	.0310	.0239
	(.0385)	(.0397)	(.0395)	(.0400)	(.0406)
volatility	.1210	.1197	.1070	.1050	.1054
	(.0362)	(.0382)	(.0374)	(.0370)	(.0370)
growth	−.4548	−.4596	−.4263	−.4496	−.4498
	(.0544)	(.0476)	(.0555)	(.0564)	(.0563)
$Career_t$	−.0622	−.0619	.0845	−.0512	−.0454
	(.0151)	(.0168)	(.0173)	(.0194)	(.0224)
Denominator of $Career_t$	.0042	.0044	.0194	.0059	.0056
	(.0019)	(.0021)	(.0021)	(.0021)	(.0021)
*volatility**$Career_t$	.0350	.0445	.0909	.1348	.1358
	(.0816)	(.0825)	(.0887)	(.0898)	(.0894)
Inverse Mills Ratio(IMR)				−.0294	−.0387
				(.0115)	(.0206)
*IMR*Time Dummies*					*Included*
gender	.0490	.0495	.0638	.0508	.0509
	(.0085)	(.0093)	(.0089)	(.0089)	(.0089)
$grad_t$	.0061	.0063	.0052	.0066	.0067
	(.0025)	(.0025)	(.0025)	(.0025)	(.0025)
$tenure_{t-1}$	−.0269	−.0274	−.0252	−.0244	−.0257
	(.0049)	(.0049)	(.0051)	(.0054)	(.0054)
$tenure^2_{t-1}$	.0015	.0015	.0013	.0012	.0014
	(.0005)	(.0005)	(.0005)	(.0005)	(.0005)
$training_t$	.0369	.0366	.0391	.0373	.0368
	(.0067)	(.0063)	(.0068)	(.0069)	(.0069)
$training_{t-1}$	−.0241	−.0241	−.0216	−.0215	−.0217
	(.0072)	(.0066)	(.0072)	(.0074)	(.0074)
$union_t$	.1412	.1406	.1455	.1447	.1445
	(.0164)	(.0139)	(.0167)	(.0165)	(.0165)
$union_{t-1}$	−.0701	−.0695	−.0425	−.0697	−.0711
	(.0119)	(.0132)	(.0122)	(.0123)	(.0123)
Δind_t	.0582	.0585	−.0941	.0369	.0395
	(.0091)	(.0093)	(.0111)	(.0127)	(.0128)
R^2	.0641	.0641		.0665	.0688
χ^2 (1)			284.24		

Note: Robust standard errors in parenthesis. In addition to the reported coefficients, we also include the following independent variables: square of growth, race, AFQT scores, local unemployment rate, if reside in SMSA, net experience, and marital status.

Table 6. Quit Regressions
(# Obs = 55,153)

Dep. Variable: $quit_t$	OLS	Probit	Probit (RE)	MLE (Selection)
Intercept	.5029 (.0140)	.4949 (.0698)	.4949 (.0660)	.4789 (.0701)
volatility	.0723 (.0264)	.3248 (.1266)	.3249 (.1285)	.3577 (.1215)
growth	−.0366 (.0306)	−.0673 (.1394)	−.0673 (.1319)	−.1953 (.1409)
*volatility*growth*	−.1120 (.0618)	−.5721 (.3058)	−.5721 (.2859)	−.6771 (.2939)
$Career_{t-1}$	−.3030 (.0088)	−1.2812 (.0448)	−1.2812 (.0334)	−1.2478 (.0448)
Denominator of $Career_{t-1}$	−.0263 (.0006)	−.1340 (.0035)	−.1340 (.0033)	−.1319 (.0035)
*volatility**$Career_{t-1}$	−.1055 (.0373)	−.5104 (.2097)	−.5104 (.1583)	−.5131 (.2079)
$wage_{t-1}$	−.0054 (.0005)	−.0310 (.0031)	−.0310 (.0025)	−.0442 (.0037)
gender	−.0244 (.0031)	−.1130 (.0166)	−.1130 (.0156)	−.0974 (.0167)
grad	.0029 (.0008)	.0141 (.0044)	.0141 (.0041)	.0174 (.0044)
$union_t$	.0097 (.0049)	−.0416 (.0252)	−.0416 (.0243)	−.0336 (.0252)
$union_{t-1}$	−.0461 (.0047)	−.2345 (.0247)	−.2345 (.0233)	−.2266 (.0246)
$tenure_{t-1}$	−.0031 (.0015)	−.0194 (.0089)	−.0194 (.0079)	−.0172 (.0089)
$tenure^2_{t-1}$	.0010 (.0001)	.0035 (.0007)	.0035 (.0007)	.0034 (.0007)
$training_{t-1}$	−.0025 (.0021)	.0043 (.0111)	.0043 (.0107)	.0149 (.0112)
Δind_t	.3838 (.0060)	1.3368 (.0193)	1.3368 (.0170)	1.3305 (.0193)
R^2	.3104	.3118		
$\bar{\chi}^2$ (1)			0.00	284.24

Note: Robust standard errors in parenthesis. In addition to the reported coefficients, we also include the following independent variables: square of growth, race, AFQT scores, local unemployment rate, if reside in SMSA, and martial status.