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Equatorial superrotation in shallow atmospheres

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[1] Simple, shallow-water models have been successful in reproducing two key observables in the atmospheres of the giant planets: the formation of robust, and fully turbulent, latitudinal jets and the decrease of the zonal wind amplitude with latitude. However, they have to date consistently failed in reproducing the strong prograde (superrotating) equatorial winds that are often observed on such planets. In this paper we show that shallow water models not only can give rise to superrotating winds, but can do so very robustly, provided that the physical process of large-scale energy dissipation by radiative relaxation is taken into account. When energy is removed by linear friction, equatorial superrotation does not develop; when energy is removed by radiative relaxation, superrotation develops at apparently any deformation radius. **Citation:** Scott, R. K., and L. M. Polvani (2008), Equatorial superrotation in shallow atmospheres, *Geophys. Res. Lett.*, *35*, L24202, doi:10.1029/2008GL036060.

1. Introduction

[2] The pronounced latitudinally aligned bands observed on the giant gas planets are the cloud-top signatures of strong alternating zonal jet streams in the so-called "weather layer", the shallow layer of stably-stratified atmosphere overlying the deeper convective region. Despite much attention over several decades, the actual dynamical processes involved in the maintenance of these jets remain controversial, to the extent that there is still debate over whether their origins lie in deep convection throughout the planetary interior [*Busse*, 1976], or rather in shallow turbulent motions within the thin atmospheric layer itself [*Williams*, 1978]. Somewhere between these two paradigms lies recent three-dimensional general circulation model studies [*Schneider and Lui*, 2008; *Yamazaki et al.*, 2005]. Quantitative predictions based on the former paradigm have been difficult to make, in part because very little is known about the planets' interior [*Guillot*, 1999], and in part because of the high cost of three-dimensional numerical integrations of convective turbulent flow. The latter paradigm is both conceptually and computationally simpler and is based upon well-known and fundamental properties of rotating, stratified flows.

[3] In the shallow rotating atmosphere, latitudinally aligned, or zonal, jets arise spontaneously due to the interaction of turbulent mixing with the background planetary differential rotation [*Rhines*, 1975; *McIntyre*, 1982]. Many studies have documented the spontaneous emergence of well defined zonal jets from a turbulent flow in the presence of a background (planetary) vorticity gradient [e.g., *Rhines*, 1975; *Maltrud and Vallis*, 1991; *Yoden and Yamada*, 1993; *Cho and Polvani*, 1996a, 1996b]. In particular, using a shallow water model with realistic physical parameters, *Cho and Polvani* [1996a, 1996b] showed that, in the absence of forcing, an initially random flow on the sphere spontaneously organizes itself into a banded configuration, with the number of bands roughly consistent with that of the four giant planets.

[4] The shallow atmosphere model has been criticized, however, because all calculations reported so far have been unable, using physically relevant parameters, to reproduce the strong, prograde, or superrotating jets found at the equators of Jupiter and Saturn. Subrotating equatorial jets have been a persistent feature of shallow water turbulence in both the freely-decaying case studied by *Cho and Polvani* [1996b] [see also *Iacono et al.*, 1999] and, more recently, in the forced-dissipative case, in which small-scale forcing represents the input of energy from random convective processes in the deeper atmosphere [*Scott and Polvani*, 2007; *Showman*, 2007]. In this paper we show that a shallow-atmosphere model is in fact perfectly able to produce strong and very robust equatorial superrotation, provided a more physically realistic large-scale energy dissipation is chosen than has typically been used to date.

[5] As we demonstrate below, the form of the large-scale energy dissipation is a determining factor in the direction of equatorial jets. In forced-dissipative calculations with simple models, linear momentum damping is commonly employed because it provides a convenient closure for the total energy in two-dimensional flow. The atmospheres of the gas giants, however, dissipate energy primarily through radiation to space [e.g., *Ingersoll et al.*, 2004; *Showman*, 2007]; the absence of a solid ground underlying the atmospheres of the giant planets obviates the usual motivation of linear momentum damping as a model for Ekman drag. Here, we focus on the effect of radiative or thermal damping and demonstrate that it leads to the spontaneous emergence of equatorial superrotation, even though the small-scale forcing is completely isotropic.

2. Methods

[6] Our model consists of the shallow water equations for a fluid of mean depth H, on the surface of a sphere of radius a, rotating at constant angular velocity Ω, and with gravity g. In terms of vorticity, ζ, divergence, δ and height $h = H + h'$, the governing equations are:

$$\zeta_t + \nabla \cdot (\mathbf{u}\zeta_a) = F - \zeta/\tau_{\mathrm{fr}} \tag{1a}$$

$$\delta_t - \mathbf{k} \cdot \nabla \times (\mathbf{u}\zeta_a) = -\nabla^2(E + gh) - \delta/\tau_{\mathrm{fr}} \tag{1b}$$

$$h_t + \nabla \cdot (\mathbf{u}h) = -h'/\tau_{\mathrm{rad}} \tag{1c}$$

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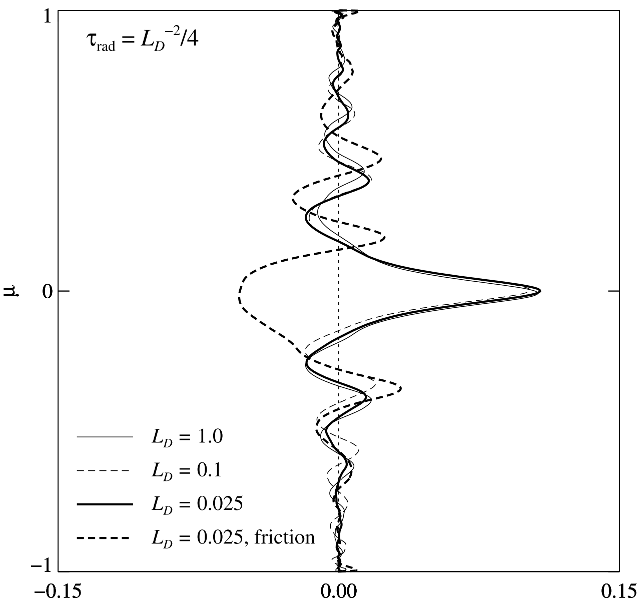

Figure 1. Instantaneous $\bar{u}(\mu)$ at t = 10000 days for cases with $\tau_{\rm rad}$ = 0.25$(L_D/a)^{-2}$ and L_D/a = 1.0, 0.1, 0.025. The case L_D/a = 0.025 with linear friction only is shown with the bold dashed curve.

where $\zeta_a = f + \zeta$ is the absolute vorticity, $f = 2\Omega\sin\phi$ is the Coriolis parameter, $\mathbf{u} = (u, v)$ is the velocity, and $E = |\mathbf{u}|^2/2$. The shallow water equations can be viewed as describing the motion of a shallow layer of rotating fluid, or, alternatively, as describing an internal vertical mode of equivalent depth H in a continuously stratified fluid. The relevant nondimensional parameters are the Rossby number $Ro = U/2a\Omega$ and Froude number $Fr = U/\sqrt{gH}$, where U is a typical velocity scale. In place of the latter we use $L_D/a = Ro/Fr$, where $L_D = \sqrt{gH}/2\Omega$ is the deformation radius, since it can be determined entirely in terms of physical parameters.

[7] The term F on the rhs of (1a) represents a weak, random small-scale forcing with energy input $\epsilon_0 = 5 \times 10^{-7}$ $a^2(\Omega/2\pi)^3$. The forcing is *spatially isotropic* with spectrum $\hat{F}(n) = \epsilon_0/4$ for $|n - n_f| \leq 2$, where n is the total wavenumber, n_f = 42, and with random phases. The forcing is white, or δ-correlated, in time. Such forcing could result, e.g., from turbulent motions in the upper levels of a deep convecting zone [*Showman*, 2007].

[8] The terms $-\zeta/\tau_{\rm fr}$, $-\delta/\tau_{\rm fr}$, and $-h'/\tau_{\rm rad}$ dissipate energy at large scales. The first two represent the effect of linear friction on timescale $\tau_{\rm fr}$, while the latter represents the effect of radiative relaxation on timescale $\tau_{\rm rad}$, i.e. cooling of the planetary atmosphere to space. In this paper, we are mostly concerned with the case in which the dissipation is purely radiative, for which $1/\tau_{\rm fr} = 0$ and $1/\tau_{\rm rad} > 0$. This choice is physically motivated and has a crucial influence on the direction of the equatorial jets.

[9] When $Fr^2 \ll Ro \ll 1$, (1) simplifies to a single evolution equation for the quasigeostrophic potential vorticity $q = (\nabla^2 - L_D^{-2})\psi$, where ψ is the quasigeostrophic streamfunction:

$$q_t + J(\psi, q) = \psi/\left(\tau_{\rm rad}L_D^2\right) - \zeta/\tau_{\rm fr}. \quad (2)$$

Radiative relaxation can therefore be considered, loosely, as a damping on the streamfunction. To meaningfully compare the flow evolution across different values of L_D but similar Ro, we scale $\tau_{\rm rad}$ with L_D^{-2} to ensure the large-scale energy dissipation is approximately independent of L_D.

[10] Equations (1a)–(1c) are integrated numerically using a standard pseudo-spectral method [*Scott and Polvani*, 2007] with a resolution of T682 (equivalent to a 2048 × 1024 longitude-latitude grid). Small-scale hyperdiffusion, $\nu\nabla^8\xi$, is included to control the enstrophy at small scales. The equations are integrated for 10^4 planetary rotations.

[11] Our choice of physical parameters is dictated by values typical of the giant planets. In particular, we are interested in the small Ro regime and we verify *a posteriori* that the zonal jet speeds that arise in our model are comparable to those of the planets (O(100) ms^{-1}). For a given forcing strength ϵ_0 the final Ro is determined by $\tau_{\rm rad}$. This leaves L_D as the main free parameter. While we are interested in how the nature of the equatorial flow changes with L_D, we are again primarily concerned with cases relevant to the giant planets, for which L_D/a is usually put in the range 0.025–0.03 [e.g., *Cho et al.*, 2001; *Ingersoll et al.*, 2004].

3. Results

[12] Figure 1 shows the instantaneous zonal mean zonal velocity $\bar{u}$ at t = 10000 days for a series of three numerical integrations with decreasing L_D/a = 1.0, 0.1, 0.025, and with radiative damping timescale $\tau_{\rm rad}$ = 0.25$(L_D/a)^{-2}$ (in all cases $1/\tau_{\rm fr} = 0$). The prominent feature, and the main result of the paper, is the strong superrotating (positive) equatorial jet, clearly visible in all cases. In contrast, when purely frictional damping is used (the case $1/\tau_{\rm rad} = 0$ and $\tau_{\rm fr}$ = 10000 is shown bold dashed) the equatorial jet is subrotating. In all cases, an alternating pattern of weaker jets is also apparent, and extends through the midlatitudes. We emphasize that these zonal jets and their structure arise *spontaneously* and despite the fact that the forcing is purely *isotropic in space and time*: there is no forcing in the zonal mean and there is no asymmetry in the forcing that might fix the sign of the jet at the equator.

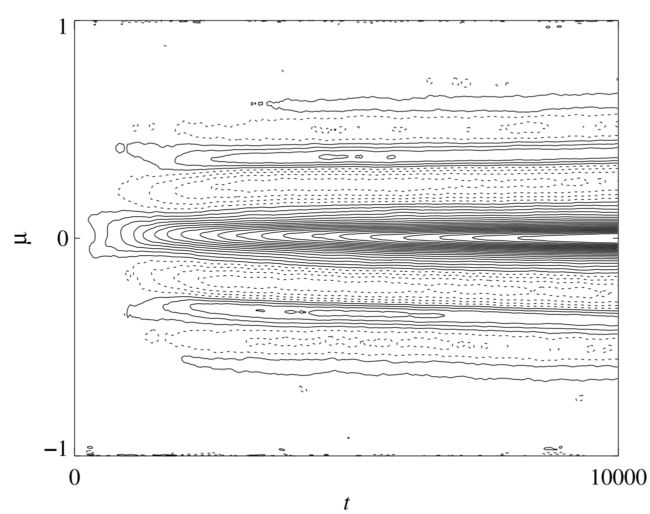

Figure 2. Zonal velocity $\bar{u}(\mu, t)$ for the case $\tau_{\rm rad}$ = 0.25$(L_D/a)^{-2}$ and L_D/a = 0.025.

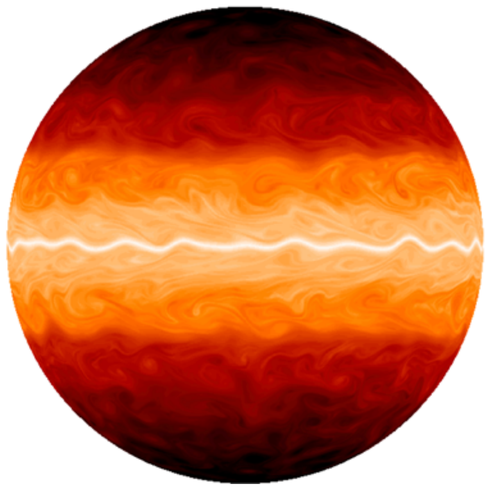

Figure 3. Instantaneous $q = \zeta_a/h$, at $t = 10000$ for the case $L_D/a = 0.025$ (corresponding to the solid bold line in Figure 1).

[13] While our model is highly idealized, we have nevertheless selected parameters that correspond, approximately, to the Jovian atmosphere. Rossby numbers are similar to Jovian values, with resulting equatorial jet speeds of approximately 200 ms^{-1}, and L_D/a ranges down to 0.025. As far as we are aware, this is the first numerical integration with physically relevant parameters in rotating shallow water to produce the observed sign of the equatorial jet. (In a two-dimensional barotropic model, that is, the shallow water model in the limit $L_D/a \to \infty$, *Dunkerton and Scott* [2008] showed that superrotating and subrotating equatorial jets emerged with roughly equal probability in an ensemble of numerical calculations with identical physical parameters. Similar behavior also emerges in the shallow water equations with linear friction for $L_D/a \gtrsim 1$, but has until now not been found for $L_D/a \ll 1$, the regime of relevance for the giant planets.)

[14] The spontaneous formation of the superrotating equatorial jet and the alternating midlatitude jets, for the case $L_D/a = 0.025$, is illustrated in Figure 2. Note that the zonal jets are very robust, despite the fact that the flow is highly turbulent, as can be seen in Figure 3, which shows the potential vorticity q at time $t = 10000$ for the same integration. Further, we have found that once the equatorial superrotation has formed it is a robust feature. Several integrations were carried out beginning from a preexisting state of superrotation, but without any forcing or dissipation; in all cases the equatorial superrotation persisted throughout these integrations (typically for thousands of days).

[15] Despite the simplicity of our model, it is worth remarking that it also captures another key *qualitative* aspect of the circulation of the giant planets. The instantaneous potential vorticity field shown in Figure 3 exhibits a mixture of zonal structures, coherent vortices and filamentary turbulence, not dissimilar to the cloud-top patterns observed on the planets (here, the potential vorticity and cloud top fields can both be approximately considered as quasi-conserved tracer). Despite the qualitative nature of such a comparison, we submit that any model that purports to capture the atmospheric circulation of the giant planets should also be able to reproduce such features.

[16] Finally, we stress that the results presented above are not fortuitous, isolated members of large ensembles of integrations: they are entirely reproducible. In fact we have performed dozens of integrations with various parameter settings (varying L_D, ϵ_0, τ_{fr} and τ_{rad}) and have found that equatorial superrotation emerges in every calculation in which radiative relaxation is the dominant form of dissipation.

[17] Equatorial superrotation can be understood diagnostically in terms of the mixing by turbulent eddies of the shallow water potential vorticity $q = \zeta_a/h$. It is consistent with angular momentum conservation provided one recognizes the role of upgradient (i.e. non-advective) potential vorticity fluxes [*McIntyre*, 1982; *Dunkerton and Scott*, 2008]. As can be seen in Figure 3, mixing of q takes place on either side of, but not across, the equator, resulting in a sharp jump at the equator (visible as the white band). Through the diagnostic relation linking the zonal mean q, u and h, the jump at the equator will necessarily be accompanied by a superrotating equatorial jet (see *Dunkerton and Scott* [2008] for details in the barotropic case).

[18] The jump in q at the equator is associated with an upgradient (non-advective) flux of q across the equator. In particular, we note that the equatorial jet here is eddy-driven, rather than forced directly by the effect of the radiative relaxation on the zonal flow, in the sense that the upgradient PV flux is an eddy flux of the form $\overline{v'q'}$. This is demonstrated in Figure 4, which shows the time averaged potential vorticity flux $\overline{v'q'}$. The eddy PV flux is related to the eddy momentum flux convergence, and hence to an acceleration of $\bar{u}$ through the well-known Taylor identity, another diagnostic relation, which, in the simplest case of barotropic motion, takes the form

$$\overline{v'q'} = -\frac{1}{a}\frac{d}{d\mu}\left(\overline{u'v'}\sqrt{1-\mu^2}\right). \qquad (3)$$

(Positive $\overline{v'q'}$ coincides with the development of positive $\bar{u}$, and vice versa.) Conceptually the situation is the same as

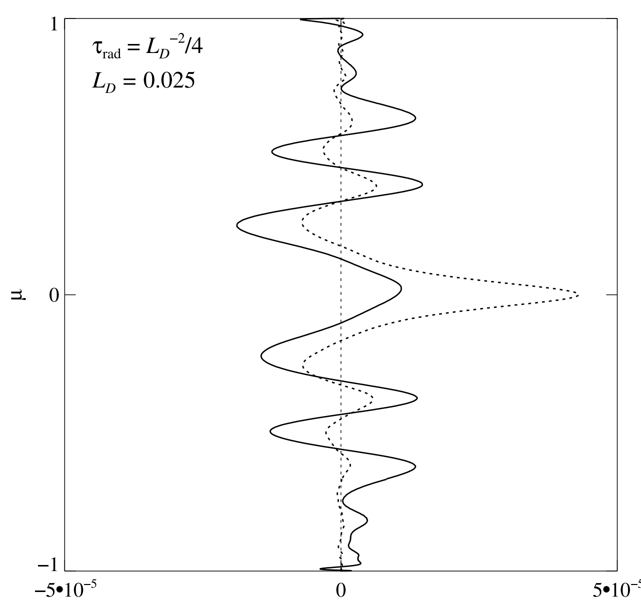

Figure 4. Time averaged $\overline{v'q'}$. Dashed line shows $\bar{u}$ ($\times 4 \times 10^{-4}$) at $t = 10000$.

that described by *McIntyre* [1982] in the case of the winter stratospheric polar vortex. Here, a modest $\overline{v'q'}$ in low latitudes results in a large zonal acceleration because of the weaker effect of thermal damping there [e.g., *Garcia*, 1987]. Note that the subrotating equatorial jets obtained in previous studies (and the frictionally damped case in Figure 1) all have mixing zones across the equator, with advective PV mixing and a corresponding downgradiant PV flux, zonal flow deceleration, and a subrotating flow at the equator.

4. Discussion

[19] In conclusion, we have shown that a simple shallow water model, with random *isotropic* forcing and a large-scale energy dissipation that crudely represents energy loss through radiation, is able to capture several of the main features of the atmospheres of the giant gas planets, specifically: (i) a turbulent flow dominated by strong, steady zonal jets; (ii) a decrease in jet amplitude with latitude; (iii) small scale filaments and vortices similar to observed cloud top features; and, most importantly, (iv) an equatorial jet that is superrotating. Further, we note that equatorial superrotation is a stable feature of this model, whose persistence does not require continued thermal damping: when the thermal damping is turned off, the equatorial jets continue to intensify (in cases where the forcing remains present) or remain steady (in cases where the forcing is also turned off).

[20] Given that they are so robust, why then have superrotating equatorial jets not been previously obtained in shallow water models? One possible reason is that in rotating shallow water anticyclones are in general more stable than cyclones [*Polvani et al.*, 1994; *Stegner and Dritschel*, 2000], an asymmetry which grows with decreasing L_D/a. Although difficult to diagnose in a fully turbulent flow, this asymmetry, coupled with the β-drift of anticyclones toward low latitudes, may account for an accumulation of anticyclonic shear, and hence a subrotating jet at the equator. Linear friction acts equally on both cyclonic and anticyclonic vorticity and so does not alter this asymmetry. In contrast it can be shown that, under certain conditions, radiative relaxation can damp anticyclones at a faster rate than cyclones (full details will be presented in a longer article), and may therefore offset the asymmetry. However, other mechanisms may also be relevant in the selection of equatorial superrotation, including the latitudinal dependence of the angular momentum changes arising from thermal damping, and the relative effects of thermal and frictional damping on mean flow changes induced by momentum flux convergences due to equatorial waves [*Andrews and McIntyre*, 1976]. Work is currently underway towards a deeper understanding of the precise mechanisms whereby the superrotation is generated.

[21] Finally we note that, although a simple shallow water model can capture many observed features in the circulation of the giant planets, including equatorial superrotation, many aspects of these circulations will require more complex physical models. For example, a common feature of shallow water integrations is that jets at high latitudes are considerably more undular than those observed on the giant planets [*Theiss*, 2004; *Scott and Polvani*, 2007]. Shallow water integrations at larger L_D produce high latitude jets that are more zonal (not shown), and this raises the question of whether estimates of L_D for the planets, based on, e.g., the phenomenology of coherent vortices [*Marcus*, 1993; *Cho et al.*, 2001], are also applicable to the jets themselves, or whether the latter might be deeper structures. Further, the equatorial jets found from shallow water integrations, such as those in Figure 1 above, are considerably narrower than those observed on Jupiter and Saturn; this latitudinal structure arises here because radiative dissipation has a weaker effect on angular momentum anomalies at low latitudes [e.g., *Garcia*, 1987]. While matching the zonal structure more closely to that of the planets might simply require more careful choices of forcing and dissipation, other dynamical processes not present in our model may also be important. For example, it has been proposed that the "double-horn" structure of Jupiter's equatorial flow may be due to a Hadley-type circulation [*Yamazaki et al.*, 2005]. And, of course, at some vertical level we expect coupling between the deep convecting interior and the shallow overlying stably stratified atmosphere. Nonetheless, it is illuminating to demonstrate, as we have done here, that something as simple as a crude radiative relaxation is sufficient to ensure the emergence of equatorial superrotation in a shallow atmosphere model, under parameters values that are relevant to the giant planets.

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