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On the DoF of two-way $2 \times 2 \times 2$ relay networks with or without relay caching

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Abstract: Two-way relay is potentially an effective approach to spectrum sharing and aggregation by allowing simultaneous bidirectional transmissions between source–destinations pairs. In this study, the two-way $2 \times 2 \times 2$ relay network, a class of four-unicast networks, where there are four source/destination nodes and two relay nodes, with each source sending a message to its destination, is studied. They show that without relay caching the total degrees of freedom (DoF) is bounded from above by $8/3$, indicating that bidirectional links do not double the DoF (it is known that the total DoF of one-way $2 \times 2 \times 2$ relay network is 2). Further, they show that the DoF of $8/3$ is achievable for the two-way $2 \times 2 \times 2$ relay network with relay caching. Finally, even though the DoF of this network is no more than $8/3$ for generic channel gains, DoF of 4 can be achieved for a symmetric configuration of channel gains.

1 Introduction

In a simple two-way relay channel, two users communicate to each other with the assistance of relays, two-unicast channels consist of two sources and two destinations communicating through a general network. The degrees of freedom (DoF) for one-way $2 \times 2 \times 2$ fully-connected two-unicast channels has been studied in [1], and further extended with interfering relays in [2]. These results were further generalised to one-way $2 \times 2 \times 2$ non-layered topology in [3, 4]. General one-way two-unicast channel has been considered in [5, 6] and it was shown in [6] that the DoF for any topology takes one of the values in $\{1, 3/2, 2\}$, depending on the topology. Two-way two-unicast channels have been studied for a single relay in [7–9]. In [10], three different achievability strategies for two-way multiple-input multiple-output (MIMO) $2 \times 2 \times 2$ fully-connected channel are proposed. A finite-field two-way two-unicast model is also studied in [11, 12].

Caching is a technique to reduce traffic load by exploiting the high degree of asynchronous content reuse and the fact that storage is cheap and ubiquitous in today's wireless devices [13, 14]. During off-peak periods when network resources are abundant, some content can be stored at the wireless edge (e.g. access points or end user devices), so that demands can be met with reduced access latencies and bandwidth requirements. There are various forms of caching, i.e. to store data at user ends, relays and so on [15]. However, using the uncoded data on devices can result in an inefficient use of the aggregate cache capacity [16]. The caching problem consists of a placement phase, which is performed offline and an online delivery phase. One important aspect of this problem is the design of the placement phase in order to facilitate the delivery phase. There are several recent works that consider communication scenarios where user nodes have pre-cached information from a fixed library of possible files during the offline phase, in order to minimise the transmission from source during the delivery phase [17, 18]. There are only a limited number of works on the DoF with caching. In particular, Han *et al.* [19, 20] study the DoF for the relay and interference channels with caching, respectively, under some assumptions and provide asymptotic results on the DoF as a function of the output of some optimisation problems.

In this paper, we study the two-way $2 \times 2 \times 2$ relay network, a class of four-unicast networks, also known as the two-way layered interference channel in the literature. We consider a general two-way $2 \times 2 \times 2$ relay network where all channel gains are chosen from the same continuous distribution. Even though the one-way $2 \times 2 \times 2$ relay network has 2 DoF, we show that the two-way $2 \times 2 \times 2$ relay network has DoF less than or equal to 8/3. Thus, the bidirectional links cannot double the DoF. In the analysis of cached $2 \times 2 \times 2$ relay network, we show the equivalence of our model to the compound multiple-input single-output (MISO) broadcast channel and use the existing results on the latter to obtain the achievable DoF of the former. Note that this is the first work relating $2 \times 2 \times 2$ relay network and compound MISO broadcast channel.

We further propose a caching strategy in relays for the two-way $2 \times 2 \times 2$ relay network based on prefetching uncoded raw bits and delivering linearly encoded messages to facilitate the transmission from relays to destinations. We show that with relay caching, the DoF of 8/3 is achievable.

Finally for a special case of two-way $2 \times 2 \times 2$ relay network where the channels exhibit symmetries, we show that the DoF is 4. This special case is interesting because (i) This shows that the $2 \times 2 \times 2$ topology allows 4 DoF for some symmetric channel gains while the DoF is outer bounded by 8/3 for generic channel gains. (ii) The non-invertibility of the symmetric channel matrix plays an important role in achieving $\text{DoF} = 4$, which does not hold for the general channel matrix. (iii) The symmetric channel is a common model for many results on interference channels [21–24]; and these results can be obtained only with such symmetric assumptions and the problems remain open otherwise.

The remainder of this paper is organised as follows. In Section 2, we give the model for the two-way $2 \times 2 \times 2$ relay network. In Section 3, the main results on the DoF of the $2 \times 2 \times 2$ relay network without relay caching is studied. The DoF of a symmetric two-way $2 \times 2 \times 2$ relay network is also investigated in this section. In Section 4, the results on the DoF of the $2 \times 2 \times 2$ relay network with relay caching is presented. Finally, Section 5 concludes this paper.

2 Channel model and related works

In this section, we first present our system model and then we discuss some recent related results in the literature.

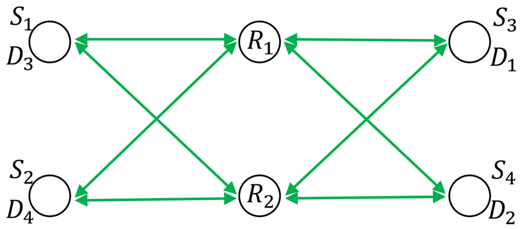

Fig. 1 *Two-way* $2 \times 2 \times 2$ *relay network*

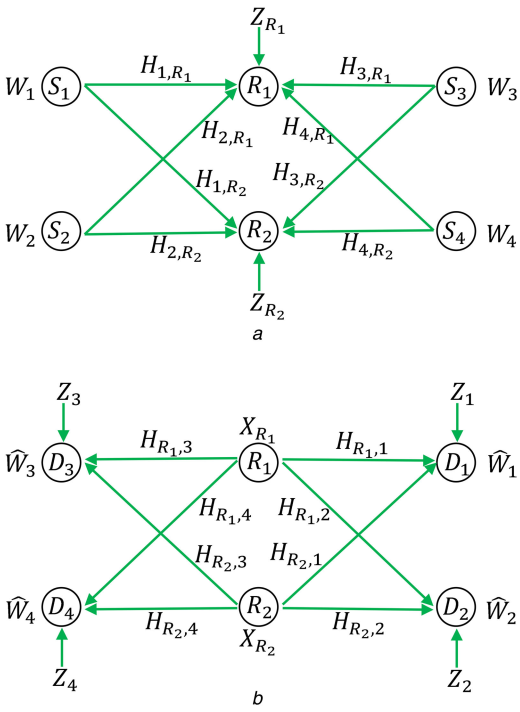

Fig. 2 *Channels from and to relays in a two-way* $2 \times 2 \times 2$ *relay network*
(a) Channels from transmitters to the relays, ***(b)*** Channels from relays to the receivers

2.1 Channel model

As shown in Fig. 1, the two-way $2 \times 2 \times 2$ relay network consists of four transmitters $S_1, \ldots, S_4$, two relays R_1, R_2, and four receivers $D_1, \ldots, D_4$. Each transmitter S_i has one message that is intended for its receiver D_i. Fig. 2 shows the two hops of this system separately. In the first hop (Fig. 2*a*), the signal received at relay R_k, $k \in \{1, 2\}$ in time slot m is

$$Y_{R_k}[m] = \sum_{i=1}^{4} H_{i,R_k}[m] X_i[m] + Z_{R_k}[m], \tag{1}$$

where $H_{i,R_k}[m]$ is the channel coefficient from transmitter S_i to relay R_k, $X_i[m]$ is the signal transmitted from S_i, $Y_{R_k}[m]$ is the signal received at relay R_k and $Z_{R_k}[m]$ is the i.i.d. circularly symmetric complex Gaussian noise with zero mean and unit variance, $i \in \{1, 2, 3, 4\}$, $k \in \{1, 2\}$. In the second hop (Fig. 2*b*), the signal received at receiver D_i in time slot m is given by

$$Y_i[m] = \sum_{k=1}^{2} H_{R_k,i}[m] X_{R_k}[m] + Z_i[m], \tag{2}$$

where $H_{R_k,i}[m]$ is the channel coefficient from relay R_k to receiver D_i, $X_{R_k}[m]$ is the signal transmitted from R_k, $Y_i[m]$ is the signal received at receiver D_i and $Z_i[m]$ is the i.i.d. circularly symmetric complex Gaussian noise with zero mean and unit variance, $i \in \{1, 2, 3, 4\}$, $k \in \{1, 2\}$. We assume that the channel coefficient values are drawn i.i.d. from a continuous distribution and they are bounded from below and above, i.e. $H_{\min} < |H_{i,R_k}[m]| < H_{\max}$ and $H_{\min} < |H_{R_k,i}[m]| < H_{\max}$ as in [25]. The relays are assumed to be full-duplex and equipped with caches. Furthermore, the relays are assumed to be causal, which means that the signals transmitted from the relays depend only on the signals received in the past and not on the current received signals and can be described as

$$X_{R_k}[m] = f(Y_{R_k}^{m-1}, X_{R_k}^{m-1}, C_{R_k}), \tag{3}$$

where $X_{R_k}^{m-1} \triangleq (X_{R_k}[1], \ldots, X_{R_k}[m-1])$, $Y_{R_k}^{m-1} \triangleq (Y_{R_k}[1], \ldots, Y_{R_k}[m-1])$, and C_{R_k} is the cached information in relay R_k. We assume that source S_i, $i \in \{1, 2, 3, 4\}$ knows only channels H_{i,R_k}, $k \in \{1, 2\}$; relay R_k, $k \in \{1, 2\}$ knows channels H_{i,R_k}, $H_{R_1,i}$ and $H_{R_2,i}$, $i \in \{1, 2, 3, 4\}$; and destination D_i, $i \in \{1, 2, 3, 4\}$ knows only channels $H_{R_k,i}$, $k \in \{1, 2\}$.

The source S_i, $i \in \{1, 2, 3, 4\}$ has a message W_i that is intended for destination D_i. $|W_i|$ denotes the size of the message W_i. The rates $\mathscr{R}_i = \log|W_i|/n$, $i \in \{1, 2, 3, 4\}$ are achievable during n channel uses by choosing n large enough, if the probability of error can be arbitrarily small for all four messages simultaneously. The capacity region $\mathscr{C} = \{(\mathscr{R}_1, \mathscr{R}_2, \mathscr{R}_3, \mathscr{R}_4) | (\mathscr{R}_1, \mathscr{R}_2, \mathscr{R}_3, \mathscr{R}_4) \in \mathscr{C}\}$ represents the set of all achievable quadruples. The sum-capacity is the maximum sum-rate that is achievable, i.e. $\mathscr{C}_\Sigma(P) = \sum_{i=1}^{4} \mathscr{R}_i^c$ where $(\mathscr{R}_1^c, \mathscr{R}_2^c, \mathscr{R}_3^c, \mathscr{R}_4^c) = \arg\max_{(\mathscr{R}_1, \mathscr{R}_2, \mathscr{R}_3, \mathscr{R}_4) \in \mathscr{C}} \sum_{i=1}^{4} \mathscr{R}_i$ and P is the transmit power at each source node. The DoF is defined as

$$\mathrm{DoF} \triangleq \lim_{P \to \infty} \frac{\mathscr{C}_\Sigma(P)}{\log P} = \sum_{i=1}^{4} \lim_{P \to \infty} \frac{\mathscr{R}_i^c}{\log P} = \sum_{i=1}^{4} d_i, \tag{4}$$

where $d_i \triangleq \lim_{P \to \infty} \mathscr{R}_i^c / \log P$ is the DoF of source S_i, for $i \in \{1, 2, 3, 4\}$. We assume that channel gains are i.i.d., chosen from the same continuous distribution, and thus the DoF is the result for almost every channel realisation (in other words, with probability 1 over the channel realisations). We denote DoF_C as the DoF for the case of with relay caching, DoF_{NC} as the DoF for the case of no relay caching.

2.2 Related works

In the literature, there has been extensive research over the last decade to characterise the DoF and the capacity region of one-way relay networks as well as two-unicast networks. However, beyond single-hop, there is much less known about the capacity of multi-flow networks. Even in the simplest case with two sources S_1 and S_2 and two destinations D_1 and D_2, there are very few results, such as [26], where the maximum flow in two-unicast undirected wireline networks is characterised. In the wireless realm, constant-gap capacity approximations for specific two-hop networks (the ZZ and ZS structures as depicted in Fig. 3) were obtained in [27]. Furthermore, it was shown that the network resulting from the concatenation of two fully-connected interference channels (the XX network as depicted in Fig. 4) admits the maximum of two DoF [1, 6]. The achievability scheme relies on the notion of real interference alignment, which was introduced in [28].

In [6], two-unicast multi-hop wireless networks with two sources S_1 and S_2 and two destinations D_1 and D_2 that have a layered structure with arbitrary connectivity are studied. It is shown that, if the channel gains are chosen independently according to continuous distributions, then, with probability 1, the DoF of the two-unicast layered Gaussian networks can be 1, 3/2 or 2. In particular, for the one-way $2 \times 2 \times 2$ relay network in Fig. 4, one DoF for each user is achievable, i.e. the total DoF is two.

There are limited number of works on the two-way $2 \times 2 \times 2$ relay network in Fig. 1. In [10], three different achievability strategies for such a network with MIMO channels are proposed. However, these schemes are considerably away from the optimum, since the achievable total DoF is only two for the SISO case, i.e. the same as the one-way network. In addition, a symmetric finite-field two-way $2 \times 2 \times 2$ relay network model is studied in [11, 12].

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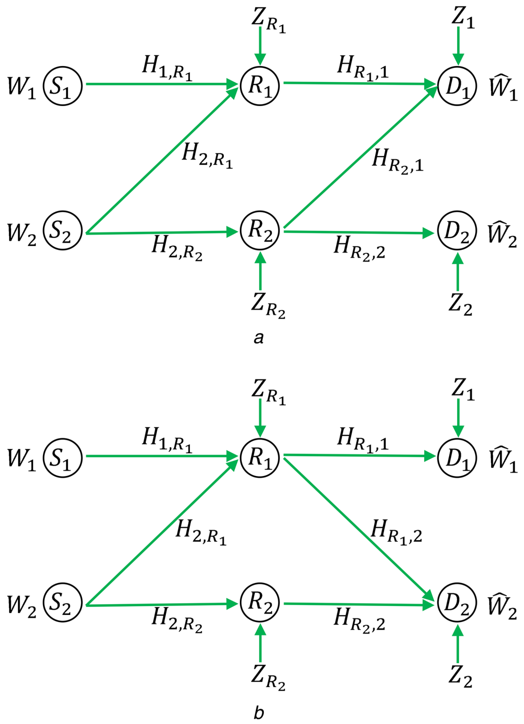

Fig. 3 *One-way ZZ and ZS networks*
(a) One-way ZZ channel, ***(b)*** One-way ZS channel

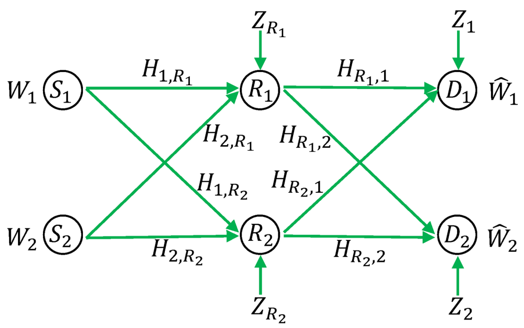

Fig. 4 *One-way* $2 \times 2 \times 2$ *relay network*

There are a few recent papers that studied the impact of caching on DoF. In particular, Han *et al.* [19, 20] analysed the DoF gain induced by caching in interference networks, and proposed a cache-induced cooperative transmission strategy. Also Naderializadeh *et al.* [29] studied some fundamental limits of the DoF for cache-aided interference networks.

3 Main results on the two-way $2 \times 2 \times 2$ relay network

In this section, we study the model in Figs. 1 and 2, with and without caching at relays.

3.1 Two-way $2 \times 2 \times 2$ relay network without caching

We assume that the channel parameters H_{j,R_k}, $H_{R_k,j}$, $j \in \{1,2,3,4\}$, $k \in \{1,2\}$ are independent and chosen from the same continuous distribution. Our result is that without caching at the relay, the total DoF of the two-way $2 \times 2 \times 2$ network is lower bounded by 2 and upper bounded by 8/3.

Proposition 1: For a two-way $2 \times 2 \times 2$ relay network, $\text{DoF}_{NC} \geq 2$.

Proof: If all nodes except for S_1, R_2 and S_3 in Fig. 1 are silent, then the channel can be seen as a two-way $1 \times 1 \times 1$ relay network formed by S_1, R_2 and S_3. This channel can achieve two DoF by simply forwarding the sum of the received signals at relay R_2, which is the sum of the two messages from S_1 and S_3. □

The next result shows that the DoF for the two-way $2 \times 2 \times 2$ relay network is upper bounded by 8/3. Thus, the DoF for the two-way network is smaller than twice the DoF for the one-way network.

Theorem 1: For a two-way $2 \times 2 \times 2$ relay network, $\text{DoF}_{NC} \leq 8/3$.

Proof: For the outer bound, we assume that there is a channel with infinite capacity between the relays. Also, suppose that a genie provides W_1 to this combined relay. Then W_3 should be decodable at the relay as it is decodable at D_3 given W_1. Following this, the messages W_2 and W_4 can be decoded if the matrix $\boldsymbol{H}$ (defined below) is full rank as (5) will suggest this mathematically, which happens with probability 1 over generic channel gains. Therefore, the combined relay should be able to decode three signals W_2, W_3 and W_4 with its two antennas (suggesting $d_2 + d_3 + d_4 \leq 2$). This is further proved in the following.
Consider n time slots of the channel use and assume that nR_i represents the maximum rate achievable for transmitter i in the total n time slots. Define $Y_i^n \triangleq (Y_i[1], \ldots, Y_i[n])$ and $X_i^n \triangleq (X_i[1], \ldots, X_i[n])$, $i = 1, \ldots, 4$. We also define $\boldsymbol{h}_{j,R} \triangleq [H_{j,R_1} H_{j,R_2}]$, $\boldsymbol{y}_R^n \triangleq [Y_{R_1}^n Y_{R_2}^n]$ and $\boldsymbol{z}_R^n \triangleq [Z_{R_1}^n Z_{R_2}^n]$, where $Y_{R_k}^n \triangleq (Y_{R_k}[1], \ldots, Y_{R_k}[n])$ and $Z_{R_k}^n \triangleq (Z_{R_k}[1], \ldots, Z_{R_k}[n])$. Then, we have

$$
\begin{aligned}
n\mathscr{R}_3 &\overset{(a)}{\leq} I(W_3; Y_3^n) + n\epsilon_n \\
&\overset{(b)}{\leq} I(W_3; Y_3^n | W_1) + n\epsilon_n \\
&\overset{(c)}{\leq} I(W_3; \boldsymbol{y}_R^n | W_1) + n\epsilon_n \\
&= h(\boldsymbol{y}_R^n | W_1) - h(\boldsymbol{y}_R^n | W_1, W_3) + n\epsilon_n \\
&= h(\boldsymbol{y}_R^n | W_1) - I(\boldsymbol{y}_R^n; W_2, W_4 | W_1, W_3) \\
&\quad - h(\boldsymbol{y}_R^n | W_1, W_3, W_2, W_4) + n\epsilon_n \\
&\overset{(d)}{=} h(\boldsymbol{y}_R^n | W_1) - I(\boldsymbol{y}_R^n; W_2, W_4 | W_1, W_3) - h(\boldsymbol{z}_R^n) + n\epsilon_n \\
&= h(\boldsymbol{y}_R^n | W_1) - I(\boldsymbol{y}_R^n; W_2, W_4 | W_1, W_3) - \log\left((2\pi e)^{2n}\right) + n\epsilon_n \\
&= h(\boldsymbol{y}_R^n | W_1) - H(W_2, W_4 | W_1, W_3) \\
&\quad + H(W_2, W_4 | W_1, W_3, \boldsymbol{y}_R^n) - 2n\log(2\pi e) + n\epsilon_n \\
&\leq h(\boldsymbol{y}_R^n | W_1) - H(W_2, W_4 | W_1, W_3) \\
&\quad + H(W_2, W_4 | \boldsymbol{y}_R^n - H_{1,R}X_1^n - H_{3,R}X_3^n) - 2n\log(2\pi e) + n\epsilon_n \\
&= h(\boldsymbol{y}_R^n | W_1) - H(W_2, W_4 | W_1, W_3) \\
&\quad + H(W_2, W_4 | H_{2,R}X_2^n + H_{4,R}X_4^n + \boldsymbol{z}_R^n) - 2n\log(2\pi e) + n\epsilon_n \\
&\overset{(e)}{=} h(\boldsymbol{y}_R^n | W_1) - H(W_2, W_4 | W_1, W_3) \\
&\quad + H(W_2, W_4 | [X_2^n X_4^n] + \boldsymbol{z}_R^n \boldsymbol{H}^{-1}) - 2n\log(2\pi e) + n\epsilon_n \\
&\overset{(f)}{=} h(\boldsymbol{y}_R^n | W_1) - H(W_2, W_4 | W_1, W_3) \\
&\quad + H(W_2, W_4 | X_2^n + z_2^{n'}, X_4^n + z_4^{n'}) - 2n\log(2\pi e) + n\epsilon_n \\
&\overset{(g)}{\leq} h(\boldsymbol{y}_R^n | W_1) - H(W_2, W_4 | W_1, W_3) \\
&\quad + H(W_2 | X_2^n + z_2^{n'}) + H(W_4 | X_4^n + z_4^{n'}) - 2n\log(2\pi e) + n\epsilon_n \\
&\overset{(h)}{=} h(\boldsymbol{y}_R^n | W_1) - H(W_2, W_4 | W_1, W_3) - 2n\log(2\pi e) + n\epsilon_n' + n\epsilon_n \\
&= h(\boldsymbol{y}_R^n | W_1) - H(W_2, W_4) - 2n\log(2\pi e) + n\epsilon_n'' \\
&\overset{(i)}{\leq} h(\boldsymbol{y}_R^n) - H(W_2, W_4) - 2n\log(2\pi e) + n\epsilon_n'' \\
&\overset{(j)}{\leq} h(Y_{R_1}^n) + h(Y_{R_2}^n) - H(W_2, W_4) - 2n\log(2\pi e) + n\epsilon_n'' \\
&\overset{(k)}{\leq} 2\left(\log\left(2\pi e(4H_{\max}^2 P + 1)\right)^n\right) - H(W_2, W_4) - 2n\log(2\pi e) + n\epsilon_n'',
\end{aligned} \tag{5}
$$

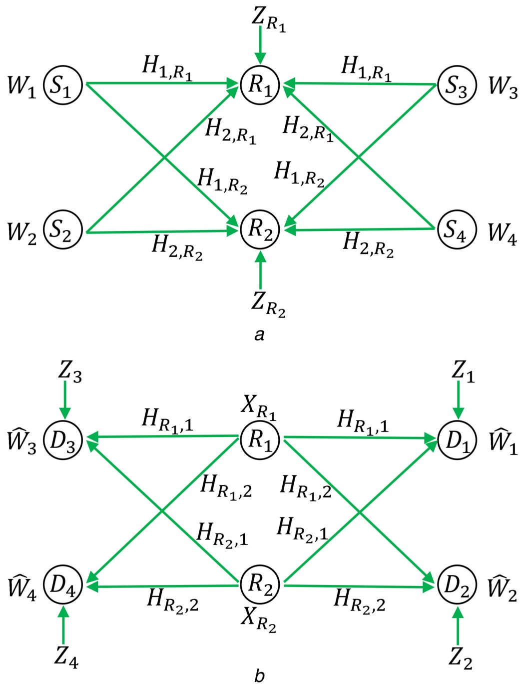

Fig. 5 *Channels from and to relays in a symmetric two-way* $2 \times 2 \times 2$ *relay network*

(a) Channels from transmitters to the relays, ***(b)*** Channels from relays to the receivers

where (a) follows since the transmission rate is less than or equal to the mutual information between the message and the received signals, and ϵ_n can be arbitrarily small by increasing n; (b) follows since $I(W_3; \boldsymbol{Y}_3^n | W_1) - I(W_3; \boldsymbol{Y}_3^n) = I(W_3; \boldsymbol{Y}_3^n; W_1) \geq -\min\{I(W_3; \boldsymbol{Y}_3^n), I(W_1; \boldsymbol{Y}_3^n), I(W_3; W_1)\} = 0$ (as $I(W_3; W_1) = 0$); (c) holds since $W_3 \to \boldsymbol{y}_R^n \to Y_3^n$; (d) follows since by subtracting the contributions of X_i^n, $i = 1, \ldots, 4$ from $\boldsymbol{y}_R^n$, we will only have Gaussian noise at the relays; (e) follows from the fact that by defining $\boldsymbol{H} \triangleq [\boldsymbol{h}_{2,R}^T \boldsymbol{h}_{4,R}^T]^T$, we obtain the following:

$$(\boldsymbol{y}_R^n - H_{1,R}X_1^n - H_{3,R}X_3^n)\boldsymbol{H}^{-1} = [X_2^n\ X_4^n]^T + z_R^n\boldsymbol{H}^{-1}, \tag{6}$$

(f) holds by defining $[z_2^{n'}\ z_4^{n'}] \triangleq z_R^n\boldsymbol{H}^{-1}$; (g) holds since conditioning decreases entropy; (h) follows from Fano's inequality and the fact that probability of error in decoding W_i given $X_i^n + z_i^{n'}$, $i = 2, 4$ goes to zero for high signal-to-noise ratio (SNR); (i) holds because conditioning decreases the entropy; (j) holds since $h(X, Y) \leq h(X) + h(Y)$; (k) holds since Y_{R_i} is in the form of (1), with $|H_{i,R_k}[m]| \leq H_{\max}$, and $X_i \sim \mathcal{CN}(0, P)$.
Dividing both sides of (5) by $n\log P$, and using $n(\mathcal{R}_2 + \mathcal{R}_4 - \epsilon_n'') \leq I(W_2; Y_2) + I(W_4; Y_4) = H(W_2) - H(W_2|Y_2) + H(W_4) - H(W_4|Y_4) \leq H(W_2) + H(W_4) = H(W_2, W_4)$, results in

$$\frac{\mathcal{R}_3}{\log P} \leq \frac{2\log\left(2\pi e(4H_{\max}^2 P + 1)\right)^n}{n\log P} - \frac{(\mathcal{R}_2 + \mathcal{R}_4)}{\log P} - \frac{2n\log(2\pi e)}{n\log P} + \frac{\epsilon_n''}{\log P}, \tag{7}$$

and with $n \to \infty$ and $P \to \infty$, we obtain the following bound:

$$d_2 + d_3 + d_4 \leq 2. \tag{8}$$

Similarly, we also have

$$d_1 + d_2 + d_3 \leq 2, \tag{9}$$

$$d_1 + d_2 + d_4 \leq 2, \tag{10}$$

$$d_1 + d_3 + d_4 \leq 2. \tag{11}$$

Summing up (8)–(11) gives the upper bound in the statement of the theorem. □

3.2 Symmetric two-way $2 \times 2 \times 2$ *relay network*

In this section, we focus on a symmetric case of the two-way $2 \times 2 \times 2$ relay network in Fig. 1, where the channel parameters are assumed to have the following symmetry: $H_{1,R_k} = H_{3,R_k}$, $H_{2,R_k} = H_{4,R_k}$, $H_{R_k,1} = H_{R_k,3}$ and $H_{R_k,2} = H_{R_k,4}$, $k \in \{1, 2\}$. The two-hop decomposition of such a symmetric two-way $2 \times 2 \times 2$ relay network is shown in Fig. 5. The following result shows that the DoF for symmetric two-way $2 \times 2 \times 2$ relay network is four. Thus, this two-way network achieves twice the DoF as compared to the one-way network studied in [1]. Essentially the symmetry in channel parameters allows for efficient alignment of the signals from the two directions at the relays that leads to the DoF of four.

Proposition 2: For the symmetric two-way $2 \times 2 \times 2$ relay network, $\text{DoF}_{NC} = 4$.

Proof: For the achievability, S_1 and S_3 can be seen as one user and S_2 and S_4 can be seen as another user from the relay nodes' perspective, due to symmetry. Based on the result for the one-way $2 \times 2 \times 2$ network in [1], each message can achieve one DoF. Since we increase the number of messages from 2 in [1] to 4 in this paper, DoF = 4 is achievable.
In addition, it can be seen that the upper bound follows from the cut-set bound. □

Remark 1: The comparison of Theorem 1 and Proposition 2 shows that, interestingly, even though without caching 8/3 is an upper bound for generic channels, for symmetric channels DoF = 4 which shows that this network topology can, in principle, allow 4 DoF. Note that step (e) in (5) for generic channel parameters requires the invertibility of $\boldsymbol{H}$, which does not hold for symmetric channels. This is what distinguishes generic channels (for which DoF $\leq 8/3$) from symmetric symmetric channels (for which DoF = 4).

4 Two-way $2 \times 2 \times 2$ network with caching

4.1 Caching and transmission strategy

In this subsection, we consider the more general model of multi-antenna relays and single-antenna source/destination nodes, where each relay R_k, $k \in \{1, 2\}$ has N_k antennas. For this model, the difference with the single-antenna case in Section 2.1 is that channels $H_{i,R_k}[m]$, $H_{R_k,i}[m]$ are $N_k \times 1$ and $1 \times N_k$ vectors, respectively, the signals to and from the relays, ($Y_{R_k}[m]$ and $X_{R_k}[m]$, respectively), are vectors of size $N_k \times 1$, and the noise $Z_{R_k}[m]$ is an $N_k \times 1$ vector, $i \in \{1, 2, 3, 4\}$, $k \in \{1, 2\}$. We assume that each relay is equipped with a cache that can store the data from the sources. Our goal is to design strategies for caching and transmission so that the sum rate of all four source–destination pairs is maximised. Similar to caching strategies in the literature [17, 18], the transmission consists of two phases. The first phase is the transmission from sources to the relays, as shown in Fig. 2*a*, which is performed offline and is known as the placement phase. The second phase is the transmission from the relays to the destinations, as shown in Fig. 2*b*, which is performed online and is known as the delivery phase. We assume that the relays decode W_i, $i = 1, \ldots, 4$ in the offline phase and save $W_1' \triangleq W_1 \oplus W_3$, $W_2' \triangleq W_2 \oplus W_4$ in their caches. Then since both relays have access to W_1' and W_2', we can consider them together as an $(N_1 + N_2)$-antenna relay, transmitting $\boldsymbol{x}_R^n = f(W_1', W_2')$, which intends to make W_1' decodable at D_1 and D_3, and W_2' decodable at D_2 and D_4 in Fig. 2*b*.

The next result, after a short review of the compound broadcast channels, shows that the DoF for the two-way $2 \times 2 \times 2$ relay

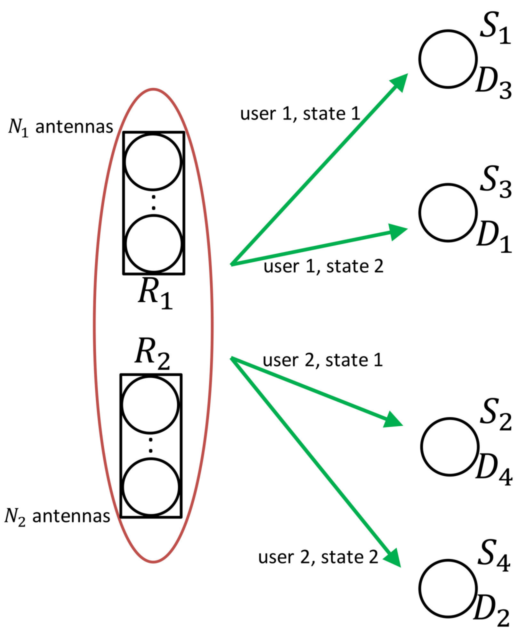

Fig. 6 *Compound MISO broadcast channel as achievability for two-way $2 \times 2 \times 2$ relay network with multiple-antenna relays and single-antenna transceivers and with relay caching*

network with multiple-antenna relays and single-antenna transceivers is lower bounded by $(4(N_1 + N_2))/(N_1 + N_2 + 1)$ under the above caching and transmission strategy.

4.2 Background on compound broadcast channel

Here, we briefly introduce the compound broadcast channel and list two lemmas that we need. The Gaussian MISO compound broadcast channel comprises one transmitter with N antennas and K single-antenna receivers. The transmitter transmits K messages, each intended for a different receiver i whose channel state is chosen from a finite set $\{1, \ldots, J_i\}$, $i = 1, \ldots, K$. In the literature there are several results on the behaviour of this channel at high SNR, i.e. the DoF. We cite the following two lemmas on the lower and upper bounds of the compound broadcast channel, respectively.

Lemma 1 [30, 31]: For the compound broadcast channel with N antennas at the transmitter, K single-antenna receivers, and $J_i \geq N$ states at receiver i, $i = 1, \ldots, K$, the total DoF of $(NK)/(N + K - 1)$ is achievable.

Lemma 2 [32]: Consider a compound broadcast channel with N antennas at the transmitter, and $K = 2$ single-antenna receivers with $J_1 = 1$, $J_2 = 2$. Then the DoF region is outer bounded by the following region:

$$\left(\frac{1}{N}\right)d_1 + d_2 \leq 1, \tag{12}$$

$$d_1 + \left(\frac{1}{N}\right)d_2 \leq 1. \tag{13}$$

4.3 DoF of two-way $2 \times 2 \times 2$ relay network with caching

In the following, we provide a result on the achievability of the two-way $2 \times 2 \times 2$ relay network with multiple-antenna relays and single-antenna transceivers with caching:

Proposition 3: Under the caching and transmission strategy given above,
$\text{DoF} \geq (4(N_1 + N_2))/(N_1 + N_2 + 1)$ for the two-way $2 \times 2 \times 2$ relay network with multiple-antenna relays and single-antenna transceivers.

Proof: In our transmission strategy, the relays amplify-and-forward the encoded data available in their caches. We treat the two relays together as a super-relay with $(N_1 + N_2)$ antennas that has access to W_1' and W_2'. The super-relay intends to make W_1' decoded at D_1 and D_3, and W_2' decoded at D_2 and D_4 since each receiver can decode the desired message by cancelling the contribution of its own message. This becomes equivalent to a compound MISO broadcast channel where message W_1' should be received at both D_1 and D_3, while message W_2' should be received at both D_2 and D_4 as depicted in Fig. 6. Thus, using Lemma 1 with $N = N_1 + N_2$ and $K = 2$, we obtain the DoF of $(2(N_1 + N_2))/(N_1 + N_2 + 1)$ which needs to be multiplied by 2 due the fact that each of the signals W_1' and W_2' is decoded by two receivers in the original channel. Hence we obtain the DoF of $(4(N_1 + N_2))/(N_1 + N_2 + 1)$ with caching. □

Now, we provide a discussion on the optimal transmission strategy for the channel with relay caching. Consider the following two transmission strategies for the relays:

• Encode W_1 and transmit with power P: It is helpful for receiver 1, has no effect for receiver 3, and is treated as interference for the other two receivers.
• Encode W_1' and transmit with power P: It is helpful for receiver 1 (exactly the same effect as in the previous case), and is also helpful for receiver 3 (in contrast to the previous case), and is treated as interference for the other two receivers similar to the previous case.

Comparison of the above two strategies suggests that if there is an achievability scheme where all of the messages are decodable for some given total transmission power at the relays, then there is also a strategy with the same power that comprises only W_1' and W_2'. Note that receivers 1 and 3 can decode their desired messages by having access to W_1' (because they have access to each other's message and can subtract it), and similar relation holds for receivers 2 and 4. With this assumption, we present the following result that gives the sum $\text{DoF} = (4(N_1 + N_2))/(N_1 + N_2 + 1)$.

Proposition 4: For the two-way $2 \times 2 \times 2$ relay network with multiple-antenna relays and single-antenna transceivers and with caching at the relays, total $\text{DoF}_C = (4(N_1 + N_2))/(N_1 + N_2 + 1)$ if the relays only use W_1' and W_2' in their transmission rather than the original individual messages.

Proof: The achievability follows from Proposition 3. Now we give the proof of the outer bound. With the assumption that the relays only transmit W_1' and W_2'. The channel can be seen as a compound broadcast channel with two receivers, where each receiver takes two possible states as in Fig. 6. We know that decreasing the number of channel states does not decrease the capacity [30]. So, we decrease the number of states in receiver 1 to only 1. Then, according to Lemma 2, the DoF region of the compound channel is bounded by (12) and (13) for $N = N_1 + N_2$, and bounds $(1/N)d_1 + d_2 \leq 1$ and $d_1 + (1/N)d_2 \leq 1$. These two bounds give a convex region with three non-zero corners of

$$(d_1, d_2) = \left(\frac{N_1 + N_2}{N_1 + N_2 + 1}, \frac{N_1 + N_2}{N_1 + N_2 + 1}\right),$$
$$(d_1, d_2) = (1, 0) \quad \text{and} \quad (d_1, d_2) = (0, 1)\,.$$

Therefore, $d_1 = d_2 = (N_1 + N_2)/(N_1 + N_2 + 1)$ results in the largest value of $d_1 + d_2 = (2(N_1 + N_2))/(N_1 + N_2 + 1)$ which as before needs to be multiplied by 2 due the fact that each of the signals W_1' and W_2' is decoded by two receivers in the original channel. Therefore, we obtain $(4(N_1 + N_2))/(N_1 + N_2 + 1)$ as a DoF upper bound. □

The above result leads to the following conjecture on the general upper bound:

Conjecture 1: For the two-way $2 \times 2 \times 2$ relay network with multiple-antenna relays and single-antenna transceivers and with caching at the relays, the total $\text{DoF}_C \leq (4(N_1 + N_2))/(N_1 + N_2 + 1)$.

Table 1 Summary of the results on the total DoF of the two-way $2 \times 2 \times 2$ relay network

	$2 \times 2 \times 2$ net	$2 \times 2 \times 2$ symmetric net	$2 \times 2 \times 2$ net w/relay caching
DoF lower bound	2 (Proposition 1)	4 (Proposition 2)	8/3 (Proposition 3)
DoF upper bound	8/3 (Theorem 1)	4 (cutset bound)	4 (cutset bound)

Remark 2: The results in Section 3.1 show that the DoF of two-way $2 \times 2 \times 2$ relay network with no relay caching is bounded as $2 \leq \mathrm{DoF}_{NC} \leq 8/3$ and Proposition 3 shows that $\mathrm{DoF}_C = 8/3$ is achievable with relay caching for $N_1 = N_2 = 1$. Hence caching can achieve the upper bound of the non-caching DoF of the two-way multiple-unicast network, thus showing that relay caching in this network could potentially improve the DoF.

Finally, the following corollary gives the DoF of symmetric $2 \times 2 \times 2$ relay networks with caching.

Corollary 1: For the symmetric two-way $2 \times 2 \times 2$ relay network with relay caching, $\mathrm{DoF}_C = 4$.

Proof: The proof follows from the following facts together:

• For the symmetric two-way $2 \times 2 \times 2$ relay network with no caching, $\mathrm{DoF}_{NC} = 4$ (Proposition 2).
• For the two-way $2 \times 2 \times 2$ relay network, $\mathrm{DoF}_C \leq 4$ due to the cut-set bound.
• $\mathrm{DoF}_C \geq \mathrm{DoF}_{NC}$.

□

5 Conclusions

We have investigated the two-way $2 \times 2 \times 2$ relay network, a class of four-unicast networks. Table 1 summarises the main results of this paper. In particular, we have shown that the total DoF is bounded from above by 8/3, indicating that bidirectional links do not double the DoF. We have also shown that DoF of 8/3 is achievable with caching at the relays. Therefore, the proposed work demonstrates that caching can achieve the outer bound of the non-caching DoF of the two-way multiple-unicast network, thus showing that relay caching in this network could be helpful in terms of DoF. Decreasing the gap between the lower and the upper bounds in Table 1 is an important problem. Finding the DoF for other models of two-way four-unicast networks as well as two-way networks with more than four source–destination pairs remains an open problem. Further, the rate analysis under finite SNR for such networks is also an open problem. Moreover, the effect of finite-size cache and message popularity in the information theoretic setting also remains to be investigated.

6 References

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