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Degrees of Freedom Region for MIMO Interference Channel with Limited Receiver Cooperation

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***Abstract*—This paper gives degrees of freedom region of two user MIMO interference channels with limited receiver cooperation. For the symmetric interference channel, we also find the amount of receiver cooperation beyond which the degrees of freedom do not improve.**

I. Introduction

Wireless networks with multiple users are interference-limited rather than noise-limited. Interference channel (IC) is a good starting point for understanding the performance limits of the interference limited communications. In spite of research spanning over three decades, the capacity of the IC has been characterized only for some special cases [1–7].

Interference channels model practical cellular networks. However, since the cellular base stations are connected via backhaul, making efficient use of the backhaul is an important practical problem. This backhaul can lead to cooperation between transmitters in the downlink and cooperation between the receivers in the uplink [8–13]. Cooperation between transmitters or receivers can help mitigate interference by forming distributed MIMO systems which provides a gain in throughput. The rate at which they cooperate, however, is limited, due to physical constraints. In this paper, we ask the fundamental question of the efficient use of limited capacity backhaul for multiple-input multiple-output (MIMO) uplink interference channels (with receiver cooperation). Recently, many results have shown that transmitter and receiver cooperation can be employed in ICs to achieve an improvement in data rates [14–21]. However, most of the existing works on ICs with cooperation are limited to discrete memoryless channels or to single-input single-output (SISO) channels. This paper analyzes the degrees of freedom region for a two-user MIMO Gaussian interference channels with limited receiver cooperation.

Interference channel with limited receiver cooperation was considered in [14] for the case of single antennas at each terminal, where the authors found the approximate capacity region with limited receiver cooperation. This paper considers the degrees of freedom region for the two user interference channel with limited receiver cooperation for the case of multiple antennas at each of the terminal. We find the degrees of freedom region improve with receiver cooperation from that of no cooperation to complete cooperation with limited cooperation. For the case of symmetric antennas, we find that the symmetric degrees of freedom improve with the amount of receiver cooperation, till the amount of receiver cooperation is $\min(N, (2M-N)^+)$, where M and N are the number of transmit and receive antennas respectively.

The symmetric degrees of freedom region formed when both the transmitters have M antennas and both the receivers have N antennas each, is a pentagon with only individual and sum degrees of freedom bound for all cases except when $N < M < 2N$. Thus, when the number of antennas at all the nodes are the same, the degrees of freedom is a pentagon. However, in the case when $N < M < 2N$, we note that the degrees of freedom region have constraints of $2d_1 + d_2$ and d_1+2d_2. These constraints are known to not hold when there is no cooperation in which case the channel model becomes interference channel with no cooperation [5], and when there is infinite cooperation in which case the channel model is equivalent to a multiple access channel [22]. In this paper, we find that the extra bounds on $2d_1+d_2$ and that on d_1+2d_2 are dominant for a finite limited cooperation (when cooperation is less than a certain amount) for $N < M < 2N$. We note that this result shows that the role of transmit and receive antennas cannot be interchanged to get the reciprocity result which exists in the case of no cooperation [5].

The remainder of the paper is organized as follows. Section II introduces the model for a MIMO IC model with limited receiver cooperation and the DoF region. Sections III describe our results on degrees of freedom region. Section IV concludes the paper.

II. Channel Model and Preliminaries

In this section, we describe the channel model considered in this paper. A two-user MIMO IC consists of two transmitters and two receivers. Transmitter i is labeled as T_i

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and receiver j is labeled as D_j for $i, j \in \{1, 2\}$. Further, we assume T_i has M_i antennas and D_i has N_i antennas, $i \in \{1, 2\}$. Henceforth, such a MIMO IC will be referred to as the (M_1, N_1, M_2, N_2) MIMO IC. We assume that the channel matrix between transmitter T_i and receiver D_j is denoted by $H_{ij} \in \mathbb{C}^{N_j \times M_i}$, for $i, j \in \{1, 2\}$. We shall consider a time-invariant or fixed channel where the channel matrices remain fixed for the entire duration of communication. We also incorporate a non-negative power attenuation factor, denoted as ρ_{ij}, for the signal transmitted from T_i to D_j. At time-instant t, transmitter T_i chooses a vector $X_i(t) \in \mathbb{C}^{M_i \times 1}$ and transmits $\sqrt{P_i}X_i(t)$ over the channel, where P_i is the average transmit power at transmitter T_i.

The received signal at receiver D_i at time instant t is denoted as $Y_i(t)$ for $i \in \{1, 2\}$, and can be written as

$$Y_1(t) = \sqrt{\rho_{11}}H_{11}X_1(t) + \sqrt{\rho_{21}}H_{21}X_2(t) + Z_1(t), \quad (1)$$
$$Y_2(t) = \sqrt{\rho_{12}}H_{12}X_1(t) + \sqrt{\rho_{22}}H_{22}X_2(t) + Z_2(t), \quad (2)$$

where $Z_i(t) \in \mathbb{C}^{N_i \times 1}$ is independent and identically distributed (i.i.d.) $\mathsf{CN}(0, I_{N_i})$ (complex Gaussian noise), ρ_{ii} is the received SNR at receiver D_i and ρ_{ij} is the received interference-to-noise-ratio at receiver D_j for $i, j \in \{1, 2\}$, $i \neq j$. A MIMO IC with limited receiver cooperation is fully described by four parameters. The first is the number of antennas at each transmitter and receiver, namely (M_1, N_1, M_2, N_2). The second is the set of channel gains, $\overline{H} = \{H_{11}, H_{12}, H_{21}, H_{22}\}$. The third is the set of average link qualities of all the channels, $\overline{\rho} = \{\rho_{11}, \rho_{12}, \rho_{21}, \rho_{22}\}$. The fourth parameter is $\overline{C} = \{C_{12}, C_{21}\}$ where C_{ji} is the capacity of the cooperation link from the other receiver of D_j to D_i. We assume that these parameters are known to all transmitters and receivers. In this paper, we assume that $\rho_{ij} = \mathsf{SNR}$ for all $i, j \in \{1, 2\}$.

The receiver-cooperative links are noiseless with finite capacities. Encoding must satisfy causality constraints in the sense that the signal transmitted from D_j at time n is a function of whatever is received over the channel, or on the cooperation link till time $n-1$. In addition, the decoded signal at D_i, $\widehat{m_i}$, is a function of the received signal from the channel, $Y_i(t)$, and the cooperation signal transmitted from receiver j to receiver i, C_{ji}, for $i \in \{1, 2\}$. Thus, the decoding functions of the two receivers are given as

$$\hat{m_i} = f_{it}(\mathsf{C}_{ji}, Y_i(t)), \quad i \in \{1, 2\}, \quad (3)$$

where f_{it} is the decoding function of D_i. Let us assume that T_i transmits information at a rate of R_i to receiver D_i using the codebook $C_{i,n}$ of length-n codewords with $|C_{i,n}| = 2^{nR_i}$. Given a message $m_i \in \{1, \ldots, 2^{nR_i}\}$, the corresponding codeword $X_i^n(m_i) \in C_{i,n}$ satisfies the power constraint mentioned before. From the received signal Y_i^n and the received cooperation from the other receiver, C_{ji}, the receiver obtains an estimate $\hat{m}_i$ of the transmitted message m_i using a decoding function. Let the average probability of error be denoted by $e_{i,n} = \Pr(\hat{m}_i \neq m_i)$.

A rate pair (R_1, R_2) is achievable if there exists a family of codebooks $C_{i,n}, i = \{1, 2\}_n$ and decoding functions such that $\max_i\{e_{i,n}\}$ goes to zero as the block length n goes to infinity. The capacity region $C(\overline{H}, \overline{\rho})$ of the IC with parameters $\overline{H}$ and $\overline{\rho}$ is defined as the closure of the set of all achievable rate pairs.

We define DoF of the user i^{th} as $d_i = \lim_{\mathsf{SNR}\to\infty} \frac{R_i}{\log \mathsf{SNR}}$ and define DoF region for the MIMO interference channel as $D = \{(d_1, d_2) \in R_+^2 : d_i = \lim_{\mathsf{SNR}\to\infty} \frac{R_i}{\log \mathsf{SNR}}\}$

III. Degrees of Freedom of MIMO Interference Channel with Feedback

In this section, we find the DoF region for the two user MIMO interference channel with limited receiver cooperation.

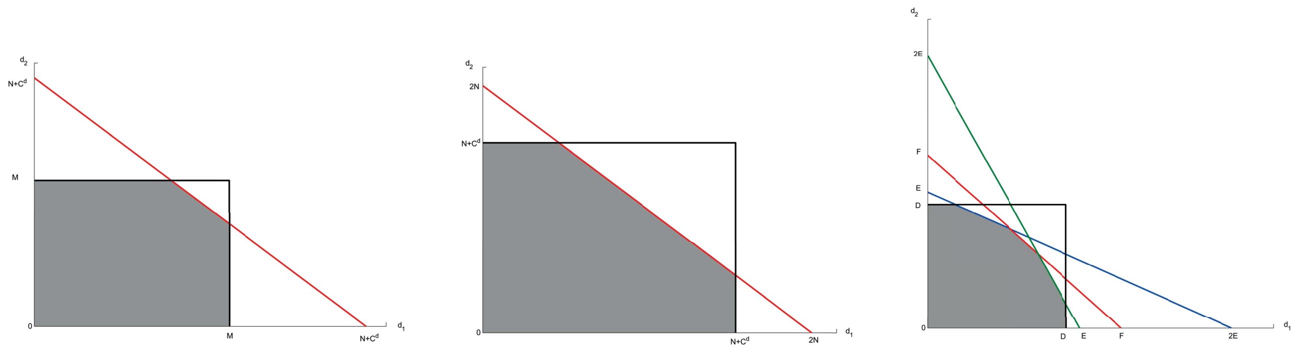
Theorem 1. *The DoF region for a general MIMO IC with limited receiver cooperation is given as follows:*

$$\begin{aligned}
d_1 &\leq \min\{M_1, N_1\} + \\
&\quad \min\{\min\{N_2, (M_1 - N_1)^+\}, C_{21}^d\}, && (4)\\
d_2 &\leq \min\{M_2, N_2\} + \\
&\quad \min\{\min\{N_1, (M_2 - N_2)^+\}, C_{12}^d\}, && (5)\\
d_1 + d_2 &\leq \min\{N_1, (M_1 - N_2)^+ + M_2\} + \\
&\quad \min\{N_2, (M_2 - N_1)^+ + M_1\} + C_{12}^d + \\
&\quad C_{21}^d, && (6)\\
d_1 + d_2 &\leq \min\{N_1, (M_1 - N_2)^+\} + \\
&\quad \min\{N_2, M_1 + M_2\} + C_{12}^d, && (7)\\
d_1 + d_2 &\leq \min\{N_2, (M_2 - N_1)^+\} + \\
&\quad \min\{N_1, M_1 + M_2\} + C_{21}^d, && (8)\\
d_1 + d_2 &\leq \min\{N_1 + N_2, M_1 + M_2\}, && (9)\\
2d_1 + d_2 &\leq \min\{N_2, (M_2 - N_1)^+ + M_1\} + \\
&\quad \min\{N_1, (M_1 - N_2)^+\} + \\
&\quad \min\{N_1, M_1 + M_2\} + C_{12}^d + C_{21}^d, && (10)\\
d_1 + 2d_2 &\leq \min\{N_1, (M_1 - N_2)^+ + M_2\} + \\
&\quad \min\{N_2, (M_2 - N_1)^+\} + \\
&\quad \min\{N_2, M_2 + M_1\} + C_{12}^d + C_{21}^d, && (11)\\
2d_1 + d_2 &\leq \min\{N_1 + N_2, M_1\} + \\
&\quad \min\{N_1, M_1 + M_2\} + C_{21}^d, && (12)\\
d_1 + 2d_2 &\leq \min\{N_1 + N_2, M_2\} + \\
&\quad \min\{N_2, M_1 + M_2\} + C_{12}^d. && (13)
\end{aligned}$$

Proof. The proof is given in Appendix A. □

For the symmetric case, the DoF region simplifies as follows.

Corollary 1. *The symmetric DoF region where $C_{12}^d = C_{21}^d = C^d$, $N_1 = N_2 = N$, and $M_1 = M_2 = M$, is given as follows:*



(a) $M \leq N$

(b) $2N \leq M$

(c) $N \leq M \leq 2N$, where $D = \min(M, N + C^d)$, $E = \frac{M+N+C^d}{2}$, and $F = \min(M + C^d, 2N)$

Fig. 1. *DoF region for symmetric MIMO IC with limited receiver cooperation (grey area).*

For $M \leq N$:

$$\begin{aligned} d_1 &\leq M, \\ d_2 &\leq M, \\ d_1 + d_2 &\leq N + C^d. \end{aligned} \tag{14}$$

For $2N \leq M$:

$$\begin{aligned} d_1 &\leq N + C^d, \\ d_2 &\leq N + C^d, \\ d_1 + d_2 &\leq 2N. \end{aligned} \tag{15}$$

For $N \leq M \leq 2N$:

$$\begin{aligned} d_1 &\leq \min\{M, N + C^d\}, \\ d_2 &\leq \min\{M, N + C^d\}, \\ d_1 + d_2 &\leq \min\{M + C^d, 2N\}, \\ 2d_1 + d_2 &\leq N + M + C^d, \\ d_1 + 2d_2 &\leq N + M + C^d. \end{aligned} \tag{16}$$

Figure 1 shows the symmetric DoF region. We note that for $C^d = 0$, we get the same degrees of freedom region as in [5]. For this case, the degrees of freedom region do not have bounds on $2d_1 + d_2$ and $d_1 + 2d_2$, and has reciprocity in M and N. However, both these properties do not hold with limited receiver cooperation. With infinite cooperation, the degrees of freedom region reduces to a MIMO MAC region as given in [22] where the bounds on $2d_1 + d_2$ and $d_1 + 2d_2$ are not dominant. We note here that for $N \leq M \leq 2N$ and $0 < C^d < min(N, (2M - N)^+)$, these bounds are dominant.

Lemma 1. *For the symmetric case when $C^d_{12} = C^d_{21} = C^d$, $N_1 = N_2 = N$, and $M_1 = M_2 = M$, DoF region with cooperation of $C^d + \epsilon$ is strictly better than that with C^d for any $\epsilon > 0$ if $0 \leq C^d < \min\{N, (2M - N)^+\}$. However, cooperation beyond $\min\{N, (2M - N)^+\}$ does not improve the DoF region which implies that the DoF region with a cooperation of $C^d = \min\{N, (2M - N)^+\}$ is the same that for a multiple access channel obtained with infinite cooperation.*

Proof. For $M \leq N$ it can be seen from (14) that the cooperation improves the DoF region until $C^d \leq (2M - N)^+ = \min\{N, (2M - N)^+\}$.

Also, for $2N \leq M$ it can be seen from (15) that the cooperation improves the DoF region until $C^d \leq N = \min\{N, (2M - N)^+\}$.

For $N \leq M \leq 2N$, we divide the proof into four different cases:

Case 1:

$$\begin{aligned} C^d &\leq M - N, \\ C^d &\leq 2N - M. \end{aligned} \tag{17}$$

In this case, the symmetric DoF region reduces to

$$\begin{aligned} d_1 &\leq N + C^d, \\ d_2 &\leq N + C^d, \\ d_1 + d_2 &\leq C^d, \\ 2d_1 + d_2 &\leq N + M + C^d, \\ d_1 + 2d_2 &\leq N + M + C^d. \end{aligned} \tag{18}$$

In this region, C^d is always less than $\min\{N, (2M - N)^+\}$ because $C^d \leq M - N \leq N = \min\{N, (2M - N)^+\}$. In this case, it is easy to see increasing the C^d always enlarges the region.

Case 2:

$$\begin{aligned} C^d &\geq M - N, \\ C^d &\leq 2N - M. \end{aligned} \tag{19}$$

In this case, the symmetric DoF region reduces to

$$\begin{aligned} d_1 &\leq M, \\ d_2 &\leq M, \\ d_1+d_2 &\leq M+C^d, \\ 2d_1+d_2 &\leq N+M+C^d, \\ d_1+2d_2 &\leq N+M+C^d. \end{aligned} \quad (20)$$

In this region, C^d is always less than $\min\{N,(2M-N)^+\}$ because $C^d \leq 2N-M \leq N = \min\{N,(2M-N)^+\}$. In this case, it is easy to see increasing the C^d always enlarges the region. According to Figure 2(c), while $C^d \leq 2N-M$, we get $2E \leq 3N$ and $F \leq 2N$ which shows none of the red, green and blue lines could include the point $(d_1,d_2) = (M,M)$ below them in this case. Also, increasing the C^d, results the increase of E and F in Figure 2(c) and as a result, enlarges the symmetric DoF region.

Case 3:

$$\begin{aligned} C^d &\leq M-N, \\ C^d &\geq 2N-M. \end{aligned} \quad (21)$$

In this case, the symmetric DoF region reduces to

$$\begin{aligned} d_1 &\leq N+C^d, \\ d_2 &\leq N+C^d, \\ d_1+d_2 &\leq 2N, \\ 2d_1+d_2 &\leq N+M+C^d, \\ d_1+2d_2 &\leq N+M+C^d. \end{aligned} \quad (22)$$

In this region, C^d is always less than $\min\{N,(2M-N)^+\}$ because $C^d \leq M-N \leq N = \min\{N,(2M-N)^+\}$. In this case, it is easy to see increasing the C^d always enlarges the region. According to Figure 2(c), while $C^d \leq M-N$, we get $D,E \leq M \leq 2N = F$ and also, increasing the C^d, results the increase of D and E in Figure 2(c) and as a result, enlarges the symmetric DoF region.

Case 4:

$$\begin{aligned} C^d &\geq M-N, \\ C^d &\geq 2N-M. \end{aligned} \quad (23)$$

In this case, the symmetric DoF region reduces to

$$\begin{aligned} d_1 &\leq M, \\ d_2 &\leq M, \\ d_1+d_2 &\leq 2N, \\ 2d_1+d_2 &\leq N+M+C^d, \\ d_1+2d_2 &\leq N+M+C^d. \end{aligned} \quad (24)$$

In this region, changing C^d only changes E in Figure 2(c). Also, we can easily see that black line and red line have intersection on $(d_1,d_2) = (M,2N-M)$. Green line includes this intersection while $C^d \geq N$ and will be below this point while $C^d \leq N$ which means increasing the C^d improves the DoF region until $C^d \leq N = \min\{N,(2M-N)^+\}$. □

.

IV. Conclusions

This paper finds the degrees of freedom region for the two user MIMO interference channel with limited receiver cooperation. We find that the degrees of freedom region improves with cooperation. For the symmetric case, we find the maximum amount of cooperation needed to get the degrees of freedom region the same as that will full cooperation. Limited receiver cooperation gives two additional bounds on $2d_1+d_2$ and d_1+2d_2, which do not exist in the cases of no cooperation as well as full cooperation.

Appendix A
Proof of Theorem 1

The degrees of freedom region is given as the limit of the capacity region divided by $\log(\mathsf{SNR})$ as SNR goes to infinity. We will first describe the approximate capacity region for the two-user MIMO IC with limited receiver cooperation.

Let $\mathcal{R}_o$ be the convex hull of the region formed by (R_1,R_2) satisfying the following constraints.

$$R_1 \leq \log\det(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger)+\min\{\log\det(I_{N_2}+\rho_{12}H_{12}H_{12}^\dagger-\rho_{12}\rho_{11}H_{12}H_{11}^\dagger(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger)^{-1}H_{11}H_{12}^\dagger),C_{21}\}, \quad (25)$$

$$R_2 \leq \log\det(I_{N_2}+\rho_{22}H_{22}H_{22}^\dagger)+\min\{\log\det(I_{N_1}+\rho_{21}H_{21}H_{21}^\dagger-\rho_{21}\rho_{22}H_{21}H_{22}^\dagger(I_{N_2}+\rho_{22}H_{22}H_{22}^\dagger)^{-1}H_{22}H_{21}^\dagger),C_{12}\}, \quad (26)$$

$$R_1+R_2 \leq \log\det(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger+\rho_{21}H_{21}H_{21}^\dagger-\rho_{11}\rho_{12}H_{11}H_{12}^\dagger(I_{N_2}+\rho_{12}H_{12}H_{12}^\dagger)^{-1}H_{12}H_{11}^\dagger)+\log\det(I_{N_2}+\rho_{22}H_{22}H_{22}^\dagger+\rho_{12}H_{12}H_{12}^\dagger-\rho_{22}\rho_{21}H_{22}H_{21}^\dagger\{(I_{N_1}+\rho_{21}H_{21}H_{21}^\dagger)^{-1}\}H_{21}H_{22}^\dagger)+C_{12}+C_{21} \quad (27)$$

$$R_1+R_2 \leq \log\det(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger-\rho_{11}\rho_{12}H_{11}H_{12}^\dagger(I_{N_2}+\rho_{12}H_{12}H_{12}^\dagger)^{-1}H_{12}H_{11}^\dagger)+\log\det(I_{N_2}+\rho_{22}H_{22}H_{22}^\dagger+\rho_{12}H_{12}H_{12}^\dagger)+C_{12}, \quad (28)$$

$$R_1+R_2 \leq \log\det(I_{N_2}+\rho_{22}H_{22}H_{22}^\dagger-\rho_{22}\rho_{21}H_{22}H_{21}^\dagger(I_{N_1}+\rho_{21}H_{21}H_{21}^\dagger)^{-1}H_{21}H_{22}^\dagger)+\log\det(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger+\rho_{21}H_{21}H_{21}^\dagger)+C_{21}, \quad (29)$$

$$R_1+R_2 \leq \log\det\left(I_{N_1+N_2}+\begin{bmatrix}\sqrt{\rho_{11}}H_{11}\\ \sqrt{\rho_{12}}H_{12}\end{bmatrix}[\sqrt{\rho_{11}}H_{11}^\dagger\ \sqrt{\rho_{12}}H_{12}^\dagger]+\begin{bmatrix}\sqrt{\rho_{21}}H_{21}\\ \sqrt{\rho_{22}}H_{22}\end{bmatrix}[\sqrt{\rho_{21}}H_{21}^\dagger\ \sqrt{\rho_{22}}H_{22}^\dagger]\right), \quad (30)$$

$$
\begin{aligned}
2R_1+R_2 \;\le\;& \log\det(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger-\rho_{11}\rho_{12}H_{11}\\
& H_{12}^\dagger(I_{N_2}+\rho_{12}H_{12}H_{12}^\dagger)^{-1}H_{12}H_{11}^\dagger)+\\
& \log\det(I_{N_2}+\rho_{22}H_{22}H_{22}^\dagger+\rho_{12}H_{12}H_{12}^\dagger\\
& -\rho_{22}\rho_{21}H_{22}H_{21}^\dagger(I_{N_1}+\rho_{21}H_{21}H_{21}^\dagger)^{-1}\\
& H_{21}H_{22}^\dagger)+\log\det(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger\\
& +\rho_{21}H_{21}H_{21}^\dagger)+C_{12}+C_{21}, \qquad (31)\\
R_1+2R_2 \;\le\;& \log\det(I_{N_2}+\rho_{22}H_{22}H_{22}^\dagger-\rho_{22}\rho_{21}H_{22}\\
& H_{21}^\dagger(I_{N_1}+\rho_{21}H_{21}H_{21}^\dagger)^{-1}H_{21}H_{22}^\dagger)+\\
& \log\det(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger+\rho_{21}H_{21}H_{21}^\dagger\\
& -\rho_{11}\rho_{12}H_{11}H_{12}^\dagger(I_{N_2}+\rho_{12}H_{12}H_{12}^\dagger)^{-1}\\
& H_{12}H_{11}^\dagger)+\log\det(I_{N_2}+\rho_{22}H_{22}H_{22}^\dagger\\
& +\rho_{12}H_{12}H_{12}^\dagger)+C_{21}+C_{12}, \qquad (32)\\
2R_1+R_2 \;\le\;& \log\det\Bigg(I_{N_1+N_2}+\begin{bmatrix}\sqrt{\rho_{22}}H_{22}\\ \sqrt{\rho_{21}}H_{21}\end{bmatrix}(I_{M_2}-\\
& \rho_{21}H_{21}^\dagger(I_{N_1}+\rho_{21}H_{21}H_{21}^\dagger)^{-1}H_{21})\\
& [\sqrt{\rho_{22}}H_{22}^\dagger\ \sqrt{\rho_{21}}H_{21}^\dagger]+\begin{bmatrix}\sqrt{\rho_{12}}H_{12}\\ \sqrt{\rho_{11}}H_{11}\end{bmatrix}\\
& [\sqrt{\rho_{12}}H_{12}^\dagger\ \sqrt{\rho_{11}}H_{11}^\dagger]\Bigg)+\log\det(I_{N_1}+\\
& \rho_{11}H_{11}H_{11}^\dagger+\rho_{21}H_{21}H_{12}^\dagger)+C_{21}, \qquad (33)\\
R_1+2R_2 \;\le\;& \log\det\Bigg(I_{N_1+N_2}+\begin{bmatrix}\sqrt{\rho_{11}}H_{11}\\ \sqrt{\rho_{12}}H_{12}\end{bmatrix}(I_{M_1}-\\
& \rho_{12}H_{12}^\dagger(I_{N_2}+\rho_{12}H_{12}H_{12}^\dagger)^{-1}H_{12})\\
& [\sqrt{\rho_{11}}H_{11}^\dagger\ \sqrt{\rho_{12}}H_{12}^\dagger]+\begin{bmatrix}\sqrt{\rho_{21}}H_{21}\\ \sqrt{\rho_{22}}H_{22}\end{bmatrix}\\
& [\sqrt{\rho_{21}}H_{21}^\dagger\ \sqrt{\rho_{22}}H_{22}^\dagger]\Bigg)+\log\det(I_{N_2}+\\
& \rho_{22}H_{22}H_{22}^\dagger+\rho_{12}H_{12}H_{12}^\dagger)+C_{12}. \qquad (34)
\end{aligned}
$$

Then, the approximate capacity region is given as follows.

Lemma 2. *The capacity region for the two-user MIMO IC with limited receiver cooperation $\mathcal{C}_{RC}$ is bounded from above and below as*

$$\mathcal{R}_o \ominus ([0,N_1+N_2]\times[0,N_1+N_2]) \subseteq \mathcal{C}_{RC} \subseteq \mathcal{R}_o, \quad (35)$$

where the inner and outer bounds are within N_1+N_2 bits.

Proof. The proof is omitted in this paper due to shortage of space, and can be seen in [23]. □

Now, we will find the limit of $\mathcal{R}_o/\log\mathsf{SNR}$ as $\mathsf{SNR}\to\infty$ to get the result as in the statement of the Theorem 1. Before going over each of the above terms and finding their high SNR limit, we first give some Lemmas that will be used for the proof.

Lemma 3 ([4]). *Let $H_1 \in \mathbb{C}^{N\times M_1}$, $H_2 \in \mathbb{C}^{N\times M_2}$,..., and $H_k \in \mathbb{C}^{N\times M_k}$ be k full rank and independent channel matrices. Then, the following holds*

$$
\begin{aligned}
& \log\det(I_N+\rho H_1H_1^\dagger+\rho H_2H_2^\dagger+...+\rho H_kH_k^\dagger)\\
=\;& \log\det(I_N+\rho[H_1\ ...\ H_k][H_1\ ...\ H_k]^\dagger)\\
=\;& \min\{N, M_1+M_2+...+M_k\}\log\mathsf{SNR}\ +\\
& o(\log\mathsf{SNR}\ ). \qquad (36)
\end{aligned}
$$

Lemma 4 ([24]). *Let $H_{ii}\in\mathbb{C}^{N_i\times M_i}$ and $H_{ij}\in\mathbb{C}^{N_i\times M_j}$ be two channel matrices with each entry independently chosen from $\mathsf{CN}(0,1)$. Then, the following holds with probability 1 (over the randomness of channel matrices).*

$$
\begin{aligned}
& \log\det(I_{N_i}+\rho H_{ii}H_{ii}^\dagger-\rho H_{ii}H_{ij}^\dagger(I_{N_j}+\rho H_{ij}H_{ij}^\dagger)^{-1}\\
& \rho H_{ij}H_{ii}^\dagger)=\min\{N_i,(M_i-N_j)^+\}\log\mathsf{SNR}\ +\\
& o(\log\mathsf{SNR}\ ). \qquad (37)
\end{aligned}
$$

Lemma 5. *Let $H_{ii}\in\mathbb{C}^{N_i\times M_i}$ and $H_{ij}\in\mathbb{C}^{N_j\times M_i}$ be two channel matrices with each entry independently chosen from $\mathsf{CN}(0,1)$. Then, the following holds with probability 1 (over the randomness of channel matrices).*

$$
\begin{aligned}
& \log\det(I_{N_j}+\rho H_{ij}H_{ij}^\dagger-\rho H_{ij}H_{ii}^\dagger(I_{N_i}+\rho H_{ii}H_{ii}^\dagger)^{-1}\\
& \rho H_{ii}H_{ij}^\dagger)=\min\{N_j,(M_i-N_i)^+\}\log\mathsf{SNR}\ +\\
& o(\log\mathsf{SNR}\ ). \qquad (38)
\end{aligned}
$$

Proof. The proof is similar to that of Lemma 4, and is thus ommitted. □

Now we find the DoF bounds equivalent to the capacity bounds that we need for the proof of the Theorem 2.

First term (4)**:** According to the first bound in $\mathcal{R}_o$, we have

$$
\begin{aligned}
& \log\det(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger)+\min\{\log\\
& \det(I_{N_2}+\rho_{12}H_{12}H_{12}^\dagger-\rho_{12}\rho_{11}H_{12}H_{11}^\dagger\\
& (I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger)^{-1}H_{11}H_{12}^\dagger),C_{21}\}\\
=\;& \log\det(I_{N_1}+\rho H_{11}H_{11}^\dagger)+\min(\log\det\\
& (I_{N_2}+\rho H_{12}H_{12}^\dagger-\rho H_{12}H_{11}^\dagger\\
& (I_{N_1}+\rho H_{11}H_{11}^\dagger)^{-1}\rho H_{11}H_{12}^\dagger),C_{21})\\
\stackrel{(a)}{=}\;& (\min\{M_1,N_1\}+\min\{\min\{N_2,(M_1-N_1)^+\},\\
& C_{21}^d\})\log\mathsf{SNR}\ +o(\log\mathsf{SNR}\ )), \qquad (39)
\end{aligned}
$$

where (a) is obtained from Lemma 3 and Lemma 5. Now, dividing both sides by $\log\mathsf{SNR}$, the first DoF bound results.

Second term (5)**:** The second bound is similar to the first bound by replacing 1 and 2 in the indices.

Third term (6)**:** According to the third bound in $\mathcal{R}_o$, we have

$$\begin{aligned}
&\log\det(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger+\rho_{21}H_{21}H_{21}^\dagger\\
&-\rho_{11}\rho_{12}H_{11}H_{12}^\dagger(I_{N_2}+\rho_{12}H_{12}H_{12}^\dagger)^{-1}\\
&H_{12}H_{11}^\dagger)+\log\det(I_{N_2}+\rho_{22}H_{22}H_{22}^\dagger+\\
&\rho_{12}H_{12}H_{12}^\dagger-\rho_{22}\rho_{21}H_{22}H_{21}^\dagger\{(I_{N_1}+\\
&\rho_{21}H_{21}H_{21}^\dagger)^{-1}\}H_{21}H_{22}^\dagger)+C_{12}+C_{21}\\
=\;&\log\det(I_{N_1}+\rho H_{11}H_{11}^\dagger+\rho H_{21}H_{21}^\dagger-\\
&\rho H_{11}H_{12}^\dagger(I_{N_2}+\rho H_{12}H_{12}^\dagger)^{-1}\rho H_{12}H_{11}^\dagger)+\\
&\log\det(I_{N_2}+\rho H_{22}H_{22}^\dagger+\rho H_{12}H_{12}^\dagger-\\
&\rho H_{22}H_{21}^\dagger(I_{N_1}+\rho H_{21}H_{21}^\dagger)^{-1}\rho H_{21}H_{22}^\dagger)+\\
&C_{12}+C_{21}\\
\overset{(a)}{=}\;&(\min\{N_1,(M_1-N_2)^++M_2\}+\\
&\min\{N_2,(M_2-N_1)^++M_1\}+\\
&C_{12}^d+C_{21}^d)\log\mathsf{SNR}\ +o(\log\mathsf{SNR}\ )), \qquad (40)
\end{aligned}$$

where (a) is obtained from Lemma 3 and Lemma 4. Now, dividing both sides by $\log\mathsf{SNR}$, the third DoF bound results.

Fourth term (7)**:** According to the fourth bound in $\mathcal{R}_o$, we have

$$\begin{aligned}
&\log\det(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger-\rho_{11}\rho_{12}H_{11}\\
&H_{12}^\dagger(I_{N_2}+\rho_{12}H_{12}H_{12}^\dagger)^{-1}H_{12}H_{11}^\dagger)+\\
&\log\det(I_{N_2}+\rho_{22}H_{22}H_{22}^\dagger+\rho_{12}H_{12}H_{12}^\dagger)\\
&+C_{12}\\
=\;&\log\det(I_{N_1}+\rho H_{11}H_{11}^\dagger-\rho H_{11}H_{12}^\dagger\\
&(I_{N_2}+\rho^\alpha H_{12}H_{12}^\dagger)^{-1}\rho H_{12}H_{11}^\dagger)+\\
&\log\det(I_{N_2}+\rho H_{22}H_{22}^\dagger+\rho H_{12}H_{12}^\dagger)+C_{12}\\
\overset{(a)}{=}\;&(\min\{N_1,(M_1-N_2)^+\}+\min\{N_2,M_1+M_2\}\\
&+C_{12}^d)\log\mathsf{SNR}\ +o(\log\mathsf{SNR}\ ), \qquad (41)
\end{aligned}$$

where (a) is obtained from Lemma 3 and Lemma 5. Now, dividing both sides by $\log\mathsf{SNR}$, the fourth DoF bound results.

Fifth term (8)**:** The fifth term is similar to the fourth term by replacing 1 and 2 in the indices.

Sixth term (9)**:** According to the sixth bound in $\mathcal{R}_o$, using Lemma 3 we have

$$\begin{aligned}
&\log\det(I_{N_1+N_2}+\begin{bmatrix}\sqrt{\rho_{11}}H_{11}\\ \sqrt{\rho_{12}}H_{12}\end{bmatrix}[\sqrt{\rho_{11}}H_{11}^\dagger\ \sqrt{\rho_{12}}H_{12}^\dagger]\\
&+\begin{bmatrix}\sqrt{\rho_{21}}H_{21}\\ \sqrt{\rho_{22}}H_{22}\end{bmatrix}[\sqrt{\rho_{21}}H_{21}^\dagger\ \sqrt{\rho_{22}}H_{22}^\dagger])\\
=\;&\log\det(I_{N_1+N_2}+\rho\begin{bmatrix}H_{11}\\ H_{12}\end{bmatrix}[H_{11}^\dagger\ H_{12}^\dagger]+\\
&\rho\begin{bmatrix}H_{21}\\ H_{22}\end{bmatrix}[H_{21}^\dagger\ H_{22}^\dagger])\\
=\;&\min(N_1+N_2,M_1+M_2)\log\mathsf{SNR}\ +o(\log\mathsf{SNR}\ ) \qquad (42)
\end{aligned}$$

Seventh term (10)**:** According to the seventh bound in R_o, we have

$$\begin{aligned}
&\log\det(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger-\rho_{11}\rho_{12}H_{11}\\
&H_{12}^\dagger(I_{N_2}+\rho_{12}H_{12}H_{12}^\dagger)^{-1}H_{12}H_{11}^\dagger)+\\
&\log\det(I_{N_2}+\rho_{22}H_{22}H_{22}^\dagger+\rho_{12}H_{12}H_{12}^\dagger\\
&-\rho_{22}\rho_{21}H_{22}H_{21}^\dagger(I_{N_1}+\rho_{21}H_{21}H_{21}^\dagger)^{-1}\\
&H_{21}H_{22}^\dagger)+\log\det(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger\\
&+\rho_{21}H_{21}H_{21}^\dagger)+C_{12}+C_{21}\\
=\;&\log\det\log\det(I_{N_1}+\rho H_{11}H_{11}^\dagger-\rho H_{11}H_{12}^\dagger\\
&(I_{N_2}+\rho H_{12}H_{12}^\dagger)^{-1}\rho H_{12}H_{11}^\dagger)+\\
&\log\det(I_{N_2}+\rho H_{22}H_{22}^\dagger+\rho H_{12}H_{12}^\dagger-\\
&\rho H_{22}H_{21}^\dagger(I_{N_1}+\rho H_{21}H_{21}^\dagger)^{-1}\rho H_{21}H_{22}^\dagger)+\\
&\log\det(I_{N_1}+\rho H_{11}H_{11}^\dagger+\rho H_{21}H_{21}^\dagger)+C_{12}^d+C_{21}^d\\
\overset{(a)}{=}\;&\min\{N_2,(M_2-N_1)^++M_1\}+\\
&\min\{N_1,(M_1-N_2)^+\}+\\
&\min\{N_1,M_1+M_2\}+C_{12}^d+C_{21}^d, \qquad (43)
\end{aligned}$$

where (a) is obtained from Lemma 3, Lemma 3 and Lemma 4. Now, dividing both sides by $\log\mathsf{SNR}$, the seventh DoF bound results.

Eighth term (11)**:** The eighth term is similar to the seventh term by replacing 1 and 2 in the indices.

Ninth term (12)**:** According to the ninth bound in $\mathcal{R}_o$, we have

$$\begin{aligned}
&\log\det(I_{N_1+N_2}+\begin{bmatrix}\sqrt{\rho_{22}}H_{22}\\ \sqrt{\rho_{21}}H_{21}\end{bmatrix}(I_{M_2}-\rho_{12}H_{21}^\dagger\\
&(I_{N_1}+\rho_{21}H_{21}H_{21}^\dagger)^{-1}H_{21})[\sqrt{\rho_{22}}H_{22}^\dagger\ \sqrt{\rho_{21}}H_{21}^\dagger]\\
&+\begin{bmatrix}\sqrt{\rho_{12}}H_{12}\\ \sqrt{\rho_{11}}H_{11}\end{bmatrix}[\sqrt{\rho_{12}}H_{12}^\dagger\ \sqrt{\rho_{11}}H_{11}^\dagger])+\log\det\\
&(I_{N_1}+\rho_{11}H_{11}H_{11}^\dagger+\rho_{21}H_{21}H_{12}^\dagger)+C_{21}\\
=\;&\log\det(I_{N_1+N_2}+\rho\begin{bmatrix}H_{22}\\ H_{21}\end{bmatrix}(I_{M_2}-\rho H_{21}^\dagger\\
&(I_{N_1}+\rho H_{21}H_{21}^\dagger)^{-1}H_{21})[H_{22}^\dagger\ H_{21}^\dagger]\\
&+\rho\begin{bmatrix}H_{12}\\ H_{11}\end{bmatrix}[H_{12}^\dagger\ H_{11}^\dagger])+\log\det(I_{N_1}+\rho H_{11}H_{11}^\dagger\\
&+\rho H_{21}H_{12}^\dagger)+C_{21}\\
=\;&(\min\{N_1+N_2,M_1\}+\min\{N_1,M_1+M_2\}+C_{21}^d)\\
&\log\mathsf{SNR}\ +o(\log\mathsf{SNR}\ ). \qquad (44)
\end{aligned}$$

Tenth term (13)**:** The tenth term is similar to the ninth term by replacing 1 and 2 in the indices.

This completes the proof for all the ten terms in the statement of the Theorem.

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