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Energy Harvesting Communication Using Limited Battery with Efficiency

Mehdi Ashraphijuo
Dep. of Electrical Eng.
Columbia University
New York, NY
Email: mehdi@ee.columbia.edu

Vaneet Aggarwal
School of Industrial Eng.
Purdue University
West Lafayette, IN
Email: vaneet@purdue.edu

Xiaodong Wang
Dep. of Electrical Eng.
Columbia University
New York, NY
Email: wangx@ee.columbia.edu

***Abstract*—We consider an energy harvesting point-to-point communication system where the transmitter is powered by an energy arrival process and is equipped with a finite battery of size $B_{\max}$, which has limited efficiencies for storing energy into the battery and withdrawing energy from the battery. We assume a discrete i.i.d. energy arrival process where at each time step, energy of size A_i is harvested with probability p_i, $\forall i \in \{1, 2, \cdots, K\}$, independent of the other time steps. We provide upper and lower bounds on the information-theoretic capacity of this channel. These bounds are within a constant gap for $K = 2$ and a special case (with perfect battery efficiencies) for $K = 3$.**

I. Introduction

Wireless communication networks composed of devices that can harvest energy from nature represent the green future of wireless. The simplest model that captures the communication scenario is a discrete time AWGN channel. In most communication systems, the transmission power is a major cost [1, 2]. This cost can be partially alleviated by using energy harvesting devices [3–5]. For transmitters that are powered by energy harvesters, typically energy is replenished by the energy harvester, while expended for communications or other processing; any unused energy is then stored in an energy storage, such as a rechargeable battery [6, 7]. However, unlike conventional communication devices that are subject only to a power constraint or a total energy constraint, transmitters with energy harvesting capabilities are subject to additional energy harvesting constraints [8, 9]. Specifically, in every time-slot, the transmitter is constrained to use at most the amount of stored energy currently available, although more energy may become available in the future slots. Thus, a causality constraint is imposed on the use of the harvested energy. This raises many interesting issues in the design of efficient energy harvesting communication schemes. In this paper, we study the bounds on the capacity of AWGN channel where the transmit node is powered with stochastic energy arrivals, and is equipped with a limited capacity battery with storage and withdrawal efficiencies.

The capacity of a single-user channel with infinite battery capacity is obtained in [10], where it is shown that the capacity equals to that of a classical AWGN channel with an average power constraint equal to the average energy harvesting rate. Despite significant recent effort [11–13] to characterize the capacity of the energy-harvesting channel in the finite battery case, there is still a lack of complete understanding. For example, [13] provides a formulation of the capacity and derives a lower bound on the capacity that is only numerically computable. However, it is difficult to obtain useful insights from numerical evaluations. Even in the case of zero-capacity battery, where [14] provides an exact single-letter characterization of the capacity in terms of an optimization problem, the corresponding optimization is difficult to solve and requires numerical evaluations. The authors of [15] investigated the case of constant input energy input with a limited battery. Recently, the approximate capacity of a single-user energy harvesting system with discrete energy arrivals has been characterized in [16], where it is assumed that the incoming energy is stored in the battery and the transmission energy is taken from the battery. Since the transmission power is only used from the battery, if battery capacity is zero, i.e. $B_{\max} = 0$, the capacity will be zero according to [16], while we assume that the unused energy is stored in the battery and thus may get a non-zero capacity. The results in [16] have been further extended to fading channels in [17, 18].

We assume that the energy arrival is a discrete random process that can take K values $A_1, \cdots, A_k$ where the incoming energy is A_i with probability p_i. The incoming energy is used in part for transmission and is in part stored into the battery, which has finite battery size $B_{\max}$, and limited efficiencies for storing energy into the battery (r_{in}) and for taking energy out of the battery (r_{out}). We provide upper and lower bounds on the information-theoretic capacity of the channel for general K when the receiver does not know the energy arrival process, and provide an approximation to the capacity region for $K = 2, 3$. For $K = 2$, we have the gap between the upper and the lower bound as 3.013 bits for any values of the system parameters involved (p_i's, A_i's, r_{in}, r_{out} and $B_{\max}$). For $K = 3$, we have the capacity within 4.818 bits when the efficiencies for storing energy into the battery and withdrawing energy from the battery are both 1 for all the other parameters (p_i's, A_i's and $B_{\max}$). Our approximate capacity characterizations provide important insights on the optimal design of energy harvesting communication systems. For lower bound, we introduce multiple strategies and choose the best for each set of system model parameters which makes our results different from that in [16] where the lower bound is based on a single strategy. For each strategy, we derive a unique energy allocation policy that is time invariant and the consumption of each arriving energy package is decreasing over time with a geometric parameter across different epochs. The proposed upper bound accounts for the flexibility of incoming harvested energy, and thus needs to decide how much incoming energy to store in battery, and how much to utilize for transmission in each time instant which makes our results more complex than that in [16] where all the incoming energy can only be stored in the battery.

The remainder of this paper is organized as follows. Section II introduces the model for a point-to-point communication system which is equipped with an energy harvesting device. Section III gives our results on the inner and the outer bounds of the capacity, which are within a constant gap in some special cases.

II. System Model

We consider a point-to-point channel with a single transmitter, which is equipped with an energy harvesting device. The energy harvesting device has a battery with a capacity of $B_{\max}$. If the battery is not full, the harvested energy is stored in part in the battery and in part used for the transmission; if the battery is full, the transmitter can directly use the harvested energy. In both the cases, some additional amount of energy from that stored in the battery can also be used for transmission. The usage of the battery is not free. The cost of storing energy into the battery is denoted through the coefficient $0 < r_{\text{in}} \leq 1$. Similarly, the cost of taking energy out the battery is denoted through the coefficient $0 < r_{\text{out}} \leq 1$. Let X_t denote the scalar real input to the channel in time step t. We consider a discrete time AWGN channel, where the output of the channel is given by $Y_t = X_t + N_t$, where $N_t \sim \mathcal{N}(0,1)$ is the additive noise. At each time slot t, the system harvests E_t energy units that is causally known at the transmitter (i.e., at time t the transmitter knows $E_t, E_{t-1}, ...$) but is not known at the receiver.

Let B_t be the available energy in the battery in time step t. We assume that at each time step, the system first harvests energy and then transmits the signal X_t. The square of $|X_t|$ is constrained by the available energy $r_{\text{out}}B_t$ (accounting for the withdrawal efficiency of the battery) plus the harvested energy E_t, i.e.,

$$|X_t|^2 \leq r_{\text{out}}B_t + E_t. \tag{1}$$

By considering the cost r_{in} of storing energy into the battery, we have that the available battery energy for transmission B_t is updated as

$$\begin{aligned} B_{t+1} &= B_t + \\ & r_{\text{in}}(E_t - |X_t|^2)^+ - \frac{1}{r_{\text{out}}}(|X_t|^2 - E_t)^+. \end{aligned} \tag{2}$$

Note that in order to not waste the harvested energy, we should choose X_t such that

$$B_t + r_{\text{in}}(E_t - |X_t|^2)^+ - \frac{1}{r_{\text{out}}}(|X_t|^2 - E_t)^+ \leq B_{\max}. \tag{3}$$

Here, we consider the case that the harvested energy E_t is a K-level i.i.d. process as

$$E_t = \{ \ A_i, \quad \text{w.p. } p_i, \quad i = 1, \cdots, K. \tag{4}$$

where $0 \leq A_1 \leq \cdots \leq A_K$, $\sum_{k=1}^{K} p_k = 1$ and $p_1, \cdots, p_K \geq 0$.

Definition 1. *The encoding functions f_t, $t = 1, \cdots, n$ and the decoding function g are defined as*

$$f_t \ : \ \mathcal{M} \times \mathcal{E}^t \to \mathcal{X}, \quad t = 1, \cdots, n, \tag{5}$$

$$g \ : \ \mathcal{Y}^n \to \mathcal{M}, \tag{6}$$

where $\mathcal{X} = \mathcal{Y} = \mathbb{R}$, $\mathcal{E} = \{A_1, \cdots, A_K\}$ and $\mathcal{M} = \{1, \cdots, M\}$ is the set of messages to be transmitted. To transmit message $w \in \mathcal{M}$, at time $t = 1, \cdots, n$, the transmitter sends $X_t = f_t(w, \{E_i\}_{i=0}^t)$. The battery state B_t is a deterministic function of $(\{X_i\}_{i=0}^t, \{E_i\}_{i=0}^t)$, therefore also of $(w, \{E_i\}_{i=0}^t)$. The functions f_t must satisfy the energy constraints (1)-(3)*:*

$$\begin{aligned} & B_t\left(w, \{E_i\}_{i=0}^t\right) + E_t - B_{\max} \\ \leq \ & \left(f_t\left(w, \{E_i\}_{i=0}^t\right)\right)^2 \leq B_t\left(w, \{E_i\}_{i=0}^t\right) + E_t. \end{aligned} \tag{7}$$

The receiver estimates $\hat{w} = g\left(\{Y_i\}_{i=0}^n\right)$. The probability of error is

$$P_e^{(n)} = \frac{1}{M}\sum_{w=1}^{M} P\left(\hat{w} \neq w | w \text{ was transmitted}\right). \tag{8}$$

The rate of an (M,n) code is $\frac{\log M}{n}$. We say rate R is achievable if for every $\delta > 0$ there exists, for all sufficiently large n, an (M,n) code with rate $\frac{\log M}{n} > R - \delta$, and error $P_e^{(n)} \to 0$. The capacity C of the above system with parameters $A_1, \ldots, A_K, p_1, \ldots, p_K$ and $B_{\max}$ is defined as the supremum of all achievable rates.

Remark 1. *For the special case of $B_{\max} = \infty$, the capacity of the above systems has been characterized in [10]. It is shown that in the optimal transmission scheme, nothing is transmitted for the first few time slots so that enough energy is accumulated. This is followed by transmission using the average harvested energy in every time slot. Hence the capacity is characterized by the average energy arrival rate. [19] also characterized the capacity with infinite buffer size using a different approach.*

Remark 2. *In [16], for the Bernoulli energy arrival, the approximate capacity is characterized under a slightly different model where energy can only be used from battery and thus even if $A_2 - A_1 > B_{\max}$, the extra energy, $(A_2 - A_1 - B_{\max})$, cannot be utilized. Further, in [16], it is assumed that $r_{\text{in}} = r_{\text{out}} = 1$ which restricts their results to model with no efficiencies.*

III. Main Results

In this section, we will first give the outer and the inner bound on the capacity. Then, the gap between the two will be found in some special cases.

A. Outer Bound

An outer bound on the capacity of discrete time AWGN channel with energy harvesting device can be given as follows.

Theorem 1. *For $0 \le A_1 \le \cdots \le A_K$, $\sum_{k=1}^K p_k = 1$, $p_1, \cdots, p_K \ge 0$, $r_{\text{in}}, r_{\text{out}} \in (0,1]$ and $B_{\max} \ge 0$, the capacity is outer bounded by*

$$\begin{aligned} &C_{ub,K} \\ &= \sum_{i=2}^K \frac{p_i}{2}\log(1+A_i - z_i) + \frac{p_1}{2}\log \\ &\left(1 + A_1 + \frac{r_{\text{in}} r_{\text{out}} \sum_{i=2}^K p_i (z_i)^+ - \sum_{i=2}^K p_i(-z_i)^+}{p_1}\right) \\ &s.t. \quad z_i \le B_{\max}, \quad \forall 1 \le i \le K, \qquad (9) \\ &\quad r_{\text{in}} r_{\text{out}} \sum_{i=2}^K p_i (z_i)^+ - \sum_{i=2}^K p_i(-z_i)^+ \ge 0, \qquad (10) \end{aligned}$$

where $\{z_2, \cdots, z_K\} = \arg\max_{z_2,\cdots,z_K}\{C_{ub,K}\}$.

Proof: The upper bound holds intuitively because on the time slots of energy arrivals A_i, $i > 1$, if the average energy that is put in the battery is z_i, then the capacity in these slots is upper bounded by $\frac{1}{2}\log(1 + A_i - z_i)$. With z_i taken out from the incoming energy on an average, an average of $r_{\text{in}} z_i$ energy is stored in the battery which can be utilized in the remaining time slots when there is A_1 energy arrival, and an average power constraint forms the upper bound.

Let $g(t) \triangleq |X_t|^2$ and $g_i(t) \triangleq g(t)(1_{E_t = A_i})$ is the power allocation strategy that maximizes the capacity. Then, the capacity is upper bounded by

$$\begin{aligned} &\lim_{N\to\infty} \inf \frac{1}{N}\sum_{t=1}^N \frac{1}{2}\log(1+g(t)) \\ &\overset{(a)}{=} \lim_{N\to\infty}\left(\sum_{i=1}^K \frac{1}{N}\sum_{t=1}^N \frac{1}{2}\log(1+g_i(t))\right) \\ &\overset{(b)}{\le} \lim_{N\to\infty}\left(\sum_{i=1}^K \frac{N_i}{2N}\log(1+\frac{1}{N_i}\sum_{t=1}^N g_i(t))\right), \end{aligned}$$

where N_i is the number of occurrences of $E_t = A_i$, and as $N \to \infty$, $N_i \approx N p_i$. In the above, (a) follows by separating the incoming energies by their level, and (b) follows since logarithm is a concave function. Suppose that an average of x_i energy is stored into the battery and y_i is withdrawn from the battery when the energy arrivals are A_i for $i > 1$, then $\mathbb{E}(g_i(t)) = p_i A_i - p_i x_i + r_{\text{out}} y_i$ for $i > 1$ and $\mathbb{E}(g_1(t)) = p_1 A_1 + \sum_{i=2}^K p_i r_{\text{out}}(r_{\text{in}} x_i - y_i)$. As $N \to \infty$, using law of large numbers, we have that the capacity is upper bounded by

$$\begin{aligned} C \le &\max_{\substack{0 \le x_i \le B_{max}, \\ 0 \le \sum_{i=2}^K p_i(r_{\text{in}} x_i - y_i)}} \sum_{i=2}^K \frac{p_i}{2}\log(1 + A_i - x_i + r_{\text{out}} y_i) \\ &+ \frac{p_1}{2}\log\left(1 + A_1 + \frac{\sum_{i=2}^K p_i r_{\text{out}}(r_{\text{in}} x_i - y_i)}{p_1}\right), \end{aligned} \qquad (11)$$

For this optimization, we find that $\min\{x_i, y_i\} = 0$ for all $i > 1$. Defining $z_i = \begin{cases} x_i, & \text{when } x_i > 0, \\ -r_{\text{out}} y_i, & \text{when } x_i = 0. \end{cases}$, we get the result as in the statement of the theorem.

■

We get the following corollaries from Theorem 1.

Corollary 1. *For the case of $K = 2$, the outer bound can be reduced to*

$$\begin{aligned} C_{ub} = &\max_{0 \le x \le B_{\max}} \frac{p_2}{2}\log(1 + A_2 - x) + \\ &\frac{p_1}{2}\log(1 + A_1 + \frac{p_2}{p_1} r_{\text{out}} r_{\text{in}} x) \end{aligned} \qquad (12)$$

The optimal value of x in (12) *is* $x^* = \min\left\{B_{\max}, \left(A_2 - A_1 - \frac{1 - r_{\text{out}} r_{\text{in}}}{r_{\text{out}} r_{\text{in}}}\right)^+ p_1\right\}$.

Remark 3. *In general, the optimization for finding* $\arg\max_{z_2,\cdots,z_K}\{C_{ub,K}\}$ *in Theorem 1 can be found recursively.*

Corollary 2. *The upper bound in* (9) *can be simplified as following in different ranges of $B_{\max}$ for $r_{\text{in}} = r_{\text{out}} = 1$.*

A) For $B_{\max} \le (A_2 - A_1)p_1$, we have

$$\begin{aligned} C_{ub,K} = &\sum_{\ell=1}^K \frac{p_\ell}{2}\log(1 + A_\ell - B_{\max}) + \\ &\left(\frac{p_1}{2}\right)\log\left(1 + A_1 + \frac{B_{\max}\sum_{j=2}^K p_j}{p_1}\right). \end{aligned} \qquad (13)$$

B) For $A_s\left(\sum_{r=1}^{s-1} p_r\right) - \sum_{i=1}^{s-1} p_i A_i \le B_{\max} \le A_{s+1}\left(\sum_{r=1}^{s} p_r\right) - \sum_{i=1}^{s} p_i A_i$, $\forall\ 2 \le s \le K-1$, we have

$$\begin{aligned} C_{ub,K} = &\sum_{\ell=s+1}^K \frac{p_\ell}{2}\log(1 + A_\ell - B_{\max}) + \\ &\left(\frac{1 - \sum_{i=s+1}^K p_i}{2}\right) \\ &\log\left(1 + A_1 + \frac{\sum_{i=2}^s p_i A_i + B_{\max}\sum_{j=s+1}^K p_j}{1 - \sum_{m=s+1}^K p_m}\right) \end{aligned} \qquad (14)$$

C) For $A_K - \sum_{i=1}^K p_i A_i \le B_{\max}$, we have

$$C_{ub,K} = \frac{1}{2}\log\left(1 + \sum_{i=1}^K p_i A_i\right). \qquad (15)$$

B. The Lower Bound

The insights developed in the upper bound can be used to give an achievability scheme, which achieves the rate as given in the following theorem.

Theorem 2. *For $0 \leq A_1 \leq \cdots \leq A_K$, $\sum_{k=1}^{K} p_k = 1$, $p_1, \cdots, p_K \geq 0$, $r_{\text{in}}, r_{\text{out}} \in (0,1]$ and $B_{\max} \geq 0$, the rate of $\max_{k=2}^{K} \max_{0 \leq x_k \leq \min\{B_{\max}, A_k\}} C^k_{lb,K}(x_k)$ can be achieved, where*

$$\begin{aligned} C^k_{lb,K}(x) = &\sum_{j=k}^{K} \frac{p_j}{2} \log\left(1 + A_j - x\right) + \\ &+ \sum_{h=1}^{k-1} \frac{p_h}{2} \log\left(1 + A_h + \frac{(\sum_{\ell=k}^{K} p_\ell) r_{\text{in}} r_{\text{out}} x}{\sum_{i=1}^{k-1} p_i}\right) \\ &- 2.013 - \log K - 0.72(1_{K>2}). \end{aligned} \tag{16}$$

The rest of the subsection proves this theorem. We show the achievability for $C^k_{lb,K}(x)$, where $k \in \{2, \cdots, K\}$ which would show the result in the statement of the theorem. This scheme considers the time slots between two consecutive energy arrivals of more than or equal to A_k at slots T_1 and T_2. Then, energy of $A_j - x$, $\forall j \in \{k, \cdots, K\}$, is used at time T_1 and the effective energy that goes into the battery is $r_{\text{in}} x$. Then, at any time step t between T_1 and T_2, the energy of $\sum_{i=k}^{K} p_i B_t$ is extracted from the battery, and thus we can use $A_t + r_{\text{out}} \sum_{i=k}^{K} p_i B_t$ energy in time t and the residual energy for slot $(t+1)$ in the battery is $B_{t+1} = (1 - \sum_{i=k}^{K} p_i) B_t$. Thus, energy usage after time T_1 is $r_{\text{in}} r_{\text{out}} (\sum_{i=k}^{K} p_i)(1 - \sum_{i=k}^{K} p_i)^{t-T_1-1} x$. We note that the inter-arrival time is a geometric random variable with parameter $\sum_{i=k}^{K} p_i$, and thus has the mean of $\frac{1}{\sum_{i=k}^{K} p_i}$ and on average all x_i's will be used. We ignore the energy left out of x at T_2, and consider the next interval starting at T_2 to find the energy usage after T_2. This policy can be evaluated to give the desired bound. Define $g^{'}(t)$ as the energy that is used in time t using this strategy.

The strategy $g^{'}(t)$ we consider is of the form $g^{'}(t) = \widetilde{g}(j)$, where $j = t - \max\{t^{'} : E(t^{'}) = A_i, i \geq k, \forall t^{'} \leq t\}$, i.e., the strategy is invariant across different epochs and the allocated energy depends on the number of time steps since the last energy arrival $A_i, i \geq k$. So, we get

$$\widetilde{g}(j) \triangleq \begin{cases} (\sum_{\ell=k}^{K} p_\ell) r_{\text{in}} r_{\text{out}} (1 - \sum_{m=k}^{K} p_m)^{j-1} x + A_1, & \text{w.p. } \frac{p_1}{\sum_{\ell=1}^{k-1} p_\ell}, \\ \cdots \\ (\sum_{\ell=k}^{K} p_\ell) r_{\text{in}} r_{\text{out}} (1 - \sum_{m=k}^{K} p_m)^{j-1} x + A_{k-1}, & \text{w.p. } \frac{p_{k-1}}{\sum_{\ell=1}^{k-1} p_\ell}, \end{cases}$$

$$\text{for all} \quad j \geq 1. \tag{17}$$

Also

$$\widetilde{g}(0) \triangleq \begin{cases} A_k - x, & \text{w.p. } \frac{p_k}{\sum_{\ell=k}^{K} p_\ell}, \\ \cdots \\ A_K - x, & \text{w.p. } \frac{p_K}{\sum_{\ell=k}^{K} p_\ell}, \end{cases}. \tag{18}$$

The idea for the achievability scheme is that if both the transmitter and receiver know at each time arrival what energy packet A_j arrives, they can agree on an energy allocation strategy ahead of time.

Communication proceeds as follows: At each time step t, the transmitter sees the realization of the energy process E_t, let $j = t - \max\{t^{'} \leq t : E_{t'} \geq A_k\}$, i.e., the number of time steps since the last time battery was recharged with energy packet arrival A_j, $j \geq k$. Let $\phi_k(i)$ denote the amount of energy allocated to transmission, i channel uses after the last time the battery was recharged via packet arrivals A_j, $j \geq k$, $i = 0, 1, \cdots$. We concentrate on an energy allocation policy $\phi_k(i)$ that is invariant across different epochs (the period of time between two adjacent packet arrivals A_j and $A_{j'}$, $j, j^{'} \geq k$). In other words, if energy A_j, $j \geq k$ arrives at the current channel use, we allocate $\phi_k(0)$ amount of energy for transmission; if energy A_j, $j \geq k$ arrived in the previous channel use but not the current channel use, then we allocate $\phi_k(1)$ amount of energy for transmission, and so on till the next arrival of energy $A_{j'}$, $j^{'} \geq k$.

Consider n consecutive time slots of energy arrivals where n is large enough. Denote $n^{(i)}$, $i \geq 0$ as the number of times slots t that their corresponding $j = t - \max\{t^{'} \leq t : E_{t'} \geq A_k\}$ is equal to i. If n goes to infinity it can be shown that $n^{(i)}$, $i \geq 0$ goes to infinity, as well. The transmitter and the receiver agree on a sequence of $M+1$ codebooks: $\mathcal{C}_k^{(0)}, \mathcal{C}_k^{(1)}, \mathcal{C}_k^{(2)}, \cdots, \mathcal{C}_k^{(M)}$ with large enough M, each codebook $\mathcal{C}_k^{(i)}$ consisting of $2^{n^{(i)} R^{(i)}}$ codewords where $R^{(i)}$ is the rate of the codebook and codebook $\mathcal{C}_k^{(i)}$ is amplitude-constrained to $\phi_k(i)$, i.e., the symbols of each codeword in $\mathcal{C}_k^{(i)}$ are such that ${X_t}^2 \leq \phi_k(i)$ if $i = t - \max\{t^{'} \leq t : E_{t'} \geq A_k\}$. This ensures that the symbol transmitted at the corresponding time will not exceed the energy constraint $\phi_k(i)$. The transmitter chooses a codeword $c_{k,i} \in \mathcal{C}_k^{(i)}$, $\forall i \in \{0, 1, \cdots, M\}$ to communicate to the receiver. More specifically, in the l^{th} occurrence of i, the transmitter sends the l^{th} symbol of codeword $c_{k,i}$, i.e., upon the arrival of the first energy packet A_j, $j \geq k$, the transmitter sends the first symbol of $c_{k,0} \in \mathcal{C}_k^{(0)}$; if there is no energy packet arrival A_j, $j \geq k$ in the next channel use, it transmits the first symbol of $c_{k,1} \in \mathcal{C}_k^{(1)}$ in the next channel use, etc. Once the second energy packet A_j, $j \geq k$ arrives, the transmitter transmits the second symbol of $c_{k,0}$, then the second symbol of $c_{k,1}$, etc. If $j > M$, the transmitter transmits zero symbol. Communication ends when the transmitter observes the arrival of the $(n^{(0)}+1)^{\text{th}}$ energy packet of energy A_j, $j \geq k$. (We assume that communication starts with the arrival of the first energy packet of energy A_j, $j \geq k$).

We assume that $H(E_t)$ bits is used to communicate the incoming energy level E_t to the receiver. The receiver can track the codebook used by the transmitter and decode each codeword separately by knowing the energy arrival E_t in the transmitter.

Let $\{T_i\}_{i=1}^{L}$ be the inter-arrival times between the i^{th} and $i+1^{\text{th}}$ energy packets of $A_i, i \geq k$, where L is the total number of packets of $A_i, i \geq k$ received by time instance N, i.e. $\sum_{i=1}^{L} T_i \leq N < \sum_{i=1}^{L+1} T_i$. Notice that T_i's are i.i.d. Geometric random variables. We can lower bound the rate achieved by $g^{'}(t)$ in terms of these new variables as

$$
\begin{aligned}
&\lim_{N\to\infty}\inf\frac{1}{N}\sum_{t=1}^{N}\frac{1}{2}\log(1+g^{'}(t))\\
\geq &\lim_{L\to\infty}\inf\frac{\sum_{i=1}^{L}\sum_{j=0}^{T_i-1}\mathbb{E}[\frac{1}{2}\log(1+\widetilde{g}(j))]}{\sum_{i=1}^{L+1}T_i}\\
=&\left(\frac{\mathbb{E}\left[\sum_{j=0}^{T_1-1}\frac{1}{2}\log(1+\widetilde{g}(j))\right]}{\mathbb{E}[T_1]}\right)\\
=&\mathbb{E}\left[\left(\sum_{\ell=k}^{K}p_\ell\right)\sum_{i=1}^{\infty}\mathbb{P}(T_1=i)\sum_{j=0}^{i-1}\frac{1}{2}\log(1+\widetilde{g}(j))\right]\\
=&\mathbb{E}\left[\left(\sum_{\ell=k}^{K}p_\ell\right)\sum_{i=1}^{\infty}\left(\sum_{\ell=k}^{K}p_\ell\right)\left(1-\left(\sum_{\ell=k}^{K}p_\ell\right)\right)^{i-1}\right.\\
&\left.\sum_{j=0}^{i-1}\frac{1}{2}\log(1+\widetilde{g}(j))\right]\\
=&\mathbb{E}\left[\sum_{r=k}^{K}p_r\sum_{j=0}^{\infty}\sum_{i=j+1}^{\infty}\left(\sum_{\ell=k}^{K}p_\ell\right)\left(1-\left(\sum_{\ell=k}^{K}p_\ell\right)\right)^{i-1}\right.\\
&\left.\frac{1}{2}\log(1+\widetilde{g}(j))\right]\\
=&\mathbb{E}\left[\sum_{j=0}^{\infty}\left(\sum_{\ell=k}^{K}p_\ell\right)\left(1-\left(\sum_{\ell=k}^{K}p_\ell\right)\right)^{j}\frac{1}{2}\log(1+\widetilde{g}(j))\right].
\end{aligned}
\tag{19}
$$

Using the discussed energy allocation strategy in this subsection and Eqn. (19), the rate in the following lemma could be achieved, which will be further lower bounded to get the statement in the Theorem.

Lemma 1. *The capacity of a system with i.i.d. energy arrival process only causally known at the transmitter, but not at the receiver, is lower bounded by*

$$
\begin{aligned}
C\geq\mathbb{E}&\left[\sum_{j=0}^{\infty}\left(\sum_{\ell=k}^{K}p_\ell\right)\left(1-\left(\sum_{\ell=k}^{K}p_\ell\right)\right)^{j}\frac{1}{2}\log(1+\widetilde{g}(j))\right]\\
&-1.04-\log K.
\end{aligned}
\tag{20}
$$

Proof: The proof follows on the same lines as [16, Lemma 2], using (19). The gap of $1.04+\log K$ is due to amplitude-constrained AWGN, and having no channel state information at the receiver. ■

The next result further lower bounds the achievable rate in Lemma 1.

Lemma 2. *We have the following inequality:*

$$
\begin{aligned}
&\sum_{j=k}^{K}\frac{p_j}{2}\log\left(1+A_j-x\right)+\\
&+\sum_{h=1}^{k-1}\frac{p_h}{2}\log\left(1+\frac{(\sum_{\ell=k}^{K}p_\ell)r_{\text{in}}r_{\text{out}}x}{\sum_{i=1}^{k-1}p_i}+A_h\right)\\
&-0.973-0.72(1_{K>2})\leq\\
&\mathbb{E}\left[\sum_{j=0}^{\infty}\left(\sum_{\ell=k}^{K}p_\ell\right)\left(1-\left(\sum_{\ell=k}^{K}p_\ell\right)\right)^{j}\frac{1}{2}\log(1+\widetilde{g}(j))\right].
\end{aligned}
\tag{21}
$$

Proof: We divide the proof into three cases. The first case is when $\frac{(\sum_{\ell=k}^{K}p_\ell)r_{\text{in}}r_{\text{out}}x}{\sum_{i=1}^{k-1}p_i}+A_1>2.85$, the second one is when $\frac{(\sum_{\ell=k}^{K}p_\ell)r_{\text{in}}r_{\text{out}}x}{\sum_{i=1}^{k-1}p_i}+A_{k-1}\leq 2.85$, and the third one is when $\frac{(\sum_{\ell=k}^{K}p_\ell)r_{\text{in}}r_{\text{out}}x}{\sum_{i=1}^{k-1}p_i}+A_{q-1}\leq 2.85\leq\frac{(\sum_{\ell=k}^{K}p_\ell)r_{\text{in}}r_{\text{out}}x}{\sum_{i=1}^{k-1}p_i}+A_q$. In the first two cases, (21) can be shown without the presence of 0.72 while in the third case, (21) can be shown with an additional 0.72. Thus, when $1=K-1$, the third case does not matter and hence the result as in the statement of the lemma follows. The detailed proof can be seen in [20]. ■

Using Lemma 1 and Lemma 2, the result in Theorem 2 follows directly.

C. Gap between the bounds

In this subsection, we show that the gap between the bounds are constant in some special cases. We first consider two-levels energy arrival process ($K=2$). From Theorems 1 and 2, the following corollary follows.

Corollary 3. *We have the following inequality that*

$$C_{ub,2}-C_{lb,2}^{2}(x_2^*)\leq 3.013\ \ \textit{bits} \tag{22}$$

for $\forall p_1,p_2,A_1,A_2,r_{\text{in}},r_{\text{out}}\in(0,1)$ *and* $B_{\max}\geq 0$.

The next result shows the gap between the inner and outer bound for three-level energy arrival process for the case of $r_{\text{in}}=r_{\text{out}}=1$.

Theorem 3. *When* $r_{\text{in}}=r_{\text{out}}=1$ *and* $K=3$*, the gap between the upper and the lower bounds is limited by 4.818 bits, i.e., we have the following inequality*

$$C_{ub,3}-\max\{C_{lb,3}^{2}(x_2^*),C_{lb,3}^{3}(x_3^*)\}\leq 4.818\ \ \textit{bits} \tag{23}$$

for $\forall p_1,p_2,p_3,A_1,A_2,A_3$ *and* $B_{\max}\geq 0$.

Proof: We note that there is a -4.318 in the statement of Theorem 2 for $K=3$, and among the remaining expression, we can show that the gap between the outer bound and one of the inner bounds is at most 1/2 bit. In order to show this gap of 1/2 bit, we divide the region of $B_{\max},A_1,A_2$ and A_3 into multiple cases. The details can be seen in [20]. ■

IV. Conclusions

We consider an energy harvesting communication system where a transmitter powered by an exogenous energy arrival process (modeled as a discrete random process) and equipped with a battery of finite capacity communicates over a discrete-time AWGN channel. The efficiency of storing energy in the battery and withdrawing energy from the battery are used to give bounds on the capacity. These bounds are shown to be within a constant gap for $K = 2$ and a special case (with perfect battery efficiencies) for $K = 3$. Extension to fading channels is a future work.

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