On the Capacity of Energy Harvesting Communication Link

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***Abstract*—We consider an energy harvesting point-to-point communication system where the transmitter is powered by an energy arrival process and is equipped with a battery of finite capacity $B_{\max}$, which could be used for saving energy for future use. We assume a discrete i.i.d. energy arrival process where at each time step, energy of amount A_i is harvested with probability p_i $\forall i \in \{1, 2, \ldots, K\}$ independent of the other time steps. We provide upper and lower bounds on the capacity of this channel. These bounds are shown to be within a constant gap for $K \leq 3$ for all parameters, and for $K > 3$ when the battery capacity $B_{\max}$ is small or large enough, where this constant does not depend on any energy or battery parameters.**

***Index Terms*—Energy harvesting, channel capacity, achievability, upper bound, energy arrival process.**

I. Introduction

WIRELESS communication networks that are composed of devices that can harvest energy from nature, represent the green future of wireless systems. The simplest model that captures the communication scenario is a discrete-time AWGN channel. In most communication systems, the transmission power is a major cost [1]–[4]. This cost can be partially alleviated by using energy harvesting devices [5]–[8]. For transmitters that are powered by energy harvesters, energy is typically replenished by the energy harvester and expended for communications or other processes; any unused energy is then stored in an energy storage, such as a rechargeable battery [9], [10]. The i.i.d. arrival process of the incoming energy in energy harvesting is a good starting point for general results and is a common assumption in literature [5]–[7], [10], [11]. However, unlike conventional communications where devices are only subject to a power constraint or a total energy constraint, transmitters with energy harvesting capabilities are subject to additional energy harvesting constraints [11]–[14]. It is noteworthy to mention other directions in the literature on characterizing the energy harvesting communication system capacity such as packet scheduling optimality [15], [16] and throughput analysis with unreliable energy sources [17], [18]. Specifically, in every time step, the transmitter is constrained to use at most the amount of stored energy that is currently available, although more energy may become available in the future slots. Thus, a causality constraint is imposed on the use of the harvested energy. This raises many interesting issues in the design of efficient energy harvesting communication schemes. In this paper, we study the bounds on the capacity of AWGN channel where the transmit node is powered by stochastic energy arrivals, and is equipped with the capability of storing and drawing energy from a finite capacity battery.

The capacity of a single-user channel with infinite battery capacity is obtained in [19], where it is shown that the capacity equals to that of a classical AWGN channel with an average power constraint equal to the average energy harvesting rate. Despite the recent significant efforts to characterize the capacity of the energy harvesting channel in the finite battery case [20]–[22], there is still a lack of complete understanding. For example, [22] provides a formulation of the capacity and derives a lower bound on the capacity that is only numerically computable. However, it is difficult to obtain useful insights from numerical evaluations. Even in the case of no battery, where [23] provides an exact single-letter characterization of the capacity in terms of an optimization problem, the corresponding optimization is difficult to solve and requires numerical evaluations. The authors of [24] investigated the case of constant input energy with a limited battery. Recently, the approximate capacity of a single-user energy harvesting system with discrete energy arrivals has been characterized in [18], where it is assumed that all the incoming energy is stored in the battery and the transmission energy is taken from the battery.

We note that [18] considers a similar energy harvesting model with the difference that the incoming energy cannot be used directly when it arrives at the transmitter and consequently if there is not enough space in the battery it will be wasted. In contrast, we assume that the incoming energy can be used for transmission and consequently increases the flexibility of the transmitter, which provides the possibility of achieving better rates. The results in [18] have been further extended to fading channels in [25], [26].

We assume that the energy arrival is an i.i.d. discrete random process that takes K values of $A_1, \ldots, A_K$, where energy A_i arrives with probability p_i. The incoming energy in part is used for transmission and in part is stored into the battery, which has a capacity $B_{\max}$. We provide upper and lower bounds on the capacity of such channel when the receiver does not

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know the energy arrival process. These bounds are within a constant gap for $K \leq 3$ (1.04 bits for $K = 1$, 2.884 bits for $K = 2$ and 4.426 bits for $K = 3$), where the constant does not depend on any energy or battery parameters. For $K > 3$, the gap between the bounds is shown to be a constant that is independent of all parameters in the cases of small enough ($B_{\max} \leq (A_2 - A_1)p_1$) and large enough ($B_{\max} \geq A_K - \sum_{i=1}^{K} p_i A_i$) battery sizes. Our approximate capacity characterizations provide important insights on the optimal design of energy harvesting communication systems. For lower bound, we introduce multiple strategies and choose the best for each set of system model parameters which makes our results different from that in [18] where the lower bound is based on a single strategy. For each strategy, we derive a unique energy allocation policy that is time invariant and the consumption of each arriving energy package is decreasing over time with a geometric parameter across different epochs. The proposed upper bound accounts for the flexibility of incoming harvested energy, and thus needs to decide how much incoming energy to store in battery, and how much to utilize for transmission in each time instant which makes our results more complex than that in [18] where all the incoming energy can only be stored in the battery.

The remainder of this paper is organized as follows. Section II introduces the model for a point-to-point communication system, equipped with an energy harvesting device. Sections III and IV give the results on the upper and lower bounds of the capacity, respectively. These bounds are shown to be within a constant gap in several cases in Section V. Finally, Section VI concludes the paper.

II. System Model

We consider a point-to-point channel with a single transmitter, equipped with an energy harvesting device that has a battery with a capacity of $B_{\max}$. If the battery is not full, the harvested energy is in part stored in the battery and in part used for transmission; if the battery is full, the transmitter can directly use all the harvested energy. In both cases, some additional amount of energy that is stored in the battery can also be used for transmission. Let X_t denote the scalar real input to the channel at time t. We consider a discrete-time AWGN channel, where the output of the channel is given by $Y_t = X_t + N_t$, where $N_t \sim \mathcal{N}(0, 1)$ is the additive white Gaussian noise. At each time t, the system harvests E_t units of energy that is causally known at the transmitter (i.e., at time t the transmitter knows $E_t, E_{t-1}, \ldots$) but is not known at the receiver.

Let B_t be the available energy in the battery at time t. We assume that at each time, the system first harvests energy and then transmits the signal X_t.

The square of X_t is constrained by the available energy B_t plus the harvested energy E_t, i.e.,

$$X_t{}^2 \leq B_t + E_t. \tag{1}$$

And the available battery energy B_t is updated as

$$B_{t+1} = \min\{B_{\max}, B_t + E_t - X_t{}^2\}. \tag{2}$$

Note that in order to not waste the harvested energy, we should choose X_t such that $B_t + E_t - X_t{}^2 \leq B_{\max}$, which leads to the lower bound on $X_t{}^2$, i.e.,

$$B_t + E_t - B_{\max} \leq X_t{}^2. \tag{3}$$

Here, we consider the case that the harvested energy E_t is a K-level i.i.d. process as

$$E_t = A_k \text{ with probability } p_k,\ k = 1, \ldots, K, \tag{4}$$

where $0 \leq A_1 < \ldots < A_K$, $\sum_{k=1}^{K} p_k = 1$ and $p_1, \ldots, p_K > 0$.

Definition 1: The encoding functions f_t, $t = 1, \cdots, n$ and the decoding function g are defined as

$$f_t : \mathcal{M} \times \mathcal{E}^t \to \mathcal{X}, \quad t = 1, \cdots, n, \tag{5}$$

$$g : \mathcal{Y}^n \to \mathcal{M}, \tag{6}$$

where $\mathcal{X} = \mathcal{Y} = \mathbb{R}$, $\mathcal{E} = \{A_1, \cdots, A_K\}$ and $\mathcal{M} = \{1, \cdots, M\}$ is the set of messages to be transmitted. To transmit message $w \in \mathcal{M}$, at time $t = 1, \cdots, n$, the transmitter sends $X_t = f_t(w, \{E_i\}_{i=0}^t)$. The battery state B_t is a deterministic function of $(\{X_i\}_{i=0}^t, \{E_i\}_{i=0}^t)$, therefore also of $(w, \{E_i\}_{i=0}^t)$. The functions f_t must satisfy the energy constraints (1) and (3):

$$\begin{aligned} &B_t\left(w, \{E_i\}_{i=0}^t\right) + E_t - B_{\max} \\ &\quad \leq \left(f_t\left(w, \{E_i\}_{i=0}^t\right)\right)^2 \leq B_t\left(w, \{E_i\}_{i=0}^t\right) + E_t. \end{aligned} \tag{7}$$

The receiver estimates $\hat{w} = g\left(\{Y_i\}_{i=0}^n\right)$. The probability of error is

$$P_e^{(n)} = \frac{1}{M} \sum_{w=1}^{M} P\left(\hat{w} \neq w | w \text{ was transmitted}\right). \tag{8}$$

The rate of an (M, n) code is $\frac{\log M}{n}$. We say rate R is achievable if for every $\delta > 0$ there exists, for all sufficiently large n, an (M, n) code with rate $\frac{\log M}{n} > R - \delta$, and error $P_e^{(n)} \to 0$. The capacity C of the above system with parameters $A_1, \ldots, A_K, p_1, \ldots, p_K$ and $B_{\max}$ is defined as the supremum of all achievable rates.

Remark 1: For the special case of $B_{\max} = \infty$, the capacity of the above system has been characterized in [19]. It is shown that in the optimal transmission scheme, nothing is transmitted for the first few time slots so that enough energy is accumulated. This is followed by transmission using the average harvested energy in every time step. Hence, the capacity is characterized by the average energy arrival rate. [3] also characterized the capacity with infinite buffer size using a different approach.

Remark 2: In [18], for the Bernoulli energy arrival, i.e., $K = 2$, $A_1 = 0$, the approximate capacity is characterized under a different battery model where the transmission energy can only be taken from the battery and thus even if $A_2 > B_{\max}$, the extra energy, $(A_2 - B_{\max})$, cannot be utilized.

Remark 3: In the remainder of the paper, we assume $K > 1$ since for $K = 1$ (i.e., constant input energy), $\frac{1}{2}\log(1 + A_1)$ is an upper bound on the capacity (since A_1 is the average power). Further, a lower bound on the capacity is $\frac{1}{2}\log(1 + A_1) - 1.04$, which can be achieved by the amplitude-constrained AWGN channel [18, Lemma 1], leading to a maximal gap of 1.04 bits between the upper and lower bounds. The authors of [24] investigated the case of constant input energy with a finite-capacity battery and obtained better bounds on capacity, where the lower bound and upper bound are quite close.

III. Upper Bound

An upper bound on the capacity of the discrete-time AWGN channel with an energy harvesting transmitter is given as follows.

Theorem 1: For $K \geq 2, 0 \leq A_1 < \ldots < A_K, \sum_{k=1}^{K} p_k = 1$, $p_1, \ldots, p_K > 0$ and $B_{\max} \geq 0$, the capacity is upper bounded by $C_{ub,K}$ which is the solution to the following optimization problem:

$$C_{ub,K} = \max_{z_2,\ldots,z_K} S_K(z_2, \ldots, z_K)$$
$$\text{subject to } z_i \leq \min\{B_{\max}, A_i\}, \forall i = 2, \ldots, K,$$
$$\sum_{i=2}^{K} p_i z_i \geq 0, \tag{9}$$

where $S_K(z_2, \ldots, z_K) \triangleq \sum_{i=2}^{K} \frac{p_i}{2} \log(1 + A_i - z_i) + \frac{p_1}{2} \log\left(1 + A_1 + \frac{\sum_{i=2}^{K} p_i z_i}{p_1}\right)$.

Proof: The claimed upper bound holds intuitively, because at times of energy arrival A_i, $i > 1$, if the average energy that is put in the battery is z_i, then the capacity in these time slots is upper bounded by $\frac{1}{2}\log(1 + A_i - z_i)$. The energy z_i taken out from the incoming energy and stored in the battery can be utilized in the time slots with energy arrival of A_1. And consequently, an average power constraint forms the upper bound.

Define $g(t) \triangleq X_t^2$ as the power allocation strategy that maximizes the long-term average transmission rate over the class of feasible online policies (1) and (2) and also define $g_i(t) \triangleq g(t)1_{E_t=A_i}$. Then, the capacity is upper bounded as

$$\begin{aligned} C &\leq \liminf_{N\to\infty} \mathbb{E}\left[\frac{1}{N}\sum_{t=1}^{N} \frac{1}{2}\log(1 + g(t))\right] \\ &\overset{(a)}{=} \liminf_{N\to\infty} \mathbb{E}\left[\sum_{i=1}^{K} \frac{1}{N}\sum_{t=1}^{N} \frac{1}{2}\log(1 + g_i(t))\right] \\ &\overset{(b)}{\leq} \liminf_{N\to\infty} \mathbb{E}\left[\sum_{i=1}^{K} \frac{N_i}{2N}\log(1 + \frac{1}{N_i}\sum_{t=1}^{N} g_i(t))\right], \\ &\overset{(c)}{=} \sum_{i=1}^{K} \frac{p_i}{2}\log\left(1 + \frac{\mathbb{E}\left[g_i(t)\right]}{p_i}\right), \end{aligned} \tag{10}$$

where N_i is the number of occurrences of $E_t = A_i$ for $t \in \{1, \ldots, N\}$, and as $N \to \infty$, $N_i \approx N p_i$. In the above, (a) follows by separating the incoming energies by their levels, (b) follows from the concavity of the log function and (c) follows from the law of large numbers. Suppose that an average of x_i energy is stored into the battery and y_i energy is drawn from the battery when the energy arrival is A_i for $i > 1$, then $\mathbb{E}(g_i(t)) = p_i\,(A_i - x_i + y_i)$ for $i > 1$ and $\mathbb{E}(g_1(t)) = p_1 A_1 + \sum_{i=2}^{K} p_i(x_i - y_i)$. As $N \to \infty$, using the law of large numbers, it can be concluded that the capacity is upper bounded by

$$\begin{aligned} C \leq \max_{\substack{0\leq x_i, y_i, x_i - y_i \leq B_{\max},\\ 0 \leq \sum_{i=2}^{K} p_i(x_i - y_i)}} &\left\{\sum_{i=2}^{K} \frac{p_i}{2}\log(1 + A_i - x_i + y_i)\right. \\ &\left. + \frac{p_1}{2}\log\left(1 + A_1 + \frac{\sum_{i=2}^{K} p_i(x_i - y_i)}{p_1}\right)\right\}. \end{aligned} \tag{11}$$

By defining $z_i \triangleq x_i - y_i$ we get the result stated in the theorem. ■

The following two results follow from Theorem 1.

Corollary 1: For the case of $K = 2$, the upper bound can be reduced to

$$\begin{aligned} C_{ub,2} = \max_{0\leq x \leq B_{\max}} &\left\{\frac{p_2}{2}\log(1 + A_2 - x)\right. \\ &\left. + \frac{p_1}{2}\log\left(1 + A_1 + \frac{p_2}{p_1}x\right)\right\}. \end{aligned} \tag{12}$$

The optimal value of x in (12) is $x^* = \min\{B_{\max}, (A_2 - A_1)p_1\}$.

The optimum solution to (9) for general K is given explicitely by the following theorem. The proof is given in Appendix A.

Theorem 2: The explicit upper bound is given as follows for different ranges of $B_{\max}$.

A) For $B_{\max} \leq (A_2 - A_1)p_1$,

$$\begin{aligned} C_{ub,K} = &\sum_{\ell=2}^{K} \frac{p_\ell}{2}\log(1 + A_\ell - B_{\max}) \\ &+ \frac{p_1}{2}\log\left(1 + A_1 + \frac{B_{\max}\sum_{j=2}^{K} p_j}{p_1}\right). \end{aligned} \tag{13}$$

B) For $A_s\left(\sum_{r=1}^{s-1} p_r\right) - \sum_{i=1}^{s-1} p_i A_i \leq B_{\max} \leq A_{s+1}\left(\sum_{r=1}^{s} p_r\right) - \sum_{i=1}^{s} p_i A_i, 2 \leq s \leq K - 1$,

$$\begin{aligned} C_{ub,K} = &\sum_{\ell=s+1}^{K} \frac{p_\ell}{2}\log(1 + A_\ell - B_{\max}) + \left(\frac{1 - \sum_{i=s+1}^{K} p_i}{2}\right) \\ &\log\left(1 + \frac{\sum_{i=1}^{s} p_i A_i + B_{\max}\sum_{j=s+1}^{K} p_j}{1 - \sum_{m=s+1}^{K} p_m}\right). \end{aligned} \tag{14}$$

C) For $A_K - \sum_{i=1}^{K} p_i A_i \leq B_{\max}$,

$$C_{ub,K} = \frac{1}{2}\log\left(1 + \sum_{i=1}^{K} p_i A_i\right). \tag{15}$$

IV. The Lower Bound

The insights developed in derivation of the upper bound can be used to give an achievability scheme, which achieves the rate given in the following theorem.

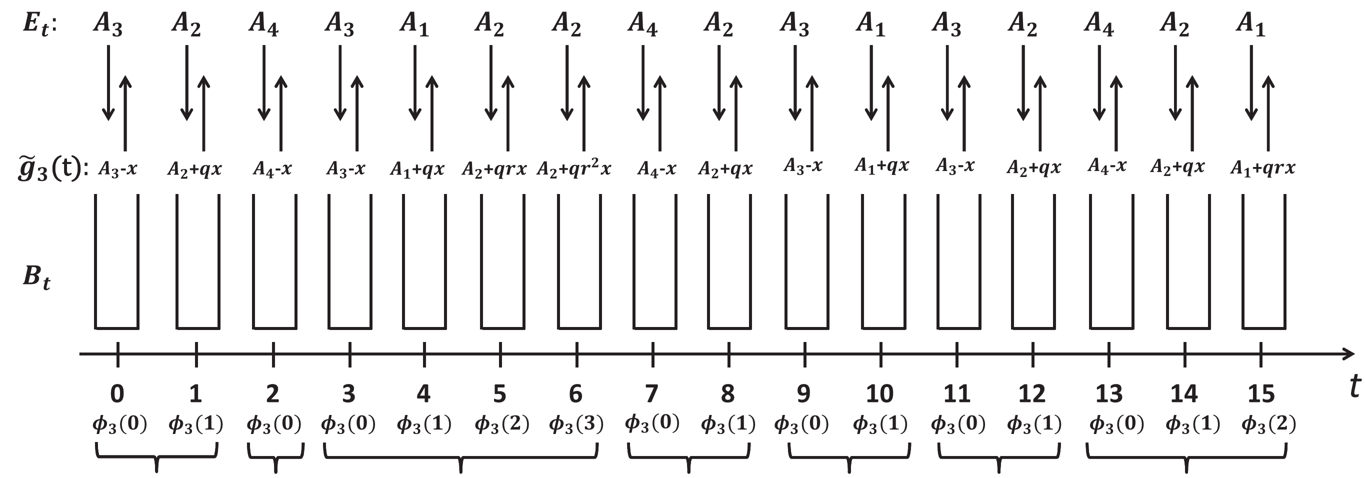

Fig. 1. Illustration of the energy allocation policy for $C^3_{lb,4}(x)$ with $q = q_3$ and $r = 1 - q_3$. Note that the energy control policy is reset each time a packet A_3 or A_4 arrives.

Theorem 3: For $K \geq 2, 0 \leq A_1 < \ldots < A_K, \sum_{k=1}^{K} p_k = 1$, $p_1, \ldots, p_K > 0$, and $B_{\max} \geq 0$ define $q_k = \sum_{i=k}^{K} p_i$. Then, the rate of $\max_{k=2,\ldots,K} \max_{0 \leq x_k \leq \min\{B_{\max}, A_k\}} C^k_{lb,K}(x_k)$ can be achieved, where

$$C^k_{lb,K}(x) = \sum_{j=k}^{K} \frac{p_j}{2} \log\left(1 + A_j - x\right) + \sum_{h=1}^{k-1} \frac{p_h}{2} \log\left(1 + A_h + \frac{xq_k}{1-q_k}\right) - 1.884 - \log K - 0.457 1_{K>2}. \quad (16)$$

The rest of this section proves this theorem. We show the achievability of $C^k_{lb,K}(x)$, for any $k \in \{2, \ldots, K\}$ and $x \in [0, \min\{B_{\max}, A_k\}]$. Let the time slots of two consecutive energy arrivals of at least A_k be T_1 and T_2, i.e., $E_{T_1}, E_{T_2} \geq A_k$ and $E_t < A_k$ for $T_1 < t < T_2$. Then, the energy of $E_{T_1} - x$ is used at time T_1 and the energy of x goes into the battery. At any time t between T_1 and T_2, the energy of $q_k B_t$ is extracted from the battery, and thus the energy of $E_t + q_k B_t$ can be used for transmission, where $E_t < A_k$, and the residual energy at time $(t+1)$ in the battery is $B_{t+1} = (1 - q_k)B_t$. Thus, the energy usage from the battery at time $t \in \{T_1 + 1, \ldots, T_2 - 1\}$ is $q_k(1 - q_k)^{t-T_1-1}x$. We note that this is a geometric random variable with parameter q_k, and thus has the mean $\frac{1}{q_k}$. We ignore the battery energy residue at T_2, and consider the next interval starting at T_2 to find the energy usage after T_2. This policy can be evaluated to give the desired bound.

The energy utilization strategy $\tilde{g}_k(t)$ proposed above is of the form $\tilde{g}_k(t) = \phi_k(j)$, where $j = t - \max\{\tau : E_\tau = A_i, i \geq k, \forall \tau \leq t\}$, i.e., the strategy is invariant across time and the allocated energy depends on the number of time steps since the last energy arrival $A_i, i \geq k$. The random variable $\phi_k(j)$ is defined as

$$\phi_k(j) \triangleq \begin{cases} q_k(1-q_k)^{j-1}x + A_1, & \text{w.p.} \frac{p_1}{1-q_k}, \\ \vdots \\ q_k(1-q_k)^{j-1}x + A_{k-1}, & \text{w.p.} \frac{p_{k-1}}{1-q_k}, \end{cases} \quad \text{for all} \quad j \geq 1, \quad (17)$$

and

$$\phi_k(0) \triangleq \begin{cases} A_k - x, & \text{w.p.} \frac{p_k}{q_k}, \\ \vdots \\ A_K - x, & \text{w.p.} \frac{p_K}{q_k}, \end{cases} . \quad (18)$$

Fig. 1 depicts an example of energy allocation policy for $C^3_{lb,4}(x)$.

The idea for the achievability scheme is that if both the transmitter and receiver know at each time arrival what energy packet A_j arrives, they can agree on an energy allocation strategy ahead of time.

Communication proceeds as follows: At each time step t, the transmitter sees the realization of the energy process E_t, let $j = t - \max\{t' \leq t : E_{t'} \geq A_k\}$, i.e., the number of time steps since the last time battery was recharged with energy packet arrival A_j, $j \geq k$. Let $\phi_k(i)$ denote the amount of energy allocated to transmission, i channel uses after the last time the battery was recharged via packet arrivals A_j, $j \geq k$, $i = 0, 1, \cdots$. We concentrate on an energy allocation policy $\phi_k(i)$ that is invariant across different epochs (the period of time between two adjacent packet arrivals A_j and $A_{j'}$, $j, j' \geq k$). In other words, if energy A_j, $j \geq k$ arrives at the current channel use, we allocate $\phi_k(0)$ amount of energy for transmission; if energy A_j, $j \geq k$ arrived in the previous channel use but not the current channel use, then we allocate $\phi_k(1)$ amount of energy for transmission, and so on till the next arrival of energy $A_{j'}$, $j' \geq k$.

Consider n consecutive time slots of energy arrivals where n is large enough. Denote $n^{(i)}$, $i \geq 0$ as the number of times slots t that their corresponding $j = t - \max\{t' \leq t : E_{t'} \geq A_k\}$ is equal to i. If n goes to infinity it can be shown that $n^{(i)}$, $i \geq 0$ goes to infinity, as well. The transmitter and the receiver agree on a sequence of $M + 1$ codebooks: $\mathcal{C}^{(0)}_k, \mathcal{C}^{(1)}_k, \mathcal{C}^{(2)}_k, \cdots, \mathcal{C}^{(M)}_k$ with large enough M, each codebook $\mathcal{C}^{(i)}_k$ consisting of $2^{n^{(i)}R^{(i)}}$ codewords where $R^{(i)}$ is the rate of the codebook and codebook $\mathcal{C}^{(i)}_k$ is amplitude-constrained to $\phi_k(i)$, i.e., the symbols of each codeword in $\mathcal{C}^{(i)}_k$ are such that ${X_t}^2 \leq \phi_k(i)$ if $i = t - \max\{t' \leq t : E_{t'} \geq A_k\}$. This ensures that the symbol transmitted at the corresponding time will not exceed the energy constraint $\phi_k(i)$. The transmitter chooses a codeword $c_{k,i} \in \mathcal{C}_k^{(i)}, \forall i \in \{0, 1, \cdots, M\}$ to communicate to the receiver. More specifically, in the l^{th} occurrence of i, the transmitter sends the l^{th} symbol of codeword $c_{k,i}$, i.e., upon the arrival of the first energy packet A_j, $j \geq k$, the transmitter sends the first symbol of $c_{k,0} \in \mathcal{C}_k^{(0)}$; if there is no energy packet arrival A_j, $j \geq k$ in the next channel use, it transmits the first symbol of $c_{k,1} \in \mathcal{C}_k^{(1)}$ in the next channel use, etc. Once the second energy packet A_j, $j \geq k$ arrives, the transmitter transmits the second symbol of $c_{k,0}$, then the second symbol of $c_{k,1}$, etc. If $j > M$, the transmitter transmits zero symbol. Communication ends when the transmitter observes the arrival of the $(n^{(0)}+1)^{\text{th}}$ energy packet of energy A_j, $j \geq k$. (We assume that communication starts with the arrival of the first energy packet of energy A_j, $j \geq k$).

We assume that $H(E_t)$ bits is used to communicate the incoming energy level E_t to the receiver. The receiver can track the codebook used by the transmitter and decode each codeword separately by knowing the energy arrival E_t in the transmitter.

Let $\{\mathcal{S}_k(\ell)\}_{\ell=1}^{L}$ be the inter-arrival times between the ℓ^{th} and $(\ell+1)^{\text{th}}$ energy arrivals of $E_t \geq A_k$, where L_k is the total number of energy arrivals of $E_t \geq A_k$ between $t = 1$ and $t = N$, i.e., $\sum_{\ell=1}^{L_k} \mathcal{S}_k(\ell) \leq N < \sum_{\ell=1}^{L_k+1} \mathcal{S}_k(\ell)$. Notice that $\mathcal{S}_k(\ell)$'s are i.i.d. geometric random variables with parameter q_k, and thus has the mean $\frac{1}{q_k}$. We can lower bound the rate achieved by $\widetilde{g}_k(t)$ in terms of these new variables as

$$\begin{aligned}
&\liminf_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \mathbb{E}\left[\frac{1}{2}\log(1+\widetilde{g}_k(t))\right] \\
&\geq \liminf_{L_k\to\infty} \frac{\sum_{\ell=1}^{L_k} \sum_{j=0}^{\mathcal{S}_k(\ell)-1} \mathbb{E}[\frac{1}{2}\log(1+\phi_k(j))]}{\sum_{\ell=1}^{L_k+1} \mathcal{S}_k(\ell)} \\
&\overset{(a)}{=} \frac{\mathbb{E}\left[\sum_{j=0}^{\mathcal{S}_k(1)-1} \mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right]\right]}{\mathbb{E}[\mathcal{S}_k(1)]} \\
&\overset{(b)}{=} q_k \sum_{i=1}^{\infty} P(\mathcal{S}_k(1)=i) \sum_{j=0}^{i-1} \mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right] \\
&= q_k \sum_{i=1}^{\infty} q_k(1-q_k)^{i-1} \sum_{j=0}^{i-1} \mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right] \\
&= \sum_{j=0}^{\infty} \sum_{i=j+1}^{\infty} q_k^2(1-q_k)^{i-1} \mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right] \\
&= \sum_{j=0}^{\infty} q_k(1-q_k)^{j} \mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right], \qquad (19)
\end{aligned}$$

where (a) follows from the law of large numbers and (b) follows from $\mathbb{E}[\mathcal{S}_k(1)] = \frac{1}{q_k}$. Using Eqn. (19) and the discussed energy allocation strategy before it, the rate in Lemma 2 below could be achieved, which will be further lower bounded to get the bound in the statement of the theorem. First, we give the following lemma which will be used in the proof of Lemma 2.

Lemma 1: [18, Lemma 1] (Lower bound on amplitude-constrained AWGN capacity)

$$\max_{p(x):X^2\leq A} I(X;Y) \geq \frac{1}{2}\log(1+A) - 1.04. \qquad (20)$$

Lemma 2: The capacity C of a system with i.i.d. energy arrival process that is only causally known at the transmitter but not at the receiver is lower bounded by

$$\begin{aligned}
C \geq &\sum_{j=0}^{\infty} q_k(1-q_k)^j \mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right] \\
&- 1.04 - H(p_1, \ldots, p_K). \qquad (21)
\end{aligned}$$

Proof: We showed that (19) is achievable for an AWGN channel with channel state information (i.e., energy arrival process) at the receiver. For the model in this paper, the gap of 1.04 is due to amplitude-constrained AWGN given in Lemma 1, and $H(p_1, \ldots, p_K)$ is due to having no channel state information at the receiver (since the energy level can be communicated to the receiver in $H(p_1, \ldots, p_K)$ bits). ■

The next result further lower bounds the achievable rate in Lemma 2. The proof is given in Appendix B.

Lemma 3: The following inequality holds for all $0 \leq x \leq \min\{B_{\max}, A_k\}$:

$$\begin{aligned}
\text{RHS of (21)} \geq &\sum_{j=k}^{K} \frac{p_j}{2}\log\left(1+A_j-x\right) \\
&+ \sum_{h=1}^{k-1} \frac{p_h}{2}\log\left(1+\frac{q_k x}{1-q_k}+A_h\right) \\
&- 1.884 - \log K - 0.457\, 1_{K>2}. \qquad (22)
\end{aligned}$$

Using Lemma 2 and Lemma 3, Theorem 3 then follows.

V. The Gap between the Bounds

In this section, we show that the gap between the upper and lower bounds is bounded by some constant in several cases, where the constant does not depend on any energy or battery parameters.

A. Case of $K = 2$

From Theorems 1 and 3, the following corollary follows.

Corollary 2: For $K = 2$, the upper and lower bounds on the capacity are within a constant gap. More formally,

$$C_{ub,2} - C_{lb,2}^2(x_2^*) \leq 2.884 \text{ bits} \qquad (23)$$

for $\forall p_1, p_2, A_1, A_2$ and $B_{\max} \geq 0$.

Proof: From Theorem 3, we get the lower bound

$$\begin{aligned}
C_{lb,2}^2(x) = &\frac{p_2}{2}\log(1+A_2-x) + \frac{p_1}{2}\log\left(1+A_1+\frac{xp_2}{p_1}\right) \\
&- 1.884 - \log 2, \qquad (24)
\end{aligned}$$

for all $0 \leq x \leq \min\{B_{\max}, A_2\}$. The upper bound is given by Corollary 1, i.e., Eq. (12) that together with (24) gives (23). ■

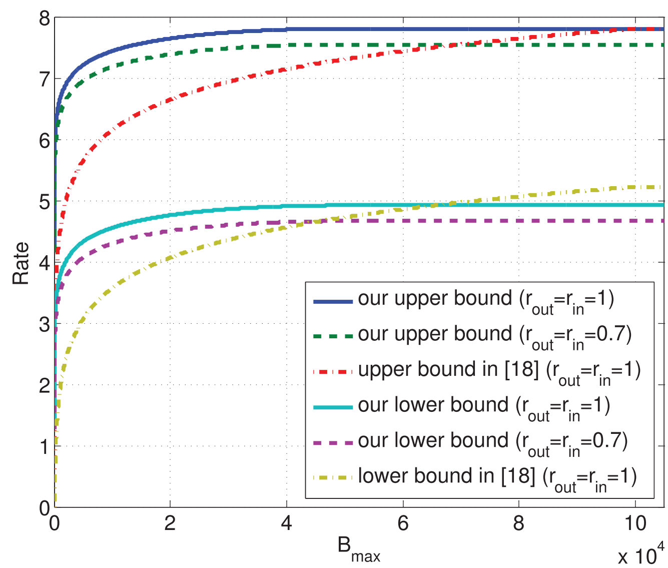

Fig. 2. Comparison of the bounds of the proposed model with those in [18] for $K = 2$.

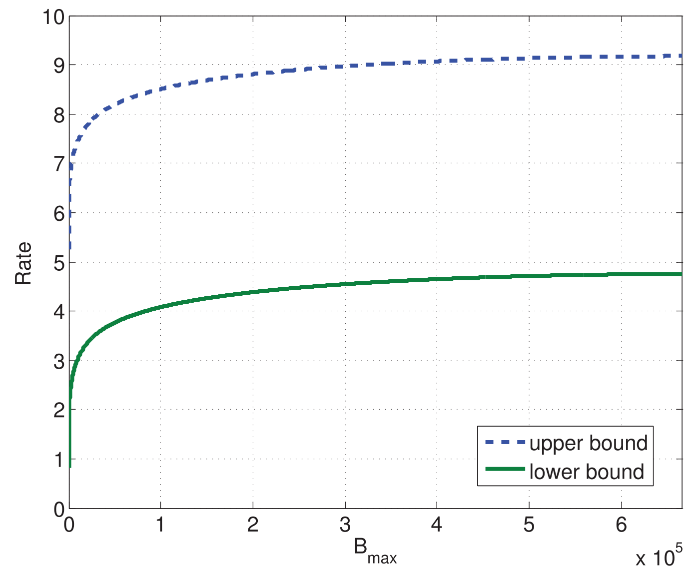

Fig. 3. Comparison of upper and lower bounds of the proposed model for $K = 3$.

We can further consider a more general case where the usage of the battery is not free. In this model, the cost of storing energy into the battery is denoted through the coefficient $0 < r_{\text{in}} \le 1$. Similarly, the cost of taking energy out the battery is denoted through the coefficient $0 < r_{\text{out}} \le 1$. Then Eqns. (1) and (2) become ${X_t}^2 \le r_{\text{out}} B_t + E_t$ and $B_{t+1} = B_t + r_{\text{in}}(E_t - {X_t}^2)^+ - \frac{1}{r_{\text{out}}}({X_t}^2 - E_t)^+$, respectively. And the upper bound in Corollary 1 and the constant gap result in Corollary 2 can be generalized as follows. The proof is given in Appendix C.

Theorem 4: For the case of $K = 2$ with the battery efficiency parameters r_{in} and r_{out}, we have the following upper bound on the capacity

$$C_{ub,2} = \max_{0 \le x \le B_{\max}} \left\{ \frac{p_2}{2} \log(1 + A_2 - x) + \frac{p_1}{2} \log\left(1 + A_1 + \frac{p_2}{p_1} r_{\text{out}} r_{\text{in}} x\right) \right\}. \quad (25)$$

where $x^* = \min\left\{B_{\max}, \left(A_2 - A_1 - \frac{1 - r_{\text{out}} r_{\text{in}}}{r_{\text{out}} r_{\text{in}}}\right)^+ p_1\right\}$. We also have

$$C_{ub,2} - 2.884 \le C \le C_{ub,2}, \quad (26)$$

For $K = 2$, assuming $p_1 = p_2 = \frac{1}{2}$, $A_1 = 0$ and $A_2 = 10^5$, in Fig. 2 we plot the upper and lower bounds for $r_{\text{in}} = r_{\text{out}} = 1$ and $r_{\text{in}} = r_{\text{out}} = 0.8$, respectively, as well as the bounds in [18] for varying values of $B_{\max}$. The bounds decrease with a decrease in efficiencies r_{in} and r_{out}. Recall that [18] assumes a different system model from the one used in this paper, where the harvested energy cannot be directly used without being stored in the battery first. Since the achievability scheme in [18] is also a feasible strategy for our model, the gap can be improved by choosing the maximum of the achievable rate in this paper and that in [18].

B. Case of $K = 3$

The next result gives the gap between the lower and upper bounds for $K = 3$.

Theorem 5: For $K = 3$, the gap between the upper and the lower bounds is bounded by 4.426 bits. More formally

$$C_{ub,3} - \max\left\{C^2_{lb,3}(x_2^*), C^3_{lb,3}(x_3^*)\right\} \le 4.426 \text{ bits} \quad (27)$$

for $\forall p_1, p_2, p_3, A_1, A_2, A_3$ and $B_{\max} \ge 0$.

Proof: We note that for $K = 3$ there is a constant of $-1.884 - \log K - 0.457\, 1_{K>2} = -3.926$ in (16), and we can show that the gap between the upper bound $C_{ub,3}$ given in Theorem 2 and the maximum of the lower bounds $C^2_{lb,3}(x_2^*)$ and $C^3_{lb,3}(x_3^*)$ given in (16) is at most 0.5 plus the above 3.926 bits. In order to show this gap of 0.5 bit, we consider 5 cases depending on the values of $B_{\max}$, A_1, A_2 and A_3. The detailed gap evaluation for these five cases is given in Appendix D. ■

For $K = 3$, assuming $p_1 = p_2 = p_3 = \frac{1}{3}$, $A_1 = 0$, $A_2 = 10^3$ and $A_3 = 10^6$, in Fig. 3 we plot the upper and lower bounds as a function of $B_{\max}$. It is seen that both bounds increase with the battery capacity; the upper bound saturates at $\frac{1}{2}\log(1 + \sum_{i=1}^{3} p_i A_i) = 9.174$ bit/s and the lower bound saturates at 4.748 bit/s.

C. Case of $K > 3$

The proposed achievability scheme can achieve a rate that is within a constant gap to the upper bound of the capacity for general K for parts A and C of Theorem 2 as shown in the following theorem.

Theorem 6: The following constant gap results for general K hold:

1) For $B_{\max} \le (A_2 - A_1)p_1$, we have

$$C_{ub,K} - C^2_{lb,K}(B_{\max}) \le 2.341 + \log K. \quad (28)$$

2) For $A_K - \sum_{i=1}^{K} p_i A_i \leq B_{\max}$, we have

$$C_{ub,K} - C_{lb,K}^{u}\left(A_u \sum_{j=1}^{u-1} p_j\right) \leq 2.341 + \frac{3}{2}\log K, \quad (29)$$

where $u \triangleq \arg\max_i A_i \sum_{j=i}^{K} p_j$.

Proof: Part 1: Follows from (13) and (16) that

$$C_{ub,K} - C_{lb,K}^{2}(B_{\max}) = 2.341 + \log K. \quad (30)$$

Part 2: Using (15) we have

$$\begin{aligned} C_{ub,K} &= \frac{1}{2}\log\left(1 + \sum_{i=1}^{K} p_i A_i\right) \\ &\leq \frac{1}{2}\log\left(1 + \sum_{i=1}^{K} A_i \sum_{j=i}^{K} p_j\right) \\ &\overset{(a)}{\leq} \frac{1}{2}\log\left(1 + K A_u \sum_{j=u}^{K} p_j\right) \\ &\leq \frac{1}{2}\log\left(1 + A_u \sum_{j=u}^{K} p_j\right) + \frac{1}{2}\log K, \end{aligned} \quad (31)$$

where (a) follows from the fact that $u = \arg\max_i A_i \sum_{j=i}^{K} p_j$. Moreover, using (16) we have

$$\begin{aligned} &C_{lb,K}^{u}\left(A_u \sum_{j=1}^{u-1} p_j\right) \\ &= \sum_{j=1}^{u-1} \frac{p_j}{2}\log\underbrace{\left(1 + A_j + \frac{\left(A_u \sum_{n=1}^{u-1} p_n\right)\sum_{m=u}^{K} p_m}{\sum_{k=1}^{u-1} p_k}\right)}_{\geq 1 + \frac{\left(A_u \sum_{n=1}^{u-1} p_n\right)\sum_{m=u}^{K} p_m}{\sum_{k=1}^{u-1} p_k}} \\ &\quad + \sum_{\ell=u}^{K} \frac{p_\ell}{2}\log\underbrace{\left(1 + A_\ell - A_u \sum_{j=1}^{u-1} p_j\right)}_{\geq 1 + A_u - A_u \sum_{j=1}^{u-1} p_j} - 2.341 - \log K \\ &\geq \frac{1}{2}\log\left(1 + A_u \sum_{j=u}^{K} p_j\right) - 2.341 - \log K. \end{aligned} \quad (32)$$

■

For $K = 5$, assuming $p_1 = \frac{1}{2}$, $p_2 = \cdots = p_5 = \frac{1}{8}$, $A_1 = 0$, $A_i = (0.5i)10^5$, $i \in \{2, \ldots, 5\}$, in Fig. 4, we plot the upper and lower bounds for a range of values of $B_{\max}$. For $B_{\max} \leq (A_2 - A_1)p_1 = 5 \times 10^4$, an upper bound is given in Case A of Theorem 2 and its gap to the achievable rate is shown to be within 4.6629 bits in Part 1 of Theorem 6. Also, for $B_{\max} \geq A_K - \sum_{i=1}^{K} p_i A_i = 1.625 \times 10^5$, an upper bound is

$$\mathbf{H} = \begin{bmatrix} \frac{-p_2}{2(1+A_2-z_2)^2} + \frac{-p_2^2}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} & \frac{-p_2 p_3}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} & \cdots & \frac{-p_2 p_K}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} \\ \frac{-p_3 p_2}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} & \frac{-p_3}{2(1+A_3-z_3)^2} + \frac{-p_3^2}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} & \cdots & \frac{-p_3 p_K}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{-p_K p_2}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} & \frac{-p_K p_3}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} & \cdots & \frac{-p_K}{2(1+A_K-z_K)^2} + \frac{-p_K^2}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} \end{bmatrix}$$

$$\overset{(a)}{\preceq} \begin{bmatrix} \frac{-p_2^2}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} & \frac{-p_2 p_3}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} & \cdots & \frac{-p_2 p_K}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} \\ \frac{-p_3 p_2}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} & \frac{-p_3^2}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} & \cdots & \frac{-p_3 p_K}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{-p_K p_2}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} & \frac{-p_K p_3}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} & \cdots & \frac{-p_K^2}{2p_1\left(1+A_1+\frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2} \end{bmatrix}$$

$$= \frac{-1}{2p_1\left(1 + A_1 + \frac{\sum_{\ell=2}^{K} p_\ell z_\ell}{p_1}\right)^2}[p_2, \ldots, p_K]^T[p_2, \ldots, p_K] \preceq 0, \quad (33)$$

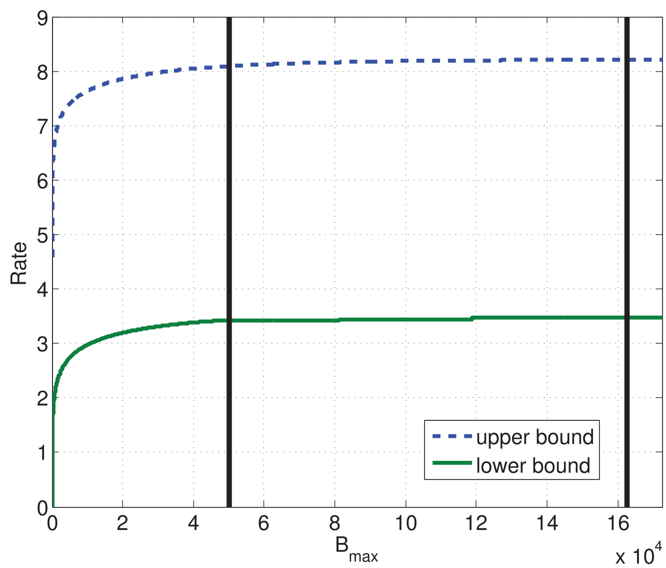

Fig. 4. Comparison of upper and lower bounds of the proposed model for $K = 5$.

given in Case C of Theorem 2 and its gap to the achievable rate is shown to be within 5.8239 bits in Part 2 of Theorem 6. In addition, for $(A_2 - A_1)p_1 < B_{\max} < A_K - \sum_{i=1}^{K} p_i A_i$, an upper bound is given in Case B of Theorem 2 and $C^3_{lb,5}\left(\min\{A_3 \sum_{j=1}^{2} p_j, B_{\max}\}\right)$ is considered as the lower bound. Further, it can be seen that both bounds increase with the battery capacity, and the upper bound saturates at $\frac{1}{2}\log(1 + \sum_{i=1}^{5} p_i A_i) = 8.4658$.

VI. Conclusions

We have considered an energy-harvesting communication system where a transmitter powered by an exogenous energy arrival process, modeled as a discrete random process, and equipped with a battery of finite capacity, communicates over a discrete-time AWGN channel. We have developed upper and lower bounds on the capacity of such systems, which are shown to be within a constant gap for $K \le 3$, and some cases of $K > 3$. Extension to fading channels is a future work.

Appendix A
Proof of Theorem 2

We first show that $S_K(z_2, \ldots, z_K)$ is concave by showing its Hessian matrix is negative-semidefinite. The Hessian is given by (33), as shown at the bottom of the previous page, where (a) follows from the fact that $\frac{-p_i}{2(1+A_i-z_i)^2} < 0$, for all $i = 2, \ldots, K$. Define the following functions corresponding to the constraints

$$g_i(z_2, \ldots, z_K) \triangleq z_i - B_{\max} \le 0, \; i = 2, \ldots, K, \quad (34)$$

$$h_i(z_2, \ldots, z_K) \triangleq z_i - A_i \le 0, \; i = 2, \ldots, K, \quad (35)$$

$$u(z_2, \ldots, z_K) \triangleq -p_2 z_2 - \ldots - p_K z_K \le 0. \quad (36)$$

We also define $\mu_{g_i}, \mu_{h_i}, \forall i \in \{2, \ldots, K\}, \mu_u$ as the corresponding Lagrangian multipliers of these constraints, respectively.

A solution $(z_2^*, \ldots, z_K^*)$ is optimum if it has the following properties:

$$\nabla S_K(z_2^*, \ldots, z_K^*) = \sum_{i=2}^{K} \mu_{g_i} \nabla g_i(z_2^*, \ldots, z_K^*) + \sum_{i=2}^{K} \mu_{h_i} \nabla h_i(z_2^*, \ldots, z_K^*) + \mu_u \nabla u(z_2^*, \ldots, z_K^*), \quad (37)$$

$$\mu_u \ge 0, \mu_{g_i} \ge 0, \mu_{h_i} \ge 0, \text{ for all } i = 2, \ldots, K,$$
$$\mu_{g_i} g_i(z_2^*, \ldots, z_K^*) = 0, \text{ for all } i = 2, \ldots, K, \quad (38)$$
$$\mu_{h_i} h_i(z_2^*, \ldots, z_K^*) = 0, \text{ for all } i = 2, \ldots, K,$$
$$\mu_u u(z_2^*, \ldots, z_K^*) = 0. \quad (39)$$

Now we prove the theorem for the given 3 cases:

A) For $B_{\max} \le (A_2 - A_1)p_1$ we show the optimality of $z_\ell^* = B_{\max}, \forall \ell \in \{2, \ldots, K\}$ which by substituting in the upper bound (9) results in (13). We have $u(z_2^*, \ldots, z_K^*) = -(1 - p_1)B_{\max} < 0$ and also for all $i = 2, \ldots, K$

$$h_i(z_2^*, \ldots, z_K^*) = B_{\max} - A_i \le (A_2 - A_1)p_1 - A_i \overset{(a)}{<} 0, \quad (40)$$

where (a) follows from the fact that $A_2 p_1 < A_2 < A_3 < \ldots < A_K$. So, we should have $\mu_{h_i} = 0, \forall i \in \{2, \ldots, K\}, \mu_u = 0$. Also, $g_i(z_2^*, \ldots, z_K^*) = 0, \forall i \in \{2, \ldots, K\}$. To show the optimality, it is sufficient to show that $\mu_{g_i} \ge 0$, for all $i = 2, \ldots, K$. Using these and (37), we get

$$\mu_{g_i} = \frac{p_i}{2}\left(\frac{1}{1 + A_1 + B_{\max}\left(\frac{1-p_1}{p_1}\right)} - \frac{1}{1 + A_i - B_{\max}}\right),$$

for all $i = 2, \ldots, K$. Moreover, we have $\mu_{g_i} \ge 0$, for all $i = 2, \ldots, K$, since

$$\frac{\mu_{g_i}}{p_i} = \frac{1}{2}\left(\frac{1}{1 + A_1 + B_{\max}\left(\frac{1-p_1}{p_1}\right)} - \frac{1}{1 + A_i - B_{\max}}\right)$$
$$= \frac{(A_i - A_1)p_1 - B_{\max}}{2p_1\left(1 + A_1 + B_{\max}\left(\frac{1-p_1}{p_1}\right)\right)(1 + A_i - B_{\max})}$$
$$\overset{(a)}{\ge} \frac{(A_2 - A_1)p_1 - B_{\max}}{2p_1\left(1 + A_1 + B_{\max}\left(\frac{1-p_1}{p_1}\right)\right)(1 + A_i - B_{\max})} \overset{(b)}{\ge} 0,$$

where (a) follows from $A_2 < A_3 < \ldots < A_K$ and (b) follows from $B_{\max} \le (A_2 - A_1)p_1$.

B) For $A_s\left(\sum_{r=1}^{s-1} p_r\right) - \sum_{i=1}^{s-1} p_i A_i < B_{\max} \le A_{s+1}\left(\sum_{r=1}^{s} p_r\right) - \sum_{i=1}^{s} p_i A_i$ we show the optimality of $z_\ell^* = A_\ell - \frac{\sum_{m=1}^{s} p_m A_m + \sum_{n=s+1}^{K} p_n B_{\max}}{\sum_{\ell=1}^{s} p_\ell}$, $\forall \ell \in \{2, \ldots, s\}$ and $z_m^* = B_{\max}$, $\forall \ell \in \{s+1, \ldots, K\}$ which by substituting in the upper bound (9) results in (14). For all $i = 2, \dots, s$ we have

$$g_i(z_2^*, \dots, z_K^*) = z_i^* - B_{\max} = A_i - \frac{\sum_{m=1}^{s} p_m A_m + \sum_{n=s+1}^{K} p_n B_{\max}}{\sum_{\ell=1}^{s} p_\ell} - B_{\max} \overset{(a)}{<} 0,$$

where (a) follows from $A_2\left(\sum_{r=1}^{s-1} p_r\right) - \sum_{i=1}^{s-1} p_i A_i < \dots < A_s\left(\sum_{r=1}^{s-1} p_r\right) - \sum_{i=1}^{s-1} p_i A_i < B_{\max}$. We also have

$$h_i(z_2^*, \dots, z_K^*) = z_i^* - A_i = -\frac{\sum_{m=1}^{s} p_m A_m + \sum_{n=s+1}^{K} p_n B_{\max}}{\sum_{\ell=1}^{s} p_\ell} < 0, \tag{41}$$

for all $i = 2, \dots, s$. It can be also seen that for all $i = s+1, \dots, K$

$$\begin{aligned} h_i(z_2^*, \dots, z_K^*) &= z_i^* - A_i = B_{\max} - A_i \\ &\le A_{s+1}\left(\sum_{r=1}^{s} p_r\right) - \sum_{i=1}^{s} p_i A_i - A_{s+1} \\ &= -A_{s+1}\left(\sum_{r=s+1}^{K} p_r\right) - \sum_{i=1}^{s} p_i A_i < 0. \end{aligned} \tag{42}$$

In addition, we have

$$\begin{aligned} u(z_2^*, \dots, z_K^*) &= -\sum_{j=1}^{K} p_j z_j^* \\ &= -B_{\max}(K - s - \sum_{j=s+1}^{K} p_j) < 0. \end{aligned} \tag{43}$$

So, we should have $\mu_{g_j} = 0, \forall j \in \{2, \dots, s\}$, $\mu_{h_i} = 0, \forall i \in \{2, \dots, K\}$, and $\mu_u = 0$. Also, $g_i(z_2^*, \dots, z_K^*) = 0, \forall i \in \{s+1, \dots, K\}$. It is sufficient to show that $\mu_{g_i} \ge 0$, for all $i = s+1, \dots, K$. Using these and (37), we get that the first $s-1$ elements of $\nabla S_K(z_2^*, \dots, z_k^*)$ are 0 and also

$$\mu_{g_i} = \frac{p_i}{2}\left(\frac{1}{1 + \frac{\sum_{m=1}^{s} p_m A_m + \sum_{n=s+1}^{K} p_n B_{\max}}{\sum_{\ell=1}^{s} p_\ell}} - \frac{1}{1 + A_i - B_{\max}}\right),$$

for all $i = s+1, \dots, K$. Moreover, we have $\mu_{g_i} \ge 0$, for all $i = s+1, \dots, K$, since (44), shown at the bottom of the page, holds where (a) follows from $A_{s+1} < A_{s+2} < \dots < A_K$ and (b) follows from $B_{\max} \le A_{s+1}\left(\sum_{r=1}^{s} p_r\right) - \sum_{i=1}^{s} p_i A_i$.

C) For $A_K - \sum_{i=1}^{K} p_i A_i \le B_{\max}$ we show the optimality of $z_\ell^* = A_\ell - \sum_{i=1}^{K} p_i A_i, \forall \ell \in \{2, \dots, K\}$ which by substituting in the upper bound (9) results (15). For this set of z_i^*'s, we get $\nabla S_K(z_2^*, \dots, z_K^*) = 0$ which shows the optimality.

APPENDIX B
PROOF OF LEMMA 3

We divide the proof into two parts; $K > 2$ and $K = 2$.

A. *Proof for $K > 2$*

For the case of $K > 2$, we divide the proof into three cases. The first case is when $\frac{q_k x}{1-q_k} + A_1 > A$, the second one is when $\frac{q_k x}{1-q_k} + A_{k-1} \le A$, and the third one is when $\frac{q_k x}{1-q_k} + A_{q-1} \le A \le \frac{q_k x}{1-q_k} + A_q$, for $1 < q < k$ and $A = 1.24$. Let choice of A is a result of performing a minimization of a function that will be mentioned in Remark 4.

I) For the case $\frac{q_k x}{1-q_k} + A_{q-1} \le A \le \frac{q_k x}{1-q_k} + A_q$, for $1 < q < k$. We have

$$\begin{aligned} &\sum_{j=k}^{K} \frac{p_j}{2} \log\left(1 + A_j - x\right) + \sum_{h=1}^{k-1} \frac{p_h}{2} \log\left(1 + \frac{q_k x}{1-q_k} + A_h\right) \\ &\le \sum_{j=k}^{K} \frac{p_j}{2} \log\left(1 + A_j - x\right) + \sum_{j=1}^{q-1} \frac{p_j}{2} \log\left(1 + A\right) \\ &\quad + \sum_{h=q}^{k-1} \frac{p_h}{2} \log\left(1 + \frac{q_k x}{1-q_k} + A_h\right), \end{aligned} \tag{45}$$

$$\begin{aligned} \frac{\mu_{g_i}}{p_i} &= \frac{1}{2}\left(\frac{1}{1 + \frac{\sum_{m=1}^{s} p_m A_m + \sum_{n=s+1}^{K} p_n B_{\max}}{\sum_{\ell=1}^{s} p_\ell}} - \frac{1}{1 + A_i - B_{\max}}\right) \\ &= \frac{A_i - B_{\max} - \frac{\sum_{m=1}^{s} p_m A_m + \sum_{n=s+1}^{K} p_n B_{\max}}{\sum_{\ell=1}^{s} p_\ell}}{2\left(1 + \frac{\sum_{m=1}^{s} p_m A_m + \sum_{n=s+1}^{K} p_n B_{\max}}{\sum_{\ell=1}^{s} p_\ell}\right)(1 + A_i - B_{\max})} \\ &= \frac{A_i(\sum_{\ell=1}^{s} p_\ell) - \sum_{m=1}^{s} p_m A_m - B_{\max}}{2\sum_{\ell=1}^{s} p_\ell\left(1 + \frac{\sum_{m=1}^{s} p_m A_m + \sum_{n=s+1}^{K} p_n B_{\max}}{\sum_{\ell=1}^{s} p_\ell}\right)(1 + A_i - B_{\max})} \\ &\overset{(a)}{\ge} \frac{A_{s+1}(\sum_{\ell=1}^{s} p_\ell) - \sum_{m=1}^{s} p_m A_m - B_{\max}}{2\sum_{\ell=1}^{s} p_\ell\left(1 + \frac{\sum_{m=1}^{s} p_m A_m + \sum_{n=s+1}^{K} p_n B_{\max}}{\sum_{\ell=1}^{s} p_\ell}\right)(1 + A_i - B_{\max})} \overset{(b)}{\ge} 0, \end{aligned} \tag{44}$$

and

$$
\begin{aligned}
&\sum_{j=0}^{\infty} q_k(1-q_k)^j \mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right] \\
&\overset{(a)}{=} \frac{q_k}{2}\sum_{n=k}^{K}\frac{p_n}{q_k}\log\left(1+A_{n-x}\right) + \sum_{j=1}^{\infty} q_k(1-q_k)^j \\
&\quad \sum_{h=1}^{k-1}\frac{p_h}{1-q_k}\frac{1}{2}\log\Bigg(\underbrace{1+q_k(1-q_k)^{j-1}x+A_h}_{\substack{> q_k(1-q_k)^{j-1}x+A_h \\ \geq q_k(1-q_k)^{j-1}x+A_h \\ (1-q_k)^j = (1-q_k)^j\left(\frac{q_k x}{1-q_k}+A_h\right)}}\Bigg)
\end{aligned} \tag{46}
$$

$$
\begin{aligned}
&\geq \underbrace{\sum_{j=1}^{\infty} q_k(1-q_k)^{j-1}}_{=1}\sum_{h=q}^{k-1}\frac{p_h}{2}\log\left(\frac{q_k}{1-q_k}x+A_h\right) \\
&\quad + \sum_{j=1}^{\infty} j q_k(1-q_k)^{j-1}\sum_{h=q}^{k-1}\frac{p_h}{2}\log\left(1-q_k\right) \\
&\quad + \sum_{n=k}^{K}\frac{p_n}{2}\log\left(1+A_{n-x}\right),
\end{aligned} \tag{47}
$$

where (a) follows from (17) and (18).

Using (45) and (46) we get

$$
\begin{aligned}
&\sum_{j=k}^{K}\frac{p_j}{2}\log\left(1+A_j-x\right) \\
&\quad + \sum_{h=1}^{k-1}\frac{p_h}{2}\log\left(1+\frac{q_k x}{1-q_k}+A_h\right) \\
&\quad - \sum_{j=0}^{\infty} q_k(1-q_k)^j\mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right] \\
&\leq \sum_{j=1}^{q-1}\frac{p_j}{2}\log\left(1+A\right) + \sum_{h=q}^{k-1}\frac{p_h}{2}\log\left(1+\frac{q_k x}{1-q_k}+A_h\right) \\
&\quad - \sum_{h=q}^{k-1}\frac{p_h}{2}\log\left(\frac{q_k x}{1-q_k}+A_h\right) \\
&\quad - \sum_{j=1}^{\infty} j q_k(1-q_k)^{j-1}\sum_{h=q}^{k-1}\frac{p_h}{2}\log\left(1-q_k\right) \\
&= \sum_{j=1}^{q-1}\frac{p_j}{2}\log\left(1+A\right) + \sum_{h=q}^{k-1}\frac{p_h}{2}\log\left(1+\frac{1}{\frac{q_k x}{1-q_k}+A_h}\right) \\
&\quad - \sum_{j=1}^{\infty} j q_k(1-q_k)^{j-1}\sum_{h=q}^{k-1}\frac{p_h}{2}\log\left(1-q_k\right) \\
&\overset{(a)}{\leq} \sum_{j=1}^{q-1}\frac{p_j}{2}\log\left(1+A\right) + \sum_{h=q}^{k-1}\frac{p_h}{2}\log\left(1+\frac{1}{\frac{q_k x}{1-q_k}+A_h}\right) \\
&\quad + 0.72 \\
&\leq \sum_{j=1}^{q-1}\frac{p_j}{2}\log\left(1+A\right) + \sum_{h=q}^{k-1}\frac{p_h}{2\ln 2}\left(\frac{1}{\frac{q_k x}{1-q_k}+A_h}\right) + 0.72
\end{aligned}
$$

$$
\leq \sum_{j=1}^{q-1}\frac{p_j}{2}\log\left(1+A\right) + \sum_{h=q}^{k-1}\frac{p_h}{2\ln 2}\left(\frac{1}{A}\right) + 0.72 \tag{48}
$$

$$
\begin{aligned}
&\overset{(b)}{\leq} \sum_{j=1}^{q-1}\frac{p_j}{2}\log\left(1+1.24\right) + \sum_{h=q}^{k-1}\frac{p_h}{2\ln 2}\left(\frac{1}{1.24}\right) + 0.72 \\
&\leq \sum_{q=1}^{k-1} p_j(0.581) + 0.72 \leq 1.301,
\end{aligned} \tag{49}
$$

where (a) follows since $-\sum_{j=1}^{\infty} j q_k(1-q_k)^j\log\left(1-q_k\right) \overset{(c)}{=} \frac{1-q_k}{2q_k}\log\left(\frac{1}{1-q_k}\right) \overset{(d)}{\leq} 0.72$, and (b) follows from $A = 1.24$. To show (c), we use the identity $\sum_{j=0}^{\infty} j q_k(1-q_k)^j = (1-q_k)\mathbb{E}[X] = \frac{1-q_k}{q_k}$, where $X \sim \text{geometric}(q_k)$, and $\sum_{j=0}^{\infty} q_k(1-q_k)^j = 1$ and (d) follows from the fact that $G(q_k) \triangleq \frac{1-q_k}{2q_k}\log\left(\frac{1}{1-q_k}\right)$ is a continuous bounded function of $q_k \in (0,1)$. Furthermore, it is monotonically decreasing and is upper bounded by $\lim_{q_k\to 0} G(q_k) = \frac{1}{2\ln(2)} = 0.72$.

Remark 4: The choice of $A = 1.24$ is made since it minimizes the RHS of (48). In other words, for $p \triangleq \sum_{j=1}^{q-1} p_j$, $\arg\min_A\left\{\max_{0\leq p\leq 1}\left\{\frac{p}{2}\log\left(1+A\right)+\frac{1-p}{2\ln 2}\left(\frac{1}{A}\right)\right\}\right\} = 1.24$, and thus choosing $A = 1.24$ results in the best bound using (48).

II) For the case $\frac{q_k x}{1-q_k}+A_1 > 1.24$, we have:

$$
\begin{aligned}
&\sum_{j=0}^{\infty} q_k(1-q_k)^j\mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right] \\
&+ \sum_{j=k}^{K}\frac{p_j}{2}\log\left(1+A_j-x\right) + \sum_{h=1}^{k-1}\frac{p_h}{2}\log\left(1+\frac{q_k x}{1-q_k}+A_h\right) \\
&- \overset{(a)}{\leq} \sum_{h=1}^{k-1}\frac{p_h}{2}\Bigg[\underbrace{\log\left(1+\frac{q_k x}{1-q_k}+A_h\right)-\log\left(\frac{q_k x}{1-q_k}+A_h\right)}_{=\log\left(1+\frac{1}{\frac{q_k x}{1-q_k}+A_h}\right)}\Bigg] \\
&- \sum_{j=1}^{\infty} j q_k(1-q_k)^{j-1}\log\left(1-q_k\right) \\
&= \sum_{h=1}^{k-1}\frac{p_h}{2}\log\left(1+\frac{1}{\frac{q_k x}{1-q_k}+A_h}\right) \\
&- \sum_{j=1}^{\infty} j q_k(1-q_k)^j\frac{1}{2}\log\left(1-q_k\right) \\
&\leq \frac{1-q_k}{2}\log\left(1+\frac{1}{\frac{q_k x}{1-q_k}+A_1}\right) \\
&\quad - \sum_{j=1}^{\infty} j q_k(1-q_k)^j\frac{1}{2}\log\left(1-q_k\right)
\end{aligned} \tag{50}
$$

$$\overset{(b)}{\leq} \frac{1-q_k}{2}\log\left(1+\frac{1}{\frac{q_k x}{1-q_k}+A_1}\right)+0.72$$
$$\leq \frac{1-q_k}{2\ln 2}\left(\frac{1}{\frac{q_k x}{1-q_k}+A_1}\right)+0.72$$
$$\leq \frac{1}{2\ln 2}\left(\frac{1}{1.24}\right)+0.72 = 1.301, \quad (51)$$

where (a) follows from (46) and (b) follows from Step (a) used in (48).

III) For the case $\frac{q_k x}{1-q_k}+A_{k-1}\leq 1.24$ we have:

$$\sum_{j=k}^{K}\frac{p_j}{2}\log\left(1+A_j-x\right)+\sum_{h=1}^{k-1}\frac{p_h}{2}\log\left(1+\frac{q_k x}{1-q_k}+A_h\right)$$
$$-\sum_{j=0}^{\infty}q_k(1-q_k)^j\mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right]$$
$$\overset{(a)}{=}\sum_{h=1}^{k-1}\frac{p_h}{2}\log\left(1+\frac{q_k x}{1-q_k}+A_h\right)$$
$$-\sum_{j=1}^{\infty}q_k(1-q_k)^{j-1}\sum_{h=1}^{k-1}\frac{p_h}{2}$$
$$\log\left(1+q_k(1-q_k)^{j-1}x+A_h\right)$$
$$\leq\sum_{h=1}^{k-1}\frac{p_h}{2}\log\left(1+\frac{q_k x}{1-q_k}+A_h\right) \quad (52)$$
$$\leq\sum_{h=1}^{k-1}\frac{p_h}{2}\log\left(1+1.24\right)$$
$$\leq\frac{1}{2}\log\left(1+1.24\right)=0.581, \quad (53)$$

where (a) follows from (46).

So, we get

$$\sum_{j=k}^{K}\frac{p_j}{2}\log\left(1+A_j-x\right)+\sum_{h=1}^{k-1}\frac{p_h}{2}\log\left(1+\frac{q_k x}{1-q_k}+A_h\right)$$
$$-\sum_{j=0}^{\infty}q_k(1-q_k)^j\mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right]\leq 1.301, \quad (54)$$

which together with $H(p_1,\cdots,p_K)\leq\log K$ completes the proof for $K>2$, i.e., by comparing this to the statement of the lemma and the fact that $1.884+\log K+0.457-1.04-H(p_1,\ldots,p_K)=1.301+\log K-H(p_1,\ldots,p_K)\geq 1.301$, the result is proven.

B. *Proof for $K = 2$*

For the case of $K=2$, we divide the proof into two cases. The first case is when $\frac{p_2 x}{p_1}+A_1>A$ and the second one is when $\frac{p_2 x}{p_1}+A_1\leq A$, where $A=2.224$. The choice of constant 2.224 is explained in Remark 5.

I) For the case $\frac{p_2 x}{p_1}+A_1>A$ we have:

$$\sum_{j=k}^{K}\frac{p_j}{2}\log\left(1+A_j-x\right)$$
$$+\sum_{h=1}^{k-1}\frac{p_h}{2}\log\left(1+\frac{q_k x}{1-q_k}+A_h\right)$$
$$-\sum_{j=0}^{\infty}q_k(1-q_k)^j\mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right]$$
$$+H(p_1,p_2)$$
$$\overset{(a)}{=}\frac{1-q_k}{2}\log\left(1+\frac{1}{\frac{q_k x}{1-q_k}+A_1}\right)$$
$$+\frac{1-q_k}{2q_k}\log\left(\frac{1}{1-q_k}\right)+H(p_1,p_2)$$
$$=\frac{p_1}{2}\log\left(1+\frac{1}{\frac{p_2 x}{p_1}+A_1}\right)$$
$$+\frac{(1-p_2)}{2p_2}\log\left(\frac{1}{1-p_2}\right)$$
$$+H(p_1,p_2)$$
$$\leq\frac{p_1}{2\ln(2)}\frac{1}{\frac{p_2 x}{p_1}+A_1}+\frac{(1-p_2)}{2p_2}\log\left(\frac{1}{1-p_2}\right)$$
$$+H(p_1,p_2)$$
$$\leq\frac{1}{2\ln(2)}\frac{1}{A}+\frac{(1-p_2)}{2p_2}\log\left(\frac{1}{1-p_2}\right)$$
$$+H(p_1,p_2)$$
$$=\frac{1}{2\ln(2)}\frac{1}{A}+\frac{(1-p_2)}{2p_2}\log\left(\frac{1}{1-p_2}\right)$$
$$+H(p_1,p_2)$$
$$=\frac{1}{2\ln(2)}\frac{1}{A}+\frac{(1-p_2)}{2p_2}\log\left(\frac{1}{1-p_2}\right)$$
$$+(1-p_2)\log\left(\frac{1}{1-p_2}\right)$$
$$+p_2\log\left(\frac{1}{p_2}\right)$$
$$\overset{(b)}{\leq}\frac{1}{2\ln(2)}\frac{1}{A}+1.52=1.844, \quad (55)$$

where (a) follows from (50), (b) is due to the fact that the expression is a continuous bounded function of $p_2\in(0,1)$. Furthermore, it is concave and attains a maximum value of $\frac{1}{2\ln(2)}\frac{1}{A}+1.52$ at $p_2=0.413$.

II) For the case $\frac{p_2 x}{p_1} + A_1 \leq A$ we have:

$$\begin{aligned}
&\sum_{j=k}^{K} \frac{p_j}{2} \log\left(1 + A_j - x\right) \\
&\quad + \sum_{h=1}^{k-1} \frac{p_h}{2} \log\left(1 + \frac{q_k x}{1-q_k} + A_h\right) \\
&\quad - \sum_{j=0}^{\infty} q_k (1-q_k)^j \mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right] + H(p_1, p_2) \\
&\overset{(a)}{\leq} \sum_{h=1}^{k-1} \frac{p_h}{2} \log\left(1 + \frac{q_k x}{1-q_k} + A_h\right) + H(p_1, p_2) \\
&\overset{(b)}{=} \frac{p_1}{2} \log\left(1 + \frac{p_2 x}{p_1} + A_1\right) + H(p_1, p_2) \\
&\leq \frac{p_1}{2} \log(1+A) + H(p_1) \\
&\leq \frac{1}{2} \log(1+A) + 1 = 1.844, \qquad (56)
\end{aligned}$$

where (a) follows from (52) and (b) follows from the fact that $K = k = 2$.

Remark 5: If the gap result in the two cases is kept as function of A (rather than substituting the value as in the proof above), $\frac{1}{2\ln(2)}\frac{1}{A} + 1.52$ (from (55)) is a strictly decreasing function of A and $\frac{1}{2}\log(1+A) + 1$ (from (56)) is a strictly increasing function of A. Since the gap is the maximum of two gaps, optimizing the maximum of the gaps gives $A = 2.224$ using which we get both the RHS of (55) and the RHS of (56) $= 1.844$ to be equal.

So, for $K = 2$ we get

$$\begin{aligned}
&\sum_{j=k}^{K} \frac{p_j}{2} \log\left(1 + A_j - x\right) + \sum_{h=1}^{k-1} \frac{p_h}{2} \log\left(1 + \frac{q_k x}{1-q_k} + A_h\right) \\
&\quad - \sum_{j=0}^{\infty} q_k (1-q_k)^j \mathbb{E}\left[\frac{1}{2}\log(1+\phi_k(j))\right] + H(p_1, p_2) \\
&\leq 1.844, \qquad (57)
\end{aligned}$$

which together with $H(p_1) \leq \log 2 = 1$ completes the proof for $K = 2$, i.e., by comparing this to the statement of the lemma and the fact that $1.884 + \log 2 - 1.04 = 1.844$, the result follows.

Appendix C
Proof of Theorem 4

We divide the proof of Theorem 4 into two parts, corresponding to the upper and lower bounds, respectively.

C. Upper Bound

Similar to the proof of Theorem 1, suppose that an average of x_2 energy is stored into the battery and y_2 energy is drawn from the battery when the energy arrival is A_2, then $\mathbb{E}(g_2(t)) = p_2(A_2 - x_2 + r_{\text{out}} y_2)$ and $\mathbb{E}(g_1(t)) = p_1 A_1 + p_2(r_{\text{out}} r_{\text{in}} x_2 - \frac{1}{r_{\text{in}}} y_2)$. As $N \to \infty$, using the law of large numbers, we see that the capacity is upper bounded by

$$\begin{aligned}
C \leq \max_{\substack{0 \leq x_2 \leq B_{max}, \\ 0 \leq p_2(r_{\text{out}} r_{\text{in}} x_2 - \frac{1}{r_{\text{in}}} y_2)}} &\left\{ \frac{p_2}{2} \log(1 + A_2 - x_2 + r_{\text{out}} y_2) \right. \\
&\left. + \frac{p_1}{2} \log\left(1 + A_1 + \frac{p_2(r_{\text{out}} r_{\text{in}} x_2 - \frac{1}{r_{\text{in}}} y_2)}{p_1}\right) \right\}. \qquad (58)
\end{aligned}$$

Now, we replace the variable x_2 by $x_2' \triangleq x_2 - \min\{x_2, y_2 r_{\text{out}} r_{\text{in}}\}$ and also the variable y_2 by $y_2' \triangleq y_2 - \frac{1}{r_{\text{out}}} \min\{x_2, y_2 r_{\text{out}} r_{\text{in}}\}$. We see that

$$\begin{aligned}
&\frac{p_2}{2} \log\left(1 + A_2 - x_2' + r_{\text{out}} y_2'\right) \\
&\quad + \frac{p_1}{2} \log\left(1 + A_1 + \frac{p_2}{p_1}\left(r_{\text{out}} r_{\text{in}} x_2' - \frac{1}{r_{\text{in}}} y_2'\right)\right) \\
&= \frac{p_2}{2} \log(1 + A_2 - x_2 + r_{\text{out}} y_2) \\
&\quad + \frac{p_1}{2} \log\left(1 + A_1 + \frac{p_2}{p_1}\left(r_{\text{out}} r_{\text{in}} x_2 - \frac{1}{r_{\text{in}}} y_2\right)\right. \\
&\quad \left. + \left(\frac{1}{r_{\text{out}} r_{\text{in}}} - r_{\text{out}} r_{\text{in}}\right) \min\{x_2, y_2 r_{\text{out}} r_{\text{in}}\}\right) \\
&\geq \frac{p_2}{2} \log(1 + A_2 - x_2 + r_{\text{out}} y_2) \\
&\quad + \frac{p_1}{2} \log\left(1 + A_1 + \frac{p_2}{p_1}\left(r_{\text{out}} r_{\text{in}} x_2 - \frac{1}{r_{\text{in}}} y_2\right)\right). \qquad (59)
\end{aligned}$$

Thus, $C_{ub,2}$ with x_2 and y_2 is less than or equal to $C_{ub,2}$ with x_2' and y_2' if $\min\{x_2, y_2 r_{\text{out}} r_{\text{in}}\} > 0$. So, for optimal x_2 and y_2, $\min\{x_2, y_2 r_{\text{out}} r_{\text{in}}\} = 0$. Since $r_{\text{out}} r_{\text{in}} x_2 - \frac{1}{r_{\text{in}}} y_2 \geq 0$, we get $y_2 = 0$ and thus the upper bound in (25) follows.

D. Lower Bound

To show the achievability given in (26), we show that the rate of $\max_{0 \leq x \leq \min\{B_{\max}, A_2\}} C_{lb,2}^2(x)$ can be achieved, where

$$\begin{aligned}
C_{lb,2}^2(x) = &\frac{p_2}{2} \log(1 + A_2 - x) \\
&+ \frac{p_1}{2} \log\left(1 + A_1 + \frac{r_{\text{in}} r_{\text{out}} x p_2}{p_1}\right) - 2.884. \qquad (60)
\end{aligned}$$

Let the time slots of two consecutive energy arrivals of A_2 be T_1 and T_2, i.e., $E_{T_1}, E_{T_2} = A_2$ and $E_t = A_1$ for $T_1 < t < T_2$. Then, the energy of $E_{T_1} - x$ is used at time T_1 and the energy of $r_{\text{in}} x$ goes into the battery is. At any time t between T_1 and T_2, the energy of $p_2 B_t$ is extracted from the battery, and thus the energy of $E_t + r_{\text{out}} p_2 B_t$ can be used for transmission, where $E_t = A_1$, and the residual energy at time $(t+1)$ in the battery is $B_{t+1} = p_1 B_t$. Thus, the energy usage from the battery at time $t \in \{T_1 + 1, \ldots, T_2 - 1\}$ is $r_{\text{in}} r_{\text{out}} p_2 p_1^{t - T_1 - 1} x$. We note that this is a geometric random variable with parameter p_2, and thus has the mean $\frac{1}{p_2}$. We ignore the battery energy residue at T_2, and consider the next interval starting at T_2 to find the energy usage after T_2.

The energy utilization strategy $\widetilde{g}(t)$ proposed above is of the form $\widetilde{g}(t) = \phi(j)$, where $j = t - \max\{\tau : E_\tau = A_2, \forall \tau \leq t\}$, i.e., the strategy is invariant across time and the allocated energy depends on the number of time steps since the last energy arrival of A_2. The random variable $\phi(j)$ is defined as

$$\phi(j) \triangleq r_{\text{in}} r_{\text{out}} p_2 p_1^{j-1} x + A_1, \text{ for all } j \geq 1, \text{ and}$$
$$\phi(0) \triangleq A_2 - x. \tag{61}$$

The rest of the proof is straightforward and in the same lines as in the case of $r_{\text{in}} = r_{\text{out}} = 1$ as in Section IV, with the only difference that in the proof of Lemma 3 in Appendix VI the choice of two cases change as follows. The first case is when $\frac{p_2 r_{\text{in}} r_{\text{out}} x}{p_1} + A_1 > 2.224$ and the second one is when $\frac{p_2 r_{\text{in}} r_{\text{out}} x}{p_1} + A_1 \leq 2.224$.

Appendix D
Proof of Theorem 5

1) Case 1: $B_{\max} \leq p_1(A_2 - A_1)$: The upper bound in (13) can be achieved within 3.926 bits using the achievability scheme $C_{lb,3}^2(B_{\max})$ in Theorem 3.

2) Case 2: $B_{\max} \geq (p_1 + p_2)(A_3 - A_1)$: We have

$$\begin{aligned} &A_1 p_1 + A_2 p_2 + A_3 p_3 \\ &\leq A_1(2p_1 + p_2) \\ &\quad + A_2(p_2 + p_3) + A_3 p_3 \\ &= (A_2 - A_1)(p_2 + p_3) + A_1 \\ &\quad + (A_3 - A_1)p_3 + A_1 \\ &\leq 2\max\{(A_2 - A_1)(p_2 + p_3) \\ &\quad + A_1, (A_3 - A_1)p_3 + A_1\}. \end{aligned} \tag{62}$$

Then using (15) we get

$$\begin{aligned} C_{ub,3} &\leq \frac{1}{2}\log(1 + A_1 p_1 + A_2 p_2 + A_3 p_3) \\ &\leq \frac{1}{2} + \frac{1}{2}\log(1 + \max\{(A_2 - A_1)(p_2 + p_3) + A_1, \\ &\quad (A_3 - A_1)p_3 + A_1\}). \end{aligned} \tag{63}$$

Also, we have the following two achievable rates

$$\begin{aligned} &C_{lb,3}^2\,(p_1(A_2 - A_1)) \\ &= \frac{p_3}{2}\log(1 + A_3 - (A_2 - A_1)p_1) \\ &\quad + \frac{p_2}{2}\log(1 + A_2 - (A_2 - A_1)p_1) \\ &\quad + \frac{p_1}{2}\log\left(1 + A_1 + \frac{(p_2 + p_3)p_1(A_2 - A_1)}{p_1}\right) - 3.926 \\ &\geq \frac{1}{2}\log(1 + (A_2 - A_1)(p_2 + p_3) + A_1) - 3.926, \end{aligned} \tag{64}$$

and

$$\begin{aligned} &C_{lb,3}^3\,((p_1 + p_2)(A_3 - A_1)) \\ &= \frac{p_3}{2}\log(1 + (A_3 - A_1)p_3 + A_1) \\ &\quad + \frac{p_2}{2}\log\left(1 + A_2 + \frac{p_3(p_1 + p_2)(A_3 - A_1)}{p_1 + p_2}\right) \\ &\quad + \frac{p_1}{2}\log\left(1 + A_1 + \frac{p_3(p_1 + p_2)(A_3 - A_1)}{p_1 + p_2}\right) - 3.926 \\ &\geq \frac{1}{2}\log(1 + (A_3 - A_1)p_3 + A_1) - 3.926. \end{aligned} \tag{65}$$

So, the maximum of the rates in (64) and (65) is achievable (because of $B_{\max} \geq (p_1 + p_2)(A_3 - A_1) \geq p_1(A_2 - A_1)$) and it is seen from (63) that their maximum is within a constant gap of 4.426 bits to the upper bound.

3) Case 3: $p_1(A_2 - A_1) < B_{\max} < (p_1 + p_2)(A_3 - A_1)$ and $(A_2 - A_1)(p_2 + p_3) + A_1 \geq (A_3 - A_1)p_3 + A_1$: For this case, similar to the previous case, we get

$$\begin{aligned} C_{ub,3} &\leq \frac{1}{2}\log(1 + A_1 p_1 + A_2 p_2 + A_3 p_3) \\ &\leq \frac{1}{2} + \frac{1}{2}\log(1 + \max\{(A_2 - A_1)(p_2 + p_3) + A_1, \\ &\quad (A_3 - A_1)p_3 + A_1\}) \\ &= \frac{1}{2} + \frac{1}{2}\log(1 + (A_2 - A_1)(p_2 + p_3) + A_1), \end{aligned} \tag{66}$$

and the rate in (64) is achievable (because of $B_{\max} \geq p_1(A_2 - A_1)$) and from (66) the maximum is within a constant gap of 4.426 bits to the upper bound.

4) Case 4: $p_1(A_2 - A_1) < B_{\max} < A_3 - \sum_{i=1}^{3} p_i A_i$ and $(A_2 - A_1)(p_2 + p_3) + A_1 < (A_3 - A_1)p_3 + A_1$: In this case for the upper bound in Theorem 1, we have $z_3^* = B_{\max}$. So,

$$\begin{aligned} C_{ub,3} &= \max_{0 \leq x_2 \leq B_{\max}} \left\{ \frac{p_2}{2}\log(1 + A_2 - x_2) \right. \\ &\quad + \frac{p_3}{2}\log(1 + A_3 - B_{\max}) \\ &\quad \left. + \frac{p_1}{2}\log\left(1 + A_1 + \frac{p_2 x_2 + p_3 B_{\max}}{p_1}\right)\right\} \\ &= \frac{p_3}{2}\log(1 + A_3 - B_{\max}) + \left(\frac{p_1 + p_2}{2}\right) \\ &\quad \log\left(1 + A_1 + \frac{p_2(A_2 - A_1) + p_3 B_{\max}}{p_1 + p_2}\right). \end{aligned} \tag{67}$$

Now, divide the achievability scheme into two parts:

A) $p_2 A_2 \leq p_3 B_{\max}$: The scheme $C_{lb,3}^3(x)$ with $x = \min\{B_{\max}, A_3\} = B_{\max}$ gives the capacity within 4.426 bits to the upper bound (67) because $C_{lb,3}^3(B_{\max})$ equates shown at the bottom of the next page.

B) $p_2(A_2 - A_1) > p_3 B_{\max}$: For this set of of parameters, we have $p_2 > p_1 p_3$. This is because if $p_2 \leq p_1 p_3$, it results

$p_1(A_2-A_1) \geq \frac{p_2(A_2-A_1)}{p_3} > B_{\max}$ which is a contradiction to the initial assumption of $p_1(A_2-A_1) \leq B_{\max}$.

Use the strategy $C^3_{lb,3}((A_2-A_1)p_1)$ for the achievability. Then, we get

$$\begin{aligned}
&C_{ub,3} - C^3_{lb,3}((A_2-A_1)p_1) - 3.926 \\
&\leq \frac{p_1+p_2}{2}\left(\log\left(1+A_1+\frac{\overbrace{p_2(A_2-A_1)+p_3B_{\max}}^{\leq 2p_2(A_2-A_1)}}{p_1+p_2}\right)\right. \\
&\quad \left. -\log\left(1+(A_2-A_1)(1-p_1)+A_1\right)\right) \\
&\leq \frac{p_1+p_2}{2}\log\left(\frac{1+A_1+\frac{2p_2(A_2-A_1)}{p_1+p_2}}{1+A_1+(A_2-A_1)(p_2+p_3)}\right) \\
&\leq \frac{p_1+p_2}{2}\log\left(\frac{1+A_1+(A_2-A_1)\left(\frac{p_2}{p_1+p_2}\right)}{1+A_1+(A_2-A_1)(p_2+p_3)}\right) \\
&\quad + \frac{p_1+p_2}{2}\log 2 \qquad (68)
\end{aligned}$$

$$\overset{(a)}{\leq} \frac{p_1+p_2}{2}\log 2 \leq \frac{1}{2}. \qquad (69)$$

where (a) follows since $\frac{\left(\frac{p_2}{p_1+p_2}\right)}{(p_2+p_3)} \leq 1$ which can be easily shown.

5) Case 5: $\sum_{i=1}^3 p_iA_i < B_{\max} < (p_1+p_2)(A_3-A_1)$ *and* $(A_2-A_1)(p_2+p_3)+A_1 < (A_3-A_1)p_3+A_1$*:* Take the upper bound $C_{ub,3} = \frac{1}{2}\log(1+p_1A_1+p_2A_2+p_3A_3)$. For achievability consider $C^3_{lb,3}(A_3-\sum_{i=1}^3 p_iA_i)$. We could see that $\frac{p_3x}{p_1+p_2} = \sum_{i=1}^3 p_iA_i - \frac{p_1}{p_1+p_2}A_1 - \frac{p_2}{p_1+p_2}A_2$, so we will have

$$\begin{aligned}
&C^3_{lb,3}\left(A_3-\sum_{i=1}^3 p_iA_i\right) = -3.926 \\
&\quad + \frac{p_3}{2}\log\left(1+\sum_{i=1}^3 p_iA_i\right) \\
&\quad + \frac{p_2}{2}\log\left(1+\sum_{i=1}^3 p_iA_i - \frac{p_1}{p_1+p_2}A_1 + \frac{p_1}{p_1+p_2}A_2\right) \\
&\quad + \frac{p_1}{2}\log\left(1+\sum_{i=1}^3 p_iA_i + \frac{p_2}{p_1+p_2}A_1 - \frac{p_2}{p_1+p_2}A_2\right),
\end{aligned}$$

and we get

$$\begin{aligned}
&C_{ub} - C^3_{lb,3}\left(A_3-\sum_{i=1}^3 p_iA_i\right) - 3.926 \\
&\leq -\frac{p_2}{2}\log\left(1+\frac{p_1(A_2-A_1)}{(p_1+p_2)(1+\sum_{i=1}^3 p_iA_i)}\right) \\
&\quad -\frac{p_1}{2}\log\left(1-\frac{p_2(A_2-A_1)}{(p_1+p_2)(1+\sum_{i=1}^3 p_iA_i)}\right) \qquad (70) \\
&\overset{(a)}{\leq} -\frac{p_2}{2}\log\left(1+\frac{p_1A_2}{(p_1+p_2)(2+p_2A_2+p_3A_3)}\right) \\
&\quad -\frac{p_1}{2}\log\left(1-\frac{p_2A_2}{(p_1+p_2)(1+p_2A_2+p_3A_3)}\right) \\
&\overset{(b)}{\leq} \lim_{A_2\to\infty}\left\{-\frac{p_2}{2}\log\left(1+\frac{p_1A_2}{(p_1+p_2)(1+p_2A_2+p_3A_3)}\right)\right. \\
&\quad \left.-\frac{p_1}{2}\log\left(1-\frac{p_2A_2}{(p_1+p_2)(1+p_2A_2+p_3A_3)}\right)\right\} \\
&= \lim_{A_2\to\infty}\left\{-\frac{p_2}{2}\log\left(1+\frac{p_1A_2}{(p_1+p_2)(p_2A_2+p_3A_3)}\right)\right. \\
&\quad \left.-\frac{p_1}{2}\log\left(1-\frac{p_2A_2}{(p_1+p_2)(p_2A_2+p_3A_3)}\right)\right\} \qquad (71) \\
&\overset{(c)}{\leq} \max_{\frac{p_2+p_3}{p_3}\leq e}\left\{-\frac{p_2}{2}\log\left(1+\frac{p_1}{(p_1+p_2)(p_2+p_3e)}\right)\right. \\
&\quad \left.-\frac{p_1}{2}\log\left(1-\frac{p_2}{(p_1+p_2)(p_2+p_3e)}\right)\right\} \qquad (72) \\
&\overset{(d)}{=} -\frac{p_2}{2}\log\left(1+\frac{p_1}{(p_1+p_2)(2p_2+p_3)}\right) \\
&\quad -\frac{p_1}{2}\log\left(1-\frac{p_2}{(p_1+p_2)(2p_2+p_3)}\right), \qquad (73)
\end{aligned}$$

where (a) follows from the fact that the function $g(x) = -\frac{p_2}{2}\log(1+p_1x) - \frac{p_1}{2}\log(1-p_2x)$ is increasing and also $\frac{(A_2-A_1)}{(p_1+p_2)(1+\sum_{i=1}^3 p_iA_i)} \leq \frac{A_2}{(p_1+p_2)(1+p_2A_2+p_3A_3)}$ and (b) follows from the fact that (71) has a positive derivative respect to A_2, (c) follows from $\frac{p_2+p_3}{p_3} \leq \frac{A_3}{A_2}$ and (d) follows from the fact that (72) has a negative derivative respect to e and $e^* = \frac{p_2+p_3}{p_3}$ in (73).

$$\begin{aligned}
&C^3_{lb,3}(B_{\max}) = -3.926 + \frac{p_3}{2}\log(1+A_3-B_{\max}) + \\
&\underbrace{\frac{p_2}{2}\log\left(1+A_2+\frac{p_3B_{\max}}{p_1+p_2}\right) + \frac{p_1}{2}\log\left(1+A_1+\frac{p_3B_{\max}}{p_1+p_2}\right)}. \\
&\geq \frac{p_1+p_2}{2}\log(1+A_1+\frac{p_3B_{\max}}{p_1+p_2}) \geq \frac{p_1+p_2}{2}\log(1+A_1+\frac{2p_3B_{\max}}{p_1+p_2}) - \frac{1}{2} \\
&\geq \frac{p_1+p_2}{2}\log(1+A_1+\frac{p_2(A_2-A_1)+p_3B_{\max}}{p_1+p_2}) - \frac{1}{2}
\end{aligned}$$

It remains to show that (73) $\leq 1/2$. We divide the proof into two parts:

A) $p_2 \leq p_1$: Let take $p_1 = m + n$, $p_2 = m$ and $p_3 = 1 - 2m - n$. Then, we get $\frac{p_2}{(p_1+p_2)(2p_2+p_3)} = \frac{m}{(2m+n)(1-n)} = \frac{m}{2m+np_3} \leq \frac{1}{2}$.

So, we have $-\frac{p_2}{2}\log\left(1+\frac{p_1}{(p_1+p_2)(2p_2+p_3)}\right) - \frac{p_1}{2}\log\left(1-\frac{p_2}{(p_1+p_2)(2p_2+p_3)}\right) \leq -\frac{p_1}{2}\log\left(1-\frac{p_2}{(p_1+p_2)(2p_2+p_3)}\right) \leq -\frac{1}{2}\log\left(1-\frac{1}{2}\right) = \frac{1}{2}$.

B) $p_2 > p_1$: Let take $p_1 = m$, $p_2 = m + n$ and $p_3 = 1 - 2m - n$. Then, we get

$$\begin{aligned}
&-\frac{p_2}{2}\log\left(1+\frac{p_1}{(p_1+p_2)(2p_2+p_3)}\right)\\
&-\frac{p_1}{2}\log\left(1-\frac{p_2}{(p_1+p_2)(2p_2+p_3)}\right)\\
\leq &-\frac{p_1}{2}\log\left(1-\frac{p_2}{(p_1+p_2)(2p_2+p_3)}\right)\\
= &-\frac{m}{2}\log\left(1-\frac{m+n}{(1+n)(2m+n)}\right)\\
= &-\frac{m}{2}\log\left(\frac{m+n(2m+n)}{(1+n)(2m+n)}\right)\\
= &\frac{m}{2}\log\left(\frac{(1+n)(2m+n)}{m+n(2m+n)}\right)\\
= &\frac{m}{2}\log\left(1+\frac{m+n}{m+n(2m+n)}\right)\\
\leq &\max_n\left\{\frac{m}{2}\log\left(1+\frac{m+n}{m+n(2m+n)}\right)\right\}.
\end{aligned} \tag{74}$$

We see that optimal n, $n^* = \sqrt{m(1-m)} - m$. Then, RHS of (74) is equal to $\frac{m}{2}\log\left(1+\frac{\sqrt{m(1-m)}}{m+m(1-m)-m^2}\right) = \frac{m}{2}\log\left(1+\frac{1}{2\sqrt{m(1-m)}}\right)$, where $0 < m \leq \frac{1}{2}$. It can be seen that $\frac{m}{2}\log\left(1+\frac{1}{2\sqrt{m(1-m)}}\right)$ is an increasing function with respect to m, and is thus $\leq \frac{1}{2}\log\left(1+\frac{1}{2\sqrt{\frac{1}{2}(\frac{1}{2})}}\right) = \frac{1}{2}$.

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