Generalized Degrees of Freedom Region for MIMO Interference Channel with Feedback

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***Abstract*—In this paper, we investigate the effect of feedback on two-user MIMO interference channels. At first, the capacity region of MIMO interference channels with feedback is characterized within a constant number of bits, where this constant is independent of the channel matrices. Further, the generalized degrees of freedom region for the MIMO interference channel with feedback is characterized.**

I. Introduction

Wireless networks with multiple users are interference-limited rather than noise-limited. Interference channel (IC) is a good starting point for understanding the performance limits of the interference limited communications [1–7]. Feedback can be employed in ICs to achieve an improvement in data rates [8–13]. However, most of the existing works on ICs with feedback are limited to discrete memoryless channels or to single-input single-output (SISO) channels. This paper analyzes multiple-input multiple-output (MIMO) Gaussian interference channels with feedback. In particular, we focus on a two-user MIMO IC with feedback.

The authors of [8] considered a SISO Gaussian IC with feedback and found its capacity region within two bits. It was shown that the capacity regions of Gaussian ICs increase unboundedly with feedback unlike the Gaussian multiple-access channel where the gains are bounded [14]. The degrees of freedom for a symmetric SISO Gaussian IC with feedback is also found in [8]. In this paper, we find an outer bound and an inner bound for the capacity region that are within a constant number of bits, and also evaluate the generalized degrees of freedom (GDoF) region for a general MIMO IC with feedback.

The first main result of the paper is the characterization of the capacity region of a MIMO IC with feedback within $N_1+N_2+\max(N_1,N_2)$ bits, where N_1 and N_2 are the numbers of receive antennas at the two receivers. The achievability strategy is based on block Markov encoding, backward decoding, and Han-Kobayashi message-splitting. This achievable rate and the outer bound are within $N_1+N_2+\max(N_1,N_2)$ bits of each other thus characterizing the capacity region of the two-user IC within constant number of bits where the constant is independent of the channel matrices. The achievability scheme that is used to prove the constant gap result assumes that the transmitted signals from the two transmitters in a time-slot are uncorrelated, unlike [8] where the signals were assumed correlated in the achievability. Thus, our achievable rate region is within 3 bits rather than 2 bits as in [8] of the capacity region of a SISO IC with feedback. An achievability scheme without correlated inputs was also shown to achieve within constant gap of the capacity region in [12] for a SISO IC with feedback. However, our gap between the inner and the outer bounds is smaller as compared to [12].

We note that the achievability strategies for a SISO IC in [8, 12] emphasize that the private part from a transmitter using the Han-Kobayashi message splitting is such that it is received at the other receiver at the noise floor. However, for a MIMO IC with feedback, it is not clear what its counterpart would be. The Han-Kobayashi message splitting used in this paper gives the notion of receiving the signal at the noise floor for a MIMO IC with feedback.

The second main result of the paper is a complete characterization of the generalized degrees of freedom (GDoF) region of a general MIMO IC with feedback when the average signal quality of each link, say ρ_{ij} for link from transmitter i to receiver j, varies with a base signal-to-noise ratio (SNR) parameter, say SNR, as $\lim_{\mathsf{SNR}\to\infty}\frac{\log\rho_{ij}}{\log\mathsf{SNR}}=\alpha_{ij}$, where α_{ij} can be different for each link with $i,j\in\{1,2\}$. In other words, the average link quality of each link can potentially have different exponents of a base SNR. As a special case, we consider a symmetric IC where the number of antennas at both transmitters is the same, the number of antennas at both receivers is the same, and the SNRs for the direct links and the cross links are SNR and SNR^α, $\alpha\geq 0$, respectively. We find the GDoF (the maximum symmetric point in the GDoF region) for a given α and show that the GDoF follows a "V"-curve rather than a "W"-curve corresponding to the GDoF without feedback as in [5]. Similar result was obtained for a SISO IC in [8] while this paper extends it to a MIMO system.

The remainder of the paper is organized as follows. Section II introduces the model for a MIMO IC model with feedback and the GDoF region. Sections III and IV describe our results on the capacity region and the GDoF region, respectively.

II. Channel Model and Preliminaries

In this section, we describe the channel model considered in this paper. A two-user MIMO IC consists of two transmitters and two receivers. Transmitter i is labeled as T_i and receiver j is labeled as D_j for $i, j \in \{1,2\}$. Further, we assume T_i has M_i antennas and D_i has N_i antennas, $i \in \{1,2\}$. Henceforth, such a MIMO IC will be referred to as the (M_1, N_1, M_2, N_2) MIMO IC. We assume that the channel matrix between transmitter T_i and receiver D_j is denoted by $H_{ij} \in \mathbb{C}^{N_j \times M_i}$, for $i, j \in \{1,2\}$. We shall consider a time-invariant or fixed channel where the channel matrices remain fixed for the entire duration of communication. We also incorporate a non-negative power attenuation factor, denoted as ρ_{ij}, for the signal transmitted from T_i to D_j. At time-instant t, transmitter T_i chooses a vector $X_i(t) \in \mathbb{C}^{M_i \times 1}$ and transmits $\sqrt{P_i} X_i(t)$ over the channel, where P_i is the average transmit power at transmitter T_i.

The received signal at receiver D_i at time instant t is denoted as $Y_i(t)$ for $i \in \{1,2\}$, and can be written as

$$
\begin{aligned}
Y_1(t) &= \sqrt{\rho_{11}} H_{11} X_1(t) + \sqrt{\rho_{21}} H_{21} X_2(t) + Z_1(t), &(1)\\
Y_2(t) &= \sqrt{\rho_{12}} H_{12} X_1(t) + \sqrt{\rho_{22}} H_{22} X_2(t) + Z_2(t), &(2)
\end{aligned}
$$

where $Z_i(t) \in \mathbb{C}^{N_i \times 1}$ is independent and identically distributed (i.i.d.) $\mathsf{CN}(0, I_{N_i})$ (complex Gaussian noise), ρ_{ii} is the received SNR at receiver D_i and ρ_{ij} is the received interference-to-noise-ratio at receiver D_j for $i, j \in \{1,2\}$, $i \neq j$. A MIMO IC is fully described by three parameters. The first is the number of antennas at each transmitter and receiver, namely (M_1, N_1, M_2, N_2). The second is the set of channel gains, $\overline{H} = \{H_{11}, H_{12}, H_{21}, H_{22}\}$. The third is the set of average link qualities of all the channels, $\overline{\rho} = \{\rho_{11}, \rho_{12}, \rho_{21}, \rho_{22}\}$. We assume that these parameters are known to all transmitters and receivers.

For MIMO IC with feedback, the transmitted signal at T_i, $X_i(t)$, is a function of the message W_i and the previous channel outputs at receiver D_i for $i \in \{1,2\}$. Thus, the encoding functions of the two transmitters are given as

$$
X_i(t) = f_i(W_i, Y_i^{t-1}), \quad i \in \{1,2\}, \tag{3}
$$

where f_{it} is the encoding function of T_i, W_i is the message of T_i and $Y_i^{t-1} = (Y_i(1), ..., Y_i(t-1))$. Let us assume that T_i transmits information at a rate of R_i to receiver D_i using the codebook $C_{i,n}$ of length-n codewords with $|C_{i,n}| = 2^{nR_i}$. Given a message $m_i \in \{1, \ldots, 2^{nR_i}\}$, the corresponding codeword $X_i^n(m_i) \in C_{i,n}$ satisfies the power constraint mentioned before. From the received signal Y_i^n, the receiver obtains an estimate $\widehat{m_i}$ of the transmitted message m_i using a decoding function. Let the average probability of error be denoted by $e_{i,n} = \Pr(\widehat{m_i} \neq m_i)$.

A rate pair (R_1, R_2) is achievable if there exists a family of codebooks $C_{i,n}, i = \{1,2\}_n$ and decoding functions such that $\max_i\{e_{i,n}\}$ goes to zero as the block length n goes to infinity. The capacity region $C(\overline{H}, \overline{\rho})$ of the IC with parameters $\overline{H}$ and $\overline{\rho}$ is defined as the closure of the set of all achievable rate pairs.

Consider a two-dimensional rate region $\mathcal{C}$. Then, the region $\mathcal{C} \oplus ([0,a] \times [0,b])$ denotes the region formed by $\{(R_1, R_2) : R_1, R_2 \geq 0, ((R_1 - a)^+, (R_2 - b)^+) \in \mathcal{C}\}$ for some $a, b \geq 0$. Similarly, the region $\mathcal{C} \ominus ([0,a] \times [0,b])$ denotes the region formed by $\{(R_1, R_2) : R_1, R_2 \geq 0, ((R_1 + a)^+, (R_2 + b)^+) \in \mathcal{C}\}$ for some $a, b \geq 0$. Further, we define the notion of an achievable rate region that is within a constant number of bits of the capacity region as follows.

Definition 1. *An achievable rate region $A(\overline{H}, \overline{\rho})$ is said to be within b bits of the capacity region if $A(\overline{H}, \overline{\rho}) \subseteq C(\overline{H}, \overline{\rho})$ and $A(\overline{H}, \overline{\rho}) \oplus ([0,b] \oplus [0,b]) \supseteq C(\overline{H}, \overline{\rho})$.*

In this paper, we will use the GDoF region to characterize the capacity region of the MIMO IC with feedback in the limit of high SNR. This notion generalizes the conventional degrees of freedom (DoF) region metric by additionally emphasizing the signal level as a signaling dimension. It characterizes the simultaneously accessible fractions of spatial and signal-level dimensions (per channel use) by the two users when all the average channel coefficients vary as exponents of a nominal SNR parameter. Thus, we assume that

$$
\lim_{\log(\mathsf{SNR}) \to \infty} \frac{\log(\rho_{ij})}{\log(\mathsf{SNR})} = \alpha_{ij}, \tag{4}
$$

where $\alpha_{ij} \in \mathbb{R}^+$ for all $i, j \in \{1,\ 2\}$. In the limit of high SNR, the capacity region diverges.

The GDoF region is defined as the region formed by the set of all (d_1, d_2) such that $(d_1 \log(\mathsf{SNR}) - o(\log(\mathsf{SNR})), d_2 \log(\mathsf{SNR}) - o(\log(\mathsf{SNR})))$[1] is inside the capacity region. Thus, the GDoF is a function of link quality scaling exponents α_{ij}. We note that since the channel matrices are of full ranks with probability 1, we will have the GDoF with probability 1 over the randomness of channel matrices.

III. Capacity Region of MIMO Interference Channel with Feedback

In this section, we will describe our result on the approximate capacity region of the two-user MIMO IC with feedback. Let $\mathcal{R}_o$ be the region formed by (R_1, R_2) satisfying the following constraints:

$$
\begin{aligned}
R_1 &\leq \log\det(I_{N_1} + \rho_{11} H_{11} H_{11}^\dagger + \rho_{21} H_{21} H_{21}^\dagger),\\
R_2 &\leq \log\det(I_{N_2} + \rho_{22} H_{22} H_{22}^\dagger + \rho_{12} H_{12} H_{12}^\dagger),\\
R_1 &\leq \log\det\left(I_{N_2} + \rho_{12} H_{12} H_{12}^\dagger\right) +\\
&\quad \log\det(I_{N_1} + \rho_{11} H_{11} H_{11}^\dagger - \sqrt{\rho_{11}\rho_{12}} H_{11} H_{12}^\dagger\\
&\quad (I_{N_2} + \rho_{12} H_{12} H_{12}^\dagger)^{-1} \sqrt{\rho_{11}\rho_{12}} H_{12} H_{11}^\dagger),\\
R_2 &\leq \log\det\left(I_{N_1} + \rho_{21} H_{21} H_{21}^\dagger\right) +\\
&\quad \log\det(I_{N_2} + \rho_{22} H_{22} H_{22}^\dagger - \sqrt{\rho_{22}\rho_{21}} H_{22} H_{21}^\dagger\\
&\quad (I_{N_1} + \rho_{21} H_{21} H_{21}^\dagger)^{-1} \sqrt{\rho_{22}\rho_{21}} H_{21} H_{22}^\dagger),
\end{aligned}
$$

[1] $a = o(\log(\mathsf{SNR}))$ indicates that $\lim_{\mathsf{SNR} \to \infty} \frac{a}{\log(\mathsf{SNR})} = 0$.

$$\begin{aligned} R_1+R_2 &\le \log\det\left(I_{N_2}+\rho_{22}H_{22}H_{22}^{\dagger}+\rho_{12}H_{12}H_{12}^{\dagger}\right)+ \\ &\quad \log\det(I_{N_1}+\rho_{11}H_{11}H_{11}^{\dagger}-\sqrt{\rho_{11}\rho_{12}}H_{11}H_{12}^{\dagger} \\ &\quad (I_{N_2}+\rho_{12}H_{12}H_{12}^{\dagger})^{-1}\sqrt{\rho_{11}\rho_{12}}H_{12}H_{11}^{\dagger}), \\ R_1+R_2 &\le \log\det\left(I_{N_1}+\rho_{11}H_{11}H_{11}^{\dagger}+\rho_{21}H_{21}H_{21}^{\dagger}\right)+ \\ &\quad \log\det(I_{N_2}+\rho_{22}H_{22}H_{22}^{\dagger}-\sqrt{\rho_{22}\rho_{21}}H_{22}H_{21}^{\dagger} \\ &\quad (I_{N_1}+\rho_{21}H_{21}H_{21}^{\dagger})^{-1}\sqrt{\rho_{22}\rho_{21}}H_{21}H_{22}^{\dagger}). \end{aligned}$$

Theorem 1. *The capacity region for the two-user MIMO IC with perfect feedback $\mathcal{C}_{FB}$ is bounded from above and below as*

$$\begin{aligned} &\mathcal{R}_o \ominus ([0,N_1+N_2]\times[0,N_1+N_2]) \\ &\subseteq \mathcal{C}_{FB} \subseteq \mathcal{R}_o \oplus ([0,N_1]\times[0,N_2]), \end{aligned} \tag{5}$$

where the inner and outer bounds are within $N_1+N_2+\max(N_1,N_2)$ bits.

Proof: We would describe the outline of achievability and the outer bound here. The detailed proof can be seen in [15].

The inner bound uses the achievable region for a two-user discrete memoryless IC with feedback as in [8]. The achievability scheme employs block Markov encoding, backward decoding, and Han-Kobayashi message-splitting. This result for a discrete memoryless channel is extended to MIMO IC with feedback using a specific message splitting by power allocation. The transmitted signal from T_i, X_i, is given as

$$X_i = X_{ip} + X_{iu}, \tag{6}$$

where X_{ip} indicate the private message of T_i, and X_{iu} is the public message of T_i. We assume that X_{ip} and X_{iu} pairs are independent for $i=1,2$.. However, these transmitted signals are correlated over time due to block Markov encoding. The private signal X_{ip} is chosen to be $X_{ip}\sim \mathsf{CN}\left(0,K_{X_{ip}}\right)$, and the public signal X_{iu} is chosen to be $X_{iu}\sim\mathsf{CN}\left(0,K_{X_{iu}}\right)$, where

$$K_{X_{ip}} = I_{M_i}-\sqrt{\rho_{ij}}H_{ij}^{\dagger}(I_{N_j}+\rho_{ij}H_{ij}H_{ij}^{\dagger})^{-1}\sqrt{\rho_{ij}}H_{ij}, \tag{7}$$

and

$$K_{X_{iu}} = I_{M_i}-K_{X_{ip}}, \tag{8}$$

for $i\in\{1,2\}$.

The power allocation is feasible since $K_{X_{ip}}\succeq 0$ and $K_{X_{iu}}\succeq 0$. Further, this message split is such that the private signal is received at the other receiver with power bounded by a constant. More specifically, we have $\rho_{ji}H_{ji}K_{X_{ip}}H_{ji}^{\dagger}\preceq I_{N_j}$ thus showing that the effective received signal covariance matrix at receiver D_j corresponding to the private signal from transmitter T_i is at the noise floor.

For the outer bound, we see that the capacity region of a two-user MIMO IC with feedback is outerbounded by the region formed by (R_1,R_2) satisfying

$$\begin{aligned} R_1 &\le h(Y_1)-h(Z_1), \\ R_2 &\le h(Y_2)-h(Z_2), \\ R_1 &\le h\left(Y_2\mid X_2\right)-h\left(Z_2\right)+h(Y_1|X_2,S_1)- \\ &\quad h(Z_1), \\ R_2 &\le h\left(Y_1\mid X_1\right)-h\left(Z_1\right)+h(Y_2|X_2,S_1)- \\ &\quad h(Z_2), \\ R_1+R_2 &\le h\left(Y_1\mid S_1,X_2\right)-h\left(Z_2\right)+h(Y_2)- \\ &\quad h(Z_1), \\ R_1+R_2 &\le h\left(Y_2\mid S_2,X_1\right)-h\left(Z_1\right)+h(Y_1)- \\ &\quad h(Z_2), \end{aligned} \tag{9}$$

where $S_i\triangleq\sqrt{\rho_{ij}}H_{ij}X_i+Z_j$. This region is further outer-bounded for MIMO channels over the choice of covariance and the cross-covariance matrices to get the result as in the statement of the Theorem. ■

The authors of [8] found the capacity region for the SISO IC with feedback within 2 bits. The above Theorems generalizes the result to find the capacity region of MIMO IC with feedback within $N_1+N_2+\max(N_1,N_2)$ bits. Note that the approximate capacity region without feedback in [4] involves bounds on $2R_1+R_2$ which do not appear in our approximate capacity region with feedback. In addition, in [8], the approximate capacity region for the SISO IC with feedback involves the cross-covariance matrix of the inputs in the inner and outer bounds, whereas our approximate capacity region for the MIMO IC with feedback does not involve cross-covariance matrix of the inputs. Further, we note that the power allocation described in the achievability is different from that given in [8] even for a SISO channel. Note that the power split levels in the achievability scheme of [8] do not sum to 1 and thus do not satisfy the total power constraint. For the special case of SISO IC with feedback, the above gives a fix to the results in [8]. This power allocation assumes uncorrelated signals transmitted by the two users at each time-slot. The authors of [12] also used uncorrelated signals for SISO but had a larger gap between the inner and outer bounds for SISO IC with feedback than that achieved by our achievability strategy.

Figure 1 gives a pictorial representation for the result of Theorem 1. The inner and the outer bounds for the capacity region for MIMO IC with feedback are within a constant number of bits from the region $\mathcal{R}_o$ and thus the inner and outer bound regions are within a constant number of bits of each other.

IV. Generalized Degrees of Freedom Region of MIMO Interference Channel with Feedback

This section describes our results on the GDoF region of the two-user MIMO IC with feedback. The GDoF gives the high SNR characterization of the capacity region. Since the inner and outer-bounds on the capacity region are within a constant gap, we characterize the exact GDoF region of the MIMO IC with feedback.

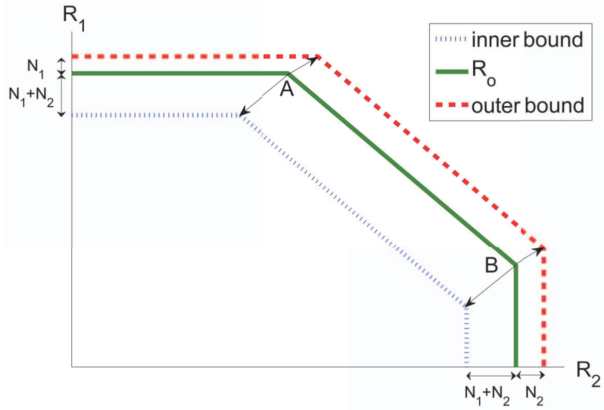


Fig. 1. Inner and outer bounds for the capacity region of MIMO IC with feedback are within a constant number of bits. The arrows from the corners A and B in $\mathcal{R}_o$ toward their respective corners on outer bound have vertical length of N_1 and horizontal length of N_2. The arrows from the corners A and B in $\mathcal{R}_o$ toward their respective corners on inner bound have the vertical and horizontal length of N_1+N_2 each.

Define

$$f(u,(a_1,u_1),(a_2,u_2)) \triangleq \tag{10}$$
$$\begin{cases} \min(u,u_1)a_1^+ + \min((u-u_1)^+,u_2)a_2^+, & \text{if } a_1 \geq a_2 \\ \min(u,u_2)a_2^+ + \min((u-u_2)^+,u_1)a_1^+, & \text{otherwise} \end{cases}.$$

The following result characterizes the GDoF for general MIMO IC with feedback for general power scaling parameters α_{ij}.

Theorem 2. *The GDoF region of the two-user MIMO IC with feedback is given by the set of (d_1,d_2) satisfying:*

$$\begin{aligned}
\alpha_{11}d_1 &\leq f(N_1,(\alpha_{11},M_1),(\alpha_{21},M_2)),\\
\alpha_{22}d_2 &\leq f(N_2,(\alpha_{22},M_2),(\alpha_{12},M_1)),\\
\alpha_{11}d_1 &\leq \alpha_{12}\min(M_1,N_2) + \\
&\quad \alpha_{11}\min\left((M_1-N_2)^+,N_1\right) + \\
&\quad (\alpha_{11}-\alpha_{12})^+(\min(M_1,N_1) - \\
&\quad \min\left((M_1-N_2)^+,N_1\right)),\\
\alpha_{22}d_2 &\leq \alpha_{21}\min(M_2,N_1) + \\
&\quad \alpha_{22}\min\left((M_2-N_1)^+,N_2\right) + \\
&\quad (\alpha_{22}-\alpha_{21})^+(\min(M_2,N_2) - \\
&\quad \min\left((M_2-N_1)^+,N_2\right)),\\
\alpha_{11}d_1+\alpha_{22}d_2 &\leq f(N_2,(\alpha_{22},M_2),(\alpha_{12},M_1)) + \\
&\quad \alpha_{11}\min\left((M_1-N_2)^+,N_1\right) + \\
&\quad (\alpha_{11}-\alpha_{12})^+(\min(M_1,N_1) - \\
&\quad \min\left((M_1-N_2)^+,N_1\right)),\\
\alpha_{11}d_1+\alpha_{22}d_2 &\leq f(N_1,(\alpha_{11},M_1),(\alpha_{21},M_2)) + \\
&\quad \alpha_{22}\min\left((M_2-N_1)^+,N_2\right) + \\
&\quad (\alpha_{22}-\alpha_{21})^+(\min(M_2,N_2) - \\
&\quad \min\left((M_2-N_1)^+,N_2\right)). \qquad (11)
\end{aligned}$$

Proof: This follows from taking large SNR limit of R_o. The details are provided in [15]. ∎

Corollary 1. *The GDoF for a two-user symmetric MIMO IC with feedback for $N \leq M$ is given as follows:*

$$GDoF_{PF} = \begin{cases} N - \frac{\alpha}{2}(2N-M)^+, & \text{if } \alpha \leq 1, \\ N(\frac{\alpha+1}{2}) - \frac{1}{2}(2N-M)^+, & \text{if } \alpha \geq 1. \end{cases} \tag{12}$$

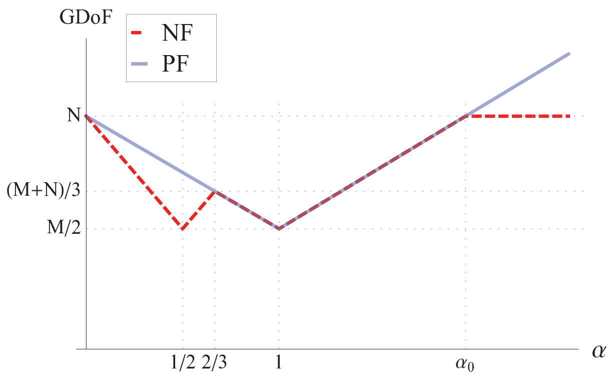
and the GDoF for $M \leq N$ follow by interchanging the roles of M and N.

The authors of [5] found the GDoF for the two-user symmetric MIMO IC without feedback as follows for $N \leq M$ (We can interchange the roles of N and M if $N > M$).

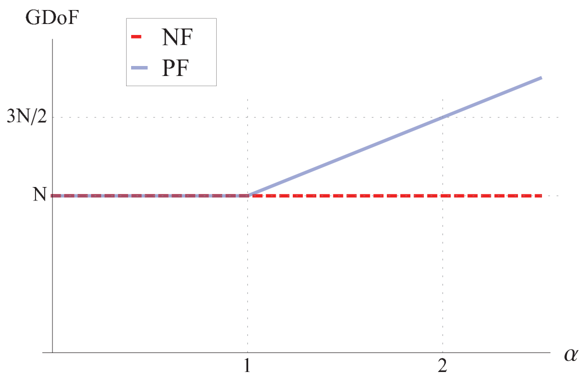
$$GDoF_{NF} = \begin{cases} N - \alpha(2N-M)^+, & \text{if } 0 \leq \alpha \leq \frac{1}{2}, \\ N - (1-\alpha)(2N-M)^+, & \text{if } \frac{1}{2} \leq \alpha \leq \frac{2}{3}, \\ N - \frac{\alpha}{2}(2N-M)^+, & \text{if } \frac{2}{3} \leq \alpha \leq 1, \\ \begin{matrix}\min\{N, N(\frac{\alpha+1}{2}) \\ -\frac{1}{2}(2N-M)^+\}\end{matrix}, & \text{if } \alpha \geq 1. \end{cases} \tag{23}$$

We note that the GDoF with and without feedback are the same for $\frac{2}{3} \leq \alpha \leq 1$. Figure 2 compares the GDoF for the two-user symmetric MIMO IC with and without feedback. In Figure 2(a), the "W"-curve obtained without feedback delineates the very weak ($0 \leq \alpha \leq \frac{1}{2}$), weak ($\frac{1}{2} \leq \alpha \leq \frac{2}{3}$), moderate ($\frac{2}{3} \leq \alpha \leq 1$), strong ($1 \leq \alpha \leq 3 - \frac{M}{N}$) and very strong ($3 - \frac{M}{N} \leq \alpha$) interference regimes. In the presence of feedback, the "W"-curve improves to a "V"-curve which delineates the weak ($0 \leq \alpha \leq 1$) and strong ($1 \leq \alpha$) interference regimes for all choices of N and M. For $\frac{M}{2} < N \leq M$, we see that the GDoF with feedback is strictly greater than that without feedback for $0 < \alpha < 2/3$ and for $\alpha > 3 - M/N$. For $N \leq M/2$, we see that the GDoF with feedback is strictly greater than that without feedback for $\alpha > 2$. The GDoF improvement indicates an unbounded gap in the corresponding capacity regions as the SNR goes to infinity.

Interestingly, from Figure 2(b) we can see that if we increase M when $N \leq \frac{M}{2}$, the GDoF does not change. This can be interpreted as that while $N \leq \frac{M}{2}$, N act as a bottle-neck and increasing M does not increase the GDoF. As a special case consider a MISO IC for which we note that the GDoF is the same for all $M \geq 2$. Thus, increasing the transmit antennas beyond 2 does not increase the GDoF. However, increasing the transmit antennas from 1 to 2 gives a strict improvement in GDoF for all $\alpha > 0$. Similar result also holds for SIMO systems where increasing the receive antennas from 1 to 2 help



(a) $\frac{M}{2} < N \le M$ $\left(\alpha_0 = 3 - \frac{M}{N} \Rightarrow 1 < \alpha_0 \le 2\right)$



(b) $N \le \frac{M}{2}$

Fig. 2. GDoF for symmetric MIMO IC with perfect feedback (PF), and no-feedback (NF) for (a) $\frac{M}{2} < N \le M$, and (b) $N \le \frac{M}{2}$.

increase GDoF while increasing the receive antennas beyond 2 does not increase the GDoF.

V. Conclusion

This paper gives the capacity region of a MIMO IC with feedback within $N_1 + N_2 + \max(N_1, N_2)$ bits. The achievability is based on block Markov encoding, backward decoding, and Han-Kobayashi message-splitting. Further, the GDoF region for general MIMO IC is characterized. It is found that for symmetric IC, the GDoF form a "V"-curve rather than the "W"-curve without feedback.

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