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On the Symmetric K-User Linear Deterministic Interference Channels with Limited Feedback

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***Abstract*—In this paper, we give an achievable scheme for a symmetric K-user linear deterministic interference channel with a $rate-limited$ feedback from each receiver to its respective transmitter. For this model, the proposed scheme achieves a symmetric rate which is the minimum of symmetric capacity with infinite feedback, and the sum of the symmetric capacity without feedback and the symmetric amount of feedback.**

I. Introduction

The interference channel (IC) has been studied in the literature since 1970's to understand performance limits of multiuser communication networks [1]. Feedback in ICs has been considered in order to achieve a possible improvement in data rates. A large body of work on ICs explores feedback strategies, where each receiver sends channel output feedback to its own transmitter [2, 3]. Recent work considers a K-user IC with infinite capacity feedback [4]. A more realistic feedback model is one where feedback links are rate-limited. Two user IC with rate-limited feedback is considered in [5]. In this paper, we consider a K-user IC with rate-limited feedback, and give novel achievable schemes.

In this paper, we will use the linear deterministic IC model proposed in [6]. This model has been used to give insights into approximate capacity results for Gaussian ICs without feedback [7–9], Gaussian ICs with unlimited feedback links [2, 4], and two user Gaussian IC with rate-limited feedback links [5]. Impact of rate-limited feedback is studied for the 2-user IC in [5], where it was shown that the maximum gain in the symmetric capacity with feedback is the amount of symmetric feedback. In this paper, we take this lead to find an achievable strategy for K-user IC with rate-limited feedback which achieves a symmetric capacity which is the minimum of symmetric capacity with perfect feedback and the sum of symmetric capacity with no feedback and the amount of rate-limited symmetric feedback for the linear deterministic model.

In order to get the maximal benefit of feedback, we use an encoding scheme which combines two well-known interference management ideas, namely, interference alignment and interference decoding as were also used in prior works [4, 5]. More precisely, the encoding at the transmitters is such that all the interfering signals are aligned at each receiver. However, a fundamental difference between our approach and the standard interference alignment approach is that we need to decode interference to be able to remove it from the received signal, while the aligned interference is usually suppressed in standard approaches. A challenge here, which makes this problem fundamentally different from the two-user inference channel [5], is that the interference is a combination of interfering messages, and decoding all of them induces strict bounds on the rate of the interfering messages. However, each transmitter does not need to decode all the interfering messages individually, instead, upon receiving feedback, it only decodes the combination of them that corrupts the intended signal of interest. We note that the scheme in [5] for two-user model cannot be directly extended to the channel with more number of users. In the proposed scheme, the receiver decodes sums of certain terms of interfering and intended signal, and send back to the transmitter. The transmitter, knowing the intended signal can help the receiver to decode the sum of interfering signals and thus helps the receiver decode the intended signal.

The rest of this paper is organized as follows. We present the channel models for the deterministic and Gaussian IC in Section II. Section III gives the symmetric rate for the deterministic model, with some examples to help to understand the main idea on achievability scheme. Section IV gives the proof of the main theorem in Section III. Section V concludes the paper.

II. Channel Model and Preliminaries

In this section, we will describe the linear deterministic IC. This model was proposed in [6] to focus on signal interactions instead of the additive noise, and obtain insights for the Gaussian model. In this model, there is a non-negative integer n_{kj} representing channel gain from transmitter k to receiver j, $j, k \in \{1, \cdots, K\}$. We assume that $n_{jk} = n$ for $j = k$ and $n_{jk} = m$ for $j \neq k$. We write the channel input to the transmitter k at time i as $X_{k,i} = [X^1_{k,i}, X^2_{k,i}, \cdots, X^q_{k,i}] \in \mathsf{F}_2{}^q$, for $k \in \{1, 2, \cdots, K\}$, such that $X^1_{k,i}$ and $X^q_{k,i}$ represent the most and the least signicant bits of the transmitted signal respectively. Also, q is the maximum of the channel gains in the network, i.e., $q = max(m, n)$. At each time i, the received signal at k^{th} receiver is given by

$$Y_{k,i} = D^{q-n} X_{k,i} + \sum_{j \neq k} D^{q-m} X_{j,i}, \tag{1}$$

where all the operations are performed modulo 2 and D is $q \times q$ shift matrix, for all $k \in \{1, 2, \cdots, K\}$. In this paper, we consider a feedback from the j^{th} receiver to j^{th} transmitter which is of capacity p. This feedback is causal and hence encodes the signal received till time i to send to the transmitter.

For a deterministic IC, a symmetric achievable rate of R_{sym} is said to be achievable if there is a strategy that all of the users can get a rate of R_{sym}. We further define $\alpha = m/n$ and $\beta = p/n$.

III. Main Result

In this Section, we describe our main result on the impact of rate-limited feedback on the symmetric rate of the linear deterministic IC.

Theorem 1. *For the linear deterministic IC with $K \geq 2$, the following symmetric rate is achievable:*

$$R_{sym} = \begin{cases} \min\{n - m + p, n - \frac{m}{2}\}, & \text{if } 0 \leq m \leq \frac{n}{2}, \\ \min\{m + p, n - \frac{m}{2}\}, & \text{if } \frac{n}{2} \leq m \leq \frac{2n}{3}, \\ n - \frac{m}{2}, & \text{if } \frac{2n}{3} \leq m < n, \\ \frac{n}{K}, & \text{if } \quad m = n, \\ \frac{m}{2}, & \text{if } n < m \leq 2n, \\ \min\{n + p, \frac{m}{2}\}, & \text{if } 2n \leq m. \end{cases} \tag{2}$$

Remark 1. *We note that the achievable symmetric rate is the minimum of*

1) the symmetric capacity with infinite feedback, $C_{sym,\infty}$, and

2) the sum of the symmetric capacity without feedback, $C_{sym,0}$, and the amount of symmetric feedback, p,

where with infinite feedback, i.e., $p = \infty$, according to Theorem 4 of [4], the symmetric capacity is

$$C_{sym,\infty} = \begin{cases} n - \frac{m}{2}, & \text{if } 0 \leq m < n, \\ \frac{n}{K}, & \text{if } \quad m = n, \\ \frac{m}{2}, & \text{if } n < m. \end{cases} \tag{3}$$

and with no feedback, i.e., $p = 0$, according to Corollary 1 of [10], the symmetric capacity is

$$C_{sym,0} = \begin{cases} n - m, & \text{if } \ 0 \leq m \leq \frac{n}{2}, \\ m, & \text{if } \ \frac{n}{2} \leq m \leq \frac{2n}{3}, \\ n - \frac{m}{2}, & \text{if } \frac{2n}{3} \leq m < n, \\ \frac{n}{K}, & \text{if } \quad m = n, \\ \frac{m}{2}, & \text{if } \ n < m \leq 2n, \\ n, & \text{if } 2n \leq m. \end{cases} \tag{4}$$

This result has been shown to be tight for $K = 2$ by the outer bound in [5], and we conjecture that this is the symmetric capacity for a general K.

Complete proof can be seen in Section IV. In this section, we present several examples of the transmission scheme that achieve the symmetric rate as claimed in Theorem 1. Let

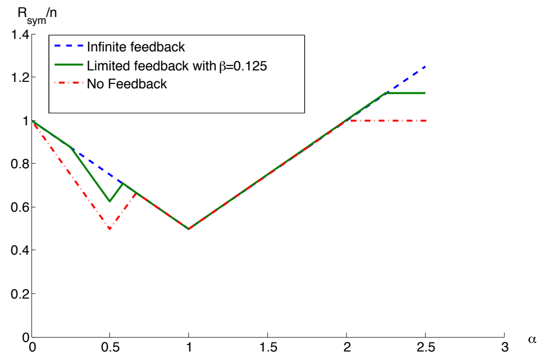

Fig. 1. Achievable symmetric rate of the deterministic IC for $\beta = 0$, $\beta = 0.125$, and $\beta = \infty$.

$\beta \triangleq p/n$. Fig. 1 illustrates the (normalized) per-user rate capacity as a function of α, for different values of $\beta = 0$ (i.e., no feedback could be achieved with an scheme similar to the one for Gaussian in [8]), and $\beta = \infty$ (i.e., infinite feedback [4]), and $\beta = 0.125$. For $\frac{2}{3} \leq \alpha \leq 2$, the sum capacity is the same for both the cases of no feedback and infinite feedback. So, any limited feedback gives the same result in this range. In the rest of the Section, we will demonstrate the proposed scheme in three examples with specific parameters, through which the basic ideas and intuition can be illustrated. Generalization of the proposed coding strategy and scheme for arbitrary n, m, and p and its analysis is presented in Section IV.

1) Very Weak Interference Regime ($\alpha \leq \frac{1}{2}$): In the very weak interference regime, the goal is to achieve a symmetric rate of $R_{sym} = \min\{n - m + p, n - \frac{m}{2}\}$ bits per user. We propose an encoding scheme that operates on a block of length 2. The basic idea can be seen from Fig. 2, wherein the coding scheme is demonstrated for $K = 3$, $n = 5$, $m = 2$ and $p = 0.5$ which implies $2R_{sym} = 7$ in a block of length 2. As shown in Fig. 2, the proposed coding scheme is able to convey seven intended symbols from each transmitter to its respective receiver in two channel uses. The information symbols intended for user one are denoted by $a_{1,1}, ..., a_{1,7}$. Each transmitter sends four fresh symbols in its first channel use. Receivers get three interference-free symbols, and one more equation, including their intended symbol as well as interference. The equation received at the least significant bit is ignored. The fourth output signals are sent to the transmitters over the feedback link, in order to be used for the next transmission. In the second channel use, each transmitter forwards the interfering parts of its received feedback on its top level. The next lower level should be empty and the three lowest levels will be used to transmit the remaining fresh symbols.

Now, consider the received signals at the first receiver

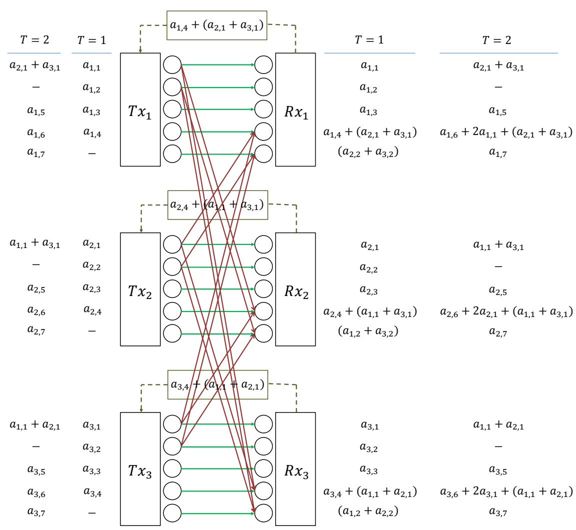

Fig. 2. Coding scheme for the linear deterministic model in the very weak interference regime ($\alpha \leq \frac{1}{2}$), for $K = 3$, $n = 5$, $m = 2$ and $p = 0.5$.

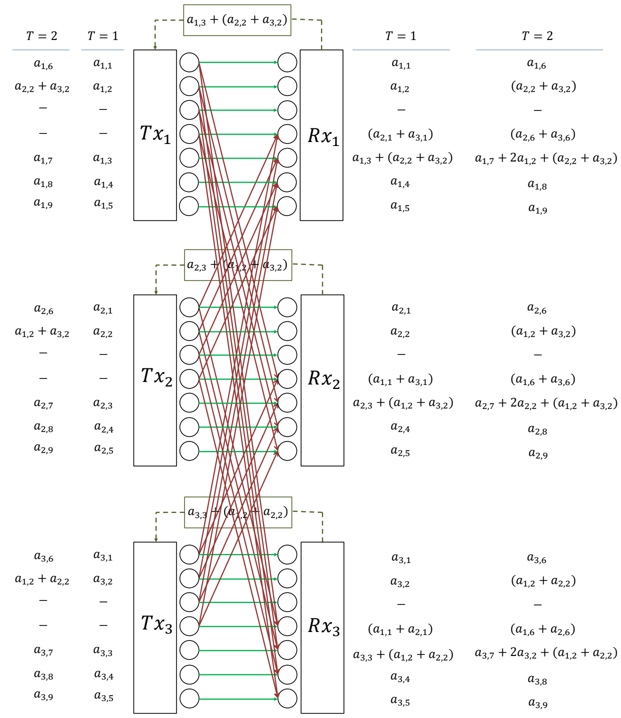

Fig. 3. Coding scheme for the linear deterministic model in the weak interference regime ($\frac{1}{2} < \alpha \leq \frac{2}{3}$), for $K = 3$, $n = 7$, $m = 4$ and $p = 0.5$.

in two channel uses. The receiver received eight linearly independent equations, involving nine variables, which seems to be unsolvable at first glance. However, we do not need to decode all the symbols. Instead, we can solve the system of linear equations in $a_{1,1}, ..., a_{1,7}$ and $a_{2,1} + a_{3,1}$ which can be solved for the intended variables. Similarly, the transmitted message can be seen to be decoded at other receivers too. Hence, a symmetric rate of $\frac{7}{2}$ symbols/channel-use is achievable with this rate-limited feedback.

2) Weak Interference Regime ($\frac{1}{2} < \alpha \leq \frac{2}{3}$)*:* In the weak interference regime, the goal is to achieve a symmetric rate of $R_{sym} = \min\{m + p, n - \frac{m}{2}\}$ bits per user. We propose an encoding scheme that operates on a block of length 2. The basic idea can be seen from Fig. 3, wherein the coding scheme is demonstrated for $K = 3$, $n = 7$, $m = 4$ and $p = 0.5$ which implies $2R_{sym} = 9$ in a block of length 2. As shown in Fig. 3, the proposed coding scheme is able to convey nine intended symbols from each transmitter to its respective receiver in two channel uses. The information symbols intended for user one are denoted by $a_{1,1}, ..., a_{1,9}$. Each transmitter sends five fresh symbols in its first channel use; two symbols on the highest two levels, nothing on the next two levels, and three more on the lowest three levels. Receivers get four interference-free symbols, and one more equation, including their intended symbol as well as interference which should be sent to the transmitters over the feedback link, in order to be used for the next transmission. In the second channel use, each transmitter forwards the interfering parts of its received feedback on its second top level. The highest level and the three lowest levels will be used to transmit the remaining fresh symbols and nothing is transmitted on the other two levels.

Now, consider the received signals at the first receiver in two channel uses. It has received twelve linearly independent equations, involving fifteen variables, which seems to be unsolvable at first glance. However, we do not need to decode all of them. Variables $a_{1,1}, a_{1,2}, a_{1,4}, a_{1,5}, a_{1,6}, a_{1,8}, a_{1,9}$ will be decoded directly from the received equations and also $(a_{2,2} + a_{3,2})$ is decoded from another level. Having $a_{1,2}$ and $(a_{2,2} + a_{3,2})$ decoded, $a_{1,3}$ and $a_{1,7}$, also, can be decoded having equations $a_{1,3} + (a_{2,2} + a_{3,2})$ and $a_{1,7} + 2a_{1,2} + (a_{2,2} + a_{3,2})$. Also, $(a_{2,1} + a_{3,1})$ and $(a_{2,6} + a_{3,6})$ will be received, too, though they are not needed in the decoding. Similar decoding is performed at the other receivers. Hence, a per-user rate of $\frac{9}{2}$ symbols/channel-use is achievable with this rate-limited feedback.

3) Very Strong Interference Regime ($\alpha > 2$)*:* In the very strong interference regime, the goal is to achieve a symmetric rate of $R_{sym} = \min\{n + p, \frac{m}{2}\}$ bits per user. We propose an encoding scheme that operates on a block of length 2. The basic idea can be seen from Fig. 4, wherein the coding scheme is demonstrated for $K = 3$, $n = 2$, $m = 6$ and $p = 0.5$ which implies $2R_{sym} = 5$ in a block of length 2. As shown in Fig. 4, the proposed coding scheme is able to convey five intended symbols from each transmitter to its respective receiver in two channel uses. The information symbols intended for user one are denoted by $a_{1,1}, ..., a_{1,5}$. Each transmitter sends three fresh symbols in its first channel

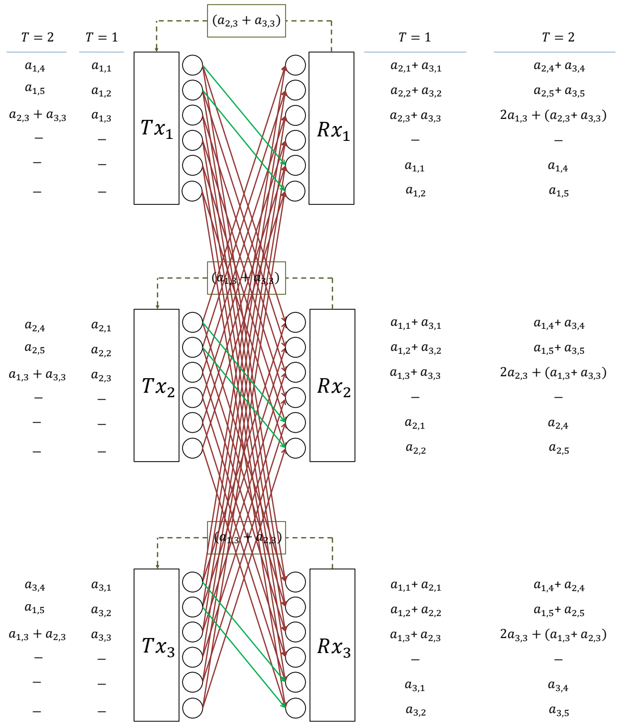

Fig. 4. Coding scheme for the linear deterministic model in the very strong interference regime ($\alpha > 2$), for $K = 3$, $n = 2$, $m = 6$ and $p = 0.5$.

use on the three highest levels and nothing on the lower three levels. Receivers get three interference equations, one empty level, and also two interference-free symbols in the lower two levels. The third output signals are sent to the transmitters over the feedback link, in order to be used for the next transmission. In the second channel use, each transmitter forwards its received feedback on its third top level. The next three lower levels are kept empty. Two highest levels will be used to transmit the remaining fresh symbols.

Now, consider the received signals at the first receiver in two channel uses. It has received ten linearly independent equations, involving fifteen variables, which seems to be unsolvable at first glance. However, we do not need to decode all of them. Variables $a_{1,1}, a_{1,2}, a_{1,4}, a_{1,5}$ will be decoded directly from the received equations and also $(a_{2,3} + a_{3,3})$ is being decoded from another level. Having $(a_{2,3} + a_{3,3})$ decoded, $a_{1,3}$, also, will be decoded having equation $2a_{1,3} + (a_{2,3} + a_{3,3})$. Similar decoding is performed at the other receivers. Hence, a per-user rate of $\frac{5}{2}$ symbols/channel-use is achievable with this rate-limited feedback.

IV. Proof of Theorem 1

In this section, we prove Theorem 1 by breaking the result into three regimes. We denote that $\overline{a_{i,j}} \triangleq \sum_{k=1,k\neq i}^{K} a_{k,j}$

Lemma 1. *For the linear deterministic IC, a symmetric rate of $n \min\{1-\alpha+\beta, 1-\frac{\alpha}{2}\}$ is achievable for $0 \leq \alpha \leq \frac{1}{2}$.*

Proof. We use a one-time $2p$ bits feedback for each two uses of the channel (rate of p). Define $l \triangleq (m-2p)^+$. With a two-round strategy, each user transmits $2n - m - l$ bits for each two uses of the channel which proves the lemma. Here we only use $m - l$ bits of the feedback ($0 \leq m - l \leq 2p$). Since the feedback does not increase the achievable rate in the statement of the Theorem beyond $p = m/2$, we only use $m/2$ bits of feedback if $p > m/2$. For the i^{th} transmitter ($i \in \{1, ..., K\}$), we transmit $a_{i,1}, ..., a_{i,2n-m-l}$ in two transmission slots. Also, $m-l$ bits of feedbacks being used after the first transmission slot.

First Round:

1. Transmission: In the first round, i^{th} transmitter sends $a_{i,1}, ..., a_{i,n-l}$ on the highest $n-l$ levels of the transmission, respectively, and nothing on the lowest l levels of the transmission.

2. Reception: i^{th} receiver receives $a_{i,1}, ..., a_{i,n-m}$ on the highest $n - m$ levels of the reception, respectively, and $a_{i,n-m+1} + \overline{a_{i,1}}, ..., a_{i,n-l} + \overline{a_{i,m-l}}$ on the next $m-l$ levels, respectively, and throws away whatever that receives on the last l levels.

Feedback:

Receiver i sends back $a_{i,n-m+1} + \overline{a_{i,1}}, ..., a_{i,n-l} + \overline{a_{i,m-l}}$ over feedback to transmitter i ($m-l$ bits). With this feedback, transmitter i decodes $\overline{a_{i,1}}, ..., \overline{a_{i,m-l}}$.

Second Round:

1. Transmission: In the second round, i^{th} transmitter sends $\overline{a_{i,1}}, ..., \overline{a_{i,m-l}}$ on the highest $m-l$ levels of the transmission, respectively, and nothing on the next lower l levels of transmission and new bits of $a_{i,n-l+1}, ..., a_{i,2n-m-l}$ on the last $n - m$ levels, respectively.

2. Reception: i^{th} receiver receives $\overline{a_{i,1}}, ..., \overline{a_{i,m-l}}$ on the highest $m - l$ levels of the reception, nothing on the next l levels of the reception, $a_{i,n-l+1}, ..., a_{i,2n-2m-l}$ on the next $n - 2m$ levels, $a_{i,2n-2m-l+1} + (K-2)\overline{a_{i,1}} + (K-1)a_{i,1}, ..., a_{i,2n-m-2l} + (K-2)\overline{a_{i,m-l}} + (K-1)a_{i,m-l}$ on the next $m - l$ levels, and $a_{i,2n-m-2l+1}, ..., a_{i,2n-m-l}$ on the lowest l levels of the reception.

Decoding:

Decoding for the i^{th} user ($i \in \{1, ..., K\}$) is performed as follows. First, $a_{i,1}, ..., a_{i,n-m}$ will be decoded from the highest $n - m$ levels of the first reception. Then, $\overline{a_{i,1}}, ..., \overline{a_{i,m-l}}$ will be decoded from the highest $m - l$ levels of the second reception. Then, having $\overline{a_{i,1}}, ..., \overline{a_{i,m-l}}$, the receiver decodes $a_{i,n-m+1}, ..., a_{i,n-l}$ from $a_{i,n-m+1} + \overline{a_{i,1}}, ..., a_{i,n-l} + \overline{a_{i,m-l}}$ on the next $m - l$ levels of the first reception. Then, the receiver decodes $a_{i,n-l+1}, ..., a_{i,2n-2m-l}$ from the $(m+1)^{\text{th}}$ to $(n-m)^{\text{th}}$ highest levels of the second reception, respectively. Then, having $a_{i,1}, ..., a_{i,m-l}$, and $\overline{a_{i,1}}, ..., \overline{a_{i,m-l}}$, i^{th} receiver decodes $a_{i,2n-2m-l+1}, ..., a_{i,2n-m-2l}$ from $a_{i,2n-2m-l+1} + (K-2)\overline{a_{i,1}} + (K-1)a_{i,1}, ..., a_{i,2n-m-2l} + (K-2)\overline{a_{i,m-l}} + (K-1)a_{i,m-l}$ on the next $m - l$ lower levels of the second reception. Finally, the receiver decodes $a_{i,2n-m-2l+1}, ..., a_{i,2n-m-l}$ from the lowest l levels of the second reception. $\square$

Lemma 2. *For the linear deterministic IC, a symmetric rate of* $n\min\{\alpha+\beta, 1-\frac{\alpha}{2}\}$ *is achievable for* $\frac{1}{2}<\alpha\leq\frac{2}{3}$.

Proof. We use a one-time $2p$ bits feedback for each two uses of the channel (rate of p). Define $l^{'}\triangleq(2n-3m-2p)^{+}$. With a two-round strategy, each user transmits $2n-m-l^{'}$ bits for each two uses of the channel which proves the lemma. Here, we only use $2n-3m-l^{'}$ bits of the feedback ($0\leq 2n-3m-l^{'}\leq 2p$). For the i^{th} transmitter ($i\in\{1,...,K\}$), we transmit $a_{i,1},...,a_{i,2n-m-l'}$ in two transmission slots. Also, $2n-3m-l^{'}$ bits of feedbacks being used after the first transmission slot.

First Round:

1. Transmission: In the first round, i^{th} transmitter sends $a_{i,1},...,a_{i,n-m-l'}$ on the highest $n-m-l^{'}$ levels of the transmission, nothing on the next lower $2m-n+l^{'}$ levels of the transmission, and $a_{i,n-m-l'+1},...,a_{i,2n-2m-l'}$ on the lowest $n-m$ levels of the transmission.

2. Reception: i^{th} receiver receives $a_{i,1},...,a_{i,n-m-l'}$ on the highest $n-m-l^{'}$ levels of the reception, nothing on the next lower $l^{'}$ levels of the reception, $\overline{a_{i,1}},...,\overline{a_{i,2m-n}}$ on the next lower $2m-n$ levels of the reception, $a_{i,n-m-l'+1}+\overline{a_{i,2m-n+1}},...,a_{i,3n-4m-2l'}+\overline{a_{i,n-m-l'}}$ on the next $2n-3m-l^{'}$ levels, and $a_{i,3n-4m-2l'+1},...,a_{i,2n-2m-l'}$ on the lowest $2m-n+l^{'}$ levels of the reception.

Feedback:

Receiver i sends back $a_{i,n-m-l'+1}+\overline{a_{i,2m-n+1}},...,a_{i,2n-3m-2l'}+\overline{a_{i,n-m-l'}}$ over feedback to transmitter i ($2n-3m-l^{'}$ bits). With this feedback, transmitter i decodes $\overline{a_{i,2m-n+1}},...,\overline{a_{i,n-m-l'}}$.

Second Round:

1. Transmission: In the second round, i^{th} transmitter sends the new bits $a_{i,2n-2m-l'+1},...,a_{i,n-l'}$ on the highest $2m-n$ levels of the transmission, $\overline{a_{i,2m-n+1}},...,\overline{a_{i,n-m-l'}}$ on the next $2n-3m-l^{'}$ levels, nothing on the next lowest $2m-n+l^{'}$ levels of the transmission, and the new bits of $a_{i,n-l'+1},...,a_{i,2n-m-l'}$ on the lowest $n-m$ levels of the transmission.

2. Reception: i^{th} receiver receives $a_{i,2n-2m-l'+1},...,a_{i,n-l'}$ on the highest $2m-n$ levels of the reception, $\overline{a_{i,2m-n+1}},...,\overline{a_{i,n-m-l'}}$ on the next $2n-3m-l^{'}$ levels, nothing on the next lower $l^{'}$ levels of the transmission, $\overline{a_{i,2n-2m-l'+1}},...,\overline{a_{i,n-l'}}$ on the next lower $2m-n$ levels of the reception, $a_{i,n-l'+1}+(K-2)\overline{a_{i,2m-n+1}}+(K-1)a_{i,2m-n+1},...,a_{i,3n-3m-2l'}+(K-2)\overline{a_{i,2n-3m-l'}}+(K-1)a_{i,2n-3m-l'}$ on the next lower $2n-3m-l^{'}$ levels of the transmission, and $a_{i,3n-3m-2l'+1},...,a_{i,2n-m-l'}$ on the lowest $2m-n+l^{'}$ levels of the transmission.

Decoding:

Decoding for the i^{th} user ($i\in\{1,...,K\}$) is performed as follows. First, $a_{i,1},...,a_{i,n-m-l'}$ will be decoded from the highest $n-m-l^{'}$ levels of the first reception. Then, $a_{i,3n-4m-2l'+1},...,a_{i,2n-2m-l'}$ will be decoded from the lowest $2m-n+l^{'}$ levels of the first reception. Further, $a_{i,2n-2m-l'+1},...,a_{i,n-l'}$ will be decoded from the highest $2m-n$ levels of the second reception, and $\overline{a_{i,2m-n+1}},...,\overline{a_{i,n-m-l'}}$ will be decoded from the next $2n-3m-l^{'}$ levels of the second reception. Moreover, $a_{i,3n-3m-2l'+1},...,a_{i,2n-m-l'}$ is decoded from the lowest $2m-n+l^{'}$ levels of the first transmission, respectively.

Then, having $\overline{a_{i,2m-n+1}},...,\overline{a_{i,n-m-l'}}$, the receiver decodes $a_{i,n-m-l'+1},...,a_{i,2n-3m-2l'}$ from $a_{i,n-m-l'+1}+\overline{a_{i,2m-n+1}},...,a_{i,2n-3m-2l'}+\overline{a_{i,n-m-l'}}$ In the first reception. Finally, having $\overline{a_{i,2m-n+1}},...,\overline{a_{i,2n-3m-l'}}$, and $a_{i,2m-n+1},...,a_{i,2n-3m-l'}$, the receiver decodes $a_{i,n-l'+1},...,a_{i,3n-3m-2l'}$ from $a_{i,n-l'+1}+(K-2)\overline{a_{i,2m-n+1}}+(K-1)a_{i,2m-n+1},...,a_{i,3n-3m-2l'}+(K-2)\overline{a_{i,2n-3m-l'}}+(K-1)a_{i,2n-3m-l'}$ In the second reception. $\square$

Lemma 3. *For the linear deterministic IC, a symmetric rate of* $n\min\{1+\beta,\frac{\alpha}{2}\}$ *is achievable for* $\alpha>2$.

Proof. We use a one-time $2p$ bits feedback for each two uses of the channel (rate of p). Define $l^{''}\triangleq(m-2n-2p)^{+}$. With a two-round strategy, each user transmits $n+m-l^{''}$ bits for each two uses of the channel which proves the lemma. Here, we only use $m-2n-l^{''}$ bits of the feedback ($0\leq m-2n-l^{''}\leq 2p$). For the i^{th} transmitter ($i\in\{1,...,K\}$), we transmit $n+m-l^{''}$ in two transmission slots. Also, $m-2n-l^{''}$ bits of feedbacks being used after the first transmission slot.

First Round:

1.Transmission: In the first round, i^{th} transmitter sends $a_{i,1},...,a_{i,m-n-l''}$ on the highest $m-n-l^{''}$ levels of the transmission, respectively, and nothing on the lower $n+l^{''}$ levels of the transmission.

2. Reception: i^{th} receiver receives $\overline{a_{i,1}},...,\overline{a_{i,m-n-l''}}$ on the highest $m-l^{''}$ levels of the reception, nothing on the next lower $l^{''}$ levels of the reception, and $a_{i,1},...,a_{i,n}$ on the lowest n levels of the reception.

Feedback:

Receiver i sends back $\overline{a_{i,n+1}},...,\overline{a_{i,m-n-l''}}$ over feedback to the i^{th} transmitter ($m-2n-l^{''}$ bits).

Second Round:

1.Transmission: In the second round, i^{th} transmitter sends new bits of $a_{i,m-n-l''+1},...,a_{i,m-l''}$ on the highest n levels of the transmission, $\overline{a_{i,n+1}},...,\overline{a_{i,m-n-l''}}$ on the next $m-2n-l^{''}$ levels, and nothing on the lower $n+l^{''}$ levels of the transmission.

2. Reception: i^{th} receiver receives $\overline{a_{i,m-n-l''+1}},...,\overline{a_{i,m-l''}}$ on the highest n levels of the reception, $(K-1)a_{i,n+1}+(K-2)\overline{a_{i,n+1}},...,(K-1)a_{i,m-n-l''}+(K-2)\overline{a_{i,m-n-l''}}$ on the next $m-2n-l^{''}$ levels, nothing on the next lower $l^{''}$ levels of the reception,

and $a_{i,m-n-l''+1},...,a_{i,m-l''}$ on the lowest n levels of the reception.

Decoding:

Decoding for the i^{th} user ($i \in \{1,...,K\}$) is performed as follows. First, $a_{i,1},...,a_{i,n}$ will be decoded from the lowest n levels of the first reception, $\overline{a_{i,n+1}},...,\overline{a_{i,m-n-l''}}$ will be decoded from the $(n+1)^{\text{th}}$ to $(m-n-l'')^{\text{th}}$ highest levels of the first reception, and $a_{i,m-n-l''+1},...,a_{i,m-l''}$ will be decoded from the lowest n levels of the second reception. Then, having $\overline{a_{i,n+1}},...,\overline{a_{i,m-n-l''}}$, the receiver decodes $a_{i,n+1},...,a_{i,m-n-l''}$ from $(K-1)a_{i,n+1}+(K-2)\overline{a_{i,n+1}},...,(K-1)a_{i,m-n-l''}+(K-2)\overline{a_{i,m-n-l''}}$ In the second reception. $\square$

V. Conclusion

This paper gives an achievable scheme for a symmetric K-user linear deterministic IC. The symmetric rate achieved by the scheme is the minimum of the symmetric capacity with infinite feedback, and the sum of the symmetric capacity without feedback and the amount of symmetric feedback. We note that the achievability scheme in [5] for two-user model cannot be directly extended for general K. The achievability in the paper lets the receiver decode sums of certain combination of interfering and intended signal, and send back to the transmitter via a limited feedback channel. We conjecture that the derived achievable rate is tight for all $K \geq 2$. The strategies propeosed in this paper can be extended to a Gaussian model [10].

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