Capacity of Two-Way Linear Deterministic Diamond Channel

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***Abstract*—In this paper, we study the capacity regions of two-way linear deterministic diamond channels. We show that the capacity of the diamond channel in each direction can be simultaneously achieved for all values of channel parameters, where the forward and backward channel parameters are not necessarily the same. We propose a relay strategy called 'reverse amplify-and-forward' strategy and show that this strategy and its variants combined with proper transmission strategies achieve the capacity of linear deterministic diamond channel.**

I. Introduction

Two-way communication between two nodes was first studied by Shannon [1]. There have been many attempts recently to demonstrate two-way communications experimentally [2, 3]. The two-way relay channel where two nodes communicate to each other in the presence of a single relay, has been widely studied [4, 5]. In this paper, we will consider the two-way diamond channel, where two nodes communicate to each other in the presence of two relays. Also, [6, 7] consider different kinds of collaboration between the nodes.

The diamond channel was first introduced in [8], and consists of one transmitter, two relays and a receiver. The two-way half-duplex K-relay channel has been studied using the amplify-and-forward strategy at the relays [9, 10]. The design of relay beamformers based on minimizing the transmit power subject to the received signal-to-noise ratio constraints was considered in [10]. Furthermore, achievability schemes using time-sharing are investigated in [11] for a symmetric reciprocal diamond channel with half-duplex nodes and the inner and outer bounds are compared using simulations. However, we show that the achievability scheme in [11] has an unbounded gap from the capacity. None of the prior works gave a capacity achieving strategy for a two-way full-duplex diamond channel.

In this paper, we consider a linear deterministic model which was proposed in [12], and has been shown to lead to approximate capacity results for Gaussian channels in [5, 13, 14]. We study the capacity region of a two-way linear deterministic diamond channel where the forward and backward channel gains are not necessarily the same. We find that the capacity in each direction can be simultaneously achieved. Thus, each user can transmit at a rate which is not affected by the fact that the relays receive the superposition of the signals.

In order to achieve the capacity in each direction separately, we develop new transmission strategies by the transmitters and the relays. The strategies proposed for the one-way diamond channel in [12] do not directly work for two-way channels. The reason is that they are dependent on the channel parameters in the forward direction; but for two-way channels we need a strategy that is optimal for both directions. For the special case when the diamond channel reduces to a two-way relay channel (channel gains to and from one of the relays are zeros), our proposed strategy reduces to a reverse amplify-and-forward strategy, where the relay reverses the order of the received signals to form the transmitted signal. The proposed strategy in this case is different from the one in [5] for two-way relay channels, since the relay strategy in [5] depends on the channel parameters, while ours simply reverses the order of the input. On the other hand, the transmission strategy at the source nodes in our approach is dependent on the channel parameters unlike that in [5]. Thus, the proposed strategy in this paper makes the relay strategy simpler by compensating in the transmission strategy at the source nodes.

For a general two-way diamond channel, we give different strategies based on the parameters of both the forward and backward channels. Depending on the forward and backward channel gains we consider four cases; these cases are further subdivided. Two special cases are Cases 3.1.2 and 4.1.2. Our first main result is that if neither the forward, nor the backward channel is of one of these two cases, then the proposed reverse amplify-and-forward strategy at the relays is optimal.

We next consider the case that exactly one of the forward and backward channels is of Case 3.1.2 or 4.1.2. Without loss of generality, we assume that the forward channel is of one of the two mentioned cases. For each of these two cases, we give four new strategies at the relay which involve various modifications to the reverse amplify-and-forward strategy, such as repeating some of the streams on multiple levels or changing the order of transmission at some levels at one of the relays. Furthermore, the transmission strategy for the forward direction is rather straightforward by simply sending capacity number of bits at the lowest levels. We show that all these modified strategies achieve the capacity in the forward direction. The choice of the strategies then depends on the parameters in the backward direction. We show that for each case of the backward channel, at least one of the four proposed strategies achieves the capacity for the backward direction. Finally, the case when both the forward and backward channels are of Case 3.1.2 or 4.1.2 is considered. Here, a modified form of the relay strategies proposed above is used to achieve the capacity in both directions.

II. Channel Model

The linear deterministic channel model was proposed in [12] to focus on signal interactions instead of the additive noise, and to obtain insights for the Gaussian channel. A two-way diamond channel consists of two nodes (denoted by A and B) who wish to communicate to each other through two relays (denoted by R_1 and R_2). We use non-negative integers n_{Ak}, n_{Bk}, n_{kA}, and n_{kB}, to represent the channel gains from node A to R_k, node B to R_k, R_k to node A, and R_k to node B, respectively, for $k \in \{1, 2\}$. In this paper, the links in the direction from A to B are said to be in the forward direction and those from B to A are in the backward direction.

Let us define $q_{AR} \triangleq \max_k\{n_{Ak}\}$, $q_{RB} \triangleq \max_k\{n_{kB}\}$, $q_{BR} \triangleq \max_k\{n_{Bk}\}$, $q_{RA} \triangleq \max_k\{n_{kA}\}$, $q_k^I \triangleq \max\{n_{Ak}, n_{Bk}\}$, and $q_k^O \triangleq \max\{n_{kA}, n_{kB}\}$ for $k \in \{1, 2\}$. Furthermore, denote the channel input at transmitter u, for $u \in \{A, B\}$, at time i as $X_{u,i} = [X_{u,i}^{q_{uR}}, \cdots, X_{u,i}^2, X_{u,i}^1]^T \in \mathsf{F}_2{}^{q_{uR}}$, such that $X_{u,i}^1$ and $X_{u,i}^{q_{uR}}$ represent the least and the most significant bits of the transmitted signal, respectively. Also, we define $X_{uk,i}^R = [X_{u,i}^{q_{uR}}, \cdots, X_{u,i}^{q_{uR}-n_{uk}+2}, X_{u,i}^{q_{uR}-n_{uk}+1}, \underbrace{0, ..., 0}_{q_k^I - n_{uk}}]^T$, for $k \in \{1, 2\}$. At each time i, the received signal at R_k is given by

$$Y_{k,i} = D_{q_k^I}^{q_k^I - n_{Ak}} X_{Ak,i}^R + D_{q_k^I}^{q_k^I - n_{Bk}} X_{Bk,i}^R \mod 2, \quad (1)$$

where $D_{q_k^I}$ is a $q_k^I \times q_k^I$ shift matrix as Eq. (9) in [12]. Also if we have $Y_{k,i} = [Y_{k,i}^{q_k^I}, \cdots, Y_{k,i}^2, Y_{k,i}^1]^T$, define $V_{k,i} = [0, \cdots, 0, Y_{k,i}^{\min(q_k^I, q_k^O)}, \cdots, Y_{k,i}^2, Y_{k,i}^1]^T$, for $k \in \{1, 2\}$, where the first $(q_k^O - q_k^I)^+$ elements of $V_{k,i}$ are zero.

Furthermore, define $T_{k,i} \triangleq f_{k,i}(V_{k,1}, ..., V_{k,i-1})$ where $f_{k,i} : \left(R^{q_k^O}\right)^{i-1} \to R^{q_k^O}$ is a function at R_k which converts $V_{k,1}, ..., V_{k,i-1}$ to the output signal at time i. We represent $T_{k,i}$'s elements as $T_{k,i} = \left[T_{k,i}^1, T_{k,i}^2, \cdots, T_{k,i}^{q_k^O}\right]^T$. Also, we define $T'_{ku,i} = [T_{k,i}^1, T_{k,i}^2, \cdots, T_{k,i}^{n_{ku}}, \underbrace{0, ..., 0}_{q_{Ru} - n_{ku}}]^T$ for $u \in \{A, B\}$. At each time i, the received signal at the receivers $u \in \{A, B\}$ is given by

$$Y_{u,i} = \sum_{k=1}^{2} D_{q_{Ru}}^{q_{Ru} - n_{ku}} T'_{ku,i} \mod 2. \quad (2)$$

Source u picks a message W_u that it wishes to communicate to $\bar{u}$ ($u, \bar{u} \in \{A, B\}$, $u \neq \bar{u}$), and transmits signal at each time i which is a function of W_u and $Y_u^{i-1} = \{Y_{u,i-1}, Y_{u,i-2}, ..., Y_{u,1}\}$. Each destination $\bar{u}$ uses a decoder, which is a mapping $g_{\bar{u}} : R^m \times |W_{\bar{u}}| \to \{1, ..., |W_u|\}$ from the m received signals and the message at the receiver to the source message indices ($|W_u|$ is the number of messages of node u that can be chosen). We say that the rate pair $(R_A \triangleq \frac{\log |W_A|}{m}, R_B \triangleq \frac{\log |W_B|}{m})$ is achievable if the probability of error in decoding both messages by their corresponding destinations can be made arbitrarily close to 0 as $m \to \infty$. The capacity region is the convex hull of all the achievable rate pairs (R_A, R_B).

III. Capacity of Two-Way Linear Deterministic Diamond Channel

In this section, we state the main result that the cut-set bound for the diamond channel in each direction can be simultaneously achieved, thus giving the capacity region for the two-way linear deterministic diamond channel. It can be seen that $\max\{n_{A1}, n_{A2}\}$ and $\max\{n_{1B}, n_{2B}\}$ are cut-set bounds on the transmissions from A and to B, respectively. Moreover, $n_{A1} + n_{2B}$ and $n_{A2} + n_{1B}$ are cut-set bounds on the sum of the two paths for the transmission from A to B. The same observation can be made for the other direction.

Theorem 1. *For the two-way linear deterministic diamond channel, the capacity region is given as follows:*

$$R_A \leq C_{AB} \triangleq \min\{\max\{n_{A1}, n_{A2}\}, \max\{n_{1B}, n_{2B}\}, n_{A1} + n_{2B}, n_{A2} + n_{1B}\}, \quad (3)$$

$$R_B \leq C_{BA} \triangleq \min\{\max\{n_{B1}, n_{B2}\}, \max\{n_{1A}, n_{2A}\}, n_{B1} + n_{2A}, n_{B2} + n_{1A}\}. \quad (4)$$

We note that the outer-bound is the cut-set bound, and thus the proof is straightforward. We will prove the achievability of the rate pair (C_{AB}, C_{BA}).

We consider four main cases and several subcases depending on the forward channel parameters as follows.

Case 1: $C_{AB} = n_{A2} + n_{1B}$.

Case 2: $C_{AB} = n_{A1} + n_{2B}$.

Case 3: $C_{AB} = \max\{n_{A1}, n_{A2}\}$. We call it **Type 1**, if $\max\{n_{A1}, n_{A2}\} = n_{A1}$, and **Type 2** otherwise. For **Type i**, where $i, j \in \{1, 2\}, i \neq j$, we have:

- **Case 3.1:** $n_{iB} < C_{AB}$. We divide it into two sub-cases:
  - **Case 3.1.1:** $n_{jB} \geq n_{Aj} + n_{iB}$.
  - **Case 3.1.2:** $n_{jB} < n_{Aj} + n_{iB}$.
- **Case 3.2:** $n_{iB} \geq C_{AB}$.

Case 4: $C_{AB} = \max\{n_{1B}, n_{2B}\}$ We call it **Type 1**, if $\max\{n_{1B}, n_{2B}\} = n_{1B}$, and **Type 2** otherwise. For **Type i**, where $i, j \in \{1, 2\}, i \neq j$, we have:

- **Case 4.1:** $n_{Ai} < C_{AB}$. We divide it into two sub-cases:
  - **Case 4.1.1:** $n_{Aj} \geq n_{jB} + n_{Ai}$.
  - **Case 4.1.2:** $n_{Aj} < n_{jB} + n_{Ai}$.
- **Case 4.2:** $n_{Ai} \geq C_{AB}$.

Similarly we divide the backward channel into four main cases and several subcases where the case definition is obtained by interchanging A and B in the forward direction cases. For instance, Case 1 in the backward direction is $C_{BA} = n_{B2} + n_{1A}$.

We divide the proof into three parts, depending on the cases in which forward and backward channel gain parameters lie. The first part is when neither the forward channel nor the backward channel is of Case 3.1.2 or 4.1.2 (Section III-A). The second part is when exactly one of the forward and backward channels is of Case 3.1.2 or 4.1.2 (Section III-B). And finally the third part is when both the forward and backward channels are of Case 3.1.2 or 4.1.2 (Section III-C).

A. Neither the forward channel nor backward channel is of Case 3.1.2 or 4.1.2

In this scenario, we use a reverse amplify-and-forward strategy in the relays to achieve the rate pair (C_{AB}, C_{BA}). Assume a particular relay (say R_i) gets n_{Ai} levels from node A and n_{Bi} levels from node B and transmits q_i^O levels. It receives $Y_{A1} = [a_{n_{Ai}}, ..., a_1]^T$ from node A and $Y_{B1} = [b_{n_{Bi}}, ..., b_1]^T$ from node B. Then it sends out the following signal to nodes A and B

$$X_{R_i} = \begin{bmatrix} a_1 \\ \vdots \\ a_{\min(n_{Ai}, q_i^O)} \\ 0_{(q_i^O - n_{Ai})^+} \end{bmatrix} + \begin{bmatrix} b_1 \\ \vdots \\ b_{\min(n_{Bi}, q_i^O)} \\ 0_{(q_i^O - n_{Bi})^+} \end{bmatrix} \mod 2. \quad (5)$$

We call this relay strategy as "Relay Strategy 0" (also called "reverse amplify-and-forward"). We will keep the strategy at the relays the same, and for different cases use different strategies for transmission at nodes A and B. Since we need to show that the rate pair (C_{AB}, C_{BA}) is achievable, it is enough to show that there is a transmission strategy for node A such that with the above relay strategy, node B is able to decode the data in a one-way diamond channel because any interference by node B on the received signal can be canceled by node B which knows the interfering signal (Showing it for one direction is enough since the same arguments hold for the other). Thus, we only consider one-way diamond channel for this case. We further consider the case when $n_{A1}, n_{A2}, n_{1B}, n_{2B} > 0$ since otherwise the diamond channel reduces to a relay channel or no connection between the nodes A and B, and in both cases it is easy to see that node A sending C_{AB} bits on the lowest levels achieves this rate in the forward direction.

It has been shown in Appendix 1 of [15] that there is a transmission strategy for each of the cases (except for Case 3.1.2 or 4.1.2) such that the above relay strategy achieves the capacity for one-way diamond channel.

B. Exactly one of the forward and backward channels is of Case 3.1.2 or 4.1.2

We assume that the forward channel is of Case 3.1.2 or 4.1.2 without loss of generality. The other case where the backward channel is of Case 3.1.2 or 4.1.2 can be proven symmetrically. Since we need to show that the rate pair (C_{AB}, C_{BA}) is achievable, we will describe a few relay strategies for which the same transmission strategy is used at node A such that node B is able to decode the corresponding message. Furthermore, we will show that at least one of these strategies is optimal for the backward channel for each case of the backward channel parameters. As before we consider the case when $n_{A1}, n_{A2}, n_{1B}, n_{2B} > 0$. In the remainder of this section, we assume that the forward channel is of Case 3.1.2. The case that the forward channel is of Case 4.1.2 is treated in Appendix 2 of [15].

When the forward channel is of Case 3.1.2, node A transmits $[a_{C_{AB}}, ..., a_1]^T$. Also, the transmission strategy for node B depends on the channel gains in the backward direction. For the relay strategy, we will choose one of the four strategies explained in the following depending on the backward channel parameters. We prove that all of these strategies are optimal for the forward channel for any set of parameters.

The parameters associated with each relay strategy proposed here are only based on the forward channel gains, and we will show that at least one of the proposed strategies is optimal for each choice of the backward channel parameters. Note that using Relay Strategy 0 in both relays, node B cannot necessarily decode the message if the forward channel is of Case 3.1.2 or 4.1.2, when the above transmission strategy is used by node A.

Remark 1. *All relay strategies in this subsection are defined with respect to the forward channel parameters (and in favor of the forward channel direction[1]) because we assumed that the forward channel is either of Case 3.1.2 or 4.1.2 and the backward channel is not of these cases. We note that Relay Strategy 0 is symmetric and is not dependent on the channel gains in any direction. In Section III-C, we will generalize some of these strategies to be based on the parameters of both the forward and backward channels.*

1) **Relay Strategy 1::** If the forward channel is of Case 3.1.2 Type i, then Relay Strategy 0 is used at R_i, and Relay Strategy 1 is used at $R_{\bar{i}}$, where $i, \bar{i} \in \{1, 2\}, i \neq \bar{i}$. Here, we define Relay Strategy 1 at R_2 (forward channel of Case 3.1.2 Type 1), while that for R_1 can be obtained by interchanging roles of relays R_1 and R_2 (interchanging 1 and 2 and forward channel of Case 3.1.2 Type 2). As shown in Figure 1, if R_2 receives a block of n_{2B} bits, first it will reverse them as in Relay Strategy 0 and then changes the order of the first $n_{1B} - (n_{A1} - n_{A2})$ streams[2] with the next $n_{A1} - n_{1B}$ streams. Node A transmits $[a_{C_{AB}}, ..., a_1]^T$. The received signals can

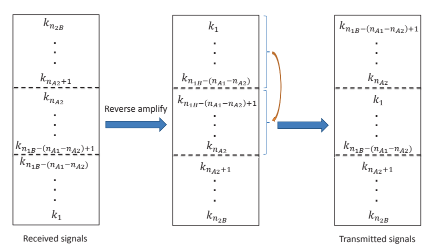

Fig. 1. Relay Strategy 1 at R_2.

be seen in Figure 2. We use (R_i, B_j) to denote block number j from R_i. Bits that are not delivered to node B from R_1 using Relay Strategy 0, $(a_{n_{1B}+1}, ..., a_{n_{A1}})$, are all sent at the highest levels from R_2 to node B and thus are decoded with no interference (block (R_2, B_1)). The remaining bits can be decoded by starting from the lowest level of reception in B ($a_{n_{1B}}$ in block (R_1, B_4)) and removing the effect of the decoded bits and going up.

[1]In the sense that the strategies are designed so that the forward communication achieves the capacity.

[2]In the following relay strategies, we divide the streams into multiple sub-streams. The number of streams in each sub-stream is a non-negative number when the forward channel is of Case 3.1.2 Type 1.

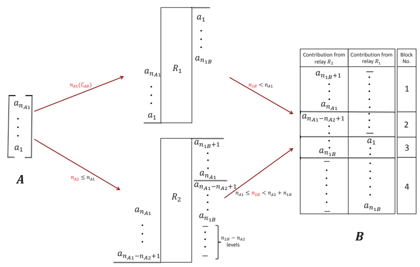

Fig. 2. Received signals by using Relay Strategy 1 when the forward channel is of Case 3.1.2 Type 1.

2) **Relay Strategy 2::** If the forward channel is of Case 3.1.2 Type i, then Relay Strategy 0 is used at R_i, and Relay Strategy 2 is used at $R_{\bar{i}}$, where $i, \bar{i} \in \{1,2\}, i \neq \bar{i}$. Here, we define Relay Strategy 2 at R_2 (forward channel of Case 3.1.2 Type 1), while that for R_1 can be obtained by interchanging roles of R_1 and R_2 (interchanging 1 and 2 and forward channel of Case 3.1.2 Type 2). It is similar to Relay Strategy 0 with the only difference that R_2 repeats a part of the top n_{A2} streams after reverse-amplify-and-forward, as explained below in nine separate scenarios based on the parameters of the forward channel. We note that the repetition of streams is based on the received signal at the relay. However, we describe only the forward direction to show that the messages can be decoded.

We partition the four-dimensional space $(n_{A1}, n_{A2}, n_{1B}, n_{2B})$ into multiple parts, and we consider one of the case below. The proof for the rest of cases is given in Section IV of [15]. This case corresponds to $\{n_{2B} + (n_{A1} - n_{A2}) \leq n_{A2} + n_{1B}, n_{1B} \leq (n_{A1} - n_{A2}) + (n_{2B} - n_{1B})\}$. Figure 3 depicts the received signal at node B (ignoring the effect of transmitted signal from B) assuming that both relays use Relay Strategy 0. The repetitions will be described below to show that messages can be decoded with the proposed strategies. R_2 repeats the

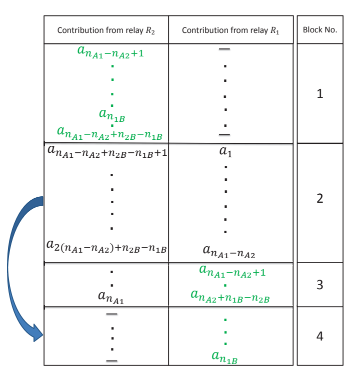

Fig. 3. The received signals at node B (ignoring the effect of transmitted signal from B) assuming that both relays use Relay Strategy 0 for channel parameters of case $\{n_{2B} + (n_{A1} - n_{A2}) \leq n_{A2} + n_{1B}, n_{1B} \leq (n_{A1} - n_{A2}) + (n_{2B} - n_{1B})\}$.

streams in block (R_2, B_2) on block (R_2, B_4). Using this strategy, block (R_2, B_1) will be decoded from the top levels of the received signal from R_2 since there is no interference from the other relay. Then, subtract the corresponding signals (blocks (R_1, B_3) and (R_1, B_4)). Furthermore, block (R_2, B_4) can be decoded from repetitions because their interference is already decoded. Then, subtract the corresponding signals (block (R_2, B_2)). Consequently, block (R_1, B_2) are decoded because their interference (block (R_2, B_2)) was decoded earlier. Finally, block (R_2, B_3) can be decoded because all its interference signals have been decoded.

3) **Relay Strategy 3::** If the forward channel is of Case 3.1.2 Type i, then Relay Strategy 0 is used at R_i, and Relay Strategy 3 is used at $R_{\bar{i}}$, where $i, \bar{i} \in \{1,2\}, i \neq \bar{i}$. Here, we define Relay Strategy 3 at R_2 (forward channel of Case 3.1.2 Type 1), while that for R_1 can be obtained by interchanging roles of relays R_1 and R_2 (interchanging 1 and 2 and forward channel of Case 3.1.2 Type 2). As shown in Figure 4, if R_2 receives a block of n_{2B} bits, first it will reverse them as in Relay Strategy 0 and then changes the order of the $n_{A2} - (n_{2B} - n_{1B})$ streams right after the first $n_{2B} - n_{1B}$ streams, with the following $n_{2B} - n_{A2}$ streams. Node A

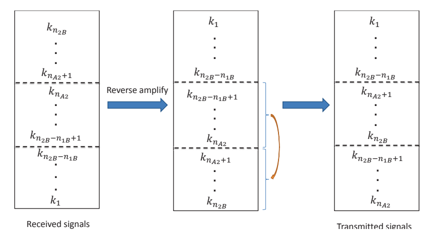

Fig. 4. Relay Strategy 3 at R_2.

transmits $[a_{C_{AB}}, ..., a_1]^T$, and it can be shown to be decoded at node B.

4) **Relay Strategy 4::** If the forward channel is of Case 3.1.2 Type i, then Relay Strategy 0 is used at $R_{\bar{i}}$, where $i, \bar{i} \in \{1,2\}, i \neq \bar{i}$, and Relay Strategy 4 is used at R_i. Here, we define Relay Strategy 4 at R_1 (forward channel of Case 3.1.2 Type 1), while that for R_2 can be obtained by interchanging roles of R_1 and R_2 (interchanging 1 and 2 and forward channel of Case 3.1.2 Type 2). As shown in Figure 5, if R_1 receives a block of n_{1B} bits, first it will reverse them as in Relay Strategy 0 and then changes the order of the first $n_{A1} - n_{A2}$ streams with the next $n_{1B} - (n_{A1} - n_{A2})$ streams. Node A

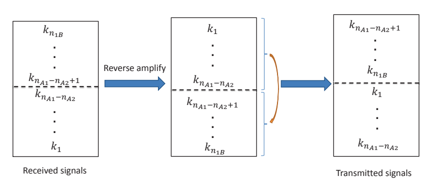

Fig. 5. Relay Strategy 4 at R_1.

transmits $[a_{C_{AB}}, ..., a_1]^T$. Bits that are not delivered to node B from R_2 using Relay Strategy 0 in the block (R_1, B_4) are decoded without any interference. The remaining bits can be decoded by starting from the highest level ($a_{n_{A1}-n_{A2}+1}$ in block (R_2, B_1)) and removing the effect of the decoded bits.

These strategies can be used at different relays to achieve the optimal rate. For instance, if the backward channel is of Case 1 and the forward channel is of Case 3.1.2 Type 1, the following strategy is used. If $n_{A2} > n_{B2}$, we use Relay Strategy 2 at R_2 and Relay Strategy 0 at R_1. Otherwise use Relay Strategy 1 at R_2 and Relay Strategy 0 at R_1. If $n_{A2} > n_{B2}$, R_2 repeats from the streams that are already decoded from the highest levels received in A, on the lower levels, and otherwise it just changes the order of some of the equations at the highest levels received in A, which does not affect the decoding.

Four new relay strategies are needed for Case 4.1.2, whose details can be seen in [15].

C. Both the forward and backward channels are either of Case 3.1.2 or 4.1.2

In Section III-B, we used Relay Strategy 2 or Relay Strategy 6 as one of the achievability strategies when the forward channel is of Case 3.1.2 or 4.1.2, respectively. In this section, we show that using a modified combination of these strategies achieve the optimal capacity region when both the forward and backward channels are either of Case 3.1.2 or 4.1.2.

We will define Relay Strategy (m_i, n_i) at R_i for $i \in \{1, 2\}$, $m_i, n_i \in \{0, 2, 6\}$. If the forward channel is of Case 3.1.2, at R_1, we use $m_1 = 0$ when the forward channel is Type 1 and $m_1 = 2$ otherwise. At R_2, we use $m_2 = 2$ when the forward channel is Type 1 and $m_2 = 0$ otherwise. If the forward channel is of Case 4.1.2, at R_1, we use $m_1 = 6$ when the forward channel is Type 1 and $m_1 = 0$ otherwise. At R_2, we use $m_2 = 0$ when the forward channel is Type 1 and $m_2 = 6$ otherwise. The value of n_i is determined the same way based on the backward channel parameters.

Relay Strategy $(m_i, 0)$ at R_i uses Relay Strategy m_i at R_i based on the forward channel parameters, and Relay Strategy $(0, n_i)$ at R_i uses Relay Strategy n_i based on the backward channel parameters. For the remaining strategies $(m_i, n_i) \in \{(2, 2), (2, 6), (6, 2), (6, 6)\}$ at R_i, we use the combination of the repetitions suggested by Relay Strategies m_i based on the forward channel parameters, and n_i based on the backward channel parameters. If these two repetitions happen at the same level, we sum these modulo 2. However, there are some modifications to account for repetitions adding to zero modulo 2, or multiple repetitions due to different strategies at the relays. The modifications are as follows.

1) If the repetitions happen in the same relay, i.e., $m_1 = n_1 = 0$ or $m_2 = n_2 = 0$: In case the repetition of a particular signal by both the forward and backward strategies is suggested at the same level, we send the repeated signal. If different repeated signals are suggested at a particular level, we send the sum of these two signals modulo two.
2) If the repetitions happen in different relays, i.e., $m_1 = n_2 = 0$ or $m_2 = n_1 = 0$, we consider two cases. In case that the repetitions of some streams from two relays are from the same level and are repeated on the same level at node B (ignoring the backward signal component) R_i skips repetitions at the corresponding levels if the forward channel is of Case 4.1.2 Type i and $R_{\bar{i}}$ skips repetitions at the corresponding levels if the forward channel is of Case 3.1.2 Type i. In case that the repetitions of some streams from two relays are from the same level and are repeated on the same level at node A (ignoring the forward signal component) R_i skips repetitions at the corresponding levels if the backward channel is of Case 4.1.2 Type i and $R_{\bar{i}}$ skips repetitions at the corresponding levels if the backward channel is of Case 3.1.2 Type i.

We use the same transmission strategy as in Section III-B for channel of both Cases 3.1.2 and 4.1.2, and the decoding of the messages at the receivers can be shown. Detailed proofs can be seen in [15].

IV. Conclusions

In this paper, we studied the capacity of the bidirectional (or two-way) diamond channel with two nodes and two relays. We used the deterministic approach to capture the essence of the problem and to determine capacity-achieving transmission and relay strategies. Depending on the forward and backward channel gains, we used either a reverse amplify-and-forward or a particular modified strategy involving repetitions, and reversing order of some streams at the relays.

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