Single Machine Scheduling: The Problem of Minimizing the Total Weighted Completion Time for Preemptive Equal-length Jobs is Pseudo-Polynomially Solvable

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Abstract: In this talk we consider the preemptive single machine scheduling problem with n jobs that have equal processing times $p \in \mathcal{N}$, different release dates $r_j \in \mathcal{N} \cup 0$ and different weights $w_j \in \mathcal{N}$. The objective is to minimize the total weighted completion time $\sum w_j C_j$. In the literature, this problem is usually abbreviated as $1|pmtn; p_j = p; r_j| \sum w_j C_j$. The computational complexity status of this problem is open. Records of the open complexity status go back for at least 25 years. We show that $1|pmtn; p_j = p; r_j| \sum w_j C_j$ is pseudo-polynomially solvable.

Outline of this talk. We formulate the scheduling problem as a Boolean Linear Programming model and show that its Linear Programming Relaxation (LPR) is a Total Dual Integral (TDI) system. This implies that for each specific problem instance there exists a polynomial-time algorithm that solves the instance to optimality. We propose an algorithm that consists of two parts. First, the LPR is solved by a general-purpose solver (like CPLEX or Xpress). If the solution is integral, the problem instance is solved. If the solution is fractional, then we propose the second part of our algorithm that transforms the fractional solution into an integral solution with the same objective function value. Since the second part has time complexity $O(n^2p^2)$, we have that the problem $1|pmtn; p_j = p; r_j| \sum w_j C_j$ is pseudo-polynomially solvable. Note that we do not necessarily need the property of TDI of the LPR to show that the problem is pseudo-polynomially solvable; the algorithm together with the LPR form a justification of pseudo-polynomial solvability of the scheduling problem themselves. We conclude our talk with a summary and some future research directions. This is joint work with Bader AlBdaiwi and Boris Goldengorin.