Colorings, Crossings, and Cliques

Date Tuesday, October 21

Time 5 pm

Location 303 Mudd

Abstract: For more than a century, the Four Color Problem has played a leading role in the development of graph theory. Now that the 4 Color Theorem is proved, it is natural to consider relaxations of planarity and how they relate to chromatic number. Three concepts are obvious choices: the genus of a graph, the thickness of a graph, and the crossing number (defined below). The relationships of the chromatic number to the genus and thickness of a graph are well understood, however little is known to relate the chromatic number to the crossing number.

The crossing number of a graph G, denoted cr(G), is the minimum number of crossings in a drawing of G. Recently, Albertson conjectured that the crossing number of every t-chromatic graph is at least the crossing number of K_t . The case t = 5 is equivalent to the 4 Color Theorem, and the case t = 6 follows from a theorem of Oporowski and Zhao. Here we prove the cases t = 7 and t = 8 and give partial results for t = 9. This is joint work with Michael Albertson.